

## Theoretical Analysis of Jet Fan Performance Using Momentum and Energy Considerations

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### ABSTRACT

Jet fans or ductless fans have been analyzed using momentum and energy considerations. This analysis is necessary for the mechanisms of mixing between the jet primary flow and the secondary or induced stream to be understood. In each case it is possible to predict performance in terms of pressure, flow ratios and hence the percentage mechanical efficiency of the whole system using equations derived from momentum and energy principles. The analysis given is suitable for situations where jet fans are utilized as area boosters in a mine ventilation system and in longitudinal vehicular tunnel ventilation. The energy and momentum equations serve as important tools for the analysis and consider all losses in the system. The derivations identified a range of flow ratios of the induced flow to the jet flow which can be used to design effective ventilation systems incorporating jet fans. Each flow ratio ( $n$ ) is associated with an optimum area ratio ( $\infty$ ) of the jet fan outlet to tunnel, or airway area, for maximum induction or entrainment of the secondary flow. It can be shown that there is an optimum set of conditions for jet fan performance which are explained in detail in the paper.

### KEY WORDS

Area Ratio, Flow Ratio, Entrainment, Momentum, Energy, Pressure, Jet Mixing Length, Mixing Velocity, Performance, Losses, and Efficiency.

### INTRODUCTION

Ductless fans or jet fans play an important role in a mine, tunnel and other ventilation systems, and when applied properly, they can significantly improve local airflow distribution. Jet fans find preference in longitudinal ventilation of road, railway and other kinds of through flow tunnels. In mine ventilation, jet fans are best suited in parallel airways stemming from the same drift and exhausting to a common one. They can be installed in an airway where there is a weak prevailing air stream and are then useful in boosting the flow, in some cases to many times the magnitude of their own inlet volume. The connecting airway length can be 15 to over 100 m long.

Jet fans can range from compressed air based air movers of 50 mm in diameter to electric driven fans of up to 200 hp and 1.5 meters in diameter. The purpose of each application can range from flashing pollutants and heat, and on a larger scale, serving as a major component of the network such as an area booster or help exhaust air to a nearby airway or shaft without a complete bulkhead or ducting.

The theoretical analysis and application of the jet fan system described, is for tunnels and mines where airways inter-

connect such as in block caving and room and pillar mining. The theory can be extended to other similar situations.

### THEORETICAL FOUNDATIONS IN JET FAN SYSTEMS

Theoretical foundations are necessary in analyzing a flow system and require an understanding of the mechanisms of jet fan performance. One way of analyzing jet fans is to perform a momentum and energy analysis by considering the losses that are encountered when two fluid streams of dissimilar velocities are mixed. Jet pump performance is often analyzed in this way, e.g. McClintock and Hood (1946), and Cunningham (1957), and (1976). The analysis for jet fans is much more complicated because of the three dimensional nature of the flow, but only a simplified theoretical approach will be developed for this study. It is reasonable to assume that density differences between the primary jet flow and the secondary induced stream are negligible. Practical data and theoretical formulations pertaining to jet fan ventilation have been described in detail by Mutama (1995), and Mutama and Hall (1996). McElroy (1945) has

studied the role of air jets in mine ventilation which effectively are similar to jet fan or unducted ventilation systems.

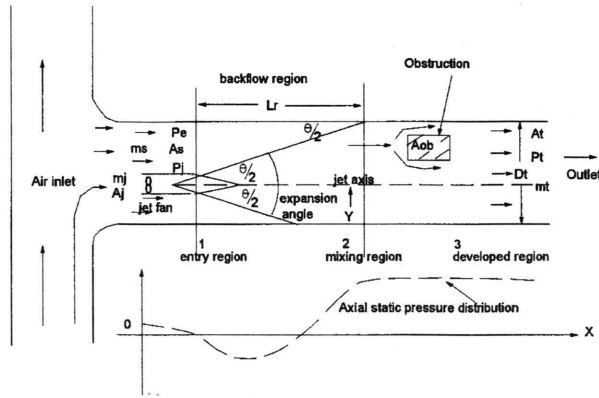


Figure 1. Schematic description of a jet fan in a through flow ventilation system.

In Figure 1 a schematic diagram of a jet fan-tunnel system is shown and the area of the cross section not occupied by the fan in the tunnel or airway can be defined as follows:

$$A_s = A_t - A_j \quad (i)$$

and from flow continuity

$$m_t = m_s + m_j \quad (ii)$$

The symbols  $A_j$ ,  $A_s$  and  $A_t$  are the area of the jet fan outlet or nozzle, the area through which the secondary air stream is introduced to the tunnel and the tunnel cross sectional area respectively. The mass flows  $m_j$ ,  $m_s$ , and  $m_t$  represent the jet flow, secondary stream and total tunnel mass flow respectively.

