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GRAVITATIONAL WAVEFORMS FROM MULTIPLE-ORBIT SIMULATIONS OF BINARY NEUTRON STARS

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Abstract. *We study the gravitational wave emission of equal-mass neutron stars in binary orbits as the stars approach the inner most last stable circular orbit. We illustrate the extraction of gravitational wave forms in a sequence of quasi-circular orbit simulations including the general relativistic hydrodynamic response of the stars. We compare the computed results with the Newtonian and post Newtonian results and show that substantial differences can arise as the orbits approach the final inspiral.*

Keywords: binary neutron stars, numerical relativity, ADM.

I. INTRODUCTION

Current interferemetric gravity wave observatories such as LIGO [1], GEO600 [2], GEO-HF [3, 4], TAMA300 [5] and VIRGO [6] have been taking data for some time [7–10], while a number of second generation observatories such as Advanced LIGO [11], Advanced VIRGO [12] and KAGRA [13] soon will be online. These observatories seek to detect gravity-wave emission from various sources, e.g. from core collapse supernovae, neutron star orbits, the stochastic cosmic background, etc. [1]. Of the many systems that emit gravitational waves, compact neutron-star and/or black-hole binaries are thought to be the best candidates for detecting gravitational radiation [14]. Indeed, the first discovery of a black hole - black hole merger has been reported. The number of neutron star - neutron star systems detectable by Advanced LIGO [15] is estimated [14, 16–21] to be of order several events per year based upon observed close binary-pulsar systems [22, 23].

To date there have been numerous attempts to calculate theoretical templates for gravity waves from compact binaries based upon numerical and/or analytic approaches (see for example [24–33]). However, most approaches utilize a combination of Post-Newtonian (PN) techniques supplemented with quasi-circular orbit calculations and then applying full GR for only the last few orbits before inspiral. In a previous paper [34] we have reported on a general relativistic hydrodynamics approach that can compute many orbits stably and efficiently from the PN regime until the last stable orbits without the need to invoke the quasi-circular orbit condition. We established the numerical stability of this approach based upon many orbit simulations of quasi-circular orbits and showed that this approach is straightforwardly scalable to evolve to the $\sim 10^4$ orbits within the LIGO frequency range. In this paper we present a first study of associated gravity waveforms from such multiple orbit simulations.

When binary neutron stars are well separated, the Post-Newtonian (PN) approximation is sufficiently accurate [35]. In the PN scheme, the stars are often treated as point masses, either with or without spin. At third order, for example, it has been estimated [36–38] that the errors due to assuming the stars are point masses is less than one orbital rotation [36] over the $\sim 16,000$ cycles that pass through the LIGO detector frequency band [14]. Nevertheless, it has been noted in many works [32, 39–50] that relativistic hydrodynamic effects might be evident even at the separations ($\sim 10 - 100$ km) relevant to the LIGO window.

Indeed, the templates generated by PN approximations, unless carried out to fifth and sixth order [36, 37], may not be accurate unless the finite size and proper fluid motion of the stars is taken into account. In essence, the signal emitted during the last phases of inspiral depends on the finite size and equation of state (EoS) through the tidal deformation of the neutron stars and the cut-off frequency when tidal disruption occurs.

Numeric and analytic simulations [51–59] of binary neutron stars have analyzed the approach to the innermost stable circular orbit (ISCO). While these simulations represent some of the most accurate to date, many simulations have only followed the evolution for a handful of orbits and are based upon an extrapolation of quasi-circular orbits. With $\sim 16,000$ cycles passing though the LIGO frequency band, it may questionable whether templates based on only a small number of orbits are sufficiently accurate to describe the full evolution of the system. Moreover, although one can obtain a solution to Einstein equations in the quasi-circular orbit condition, there is no guarantee that the true dynamical evolution actually passes through a given set quasi-circular solutions.

Accurate templates may eventually require the ability to calculate many orbits, including the radiation back reaction and relativistic hydrodynamic effects. Ideally, one would like to calculate from the post-Newtonian regime to near the inner most stable circular orbit (ISCO).

