

## CONSTRAINTS ON EARLY STAR FORMATION AND MIXING FROM NUCLEOSYNTHESIS IN PROTOGALACTIC CLOUDS

NGUYEN QUYNH LAN, NGUYEN THU GIANG  
*Hanoi National University of Education*

GRANT J MATHEWS  
*Center for Astrophysics, Department of Physics,  
University of Notre Dame, Notre Dame, IN 46556, U.S.A.*

LAMYA SALEH  
*Department of Physics & Astronomy,  
Northwestern University, Evanston, IL 60208, U.S.A.*

**Abstract.** *We calculate the stochastic chemical evolution of pre-galactic clouds as a means to constrain the degree of mixing in the early Galaxy. We employ a model based upon the cold-dark-matter paradigm for hierarchical galaxy formation. We argue that a significant dispersion in the metallicity for the alpha elements as a function of heavy elements is unavoidable for models with realistic stellar nucleosynthesis yields and a reasonable initial mass function. Although the calculated scatter in light-element abundance can be reduced by restricting the stellar initial mass function. The observed lack of dispersion inevitably requires that extensive mixing of the supernova ejecta containing alpha elements to have occurred. We use the difference between the observed and calculated dispersion of alpha to determine the degree of mixing of the early Galaxy. We argue that the large dispersion for heavy r-process elements suggests that they arise from rare events which did not have time to mix during the formation of the Galactic halo.*

### I. INTRODUCTION

The oldest and most metal-poor stars in the Galaxy reside in the Galactic halo. As such, these stars are an important probe of the early evolution of the Galaxy. For this reason many studies have investigated the trends in the chemical, spatial, and kinematic properties of low-metallicity halo stars [1–16]

The Galaxy is thought to have assembled from cold dark matter haloes (e.g., [17]) that originated from primordial density fluctuations. These accreted primordial gas began to merge and form the stars that make up the present Galactic halo. Indeed, such Cold dark matter (CDM) models are consistent with the observed kinematics of the Galactic halo [18, 19]. As the mergers continued, material also collected into the Galactic disk where continuous star formation and stellar nucleosynthesis has evolved the elemental abundances to those of the present Galaxy. Although this scenario is widely accepted, the extent of mixing and star formation in the early Galaxy is still unknown. Here, we propose that variations in the dispersion of heavy-element abundances can be used to probe not

only the degree of star formation and mixing in the early Galaxy, but also to identify the stellar site for the nucleosynthesis of some elements.

The basic trends of metallicity  $[Z/H]$  as a function of  $[Fe/H]$  can be understood in terms of relatively simple galactic chemical evolution models [20]. However, the dispersions in metallicity for stars with the same  $[Fe/H]$  is a more challenging problem. Observations indicate variations in the observed dispersion for different elements that range from almost no dispersion for many alpha and iron-group elements, to a dispersion of more than two orders of magnitude for the heavy r-process elements at very low metallicity  $[Fe/H] < -3$ ). The existence of this latter large dispersion has led to the suggestion that the lowest-metallicity stars were the result of mixing of the ejecta from single, or at most a few, supernovae, with a limited amount of gas in the parent clouds [3, 21–24]. On the other hand, the small very dispersion among  $\alpha$ -elements like Mg/Fe [13] would seem to indicate the occurrence efficient mixing of the stellar ejecta among the different regions in which these elements were synthesized in the early Galaxy [10, 12, 14]. In the present work we seek to develop a unified framework in which the degree of this mixing can be quantified and the large dispersion for some elements explained.

Chemical evolution models that have treated these early stages still lack a complete picture that includes all chemical and dynamical effects. There are, however, some phenomenological stochastic models [7, 8, 15, 16, 25–31, 34, 35] which can reproduce some of the observed dispersion in elemental abundances. In this paper we make use of the results from the realistic model of [15, 16] which incorporates the metallicity dependent stellar ejecta of [36] and finite stellar lifetimes in a stochastic star-formation scenario.

