Practical Values of Friction Factors

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ABSTRACT

Over the past fifteen years, engineers from Mine Ventilation Services, Inc. (MVS) have measured numerous friction factors at many different types of mining operations. The results of these measurements indicate that standardized friction factors referenced in most ventilation textbooks are greater than those measured in the field for similar airway support systems. Many referenced friction factors are still based on G. E. McElroy's classic paper "Engineering Factors in the Ventilation of Metal Mines" published in 1935. Most mechanized mines now incorporate airways that are larger, have more advanced support systems, and more uniform openings. This paper describes the measurement techniques and results from friction factor measurements taken during ventilation surveys at various mines with differing support systems. A comparison between textbook and measured values is also presented.

KEYWORDS

Friction Factors, Resistance, and Atkinson.

INTRODUCTION

The Atkinson friction factor has long been a primary component in calculating airway resistance for ventilation planning purposes. One of the original publications concerning friction factors in mines was published in 1935 by the former U.S. Bureau of Mines (McElroy, 1935). Subsequently, numerous papers, articles, and texts have been published on friction factors including Kharkar *et al.*, (1974), Hall (1981), Wala (1991), McPherson (1992), and Hartman, *et al.*, (1997). Accurate values of friction factor are critical in ventilation planning exercises. No computer simulation is meaningful if the airway resistances throughout a mine are not accurately assessed. For proposed underground airways, the only way to develop a ventilation model for planning purposes is by the estimation of friction factors. For existing underground mines, it is recommended that a proper ventilation survey of the mine ventilation infrastructure be conducted prior to ventilation planning exercises. However, the reality is that many mines lack both the time and resources to conduct these thorough investigations. This results in a reliance upon published friction factor data for estimation of airway resistances.

Mine Ventilation Services, Inc. (MVS) engineers have been involved in the measurement, classification and planning of ventilation systems for over fifteen years and have consequently built a substantial library of measured friction factors. A review of these measurements indicates that standardized friction factors referenced in many articles and textbooks on ventilation appear greater than those MVS has measured in the field for similar airway support systems. A comparison of friction factors is described in this paper for both hard rock and coal mining operation.

GENERAL THEORY

The determination of frictional pressure drop (p) in mine airways may be obtained from the following relationship:

$$p = fL\frac{Per}{A}\rho\frac{u^2}{2}, \text{ Pa} \qquad (1)$$

f = coefficient of friction (dimensionless)
ρ = Air density (kg/m^3)
Per = Airway perimeter (m)
u = Air velocity (m/s)

A = Area (m^2) L = Length (m)

This is a form of the Chezy-Darcy Equation, and is applicable to circular and non-circular airways and ducts. The Chezy-Darcy coefficient of friction (dimensionless) varies with respect to Reynolds Number, the trend of which is plotted on the Moody diagram. The Chezy-Darcy equation was adapted by Atkinson to give the following, commonly used, Atkinson Equation:

$$p = kL\frac{Per}{A}u^2 \text{ , Pa} \qquad (2)$$

The Atkinson friction factor (k) is a function of air density, and is computed as the product of the Chezy-Darcy coefficient of friction and the air density, divided by a factor of two. Since the Chezy-Darcy coefficient of friction is dimensionless, the Atkinson friction factor has the units of density (kg/m^3). The Atkinson Equation may be expressed in terms of the Atkinson resistance (R) for the airway, where:

$$R = \frac{p}{Q^2} = kL\frac{Per}{A^3} \quad Ns^2/m^8 \qquad (3)$$

The first part of this equation, relating frictional pressure drop and quantity to resistance, is known as the Square Law. This important relationship is used to establish resistance from measured pressure and quantity data. The second part of the equation is used to determine resistance from typical Atkinson friction factors, and known or proposed airway geometry. It should be noted that the frictional pressure drop term in the Square Law is directly proportional to air density, as is the Atkinson friction factor. Hence, the Atkinson friction factor that is applied must be adjusted for actual mine air density.