Toward that end, we have developed an approach [34] based upon the general relativistic hydrodynamics formalism developed in [40, 42, 60] that can evolve from the post-Newtonian to ISCO regimes in a single calculation. We showed [34] that it is straightforwardly scalable to the computation of the continuous evolution through the $\sim 10^4$ orbits in the LIGO window. Here, we illustrate an application of the formalism to estimate of the emergent gravity wave signal. The method for including the radiation back reaction and the extraction of the outgoing gravity wave template is described in Section 2 along with an illustrative extraction of the gravity wave parameters. Conclusions are presented in Sections 3.

II. GRAVITATIONAL WAVES

The physical processes occurring during the last orbits of a neutron star binary are currently a subject of intense interest. As the stars approach their final orbits it is expected that the coupling of the orbital motion to the hydrodynamic evolution of the stars in the strong relativistic fields could provide insight into various physical properties of the coalescing system [34, 45, 61]. In this regard, careful modeling is needed which includes both the nonlinear general relativistic and hydrodynamic effects as well as a realistic neutron star equation of state.

Because our method of solving the field equations [34, 40, 60] does not explicitly evolve gravitational radiation we use a multipole expansion originally developed in [62] as described in [40, 60]. The gravitational radiation signal is derived via a multipole expansion of the metric perturbation to the hexadecapole ($l = 4$) order including both mass and current moments and a correction for the slow motion approximation. In [34], we computed quasi-equilibrium circular orbit conditions for two neutron stars initially of gravitational mass 1.44 $M_{\odot}$. We then considered configurations as a function of total angular momentum from the post-Newtonian regime up to the point that the stars enter the inspiral phase [34]. In this paper we perform an illustrative calculation of the angular momentum and power loss rate and reconstruct the gravitational wave form. We also summarize how to infer the possible signal to noise in the LIGO sensitivity band.

In general it is possible to express the emission of gravitational radiation in terms of an "exact" expansion of multipole moments of the effective stress energy tensor, including corrections for the so-called "slow motion" approximation [62]. It is important to appreciate that these formulae can apply to strong-field sources as well as to weak field sources [62] as long as the relevant components of the effective stress energy tensor can be identified.

Since in this paper, we are only concerned with orbital motion of equal mass binaries, the multipole expansions reduce to only a few nonzero terms. These we evaluate and test for convergence of the expansion.

In any asymptotically flat coordinate system (such as the one we are using here) in which the gravity waves far from the source can be characterized as linear metric perturbations propagating on a flat background, the transverse traceless part of the metric perturbation characterizes the radiation completely. This metric perturbation then can be expressed [40, 60, 62] in terms of the mass-multipole (I^{lm}) and current-multipole (S^{lm}) moments as

$$h_{jk}^{TT} = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \left[r^{-1}\ {}^{(l)}I^{lm}(t-r) T_{jk}^{E2,lm} + r^{-1}\ {}^{(l)}S^{lm}(t-r) T_{jk}^{B2,lm} \right] , \tag{1}$$

where the superscript TT denotes the transverse traceless part of the metric perturbation and the notation ${}^{(l)}I^{lm}$ and ${}^{(l)}S^{lm}$ denotes the l^{th} time derivative of the respective moments. As usual, the gravitational wave strain can be given in terms of two polarization $h_{\times}$ and h_{+}.

From this, the general expression for energy loss is

$$\frac{dE}{dt} = \frac{1}{32\pi} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \langle |{}^{(l+1)}I^{lm}|^2 + |{}^{(l+1)}S^{lm}|^2 \rangle , \tag{2}$$

where the brackets denote averages over several wavelengths. Angular momentum loss can similarly be written

$$\frac{dJ}{dt} = \frac{i}{32\pi}\sum_{l=2}^{\infty}\sum_{m=-l}^{l}\langle {}^{(l)}I^{lm*}m^{(l+1)}I^{lm}\rangle + \langle {}^{(l)}S^{lm*}m^{(l+1)}S^{lm}\rangle \ , \tag{3}$$

where the expression in Eq. (3) assumes an alignment of the angular momentum vector with the z axis.

The radiation reaction potential for the loss of orbital momentum can also be written [40] in terms of these moments.

$$\chi = \frac{1}{32\pi}\sum_{l=2}^{\infty}\sum_{m=-l}^{l} x_i x_j \langle |{}^{(l+1)}I^{lm}|^2 + |{}^{(l+1)}S^{lm}|^2\rangle \ , \tag{4}$$

The derivation of the relevant mass and current moments in our coordinates is straightforward as described in detail in Refs. [40, 60] and need not be repeated here. The contribution from both the current moments and slow-motion correction is expected to be small. We compute terms out to ω^{10}, which includes mass multipoles out to $l = 4$, current multipoles out to $l = 3$ and the leading correction for the slow motion correction.