$$\text{Let } A_j/A_s = \alpha \quad (iii)$$

$$\text{Then } \frac{A_j}{A_t} = \frac{\alpha}{1 + \alpha} \quad (iv)$$

$$\text{Let } \frac{m_s}{m_j} = n \quad (v)$$

$$\text{Then } \frac{m_t}{m_j} = \frac{m_j + m_s}{m_j} = 1 + n \quad (vi)$$

The velocity of the secondary stream  $u_s$ , and tunnel flow  $u_t$  can be expressed in terms of the jet discharge velocity  $u_j$  as follows:  $u_s = \alpha n u_j$ ,  $(vii)$

$$u_t = \frac{\alpha(1+n)}{1+\alpha} u_j \quad (viii)$$

The axial static pressure of the flow at the initial mixing of the primary jet and secondary stream and at the final mixed stage is given by the following equation if zero friction loss is assumed,

$$(p_t - p_e)A_t = m_j u_j + m_s u_s - m_t u_t \quad (1)$$

By using equations (i) to (viii), the above equation reduces to:

$$p_t - p_e = \alpha \rho u_j^2 (1 - 2\alpha n + \alpha^2 n^2)$$

$$p_t - p_e = \alpha \rho u_j^2 (1 - \alpha n)^2 \quad (2)$$

The above equation is valid only when there are no losses in the system which is not the case in reality. A number of equations can be written for each part of the jet fan ventilation system.

#### JET FAN NOZZLE ENERGY EQUATION

$$\frac{p_i}{\rho} + \frac{u_{ni}^2}{2} = \frac{p_e}{\rho} + \frac{u_j^2}{2} + \frac{P_{fl}}{\rho}$$

$P_{fl}/\rho$  represents the nozzle energy loss. Let  $P_j = p_i + \rho u_{ni}^2/2$  the total pressure of the jet and  $P_{fl} = \zeta_n \rho \frac{u_j^2}{2}$

The jet fan nozzle equation is then

$$P_j - p_e = (1 + \zeta_n) \frac{\rho u_j^2}{2} \quad (3)$$

#### Secondary or entrained flow energy equation

An equation similar to (3) can be written for the secondary stream

$$P_j - p_e = (1 + \zeta_s) \frac{\rho u_s^2}{2},$$

$P_s$  and  $p_e$  are the suction pressure at entry and start of mixing point respectively.  $\zeta_n$  is a suction loss coefficient. The above equation can now be expressed as

$$P_j - p_e = (1 + \zeta_s) \alpha^2 n^2 \rho u_j^2 \quad (4)$$

**MOMENTUM BALANCE IN THE TUNNEL**

Momentum balance in the tunnel is the form of equation (1) but all possible losses are considered. The momentum equation can be written as follows;

$$m_j u_j + m_s u_s - m_t u_t - \zeta_t A_t \frac{\rho u_t^2}{2} = (p_t - p_e) A_t \quad (5)$$

If significant obstructions are present which might contribute to the overall momentum loss, then equation (5) can be expressed as follows;

$$m_j u_j + m_s u_s - m_t u_t - \zeta_t A_t \frac{\rho u_t^2}{2} - \zeta_{ob} f_{ob} A_t \frac{\rho u_t^2}{2} = (p_t - p_e) A_t \quad (6)$$

The symbol  $\zeta_t$  is the friction loss due to tunnel or confining walls,  $\zeta_{ob}$  is a drag or a resistance coefficient due to the presence of obstructions and  $f_{ob}$  is the fraction of tunnel area occupied by obstructing objects giving  $f_{ob} A_t$  to be the frontal area of these objects.

The tunnel experiences a significant amount of recirculation in the mixing section but momentum is not lost. This momentum is equivalent to  $m_r u_r$ , where  $m_r$  and  $u_r$  are the back or recirculating mass flow and average backflow velocity throughout the tunnel respectively.

By using equations (2) to (5), equation (6) becomes

$$-(\zeta_t + \zeta_{ob} f_{ob}) A_t \frac{\alpha^2 (1+n)^2 \rho u_j^2}{2(1+\alpha)^2} = (p_t - p_e) A_t \quad (7)$$

By making the substitution  $m_j u_j = A_j \rho u_j^2 = \frac{\alpha A_t}{1+\alpha} \rho u_j^2$  in the above equation, the pressure drop in the tunnel is given by

$$p_t - p_e = \frac{1}{A_t} \left( \frac{\alpha A_t}{(1+\alpha)} \rho u_j^2 + \frac{\alpha^2 n^2 A_t}{1+\alpha} - \frac{\alpha^2 (1+n)^2}{(1+\alpha)^2} A_t \rho u_j^2 - (\zeta_t + \zeta_{ob} f_{ob}) A_t \frac{\alpha^2 (1+n)^2}{2(1+\alpha)^2} \rho u_j^2 \right) \quad (8)$$