When using the Atkinson friction factor it is important to remember that the factor is not constant for a given airway, but varies with Reynold's Number. However, in mine ventilation it is normal to assume that the Atkinson friction factor is relatively constant, regardless of the flow regime. This is because for fully turbulent flow (which is typically the case in mine ventilation) the friction factor is a function only of the relative roughness of the airway. Roughness can be defined as the height of the airway aspiraties (e) divided by the hydraulic mean diameter (d = 4A/Per). The Von Kármán equation gives the relationship for Atkinson friction factor and relative roughness for fully turbulent flow:

$$f = \frac{2k}{\rho} = \frac{1}{4\left[2\log_{10}\left(\frac{d}{e}\right) + 1.14\right]^2} \qquad (4)$$

From this equation, it is apparent that the Atkinson friction factor will vary for airways with the same surface roughness (asperity height), but different hydraulic mean diameters. Hence, as the airway hydraulic mean diameter increases, and all other conditions remain the same, both the relative roughness and the Atkinson friction factor will decrease. However, this change in Atkinson friction factor is usually small, and is often not discernible in field measurements. For example, an airway with an average asperity height of 50 mm (0.16 ft), and dimensions of 2 m (6.6 ft) by 6 m (19.7 ft), the Atkinson friction factor at standard density is 0.0068 kg/m^3 (36.7 $lbfmin^2/ft^4 \times 10^{-10}$). If a second airway is considered which has the same surface asperity height, but has dimensions of 3 m (9.8 ft) by 6 m (19.7 ft), the Atkinson friction factor drops to 0.0061 kg/m^3 (32.9 $lbfmin^2/ft^4 \times 10^{-10}$). Hence, for this example a 50% increase in flow area results in only 10% change in the Atkinson friction factor. It is difficult to measure this difference in the field due to the numerous factors that are required to compute friction factors. For this study, variations in friction factor as a function of airway size were not considered due to the considerable scatter in the measured Atkinson friction factors for various entry types.

FRICTION FACTOR MEASUREMENT TECHNIQUES

The friction factors measured by MVS were conducted during the course of ventilation surveys at numerous mining operations. For each mine, measured frictional pressure drops and airflow data were used to develop ventilation networks. To determine accurate friction factors, airways were selected which minimized shock losses. The air quantities were measured by determining the mean air velocities and airway cross-sectional areas at predetermined locations in the airways of interest. Rotating vane anemometers attached to extendible rods were used to traverse the airways for measurement of the mean air velocities. Traverses were repeated until two readings were obtained within ±5%. The airway cross-sectional areas were measured using steel tapes. The air quantities at each station were computed as the product of the air velocity and the airway cross-sectional area.

Frictional pressure drops through the airway were determined using the gauge-and-tube technique. The gauge-and-tube (or trailing hose) method allows direct measurement of frictional pressure differentials using a digital manometer connected to a length of tubing, the ends of which were connected to the total pressure ports of pitot-static tubes. Psychrometric properties of the air were also measured in the airways in order to determine the air densities so that friction factors could be reported on a standardized basis.

The calculation of Atkinson friction factor (k) is conducted by re-writing the Atkinson resistance equation so that:

$$k = R\frac{A^3}{LPer} \qquad (kg/m^3) \qquad (5)$$

Where R is determined by the square law, $R = p/Q^2$. Airway lengths were evaluated either from the known length of the pressure tube, or were measured with a nylon tape. The cross-sectional area and perimeter used in this equation were averaged from two to four measurements taken along the length of drift used in computing the friction factor. When measurements are taken in the field they are measured and recorded at the actual mine air density. When reporting friction factors against published information they must be calculated on a standardized basis as follows:

$$k_{std} = k_{act}\frac{\rho_{std}}{\rho_{act}} \ kg/m^3 \qquad (6)$$

k_{std} = Standardized friction factor (kg/m^3)
k_{act} = Actual friction factor (kg/m^3)
ρ_{act} = Actual air density (kg/m^3)
ρ_{act} = Standard Air density (kg/m^3)

All measurements presented in this paper are reported at a standardized density of 1.2 kg/m^3 (0.075 lb/ft^3). When standardized friction factors are used in future ventilation modeling, it is imperative that they be corrected for the air density expected at the mine. For example, a proposed mine at 1,830 m (6,000 ft) above sea level may have an air density of 0.960 kg/m^3 (0.060 lb/ft^3). If a friction factor of 0.0093 kg/m^3 (50 $lbf.min^2/ft^4 \times 10^{-10}$) was selected from a standardized reference table, then for computer modeling a corrected friction factor of 0.0074 kg/m^3 (40 $lbf.min^2/ft^4 \times 10^{-10}$) should be used.

