

AN ALTERNATIVE METHOD FOR DETERMINING THE COEFFICIENT OF THERMAL EXPANSION OF COMPOSITE MATERIAL OF SPHERICAL PARTICLES

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Abstract. Composite material is widely used in modern structures. Many researchers have been involved in studying, developing and applying this kind of material. The composite material of spherical particles is a material composed of continuous matrix phase and spherical particles. In modern technique, it is very necessary to consider the influence of temperature on toughness and stability of structures. Therefore, determining the coefficient of thermal expansion of composite as a function of the coefficients and volume fractions of matrix and particle phases is a practical requirement. In this paper, we would like to introduce an alternative derivation method in order to obtain the thermal expansion coefficient of two-phase composite of spherical particles. Our results are the same as Vanin's and other authors'.

1. INTRODUCTION

Thermal expansion behaviour can be important when composite materials are used in conjunction with other materials and when it is necessary to match the thermal expansion coefficient of one structural component with another.

Among the kinds of composite material, particle-reinforced material is an important one. Many researchers have been involved in determining the effective thermal expansion coefficient of this kind of composite, especially composite of spherical inclusions [7].

Levin [8] showed that a simple relationship between the effective expansion coefficients and the effective elastic moduli could be derived for two-phase materials using the thermo-elastic stress-strain relations. For an isotropic composite with two isotropic phases the basic relationship can be written in form

$$\alpha^* = \bar{\alpha} + \frac{\alpha_1 - \alpha_2}{\left[\frac{1}{K_1} \right] - \left[\frac{1}{K_2} \right]} \left[\frac{1}{K^*} - \left(\frac{1}{K} \right) \right], \quad (1.1)$$

their bulk moduli, K^* and α^* the effective bulk modulus and effective thermal expansion coefficient of composite, and the bars over the symbols indicate volume averages.

If one of the phases is a fluid (or a low shear modulus matrix) and the other consists of rigid particles, mechanical interactions represented by the final term of (1.1) can be

neglected

$$\frac{1}{K^*} \simeq \overline{\left(\frac{1}{K}\right)} \equiv \frac{1}{K_1}V_1 + \frac{1}{K_2}V_2, \quad (1.2)$$

where V_1 , V_2 are the volume fractions of phases and it follows from equation (1.1) the simple rule of mixture

$$\alpha^* \simeq \bar{\alpha} \equiv \alpha_1V_1 + \alpha_2V_2. \quad (1.3)$$

For a dilute suspension in which spherical particles with bulk modulus K_1 are dispersed in a matrix with bulk modulus K_2 and shear modulus μ_2 , we can use Hashin's relation for effective bulk modulus as follows [1].

$$K^* = K_2 + (K_1 - K_2) \frac{3K_2 + 4\mu_2}{3K_1 + 4\mu_2} V_1. \quad (1.4)$$

This relation may be expected to apply when the volume fraction of particles V_1 does not exceed a few percent.

From equations (1.1) and (1.4) we obtain

$$\alpha^* = \alpha_2 + \frac{(\alpha_1 - \alpha_2)K_1(3K_2 + 4\mu_2)V_1}{K_2(3K_1 + 4\mu_2) + (K_1 - K_2)(3K_2 + 4\mu_2)V_1}. \quad (1.5)$$

For higher particle concentrations, Hashin obtained the exact relation for the effective bulk modulus of this system [1]

$$K^* = K_2 + (K_1 - K_2) \frac{(3K_2 + 4\mu_2)}{3K_1 + 4\mu_2 + 3(K_2 - K_1)V_1}. \quad (1.6)$$

Substituting (1.6) in Levin's equation (1.1), we obtain the relation

$$\alpha^* = \alpha_2 - \frac{(\alpha_2 - \alpha_1)K_1(3K_2 + 4\mu_2)V_1}{K_2(3K_1 + 4\mu_2) + 4(K_1 - K_2)\mu_2V_1}. \quad (1.7)$$

This relation can also be obtained from the expressions given by Kerner [7].

Using the relationships

$$K = \frac{E}{3(1 - 2\nu)}, \quad \mu = \frac{E}{2(1 + \nu)}$$

equation (1.7) can be written in the form

$$\alpha^* = \alpha_2 - (\alpha_2 - \alpha_1) \frac{3(E_1/E_2)(1 - \nu_2)V_1}{(E_1/E_2)[2V_1(1 - 2\nu_2) + (1 + \nu_2)] + 2(1 - 2\nu_1)V_2} \quad (1.8)$$

which corresponds with the relations obtained by Fahmy and Ragai [6]. These two authors derived equation (1.8) from a consideration the difference between linear strains at interfaces in a composite sphere.