## II. THE MODEL

Details of the stochastic chemical evolution model have been presented in [15, 16]. Here we summarize the main features for completeness. The main features of hierarchical galaxy formation are incorporated along with several key assumptions. These include: 1) In the first zero-metallicity clouds, the first-generation stars are born as population III (Pop III) stars. These explode and the clouds that survive are enriched by their expelled yields; 2) Star formation is induced by supernovae, mainly Type II (SNeII) based upon the model of [31]; 3) Each individual supernova is taken to form a shell which expands with an average velocity and triggers star formation with an average efficiency  $\epsilon$  over a timescale  $\Delta t$ ; 4) The mass of the shells is the sum of the whole SNII gas and the swept-up surroundings. The metallicity is then given by a complete mixture of both metal contents; 5) The metallicity-dependent SN II yields are taken from the stellar models of [36]; 6) The stellar initial mass function (IMF) can differ according to various conditions within the cloud.

The chemical evolution of the baryon component of individual clouds in the mass range of  $10^5$  to  $10^8 M_\odot$  is treated as a one-zone closed box [32] assuming that all stars initially form within the clouds by SN-induced star formation (e.g., [31]). For a given cloud, at  $t = 0$ , the initial rate at which mass goes into stars is

$$\psi(t = 0) = \epsilon M_{sh}(m, 0) / \Delta t, \quad (1)$$

where  $\epsilon$  is the star formation efficiency denoting the mass fraction of the shell that is used up in forming stars during the interval  $\Delta t$ . The  $\epsilon$  parameter was adjusted in [15] to reproduce the observed [2] shift in slope of the elemental abundance ratios for three iron-group elements below  $[\text{Fe}/\text{H}] \sim -2.4$  and also the age-metallicity distribution for extreme metal-poor stars. The quantity  $M_{sh}(m, t)$  is the mass of the supernova shell formed at time  $t$ , from a progenitor star of mass  $m$ . It is given by :

$$M_{sh}(m, t) = E_j(m, Z) + M_{sw} , \quad (2)$$

where  $M_{sw} \sim 5 \times 10^4 M_\odot$  [3, 31, 33] is the mass of the interstellar gas swept up by the expansion.

$E_j(m, Z)$  is the mass of ejected material from SNe of progenitor mass  $m$  and metallicity  $Z$  of the shell from which it was formed. The metallicity in the ejected shell  $Z_{sh}(m, t)$  is given by:

$$Z_{sh}(m, t) = 1 - [x_{sh}^H(m, t) + x_{sh}^{He}(m, t)] \quad (3)$$

with  $x_{sh}^H$  and  $x_{sh}^{He}$  denoting the fractions of H and He within the shell, given by,

$$x_{sh}^H(m, t) = [y^H(m, t) + x_{gas}^H(t)M_{sw}]/M_{sh}(m, t) , \quad (4)$$

$$x_{sh}^{He}(m, t) = [y^{He}(m, t) + x_{gas}^{He}(t)M_{sw}]/M_{sh}(m, t) , \quad (5)$$

where  $y^H$  and  $y^{He}$  are the mass of ejected H and He, respectively, from the explosion. We will refer to these quantities as the ‘‘yields’’. They are metallicity and time dependent. The quantities  $x_{gas}^H$  and  $x_{gas}^{He}$  are the fractions of H and He in the interstellar gas at the time of formation of the shell  $t$ , respectively. This fraction is given for any element  $x_{gas}^i$  by:

$$x_{gas}^i(t) = M_i(t)/M_g(t) . \quad (6)$$

The star formation rate (SFR),  $\psi(t)$ , is derived from a sum over shells that form at time  $t$ . It thus depends on the SFR at the time the progenitor star formed,  $\psi(t - \tau_m)$ , where  $\tau_m$  is the lifetime of the progenitor.