In Table 1 we summarize the relative contributions of various moments to the energy and angular momentum loss rates for a few values of total angular momentum J based upon the fiducial multiple orbit simulation or Ref. [34] utilizing the MW equation of state for two equal mass neutron stars of gravitational mass $M_g = 1.44$ $M_\odot$. The orbit parameters for various fixed angular momenta are summarized in Table 2 from Ref. [34]. As expected, the quadrupole term dominates by more than an order of magnitude. The next largest term is the slow motion correction which contributes only a few percent to the gravitational radiation and tends to decrease the loss rate. Hence, we conclude that the moment expansion indeed converges quickly.

Table 1. Contribution of various moments to energy and angular momentum loss rates

J	$\dot{E}$	$l = 2, m = 2$	Slow Motion Correction	$l = 4, m = 2$	$l = 4, m = 4$
2.7×10^{11}	5.06×10^{-9}	2.63×10^{-9}	-9.86×10^{-11}	5.56×10^{-17}	1.02×10^{-13}
2.8×10^{11}	3.57×10^{-9}	1.85×10^{-9}	-6.76×10^{-11}	3.51×10^{-17}	6.32×10^{-14}
3.0×10^{11}	1.72×10^{-9}	8.90×10^{-10}	-3.08×10^{-11}	1.29×10^{-17}	2.32×10^{-14}
J	$\dot{J}$	$l = 2, m = 2$	Slow Motion Correction	$l = 4, m = 2$	$l = 4, m = 4$
2.7×10^{11}	1.26×10^{-1}	1.17×10^{-1}	-4.40×10^{-3}	2.52×10^{-9}	4.53×10^{-6}
2.8×10^{11}	1.77×10^{-1}	9.20×10^{-2}	-3.36×10^{-3}	1.75×10^{-9}	3.14×10^{-6}
3.0×10^{11}	1.07×10^{-1}	5.53×10^{-2}	-1.92×10^{-3}	8.04×10^{-10}	1.45×10^{-6}

Table 2. Orbital parameters for each EoS

EoS	$J(cm^2)$	$\omega(rad\ s^{-1})$	$d_p(km)$	$d_c(km)$	$M_{ADM}(M_\odot)$	$\rho_c(g\ cm^{-3})$
MW	2.6×10^{11}	780.92	65.22	51.52	1.391	1.67×10^{15}
	2.7×10^{11}	671.85	71.18	57.24	1.393	1.62×10^{15}
	2.8×10^{11}	602.80	76.94	61.86	1.394	1.60×10^{15}
	3.0×10^{11}	482.30	86.91	72.36	1.396	1.55×10^{15}
	3.5×10^{11}	300.46	116.13	100.8	1.399	1.44×10^{15}
	3.8×10^{11}	235.72	136.93	119.74	1.401	1.39×10^{15}

The sensitivity of the gravity wave frequency to the equation of state was summarized in Ref. [34]. Here we analyze the gravity wave characteristics based upon one representative equation of state. Fig. 1 shows $f,h,\dot{E},\dot{J}$ as a function of time to inspiral for a simulation based upon the fiducial MW EoS. For this plot we adopt $h\equiv(h_+ + h_\times)/2$. In Fig. 2 we show various parameters, i.e. $f,h,\dot{E},\dot{J}$ characterizing the gravity wave signals as a function of total angular momentum J for calculations based upon the MW equation of state of Ref. [68]. In this figure, the points are the numerical results. The lines drawn are polynomial fits to these computational results. These analytic functions are summarized in Table 3.

Table 3. Polynomial fits to $f,h,\dot{E},\dot{J}$

	a_0	a_1x	a_2x^2	a_3x^3	a_4x^4
f	42.3	-32.65	8.77	-0.802	
h	44.01	-29.48E+02	7.221	-0.6144	
$\dot{E}$	3752.90	-4488.11	2009.89	-399.21	29.66
$\dot{J}$	14.14	-12.00	3.42	-0.325	

Having analytic fits to the various parameters as a function of J we can convert them to time in the limit that only gravity waves affect the orbit decay time by integrating the angular momentum loss timescale, i.e.

$$t=\int_{J_{coll}}^{J}\frac{dJ'}{\dot{J}} \tag{5}$$

where J_{coll} is the minimum J value before collapse of the orbit.