$$p_t - p_e = \rho u_j^2 \left( \frac{\alpha}{(1+\alpha)} + \frac{\alpha^2 n^2}{1+\alpha} - \frac{\alpha^2 (1+n)^2}{(1+\alpha)^2} - (\zeta_t + \zeta_{ob} f_{ob}) \frac{\alpha^2 (1+n)^2}{2(1+\alpha)^2} \right) \quad (9)$$

$$p_t - p_e = \frac{2\alpha}{+\alpha} \frac{\rho u_j^2}{2} \left[ 1 + \alpha n^2 - \frac{\alpha(1+n)^2}{1+\alpha} - (\zeta_t + \zeta_{ob} f_{ob}) \frac{\alpha(1+n)^2}{2(1+\alpha)} \right] \quad (10)$$

$$p_t - p_e = \frac{\rho u_j^2}{(1+\alpha)^2} (\alpha(1+\alpha) + \alpha^2 n^2 (1+\alpha) - \alpha^2 (1+n)^2 - (\zeta_t + \zeta_{ob} f_{ob}) \frac{\alpha^2 (1+n)^2}{2}) \quad (11)$$

$$p_t - p_e = \frac{\rho u_j^2}{2(1+\alpha)^2} (2\alpha(1-\alpha n)^2 - \alpha^2 (1+n)^2 (\zeta_t + \zeta_{ob} f_{ob})) \quad (12)$$

The pressure drop of the jet flow is given by subtracting equation (12) from (3) and obtaining

$$P_j - p_t = (1 + \zeta_n) \frac{\rho u_j^2}{2} - \frac{\rho u_j^2}{2(1+\alpha)^2} (2\alpha(1-\alpha n)^2 - \alpha^2 (1+n)^2 (\zeta_t + \zeta_{ob} f_{ob})) \quad (13)$$

and can be further simplified to

$$P_j - p_t = \frac{\rho u_j^2}{2(1+\alpha)^2} ((1+\alpha)^2 (1+\zeta_n) - 2\alpha(1-\alpha n)^2 + \alpha^2 (1+n)^2 (\zeta_t + \zeta_{ob} f_{ob})) \quad (14)$$

**JET FAN PERFORMANCE EFFICIENCY**

A jet fan performance parameter can now be described as the ratio of energy output to energy input

$$E_{out} = \frac{m_s}{\rho} (p_t - p_e) = \frac{nm_j}{\rho} (p_t - p_e) \quad (15)$$

$$E_{in} = \frac{m_j}{\rho} (P_t - p_e) \quad (16)$$

$$\eta = E_{out} / E_{in} = n(p_t - p_e) / (P_j - p_t) \quad (17)$$

$$\eta = n \frac{2\alpha(1-\alpha n)^2 - \alpha^2(1+n)^2(\zeta_t + \zeta_{ob} f_{ob})}{(1+\alpha)^2(1+\zeta_n) - 2\alpha(1-\alpha n)^2 + \alpha(1+n)^2(\zeta_t + \zeta_{ob} f_{ob})} \quad (18)$$

Equation (18) shows that the efficiency of the jet fan depends on the entrainment or the mass flow ratio n i.e. the

secondary stream to the primary jet fan mass flow ratio  $m_s/m_j$  and the area ratio  $\alpha$  which is the ratio of the area occupied by the jet fan at the tunnel entry to the area available for the secondary flow to enter the tunnel  $A_j/A_s$ . The loss coefficients  $\zeta_t$  and  $\zeta_{ob}$  can be determined directly or indirectly. The friction coefficient  $\zeta_t$  can be obtained from wall shear stress measurements. The resistance caused by obstructions  $\zeta_{ob}$  can be estimated fairly accurately. If the pressure drop for a clear tunnel is known, the differences can be attributed to the presence of obstructions and therefore the coefficient  $\zeta_{ob}$  can be found. If the pressure drop in the tunnel or opening and the flow ratio  $n$  are known then the loss coefficients can be estimated by solving equation (14). Both  $n$  and the recirculation or backflow fraction  $\zeta_r$  are a function of the jet fan position  $Y/D_t$  from the confining walls.

#### THEORETICAL ESTIMATION OF THE BACKFLOW FRACTION

A jet fan of diameter  $D_j$  is situated at a distance  $Y$  metres from the tunnel wall to its axis. The tunnel is of hydraulic diameter  $D_t$ . Since from experimental observation it is known that recirculation takes place on one side of the tunnel when the jet fan is situated at distance  $\frac{Y}{D_t} < 0.5$ , an estimation of the volume occupied by the body of the backflow can be carried out by assuming that the jet develops like a free jet until it reaches the tunnel walls. The jet expands with angle  $\theta$  from the nozzle and the distance it takes to reach one side of the tunnel wall is  $L_r$  the recirculation length.