FRICTION FACTOR MEASUREMENTS IN METAL MINES

Measurements of airway friction factors were obtained during ventilation surveys of thirteen metal mines around the world. The majority of these mines employed traditional jumbo drill and blast development techniques to drive the main airways for which these friction factors are representative. Friction factor data were taken wherever possible for varying airway sizes, ramps, and bored and alimak raises. Figure 1 shows the measured k-factors taken in a number of metal mines along a straight level drift. In general, these drifts were arched with rock bolts and mesh. The results indicate that for the 40 measurements taken of friction factor for a level airway mined by drill and blast techniques, the mean value is approximately 0.009 kg/m^3 (49 $lbf.min^2/ft^4 \times 10^{-10}$). However, it is important to note that the standard deviation for this average is 0.00239 kg/m^3 (12.9 $lbf.min^2/ft^4 \times 10^{-10}$). This gives a statistical range of 0.0066 kg/m^3 (36 $lbf.min^2/ft^4 \times 10^{-10}$) to 0.011 kg/m^3 (62 $lbf.min^2/ft^4 \times 10^{-10}$). Because of the statistical scatter in the readings (particularly in the upper range), it is recommended that for design purposes a friction factor of approximately 0.010 kg/m^3 (60 $lbf.min^2/ft^4 \times 10^{-10}$) be used. This value should provide some conservatism in the design. Table 1 shows friction factor measurements for other types of airways in a metal mine.

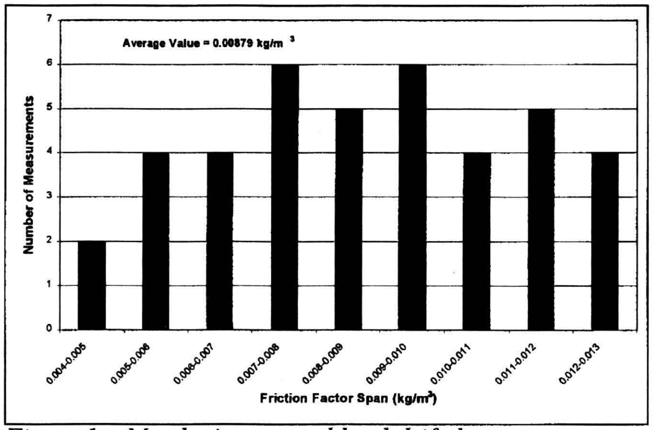

Figure 1. Metal mine general level drift data.

The measurements taken for conveyor drifts, alimak raises, bored raises and ramps show a relatively wide range of values. This spread is mainly due to the way each individual airway is constructed and any shock losses present due to entry or exit losses and constrictions or expansions. For example, the friction factor for a ramp is directly dependant on how tightly spiraled the ramp is constructed. The measurements given in the table all take into account the shock losses encountered in the airway due to bends (airway spiral). For each airway type listed on Table 1, it is important to note the standard deviation computed for the range of data presented. The data suggests that it is probably prudent to use the mean values presented plus one half to one standard deviation in order to be conservative.

FRICTION FACTOR MEASUREMENTS IN COAL AND SOFT ROCK MINES

Measurements of airway friction factors were obtained during the ventilation surveys of fourteen coal and soft rock mines from both the east and west coast regions of the United States. Sufficient data were measured to determine characteristic friction factors for both intake and return airways, however, a lack of data for belt and cribbed entries was noted. In most coal mines the airflow in beltways is kept to a minimum which results in difficult conditions for

Table 1. Standardized friction factors for metal mine airways.