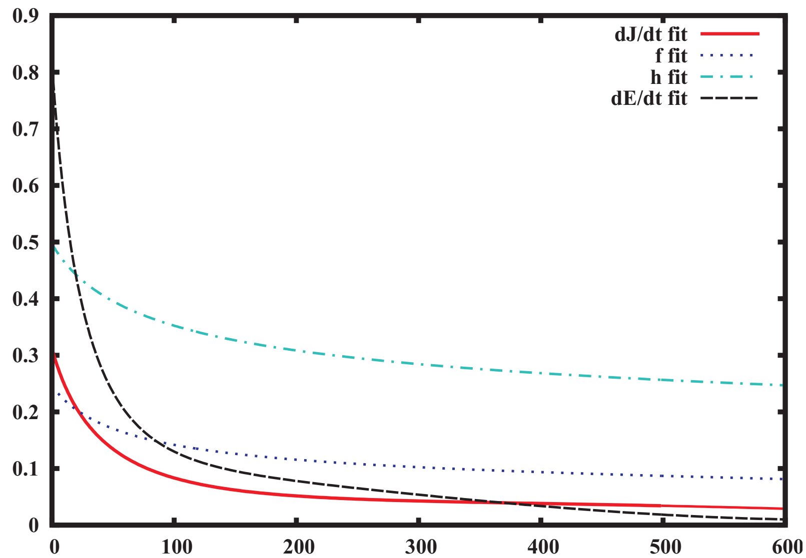

Fig. 1. Plot of f , h, $\dot{E}$, and $\dot{J}$ versus time to inspiral

For comparison the expected increase in gravity wave frequency from the post-Newtonian and Newtonian estimate is compared with our calculations on Fig. 3. We see the chirp as the stars approach. The fact that there is less increase in frequency as the stars approach implies that there are more cycles per time bin so that the inspiral may be easier to detect [69].

Ultimately, however, one wishes to know the actual signal to noise response in an gravity wave detector. The detector response in real applications involves a fourier transform integrated over many orbits within the LIGO frequency band. To estimate of the effect of these orbits on the detector signal to noise one must simply fourier transform the observed time-dependent gravity wave form given above. That is, the time dependent detector strain is given by,

$$h_t(t) = A \times h \cos \int 2\pi f dt \ , \tag{6}$$

where A is a parameter that depends upon detector inclination and response, while h is defined above and given in Fig. 1. The detailed time evolution of $h_\times$ and h_+ are shown in Fig. 4 and Fig. 5 for the last 200 ms of inspiral.

The fourier transform is then written,

$$\tilde{h}(f) = \int e^{2\pi f t} h_t(t) dt \tag{7}$$

The signal to noise per unit frequency is then,

$$\frac{d(S/N)^2}{df} = \frac{4|\tilde{H}^2|}{S_n} \tag{8}$$

where S_n is the detector spectral density function. For our purposes S_n can be taken from the analytic approximations of Ref. [70].

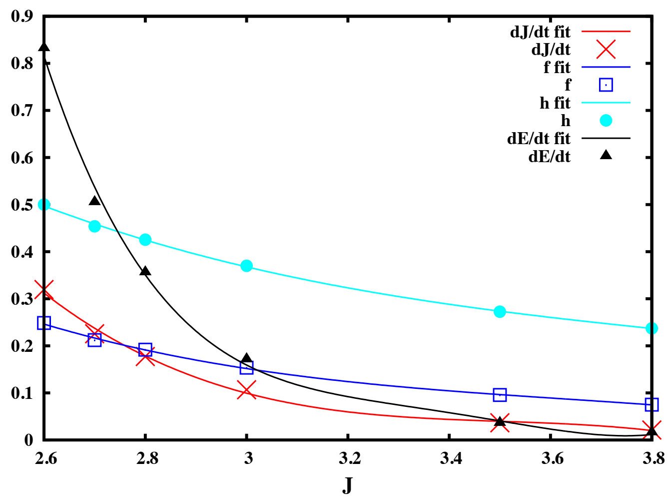

Fig. 2. Plot of f, h, $\dot{E}$, and $\dot{J}$ versus J. The points are calculated. The lines are the polynomial fits given in Table 3