$$\tan \frac{\theta}{2} = \frac{D_t - Y - 0.5D_j}{L_r} \quad (19)$$

the backflow length is then given by

$$L_r = \frac{D_t - Y - 0.5D_j}{\tan \theta/2} \quad (20)$$

The area occupied by the backflow eddy, if assumed to be approximated by a triangular shape, can be expressed as

$$A_r = 0.5(D_t - Y - 0.5D_j) \frac{D_t - Y - 0.5D_j}{\tan \theta/2}$$

$$A_r = 0.5(D_t - Y - 0.5D_j)L_r \quad (21)$$

If an average backflow velocity  $u_r$  is assumed within this area then the recirculation fraction can be calculated as

$$\zeta_r = \frac{0.5(D_t - Y - 0.5D_j)u_r L_r}{L_r u_t D_t} = \frac{0.5(D_t - Y - 0.5D_j)u_r}{u_t D_t} \quad (22)$$

and in terms of the jet discharge velocity

$$\zeta_r = \frac{0.5(1+\alpha)(D_t - Y - 0.5D_j)u_r}{\alpha(1+n)u_j D_t} \quad (23)$$

#### JET FAN ANALYSIS FROM ENERGY CONSIDERATIONS

The jet fan performance can be analyzed from energy considerations by carrying out an energy balance in the tunnel. This is done by first assessing all possible losses and taking into account the energy input and output. The losses are accounted as follows.

##### Loss Consideration

$$1. (i) \text{ Secondary stream energy loss} = \zeta_s m_s \frac{u_s^2}{2} \quad (24)$$

$$(ii) \text{ Jet fan nozzle loss} = \zeta_n m_j \frac{u_j^2}{2} \quad (25)$$

$$(iii) \text{ Tunnel friction loss} = \zeta_t m_t \frac{u_t^2}{2} \quad (26)$$

##### 2. Backflow (recirculation) energy

$$\text{loss } E_{rl} = m_r \frac{u_r^2}{2} = \zeta_r m_t \frac{u_t^2}{2} \quad (27)$$

3. The jet experiences a loss during its discharge from the fan. This loss in energy is

$$E_{jl} = \frac{m_j}{\rho} (p_e - p_a) = \frac{m_j}{\rho} \rho \frac{u_s^2}{2} = m_j \frac{u_s^2}{2} \quad (28)$$

##### 4. Mixing Energy Losses

In addition to the other losses mixing energy losses occur because two streams of different velocities are brought together. The mixing loss is

$$E_{ml} = m_t \frac{u_{mix}^2}{2} \quad (29)$$

In the above equation  $m_t$  is the tunnel mass flow and  $u_{mix}$  is the mixing velocity which will be dealt with later.

An overall energy balance can be written as follows

$$E_{in} = W_t + E_{rl} + E_{ml} + E_{fl} + E_{jl} \quad (30)$$

The quantity  $W_t = E_{out}$  i.e. energy out of the tunnel.

The total energy loss is therefore

$$E_{loss} = E_{rl} + E_{ml} + E_{fl} + E_{jl}$$

$E_{fl}$  is the sum of all frictional energy losses and is

$$E_{fl} = \zeta_s m_s \frac{u_s^2}{2} + \zeta_n m_j \frac{u_j^2}{2} + \zeta_t m_t \frac{u_t^2}{2} \quad (31)$$

$$E_{fl} = \zeta_s \alpha^2 n^3 m_j \frac{u_j^2}{2} + \zeta_n m_j \frac{u_j^2}{2} + \alpha^2 \frac{(1+n)^3}{(1+\alpha)^2} \zeta_t m_j \frac{u_j^2}{2} \quad (32)$$

$$E_{fl} = m_j \frac{u_j^2}{2} \left( \alpha^2 n^3 \zeta_s + \zeta_n + \alpha^2 \zeta_t \frac{(1+n)^3}{(1+\alpha)^2} \right) \quad (33)$$

The recirculation loss  $E_{rl}$  is

$$E_{rl} = \zeta_r m_r \frac{u_r^2}{2} = \zeta_r \alpha^2 \frac{(1+n)^3}{(1+\alpha)^2} \zeta_t m_t \frac{u_t^2}{2} \quad (34)$$

Concept of Mixing Velocity to Define Mixing Energy Loss

In order to define the mixing energy loss the mixing velocity has to be defined and determined. The following reasoning can be applied. First it is necessary to assume that the jet once past the nozzle of the fan develops like an axisymmetric free jet until it reaches the confining walls. The centerline velocity of an axisymmetric jet is given by

$$u_m = 12 \frac{r_o}{x} u_j \quad (35)$$

The mixing velocity is given by

$$u_{mix} = \frac{1}{x_l - x_0} \int_{x_0}^{x_l} (u_m - u_t) dx \quad (36)$$

The limits of integration  $x_0$  and  $x_l$  are the end of the jet potential core and the point where jet centerline velocity  $u_m = u_t$  respectively. At this point the body of the tunnel airflow is assumed to be fully mixed (see Figure 2).