$$\psi(t > 0) = \int_{\max(m_t, 10)}^{m_u} \epsilon M_{sh}(m_i, 0) \left[ \frac{\phi(m)}{m} \right] \psi(t - \tau_m) dm \quad (7)$$

where  $\phi(m)$  is the IMF normalized in the usual way

$$\int_{m_l}^{m_u} \phi(m) dm = 1 . \quad (8)$$

The quantities  $m_l = 0.1$  and  $m_u = 40 M_\odot$ , denote the lower and upper mass limits for the IMF. The lower mass limit for stars that produce type-II SNe is taken to be  $10 M_\odot$ . The quantity  $m_t$  in the lower integration limit of Eq. (7) is the mass of a star with lifetime  $t$ , as measured from the time of formation of the first shell. Since the ejected mass from SNe changes with progenitor metallicity,  $E_j(m, Z_{sh})$  in Eq. (2) must be replaced with an average over stars of mass  $m$  and different values of  $Z_{sh}$ , which explode at time  $t$ :

$$\langle M_{ej}(m, t) \rangle = \int_{\max\{m(t-\tau_m), 10\}}^{m_u} dm' \left[ (\phi(m')/m') m_{ej}(m, Z_{sh}(m', t - \tau_m)) \right] , \quad (9)$$

where,  $m'$  is the mass of the progenitor which produces the shell from which  $m$  is formed, and  $m_{ej}(m, Z_{sh}(m', t - \tau_m))$  is the mass ejected from the star with mass  $m$  and metallicity  $Z_{sh}(m', t - \tau_m)$ . The quantity  $(t - \tau_m)$  is the time of formation of a star of mass  $m$  and lifetime  $\tau'_m$ . Similarly, the yields of an individual  $y_i$  is

$$\langle y_i(m, t) \rangle = \int_{\max\{m(t-\tau_m), 10\}}^{m_u} dm' u \left[ \phi(m')/m' \right] y_i^{ej}(m, Z_{sh}(m', t - \tau_m)), \quad (10)$$

and  $y_i^{ej}(m, Z_{sh}(m', t - \tau_m))$  is the mass of the element  $i$  ejected from stars with progenitor mass  $m$  and metallicity  $Z_{sh}(m', t - \tau_m)$ .

The change in gas mass with time is:

$$\frac{dM_g}{dt} = -\psi(t) + \int_{\max(m_t, m_i)}^{m_u} dm [\phi(m)/m] \times M_{ej}(m, t) \psi(t - \tau_m). \quad (11)$$

Here, the first term in this equation is the SFR, equal to the amount of gas going into stars at time  $t$ . It is derived from Eq. (7). The second term accounts for the enrichment of the ISM by ejecta from stars whose life ends at time  $t$ . The change in the mass of element  $i$  in the gas is:

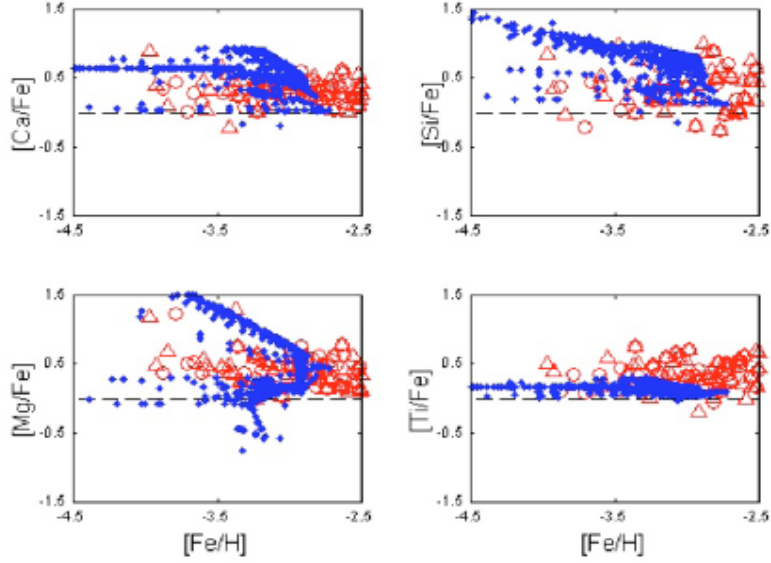
$$\begin{aligned} \frac{dM_i}{dt} = & - \int_{\max(m_t, 10)}^{m_u} dm [\phi(m)/m] \times \epsilon M_{sh}(m, t) x_i(m, t) \psi(t - \tau_m) \\ & + \int_{\max(m_t, m_i)}^{m_u} dm [\phi(m)/m] \times y_i(m, t) \psi(t - \tau_m) ., \end{aligned} \quad (12)$$

where the first term is the rate at which element  $i$  is incorporated into stars at time  $t$ , and the second term is the rate at which it is added to the ISM as stellar ejecta.