Assuming homogeneous strain throughout the composite and using a balance of internal average stresses, Turner suggested thermal expansion coefficient of two-phase composite taking into account the stress interaction between phases as follows.

$$\alpha^* = \frac{K_1\alpha_1V_1 + K_2\alpha_2V_2}{K_1V_1 + K_2V_2} = \overline{\frac{K\alpha}{K}}. \quad (1.9)$$

Employing extremum principles of thermoelasticity and taking into account the stress interaction between components, Schapery has derived the following relation for the thermal expansion coefficient of composite material

$$\alpha^* = \alpha_1 + (\alpha_2 - \alpha_1) \frac{(1/K^*) - (1/K_1)}{(1/K_2) - (1/K_1)}. \quad (1.10)$$

Finally, we discuss about Vanin's derivation [2]. According to Eshelby's formula [1], the energy of composite material of spherical particles is given as follows

$$U = U_0 + \frac{1}{2} \iint_{S_i} (\sigma_i u_i^0 - \sigma_i^0 u_i) dS, \quad (1.11)$$

where S_i is the surface of spherical particles, (σ_i, u_i) - the stresses and displacements of composite and (σ_i^0, u_i^0) the stresses and displacements of homogeneous elastic material. Fundamentally, using this energy relation Vanin obtained the thermal expansion coefficient of two-phase composite material as follows

$$\beta^* = \beta_2 - (\beta_2 - \beta_1) \frac{\xi K_1 (3K_2 + 4G_2)}{3K_1 K_2 + 4G_2 [\xi K_1 + (1 - \xi) K_2]}. \quad (1.12)$$

Nevertheless, the displacement field selected by Vanin is relatively complex.

At present work, we would like to introduce an alternative method of derivation in order to give the effective thermal expansion coefficient of two-phase composite of spherical particles. Our derivation is based on the theory of thermo-elasticity and broadening results of Lamé problem taking into account the influence of temperature [3]. Our results coincide with relations obtained by Vanin, Hashin, Kerner, Fahmy and Ragai.

2. THE PROBLEM

Investigating composite material of spherical particles is derived by considering a representative volume element having form of a sphere contained in a cube. According to composite spheres model suggested by Hashin [1], the outer cube is estimated by a sphere having the same volume. This model give us a solid sphere covered by a spherical shell. Inner solid sphere represents particle phase, whereas outer spherical shell- matrix phase. We assume that particle and matrix phases are made of isotropic homogeneous elastic materials, moreover, the radii of particles are the same. Consequently, present problem can be posed as follows.

Let us consider a heterogeneous sphere that core sphere ($0 \leq r \leq a$) and spherical shell ($a \leq r \leq b$) are homogeneous materials of properties (λ_1, μ_1) , (λ_2, μ_2) and thermal expansion coefficients (α_1, α_2) , respectively. It is supposed that the heterogeneous sphere is subjected to a hydrostatic stress p_2 on the boundary $r = b$, at the same time, is warmed up by temperature T (in fact, temperature interval $\Delta T = T - T_0$ represents the difference of temperatures and T_0 is initial temperature). There exists a problem to think of proposed inhomogeneous sphere as a composite sphere and determine the effective thermal expansion coefficient of composite as a function of coefficients and elastic moduli of constituent phases.

3. GOVERNING RELATIONS

Some governing relations of proposed problem are given in the spherical coordinate system as follows [3].

Because of symmetry, it is clear that the only non-zero displacement component is radius displacement u_r . Moreover, it is a function of radius r .

$$u_r = u_r(r), \quad u_\theta = u_\varphi = 0. \quad (3.1)$$

By theory of thermoelasticity [3], we express the components of stress tensor in terms of displacements.

$$\begin{aligned} \sigma_{rr} &= \lambda\theta + 2\mu\varepsilon_{rr} - (3\lambda + 2\mu)\alpha\Delta T, \\ \sigma_{\theta\theta} = \sigma_{\varphi\varphi} &= \lambda\theta + 2\mu\varepsilon_{\varphi\varphi} - (3\lambda + 2\mu)\alpha\Delta T. \end{aligned} \quad (3.2)$$

Here $\theta = du_r/dr + 2u_r/r$ and $\Delta T = T - T_0$ considered as a constant in present work.