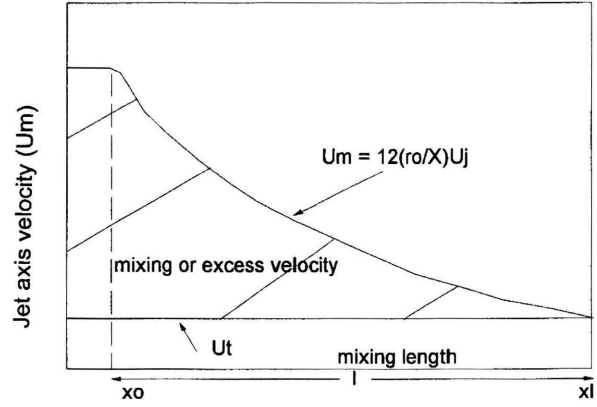


Figure 2. Illustration of velocity decay of an axisymmetric jet and mixing velocity concept.

$$\begin{aligned} u_{mix} &= \frac{1}{x_l - x_0} \left( 12r_o u_j \int_{x_0}^{x_l} \frac{dx}{x} - \int_{x_0}^{x_l} u_t dx \right) \\ &= \frac{12r_o}{x_l - x_0} u_j \ln \frac{x_l}{x_0} - u_t \end{aligned} \quad (37)$$

The mixing energy loss is then

$$\begin{aligned} E_{ml} &= \frac{m_t}{2} \left[ \frac{12r_o}{x_l - x_0} u_j \ln \frac{x_l}{x_0} - u_t \right]^2 \\ &= (1+n) \frac{m_j}{2} \left[ \frac{12r_o}{x_l - x_0} u_j \ln \frac{x_l}{x_0} - \frac{\alpha(1+n)}{1+\alpha} u_t \right]^2 \\ E_{ml} &= (1+n) m_j \frac{u_j^2}{2} \left[ \frac{12r_o}{x_l - x_0} u_j \ln \frac{x_l}{x_0} - \frac{\alpha(1+n)}{1+\alpha} \right]^2 \end{aligned} \quad (38)$$

The total energy loss can be determined by summing all the losses

$$\begin{aligned} E_{loss} &= \zeta_s \alpha^2 n^3 m_j \frac{u_j^2}{2} + \alpha^2 \frac{(1+n)^3}{(1+\alpha)^2} \zeta_t m_j \frac{u_j^2}{2} \\ &+ (1+n) m_j \frac{u_j^2}{2} \left( \frac{12r_o}{x_l - x_0} \ln \frac{x_l}{x_0} - \frac{\alpha(1+n)}{1+\alpha} \right)^2 + \zeta_r \frac{\alpha^2 (1+n)^3}{(1+\alpha)^2} m_j \frac{u_j^2}{2} \\ E_{loss} &= m_j \frac{u_j^2}{2} \left( \zeta_s \alpha^2 n^3 + \zeta_n + \alpha^2 \frac{(1+n)^3}{(1+\alpha)^2} \zeta_t \right. \\ &\left. + (1+n) \left( \frac{12r_o}{x_l - x_0} \ln \frac{x_l}{x_0} - \frac{\alpha(1+n)}{1+\alpha} \right)^2 + \zeta_r \frac{\alpha^2 (1+n)^3}{(1+\alpha)^2} \right) \end{aligned} \quad (39)$$

$$E_{ml} = \frac{m_j}{\rho} (P_j - p_t) \quad (40)$$

$$E_{out} = \frac{m_s}{\rho} (P_j - p_t) = \frac{nm_j}{\rho} (P_j - p_t) \quad (41)$$

$$E_{in} - E_{out} = E_{loss} \quad (42)$$

$$(P_j - p) - n(p_t - p_e) = \frac{\rho u_j^2}{2} (\zeta_s \alpha^2 n^3 + \zeta_n + \alpha^2 \frac{(1+n)^3}{(1+\alpha)^2} \zeta_t) + (1+n) \left( \frac{12r_o}{x_l - x_o} \ln \left( \frac{x_l}{x_o} \right) - \frac{\alpha(1+n)}{(1+\alpha)} \right)^2 + \zeta_r \frac{\alpha^2 (1+n)^3}{(1+\alpha)^2} \quad (43)$$

$$(P_j - p_t) - n(p_t - p_e) = k_\phi \rho \frac{u_j^2}{2} \quad (44)$$

$$k_\phi = \zeta_s \alpha^2 n^3 + \zeta_n + \alpha^2 \frac{(1+n)^3}{(1+\alpha)^2} \zeta_t + (1+n) \left( \frac{12r_o}{x_l - x_o} \ln \left( \frac{x_l}{x_o} \right) - \frac{\alpha(1+n)}{(1+\alpha)} \right)^2 + \zeta_r \frac{\alpha^2 (1+n)^3}{(1+\alpha)^2} \quad (45)$$