This model utilized the stellar yields of [36] for high- mass stars that are more-or-less consistent with the yields of [37] [38] and [39]. Therefore this model complementary to the models of [7, 8] who made a similar stochastic study. For intermediate stellar masses the standard nucleosynthesis yields of Renzini and Voli [40] were used along with the Type-Ia SN yields of [42]. A variety of initial mass functions were studied in [15]. We have found, however, that the elemental dispersions of relevance the present discussion are largely insensitive to the adopted stellar yields or IMF employed unless an arbitrary cut off if the IMF is added at high mass as described below.

### III. RESULTS AND DISCUSSION

Simulations in [15] were evolved for for 5 Gyr with SNIa contributing after a delay of 1 Gyr to allow for the usual progenitor and binary evolution time. Stochastic elemental abundances were recorded for every shell. The model produces a small number of stars at low metallicity, and without mixing, shows a considerable dispersion in the  $[X/Fe]$  ratios. There are two problems with a straightforward application of the model. For one, the observed enhancement of alpha elements relative to iron at low metallicity is not achieved. The other is that the observed narrow dispersion of the alpha elements is not reproduced. To some extent both of these issues can be improved, as pointed out in [15, 16], by reducing the iron yields by about a factor of two. This is a reasonable approach as the cut-off mass shell within the supernova core is not known and mainly affects the ejected iron abundance.



**Fig. 1.** Abundances of some alpha elements from the model of [15] for a model in which the ejected Fe yields are reduced by a factor of 2 from those of model C in [36]. Data are from [3] (triangles) and [41] (circles). The small (dots) show the distribution of model stars.

Figure 1 shows some results for  $[\alpha/\text{Fe}]$  vs.  $[\text{Fe}/\text{H}]$  from simulations described in [15] that were run for a model in which the Fe yield is reduced by a factor of 2 (small dots). These are compared with two sets of observational data, Ryan et al. [3] (triangles) and Norris et al. [41] (circles). The observed values show an over-abundance relative to solar, and a scatter that is significant at all values of  $[\text{Fe}/\text{H}]$ , larger at the lowest abundances. There is also a peculiar trend in the data whereby some stars which appear to have increasing  $[\alpha/\text{Fe}]$  with decreasing  $[\text{Fe}/\text{H}]$  which shows up as an extension in the data. This behavior is apparent in the abundances of Mg, Ca, and Si. On the other hand, Ti shows a scatter that is almost constant over the metallicity range.

The narrow band of elements with high  $[\text{Mg}/\text{Fe}]$  were affected by the ejecta of the most massive stars. It has been shown [16] that this can be removed by limiting the IMF to stars with  $M < 35 M_{\odot}$ . Even after reducing the range of the IMF, however, we find that the dispersion in the alpha elements is too large from the simulations. For example,  $[\text{Mg}/\text{Fe}]$  near  $[\text{Fe}/\text{H}] \sim -3$  shows a calculated dispersion  $\sigma$  which reduces from 0.35 to 0.20 when the most massive stars are removed. This, however, is still much larger than the observed dispersion [14] of  $\sigma = 0.10$ . This result is similar to other studies (e.g., [8] which found that such stochastic models tend to predict too much scatter (e.g. for O and Mg) and fail to reproduce the trends of other elements.