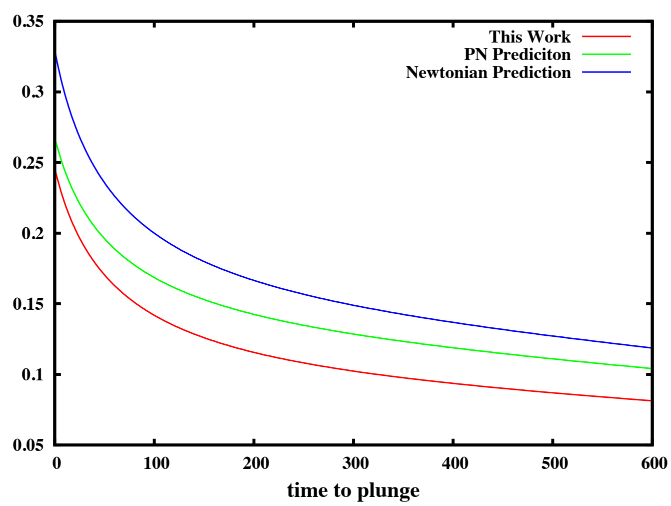

Fig. 3. Plot of gravity wave frequency f versus time to inspiral from this work compared with the the PN prediction and the Newtonian prediction

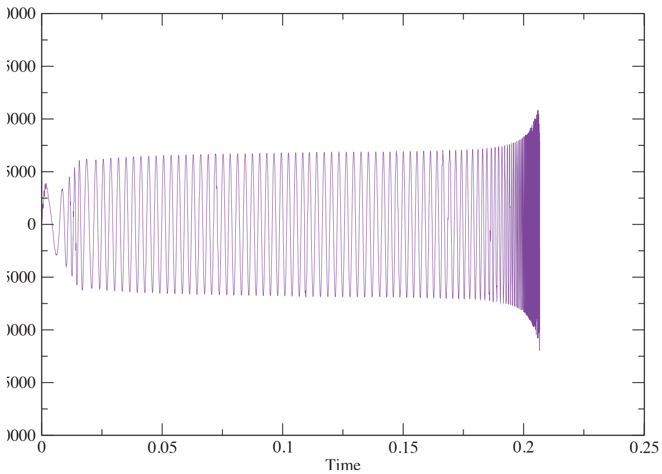

Fig. 4. Plot of the strain $h_\times$, versus time for the last 200 ms before inspiral.

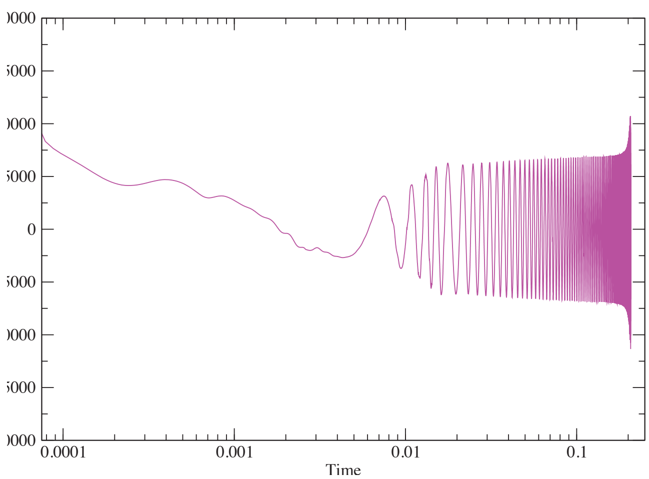

Fig. 5. Plot of the strain h_+, versus time for the last 200 ms before inspiral.

III. CONCLUSIONS

We have computed the emergent gravity wave signal from a multiple orbit simulation of an equal-mass binary neutron star system. The gravitational wave form was obtained via a multiple expansion up to $l = 4$ and including the corrections to the slow motion approximation. Although our approach employs a number of approximations including a conformally flat metric, a multipole expansion, and quasi-stationary orbits, we have shown that this calculation can be performed efficiently with limited computer resources. Hence, we suggest that the templates developed by this method should be applied in current gravity wave observatories.

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