Efficiency can now be defined as

$$\eta = 1 - \frac{E_{loss}}{E_{in}} \quad (46)$$

$$\eta = 1 - \frac{k_\phi \rho u_j^2 / 2}{(P_j - p_t)} \quad (47)$$

#### ANALYSIS OF THE PERFORMANCE $\eta$

Equation (47) describes performance of the jet fan but its use is limited unless the loss coefficient  $k_\phi$  can be determined fairly accurately. This requires the recirculation fraction ( $\zeta_r$ ), the wall friction loss ( $\zeta_w$ ), the nozzle loss ( $\zeta_n$ ), and entrained or secondary fluid loss coefficients ( $\zeta_s$ ) to be determined for each flow ratio  $n$  and area ratio  $\alpha$  ( $A_j/A_s$ ). The mixing distance is defined by point  $x_o$  at jet discharge to  $x_l$  where the jet flow is fully mixed with the secondary flow, corresponds to velocities  $u_j$  and  $u_t$  respectively and has to be determined for each  $n$ , in order to determine the mixing energy loss contribution to the loss coefficient (equation 45). The jet discharge loss included in the nozzle, contributes significantly to the loss coefficient. Careful considerations should be taken in comparing equations (18) and (47). The difficulties are manifested in the flow physics itself because the individual loss factors in equation (45) change for every different flow ratio  $n$ . Fortunately equation (18)

can easily be plotted to determine the theoretical performance of a jet fan for a variety of conditions.

The dependency of the performance parameter  $\eta$  on flow ratio ( $n$ ) and area ratio ( $\alpha$ ) from momentum considerations can be evaluated by plotting equation (18) graphically. Equation (18) is a product of flow ratio ( $n$ ) and the pressure ratio  $(p_t - p_e) / (P_j - p_t)$ . The numerator ( $p_t - p_e$ ) is the tunnel pressure rise due to the jet fan and  $(P_j - p_t)$  is the driving pressure drop of the jet fan. For ease of understanding, equation (18) is repeated below as equation (48)

$$\eta = n \frac{2\alpha(1-\alpha n)^2 - \alpha^2(1+n)^2(\zeta_t + \zeta_{ob} f_{ob})}{(1+\alpha)^2(1+\zeta_n) - 2\alpha(1-\alpha n)^2 + \alpha(1+n)^2(\zeta_t + \zeta_{ob} f_{ob})} \quad (48)$$

To simplify the problem, it is assumed that there are no obstructions in the tunnel and therefore the quantity  $\zeta_{ob} f_{ob} = 0$ . Equation 18 becomes

$$\eta = n \frac{2\alpha(1-\alpha n)^2 - \alpha^2 \zeta_t (1+n)^2}{(1+\alpha)^2(1+\zeta_n) - 2\alpha(1-\alpha n)^2 + \alpha^2 \zeta_t (1+n)^2} \quad (49)$$

Equation (49) is shown in Figure 3 as a plot of ( $\eta$ ) vs. flow ratio  $n$  ( $= m_s/m_j$ ) for constant values of the friction loss factor  $\zeta_t$ . The area ratio ( $\alpha = A_j/A_s$ ) is the area that is occupied by the fan to that of the airway where the secondary flow enters the tunnel ( $A_s = A_t - A_j$ ). A friction loss term  $\zeta_t = 0.01$  is used in this example for a Reynolds number of 6,000. A jet fan nozzle loss coefficient of  $\zeta_n$  of 0.01 is used. The jet fan nozzle loss can be as high as 0.4 but in this case it will be important to see how the jet fan efficiency ( $\eta$ ) behaves as the area ratio ( $\alpha$ ) changes for various flow ratios ( $n$ ).

In Figure 3, six area ratios,  $\alpha$  ( $A_j/A_s$ ) of 0.01, 0.05, 0.15, 0.2, 0.3 and 0.5 are used to determine jet fan efficiency variation with flow ratio  $n$  (equation 49). The range of flow ratio used is 0 to 5 to obtain meaningful values of efficiency for each area ratio. The smallest area ratio used of 0.01 shows that the efficiency rises slowly as flow ratio  $n$  increases and although not shown for the area ratios of 0.01 and 0.05, there is a peak jet fan efficiency in each case. The peak efficiencies are clearly shown for area ratios of 0.15, 0.20, 0.3 and 0.5 and are 29 %, 19 %, 27% and 16 % at flow ratios of 2, 1.4, 1.2 and 0.5 respectively. The larger the area ratio the smaller the peak efficiency, which will occur at decreasing flow ratio  $n$ .