Putting (3.2) into equilibrium equation

$$\frac{d\sigma_{rr}}{dr} + \frac{2}{r}(\sigma_{rr} - \sigma_{\varphi\varphi}) = 0, \quad (3.3)$$

we obtain the following differential equation for u_r

$$\frac{d^2u_r}{dr^2} + \frac{2}{r}\frac{du_r}{dr} - 2\frac{u_r}{r^2} = 0. \quad (3.4)$$

4. SOLUTION METHOD

As was mentioned above, the governing idea for solving recommended problem is that widening results of Lamé problem when consider the influence of temperature. Firstly, we separately define the state of displacement and stress of particle and matrix phases of heterogeneous sphere. Then we define displacement and stress fields of composite sphere. By a fact that displacement on boundary of composite sphere is the same as that on boundary of matrix spherical shell, we obtain the objective of problem.

4.1. Part of matrix phase

In part of matrix phase ($a \preceq r \preceq b$) the displacement and stress are in forms

$$u_r^{(2)} = A_2r + \frac{B_2}{r^2}, \quad (4.1)$$

$$\sigma_{rr}^{(2)} = (3\lambda_2 + 2\mu_2)A_2 - \frac{4B_2\mu_2}{r^3} - (3\lambda_2 + 2\mu_2)\alpha_2\Delta T. \quad (4.2)$$

Introduction of $\sigma_{rr}^{(2)}$ into boundary and surface conditions

$$\sigma_{rr}^{(2)}\Big|_{r=b} = -p_2, \quad \sigma_{rr}^{(2)}\Big|_{r=a} = -p, \quad (4.3)$$

(in fact, p is interaction stress between two phases), we define integration constants as well as displacement and stress in the matrix phase

$$u_r^{(2)} = \left[\frac{pa^3 - p_2b^3}{b^3 - a^3} \frac{1}{3\lambda_2 + 2\mu_2} + \alpha_2\Delta T \right] r + \frac{(p - p_2)a^3b^3}{(b^3 - a^3)4\mu_2} \frac{1}{r^2}, \quad (4.4)$$

$$\sigma_{rr}^{(2)} = \frac{pa^3 - p_2b^3}{b^3 - a^3} + \frac{(p_2 - p)a^3b^3}{b^3 - a^3} \frac{1}{r^3} + (3\lambda_2 + 2\mu_2)\alpha_2\Delta T. \quad (4.5)$$

4.2. Part of particle phase

In this part ($0 \leq r \leq a$) displacement and stress fields have form

$$u_r^{(1)} = A_1 r, \quad (4.6)$$

$$\sigma_{rr}^{(1)} = (3\lambda_1 + 2\mu_1)A_1 - (3\lambda_1 + 2\mu_1)\alpha_1 \Delta T. \quad (4.7)$$

By the continuity conditions of displacement and stress at interface $r = a$

$$u_r^{(+)} = u_r^{(-)}, \quad \sigma_{rr}^{(+)} = \sigma_{rr}^{(-)},$$

we obtain the following relations of A_1 and interaction stress p .

$$A_1 = -\frac{(K_2 + 4\mu_2/3)p_2 b^3 - \left[(3K_2 b^3 + 4\mu_2 a^3)K_1 \alpha_1 + 4\mu_2 (b^3 - a^3)K_2 \alpha_2 \right] \Delta T}{(3K_2 b^3 + 4\mu_2 a^3)K_1 + 4\mu_2 (b^3 - a^3)K_2}, \quad (4.8)$$

$$p = -K_1 \frac{(3K_2 + 4\mu_2)p_2 b^3 - 12\mu_2 K_2 (b^3 - a^3)(\alpha_2 - \alpha_1) \Delta T}{(3K_2 b^3 + 4\mu_2 a^3)K_1 + 4\mu_2 (b^3 - a^3)K_2}, \quad (4.9)$$

where $K_i = \lambda_i + 2\mu_i/3$ ($i = 1, 2$).

4.3. Equivalent homogeneous sphere

Now we consider equivalent homogeneous sphere having radius $r = b$, properties (λ, μ) and thermal expansion coefficient α , such a sphere is referred to as a composite sphere. Composite sphere is subjected to a hydrostatic stress p_2 on boundary $r = b$ and is warmed up by temperature T .