	Level Drift	Ramp	Alimak Raise	Bored Raise	Beltway	TBM Drift
Average Value	0.00879 (47.4)	0.01158 (62.4)	0.01126 (60.7)	0.00466 (25.1)	0.01399 (75.4)	0.00440 (23.7)
Maximum Value	0.01284 (69.2)	0.01739 (93.7)	0.01579 (85.1)	0.00698 (37.6)	0.01664 (89.7)	0.00560 (30.2)
Minimum Value	0.00468 (25.5)	0.00698 (37.6)	0.00874 (47.1)	0.00230 (12.4)	0.01228 (66.2)	0.00341 (18.4)
Std. Deviation	0.00239 (12.9)	0.00310 (16.7)	0.00330 (17.8)	0.00152 (8.2)	0.00184 (9.9)	0.00111 (6.0)
# of Measurements	40	20	5	10	5	3

Note: Atkinson's Friction Factor in kg/m³ (lbf.min²/ft⁴ × 10^{-10})

the measurement of frictional pressure differentials. The variance in resistance encountered in the cribbed drifts was extreme. The friction factor for these drifts will vary based upon the cribbing spacing, construction, layout in the drift, and the aerodynamic properties of the construction. There were insufficient field data recorded to adequately describe these factors for cribbed entries. In general the mean friction factor for return entries appears to be higher than that of intake entries for the same roof support type. This is due to the intake entries being better maintained and generally cleaner than the return entries. For this paper an intake airway is defined as a clean rectangular entry with roof bolts and limited mesh lining. A return airway is described as a rectangular airway with some irregularities (sloughing), roof bolts, and limited mesh.

Figures 2 and 3 show the measurements taken in coal and soft rock mines for both typical intake and return airways. Table 2 shows the friction factor data for intake, return, conveyor and cribbed airways. In general, the airways measured were rectangular. The results of these measurements show that the mean value of friction factor computed for intake and return airways is a reasonable value to use for future ventilation planning purposes. The standard deviation is not too significant for these types of airways and the data does not appear to be skewed above or below the average. However, there is a significant spread of data for conveyor and cribbed airways. For these airways it is suggested that care be used in implementing these data in a design. It may be prudent to use the average value presented plus one half standard deviation to provide some conservatism in the design. The reason for this is that the friction factor is very dependent on the geometry and size of the conveyor belt, and on the cribbing material and spacing.

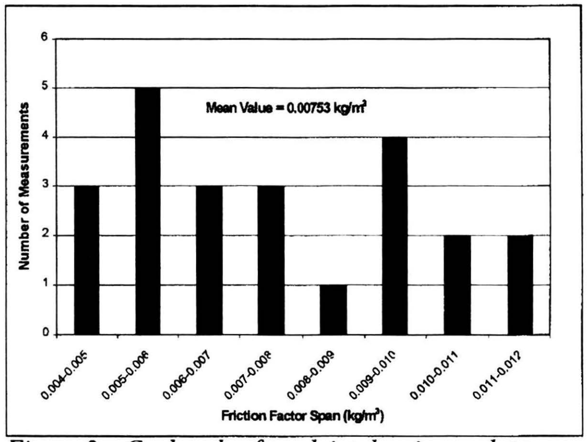

Figure 2. Coal and soft rock intake airway data.

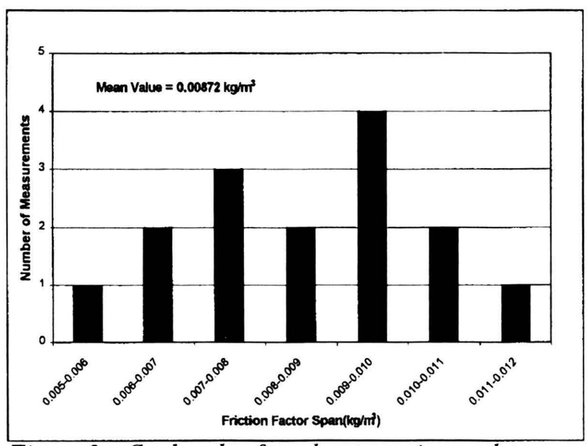

Figure 3. Coal and soft rock return airway data.

Table 2. Standardized friction factors for coal mine airways.

