A Mathematical Model of Heat Transfer in Partially Insulated Airways in Deep, Frozen Ground Placer Mines

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ABSTRACT

Large seasonal variations in the temperature of the ventilating air in mines in the arctic cause changes in the original thermal field through heat and mass exchanges between the air and the surrounding medium. These thermal interactions have major influence on climatic quality as well as on the stability of the mine openings.

Thawing of walls and roof in mine airways can be reduced by various types of thermal-insulation. Application of thermal-insulation prevents deep thawing of the rockmass surrounding an airway. In this case, the mechanism of heat transfer around a frozen, underground airway would be much different. A model of heat transfer in a deep, partially insulated airway has been developed and analyzed using finite element methods.

Results of the analysis show that without any thermal control, there will be stable change in temperature around the mine airway. With different insulations on the walls of the airway, roof thawing can be reduced and in certain cases, completely eliminated.

KEYWORDS

Heat Transfer, Airway, Heat Exchange, ABAQUS, Thawing, Permafrost, and Finite Element Method.

INTRODUCTION

The nature and intensity of heat exchange between the air and the walls of an underground mine in permafrost are functions of a number of independent variables, including the thermal properties of the permafrost and air, the velocity and distribution of air stream, the size and cross-section of the airway, the ice content and the temperature of the permafrost, the temperature and moisture content of the ventilation air, time of ventilation, and methods of thermal regime control.

Permafrost, by definition, is any soil that has been continuously frozen for at least two years. The physical characteristics of permafrost vary with soil type, moisture content, and temperature.

Large seasonal variations in the temperatures of the ventilating air cause changes in the original thermal field around a mine airway, through heat and mass exchange between the air and the surrounding medium. This is especially true in the summer, when the temperatures between the air entering the mine is far above the freezing point of water, causing thawing of the ice contained in the surrounding rock. In the winter, in order to avoid icing up of the intake airway and to protect mine workers from cold air, the intake air is often heated, and the velocity of the heated air must be kept fairly high. The most complex problem is insuring stability of the roof when underground operations are performed in "warm" permafrost with a temperature of 0°C to -2°C. When the permafrost is warm temperature passing of heated ventilation air through the underground workings invariably establishes a thermal stream between the ventilating air and the surrounding rock, and this leads to a general increase in the temperature of the frozen ground and to a decrease in its ice content. These thermal interactions have major influence on climatic quality as well as on the stability of the mine openings (Bandopadhyay, *et al.*, 1996).

The thawing of walls and roof in mine airways can be reduced by various types of thermal-insulation. Application of thermal-insulation prevents deep thawing of the rockmass surrounding an airway. Analysis of the effectiveness of various types of thermal-insulation, and optimum thickness which will reduce the heat exchange between the ventilation air and the surrounding rockmass (Bandopadhyay, *et al.*, 1997), is presented in this paper.

MODELLING OF HEAT TRANSFER IN DEEP MINE AIRWAYS

A physical model of heat transfer processes between ventilating air and the frozen rockmass around a mine opening that undergoes periodic freezing and thawing can be described as follows: temperature of the ventilating air (T_a) changes along the length of the airway (z) of the opening and time (t) as a result of the heat and mass exchange with the frozen rockmass. The heat exchange between the ventilating air and the rockmass takes place at changing rate and periodically alternates its direction. The temperature of the rockmass (T_g) changes with time and with increasing distance from the surface of the mine opening, and as well as along the airway length (Bandopadhyay and Zhang, 1990).

Temperature regime of rockmass and intensity of heat conduction, depth of thawing and freezing can be determined based on the thermo-physical properties of rock, its ice content, initial characteristics of temperatures, and the initial heat regime of the mine. The mathematical formulation of heat conduction problems involving phase change can be expressed by the "Stephan Equation" is as follows:

$$\frac{\partial}{\partial x}(k_x\frac{\partial T}{\partial x})+\frac{\partial}{\partial y}(k_y\frac{\partial T}{\partial y})+\frac{\partial}{\partial z}(k_z\frac{\partial T}{\partial z})$$

$$+a_0 Q = [\rho C_p + L\delta(T-T_0)]\frac{\partial T}{\partial t} \qquad (1)$$

The temperature, T in Equation (1) is the single value unknown function of temperature at any point (x, y, z) at any time t; Q is the rate of heat generation per unit volume; a_o is the coefficient of heat dissipation; k_x, k_y and k_z are anisotropic thermal conductivities; C_p is the specific heat; ρ is the density; L is the latent heat of phase change; and $\delta\ (T-T_a)$ is the Dirac Delta function:

$$\delta(y)=\begin{cases}1 & y\geq 0\\ 0 & y<0\end{cases}$$

In general, the parameters k_x, k_y, k_z, Q, ρ and C_p in Equation (1) may be functions of time, either through a dependence on local temperature or through a change in local material (thawed or frozen) state or condition with time. The a_o is a constant.

In Equation (1), the first set of terms accounts for thermal conduction due to temperature gradients while the second set represents the heat source (or sink) due to ice-water phase transition.