Because the composite sphere is solid, solution form is similar to that of particle phase.

$$u_r^* = A^* r, \quad (4.10)$$

$$\sigma_{rr}^* = (3\lambda + 2\mu)A^* - (3\lambda + 2\mu)\alpha \Delta T. \quad (4.11)$$

The displacement field is defined from boundary condition

$$\sigma_{rr}^* \Big|_{r=b} = -p_2,$$

as follows

$$u_r^* = \left[\frac{-p_2}{3\lambda + 2\mu} + \alpha \Delta T \right] r. \quad (4.12)$$

In fact, the displacements of composite sphere and spherical matrix shell are the same, specifically

$$u_r^{(2)} \Big|_{r=b} = u_r^* \Big|_{r=b}. \quad (4.13)$$

Putting equations (4.4) and (4.12) into equation (4.13) taking into account (4.9), we obtain the following equation

$$p_2 \left[(K + 4\mu_2/3)(K_1 - K_2)a^3 - (K - K_2)(K_1 + 4\mu_2/3)b^3 \right] + \left[K K_2 (3K_1 + 4\mu_2)(\alpha - \alpha_2)(a^3 - b^3) - K K_1 (3K_2 + 4\mu_2)(\alpha - \alpha_1)a^3 \right] \Delta T = 0. \quad (4.14)$$

Equation (4.14) is satisfied for arbitrary p_2 , therefore, its coefficients must be zero.

Equating to zero the coefficient of p_2 gives us relation

$$K = K_2 + \frac{(K_1 - K_2)\xi}{1 + (K_1 - K_2)(K_2 + 4G_2/3)^{-1}(1 - \xi)}, \quad (4.15)$$

which is the same as Hashin's formula introduced in Christensen's work [1] for determining the effective bulk modulus of composite of spherical particles.

Then making the term independent of p_2 to be zero, we obtain relation

$$\alpha = \alpha_2 + (\alpha_1 - \alpha_2) \frac{K_1 (3K_2 + 4G_2) \xi}{K_2 (3K_1 + 4G_2) + 4(K_1 - K_2)G_2\xi}, \quad (4.16)$$

where $\xi = \frac{a^3}{b^3}$ is the volume fraction of particle phase.

We readily recognize that relation (4.16) is the same as Vanin's formula (1.12) for calculating the effective thermal expansion coefficient of composite material of spherical particles. It is a function of the coefficients, elastic moduli and volume fractions of two phases.

5. CONCLUSIONS

By composite spheres model and using the theory of thermoelasticity, we have determined the effective thermal expansion coefficient of two-phase composite material of spherical reinforced particles. Our formula coincide with result obtained by Vanin and several other authors such as Hashin, Kerner, Fahmy and Ragai. Mathematically, we think that our derivation method is more simple than Vanin's and other authors'. Moreover, our results exactly represent the nature of heat transfer, specifically, the effective bulk modulus of composite is obtained as a corollary. Finally, this approach can be broadened to composite cylinders model in order to reach the thermal expansion coefficients of fibre-reinforced composite material.

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MỘT PHƯƠNG PHÁP KHÁC ĐỂ XÁC ĐỊNH HỆ SỐ GIÃN NỞ NHIỆT CỦA VẬT LIỆU COMPOSITE VỚI CÁC HẠT ĐƠN CẦU

Vật liệu composite được sử dụng rộng rãi trong nhiều kết cấu hiện đại nhờ tính ưu việt của nó. Việc nghiên cứu, phát triển và ứng dụng loại vật liệu này nhận được sự quan tâm của nhiều nhà nghiên cứu. Vật liệu composite với các hạt đơn cầu là một loại vật liệu đẳng hướng được cấu thành từ pha nền liên tục và các hạt đơn cầu. Trong kỹ thuật hiện đại, việc xem xét ảnh hưởng của nhiệt độ đến độ bền và độ ổn định của các kết cấu là rất cần thiết. Do đó việc xác định hệ số giãn nở nhiệt của composite như một hàm của các hệ số giãn nở nhiệt và tỷ lệ thể tích của các vật liệu thành phần là một yêu cầu rất thực tế. Trong bài báo này chúng tôi giới thiệu một phương pháp khác nhằm thu được hệ số giãn nở nhiệt của composite hai pha đơn các hạt cầu. Kết quả của chúng tôi trùng hoàn toàn với kết quả của Vanin và một số tác giả khác.