	Intake Drift	Return Drift	Belt Drift	Cribbed Drift
Average Value	0.00753 (40.6)	0.00872 (47.0)	0.01058 (57.0)	0.06781 (365.5)
Maximum Value	0.01148 (61.9)	0.01133 (61.1)	0.01757 (94.7)	0.14409 (776.6)
Minimum Value	0.00482 (26.0)	0.00566 (30.5)	0.00459 (24.3)	0.04522 (243.7)
Std. Deviation	0.00219 (11.8)	0.00176 (9.5)	0.00636 (34.3)	0.02516 (135.6)
# of Measurements	23	15	5	7

Note: Atkinson's Friction Factor in kg/m³ (lbf.min²/ft⁴ × 10^{-10})

CONCLUSIONS AND COMPARISONS WITH VENTILATION TEXTS

The mean and recommended standardized friction factors presented in this paper were compared with standardized friction factors for similarly described airways in several articles and ventilation texts. This comparison is shown on Table 3. It was noted that a number of ventilation textbooks reference metal mine friction factor data originally computed by McElroy (1935). These texts include; "Mine Ventilation and Air Conditioning" Hartman (1997), "Mining Engineering Handbook" Hartman (1992), and "Mine Ventilation Engineering" Hall (1981). Table 3 only lists "Mine Ventilation and Air Conditioning" by Hartman *et al.*, (1997) since the other texts reference the same source for metal mine friction factors. This text also references Kharkar, *et al.*, (1974) for coal mine entries. In general, the recommended MVS values are consistently lower than the values quoted in the ventilation texts. For coal mines, the friction factors listed by McPherson, 1993 and Hartman *et al.*, (1997), are very close to the factors measured by MVS. *However, friction factors based on McElroy's work for airways driven in igneous rocks (metal mine airways) are over 100% higher than what was measured by MVS.* One possible explanation for this discrepancy is the modern techniques and equipment used to drive drifts in metal mines today. These modern mining techniques may provide for a larger, smoother, and more regular airway, which would consequently have a lower friction factor. MVS did not measure a single friction factor as high as those referenced by McElroy (1935). It can be seen that if McElroy's values of friction factor are used for mine planning, an unnecessarily high mine resistance will be built into the design. This could result in over sizing main fans and possibly result in unnecessary developments. Comparison of the MVS recommended friction factors with McPherson (1993) showed reasonably close results.

In reviewing engineering work conducted by others, MVS personnel have observed that a common mistake is made by not adjusting the friction factor for actual mine density. As mentioned previously, certain operations where the air density is significantly higher or lower than standard air density not adjusting the friction factor could have a significant impact on the total mine resistance.

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Table 3. Comparison of standardized MVS measured k factors with published data.

Airway Type	Mean MVS Measured Data	Suggested MVS Value	McPherson (1993)	Hartman, *et al.*, (1997)
Rectangular Airway – Clean Airway (coal or soft rock with rock bolts limited mesh)	0.0075 (41)	0.0075 (41)	0.009 (49)	0.0080 (43)
Rectangular Airway – Some Irregularities (coal or soft rock with rock bolts limited mesh)	0.0087 (47)	0.0087 (47)	0.009 (49)	0.0091 (49)
Metal Mine Drift (arched and bolted with limited mesh)	0.0088 (47)	0.010 (60)	0.0120 (65)	**0.0269 (145)**
Metal Mine Ramp (arched and bolted with limited mesh)	0.0116 (62)	0.013 (71)	-n/a-	**0.0297 (160)**
Metal Mine Beltway (large area, rock bolted with mesh)	0.0140 (75)	0.015 (80)	-n/a-	-n/a-
Bored Circular Raise (contains entry/exit loss)	0.0047 (25)	0.0050 (27)	0.004 (22)	0.0028 (15)
Rectangular Alimak Raise (un-timbered with rock bolt and mesh)	0.01126 (61)	0.0129 (70)	0.014 (75)	-n/a-
TBM Drift (rock bolts with mesh)	0.0044 (24)	0.050 (26)	0.0055 (30)	0.0037 (20)

Note: Atkinson's Friction Factor in kg/m^3 ($lbfmin^2/ft^4 \times 10^{-10}$). Bold indicates large discrepancy with MVS measured values.