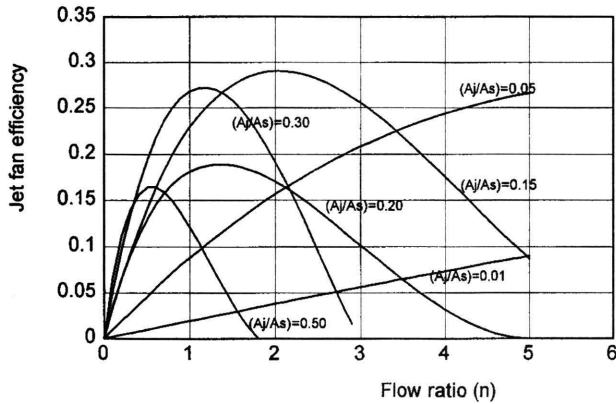


Figure 3. Plot of theoretical jet fan efficiency ( $\eta$ ) versus flow ratio ( $n = m_s/m_j$ ) for various area ratios ( $\alpha = A_j/A_s$ ).

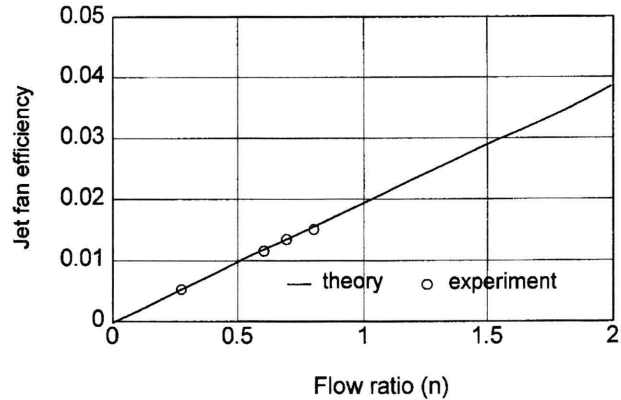


Figure 4. Comparison of theoretical performance with measured data for the area ratio of  $\alpha = 0.01$ .

In Figure 4, a comparison is made between experimental values of jet fan efficiency with that obtained from equation (49) for the same area ratio of 0.01. The agreement is quite remarkable and validates equation (49). This shows that if a practical situation involving a jet fan system needs to be set up, equation (49) will be a useful tool for determining the required efficiency for the design area ratio  $\alpha$ , for a given jet fan discharge velocity. Useful jet discharge velocities ( $u_j$ ) range from 20 to 40 m/s for larger systems. In an operating mine where a jet fan type set up was used to exhaust 71 m<sup>3</sup>/s (150,000 cfm) into a nearby shaft, the area ratio was 0.55,  $u_j = 46$  m/s and the flow ratio ( $n$ ) obtained was 0.25. The jet fan moved an additional 14 m<sup>3</sup>/s (30,000 cfm) while it was handling 56.6 m<sup>3</sup>/s (120,000 cfm) through itself to make a total of 71 m<sup>3</sup>/s (150,000 cfm). A value of  $n = 0.25$  and  $\alpha = 0.55$  will give a performance efficiency of 12% using Figure 3 assuming the area ratio of 0.55 is close enough to 0.5.

Figure 5 serves to show for small area ratios much smaller than 0.1 the range of flow ratio over which efficiency is defined increases. For example for the area ratio  $\alpha = 0.05$  the flow ratio ( $n$ ) varies from 0 to 20 and for  $\alpha = 0.01$  the flow ratio range is even larger i.e. from 0 to 50. No matter how small, each area ratio is associated with a peak efficiency at a certain flow ratio. If the area ratio is reduced to even smaller values e.g. in some air movers which use compressed air jets to entrain more secondary flow, where  $\alpha = 0.001$  or less the range of flow of flow ratio ( $n$ ) becomes even larger. Values of flow ratio greater than 80 are quite common depending on how small  $\alpha$  is. These systems are still quite effective to move air in many practical situations especially where heat, dust and other pollutants need to be flushed out.

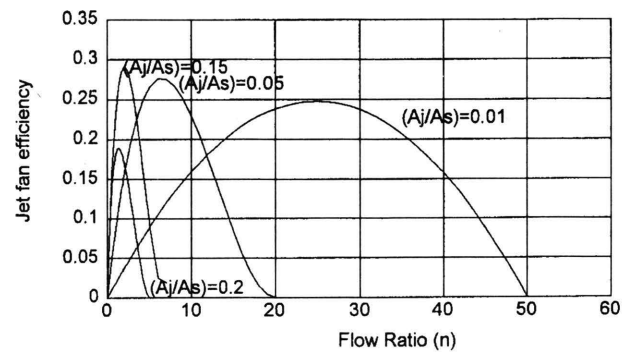


Figure 5. Plot showing full range of flow ratio ( $n$ ) for area ratio  $\alpha = 0.01$  compared to  $\alpha = 0.05, 0.15$  and  $0.2$

As larger nozzle and friction loss factors are introduced to equation (49), the peak efficiency decreases significantly depending on their magnitudes. Therefore the higher the nozzle loss factor and friction in the air passage the lower the performance value  $\eta$ . It is important to note that the jet fan performance values will always be low (less than 30% in all cases).