Here, we argue, however, that this excess calculated dispersion can be used to estimate the extent of mixing in the proto-galactic clouds. Consider a simple model in which some fraction  $f_{mix}$  of the clouds are completely mixed so that their abundance is the average abundance and their dispersion is reduced to zero. In this case the cloud variance  $\sigma_{clouds}^2 = \sum [X_i^2 - \bar{X}_i^2]$  reduces to an observed variance of  $\sigma_{obs}^2 = \sum X_i^2(1 - f_{mix}) + f_{mix}\bar{X}_i^2 - \bar{X}_i^2$ . Then the dispersion of the resultant distribution will be reduced to

$$\sigma_{obs} = \sqrt{(1 - f_{mix})\sigma_{clouds}^2} , \quad (13)$$

where  $\sigma_{clouds}$  is the dispersion calculated without mixing. For the present case in point, then the observed reduction in the dispersion by a factor of 2 to 3 would require rather extensive mixing of  $f_{mix} \sim 70 - 90\%$ .

Even though the alpha elements are observed to have a narrow dispersion, one must still take into account the fact that other elements, such as the r-process elements, show a broad dispersion [43]. This can be attributed to the fact that these are rare events and therefore do not occur in every supernova and do not have a chance to mix. This modifies the mixing fraction such that for elements which are only produced by progenitor stars in the limited range from  $m_1$  to  $m_2$  the effective mixing parameter becomes:

$$f_{eff}^i = f_{mix} \left[ \frac{\int_{m_1}^{m_2} m\phi(m)dm}{\int_{10}^{35} m\phi(m)dm} \right] . \quad (14)$$

In this way the actual dispersion observed changes to

$$\sigma_{obs}^i = \sqrt{(1 - f_{eff})\sigma_{clouds}^2} , \quad (15)$$

If the r-process elements are only produced in a narrow range of stars, e.g. for  $m = 10 - 11 M_\odot$ , then the mixing efficiency reduces to only about 2%. In other words, even though every cloud will have alpha elements, not every cloud will have r-process ejecta, so that even after 70-90% of the clouds mix, a large dispersion in r-process elements will remain.

Hence, we conclude that the observed dispersion or lack thereof (for alpha elements) can be used to deduce not only the degree of mixing in the the early galaxy, but also the rang of masses which can lead to the production of rare events like the r-process.

## ACKNOWLEDGMENT

Work at the University of Notre Dame supported by the U.S. Department of Energy under Nuclear Theory Grant DE-FG02-95-ER40934.

## REFERENCES

- [1] G. J. Mathews, G. Bazan, and J. J. Cowan, *ApJ***391** (1992) 719
- [2] A. McWilliam, G. W. Preston, C. Sneden, L. Searle , *AJ* **109** (1995) 2757
- [3] S. G. Ryan, J. E. Norris, and T. C. Beers, *ApJ* **471** (1996) 254
- [4] J. R. King, *AJ* **113** (1997) 2302
- [5] B. W. Carney, J. S. Wright, C. Sneden, J. B. Laird, L. A. Aguilar, and D. W. Latham, *AJ* **114** (1997) 363
- [6] G. W. Preston and C. Sneden, *AJ* **120** (2000) 1014
- [7] D. Argast, M. Samland, O. E. Gerhard, F.-K. Thielemann, *A&A*, **356** (2000) 873
- [8] D. Argast, M. Samland, F.-K. Thielemann and O. E. Gerhard, *A&A* **388** (2002) 842

- [9] T. Kinman, F. Castelli, C. Cacciari, A. Bragaglia, D. Harmer, F. Valdes, *A&A* **364** (2000) 102
- [10] E. Carretta, R. Gratton, J. G. Cohen, T. C. Beers, N. Christlieb, *AJ* **124** (2002) 481
- [11] J. G. Cohen, N. Christlieb, T. C. Beers, R. Gratton, and E. Carretta, *AJ* **124** (2002) 470
- [12] J. G. Cohen, C. Norbert, A. McWilliam, S. Stechman, I. Thompson, G. J. Wassweburg, I. Ivans, M. Dehn, T. Karlsson, and J. Melendez, *ApJ* **612** (2004) 1107
- [13] Arrnone, E. Ryan, S. G., Argast, D. Norris, J. E. & Beers, T. C. 2004, *Astron. Astrophys*, **430**, 507
- [14] R. Cayrel *et al.* *A&A* **416** (2004) 1117
- [15] L. Saleh, T. C. Beers, and G. J. Mathews, *J. Phys. G.* **32** (2006) 581
- [16] G. J. Mathews, L. Saleh, T. C. Beers, and G. J. Mathews, *Proceedings of the International Symposium on Nuclear Astrophysics "Nuclei in the Cosmos - IX"*, CERN, Geneva, June 25-30, 2006; *Proceedings of Science*, PoS(NIC-IX)194, (2007)
- [17] J. Peacock, *Proc. of the MPA- ESO Cosmology Conference*, Garching, Germany, *Evolution of Large Scale Structure: from Recombination to Garching*, edited by A. J. Banday, R. K. Sheth, L. N. da Costa. Garching, Germany: European Southern Observatory, (1999) p.64
- [18] M. Chiba and T. C. Beers, *AJ* **119** (2000) 2843
- [19] A. S. Font, K. V. Johnston, J. S. Bullock, and B. Robertson, *ApJ* **638** (2006) 585
- [20] F. X. Timmes, S. E. Woosley, and T. A. Weaver, *ApJS* **98** (1995) 617
- [21] Audouze and Silk, *ApJ* **451** (1995) L49
- [22] A. McWilliam, *ARA&A* **35** (1997) 503
- [23] A. McWilliam, *AJ* **115** (1998) 1640
- [24] J. E. Norris, T. C. Beers, and S. G. Ryan, *ApJ* **540** (2000) 456
- [25] G. Malinie, D. H. Hartmann, D. D. Clayton and G. J. Mathews, *ApJ* **413** (1993) 633
- [26] Y. Ishimaru and S. Wanaajo, *ApJL* **511** (1999) L33
- [27] T. Karlsson and B. Gustafsson, *Astron. Astrophys.* **436** (2005) 879
- [28] T. Karlsson, *ApJ* **439** (2005) 93
- [29] A. McWilliam and L. Searle, *Ap&SS* **265** (1999) 133
- [30] C. M. Raiteri, M. Villata, R. Gallino, M. Busso, and A. Cravanzola, *ApJ* **518** (1999) L91
- [31] T. Tsujimoto, T. Shigeyama, and Y. Yoshii, *ApJ* **519** (1999) L63
- [32] B. M. Tinsley, *Fundamentals of Cosmic Physics* **5** (1980) 287
- [33] T. Shigeyama and T. Tsujimoto, , *ApJ* **507** (1998) L135
- [34] C. Travaglio, A. Burkert, and D. Galli, *The Galactic Halo: From Globular Clusters to Field Stars*, *Proceedings of the 35th Liege International Astrophysics Colloquium*, Eds. A. Noels, P. Magain, D. Caro, E. Jehin, G. Parmentier, and A. A. Thoul. Liege, Belgium : Institut d'Astrophysique et de Geophysique, (2000), p.135
- [35] M. S. Oey, *MNRAS* **339** (2003) 849
- [36] S. E. Woosley and T. A. Weaver, *ApJS* **101** (1995) 181 (WW95)
- [37] L. Portinari, C. Chiosi, and A. Bressan, *A&A* **334** (1998) 505
- [38] F.-K. Thielemann, K. Nomoto, and M. Hashimoto, *ApJ* **460** (1996) 408
- [39] K. Nomoto, K. Iwamoto, and N. Kishimoto, *Sci.* **276** (1997) 1378
- [40] A. Renzini and M. Voli, *A&A* **94** (1981) 175
- [41] J. E. Norris, S. G. Ryan, and T.C. Beers, *ApJ* **561** (2001) 1034
- [42] F.-K. Thielemann, K. Nomoto, K. Yokoi, *A&A* **158** (1986) 17
- [43] C. Sneden, J. J. Cowan, D. L. Burris, and J. W. Truran, *ApJ* **496** (1998) 235

*Received 15 August 2009.*