In Figure 6 the performance parameter ( $\eta$ ) is plotted against area ratio ( $\alpha$ ) for a family of flow ratio  $n$  values of 0.1, 0.3 and 0.7. At each flow ratio, there is an area ratio ( $\alpha$ ) that gives the maximum performance. The larger the flow ratio e.g.  $n = 0.7$  the higher the performance parameter ( $\eta = 0.19$ ) and the smaller the optimum area ratio ( $\alpha = 0.3$ ). The optimum area ratios for smaller  $n$  values are larger e.g.  $\alpha = 0.48$  for  $n = 0.3$ ,  $\alpha = 0.78$  for  $n = 0.1$ . Figure 7 shows the optimum area ratio plot for various flow ratios  $n$ . The value of optimum area ratio declines rapidly with increasing flow ratio. In most mining applications the optimum values of area ratios may not be used because the sizes of the passages might be determined primarily by mining considerations with ventilation playing a secondary role. Fans are of

limited range in diameter and therefore on average, area ratios  $A_j/A_s$  encountered in mining are less than 0.5. However a higher area ratio can be achieved by building a partial bulkhead next to the fan. Adjustable doors can also be fitted to control the entry area of the secondary flow.

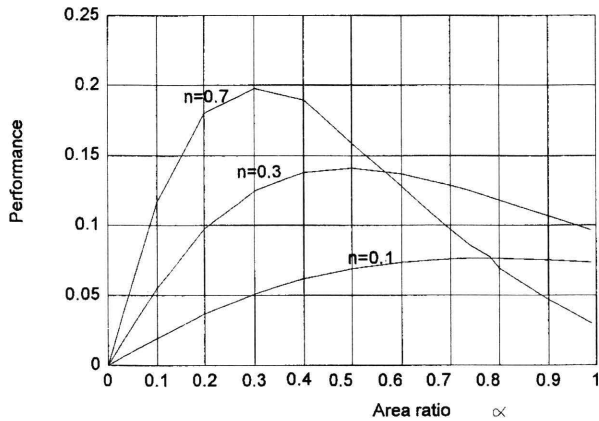


Figure 6. Performance  $\eta$  versus area ratio  $\alpha$  for various flow ratios  $n$ .

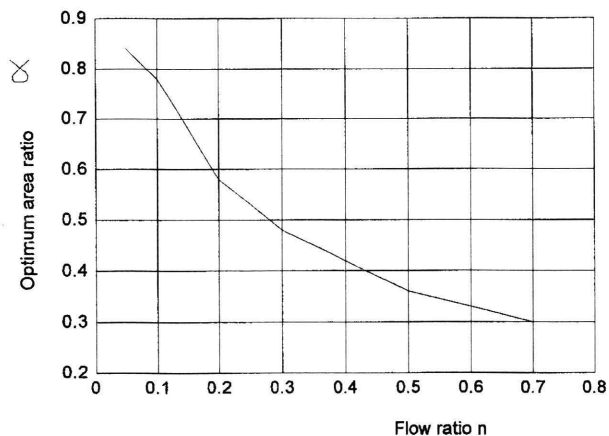


Figure 7. Flow ratio ( $n$ ) versus optimum area ratio ( $\alpha$ ).

The flow ratio in practical situations depends on the discharge velocity of the jet, available secondary fluid, fan position and also the size of the fan with respect to the flow passage.

## CONCLUSIONS

Jet fan performance can be analyzed successfully using momentum and energy considerations. Efficiency of a jet fan ventilation system depends on the area ratio ( $\alpha$  or  $A_j/A_s$ ) of the fan to the area available for secondary flow to be introduced and the flow ratio ( $n$ ) of the amount of entrained

secondary flow to the jet fan flow (i.e.  $n = m_j/m_s$ ). It has been shown that each area ratio has a range of flow ratios that can be achieved. There is a maximum efficiency for each area ratio at a certain flow ratio. Equation (49) is a powerful tool in designing an optimum jet fan ventilation system. Accurate values of loss coefficients due to friction, nozzle or jet discharge and can be determined by a combination of empirical relations and practical measurements. Performance of a jet fan using energy considerations gives rise to an equation such as (47). The successful analysis of equation (47) unlike (49) is not an easy task, it depends on the computation of the loss coefficient  $k_\phi$  for each flow ratio  $n$ . A number of individual loss factors are embedded in  $k_\phi$  (equation 45), and these loss factors are due to friction, recirculation, mixing, suction (entry) or entrainment, jet nozzle shock, discharge or expansion loss. Energy loss considerations are still useful in considering certain losses such as recirculation and jet nozzle shock losses upon discharge. This paper has introduced the concept of a jet mixing velocity in order to quantify or assess mixing losses. How the available energy of the jet fan is utilized in driving the secondary flow determines very much the final efficiency of the system. Geometry plays a big part in the momentum energy exchange process.

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