There are two specific heat sources which impose some degree of impact on the thermal regime around a shallow mine airway. One is the heat mass in the ventilating air, the other is due to the seasonal air temperature variations on the surface. When a mine airway is excavated in frozen ground at a considerable depth, temperature fluctuations of air on the surface may not have any significant influence on the surrounding rockmass of the airway (Bandopadhyay, *et al.*, 1998). Most parts of the ventilation airways for deep underground placer mines will be located at a considerable depth in the frozen ground. Therefore, the analysis of unsteady heat transfer with fixed temperatures and insulated boundary conditions is a more general situation in predicting and analyzing the temperature redistribution of the rockmass surrounding a mine airway.

HEAT TRANSFER IN DEEP MINE UNINSULATED AIRWAY

A specific application of the general heat transfer equation is determined by specifying the geometry, the boundary conditions, the physical properties, and the initial conditions. In this study, the geometry of the selected mine airway is a rectangular opening (3 m x 2 m). The problem domain selected for the analysis is also a rectangular area (11.5 m x 27 m) located at a considerable depth in the frozen ground (Figure 1). Base on the domain defined before, the boundary conditions for heat transfer most often considered are:

The initial condition:

$$T(x,y,t)\big|_{t=0} = T_0, \qquad (2)$$

The boundary conditions:

(1) On the inside wall 'A-B', the convective boundary is defined as:

$$-k\frac{\partial T}{\partial y}\bigg|_{y=d_1+b_1} = h(T_a - T), \quad 0 < x < \frac{b_2}{2} \qquad (3)$$

(2) On the inside wall 'B-C', the convective boundary is defined as:

$$-K\frac{\partial T}{\partial x}\bigg|_{x=\frac{b_2}{2}} = h(T_a - T), \quad d_1 < y < d_1 + b_1 \qquad (4)$$

(3) On the inside wall 'C-D', the convective boundary is defined as:

$$k\frac{\partial T}{\partial y}\Big|_{y=d_1} = h(T_a - T), \quad 0 < x < \frac{b_2}{2} \tag{5}$$

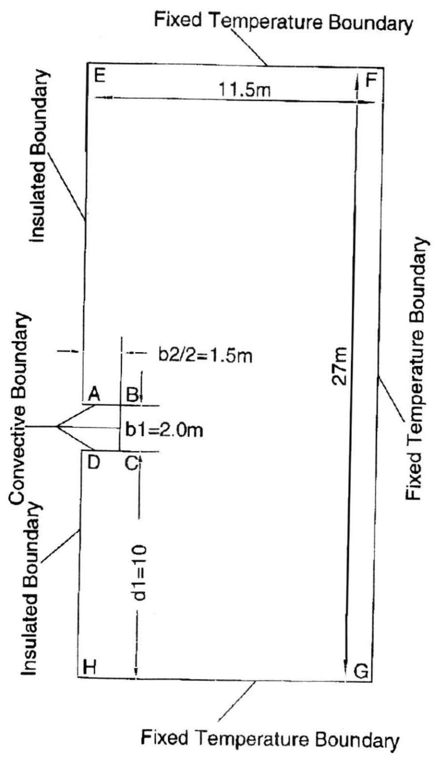

Figure 1. Domain of heat transfer around a mine airway.

(4) On the top surface 'E-F', the fixed temperature boundary is defined as:

$$T(x,y,t)\Big|_{y\to\infty} = T_g \tag{6}$$

(5) On the far right outside surface 'F-G', the fixed temperature boundary is defined as:

$$T(x,y,t)\Big|_{x\to\infty} = T_g \tag{7}$$

(6) On the bottom surface 'G-H,' the fixed temperature boundary is defined as:

$$T(x,y,t)\Big|_{y\to 0} = T_g \tag{8}$$

(7) On the left symmetrical surfaces 'H-D' and 'A-E', the insulated boundary is defined as:

$$k\frac{\partial T}{\partial x}\Big|_{y=0} = 0 \tag{9}$$

Because of the complex configurations, non-linear boundary conditions, a close form solution to Equation (1) combined with the boundary conditions (Equations (2) - (9)) are not readily available.

Table 1. The time to thaw a 2-meter depth in the mine.

Heat Exch. Co-eff.	Material	Thawing Depth	Time
		Roof Dist.	
W/(sq.m-deg.C)		m	year
17	Silt	2	9.31
17	Gravel	2	5.67
23	Silt	2	8.44
23	Gravel	2	4.77
28	Silt	2	8.42
28	Gravel	2	4.35

Numerical methods such as finite difference methods or finite element methods are often used to solve the problems defined by the Equation (1). Bandopadhyay *et al.*, (1995) treated the two dimensional heat transfer problem in mine ventilation with non-linear boundary conditions as two, local, one-dimensional problems with associated boundary conditions and solved it by using finite difference approximation. Bandopadhyay *et al.*, (1995) also presented a finite element method for solution of two dimensional heat flow with non-linear boundary conditions in an extensive study of temperature variations within mine roof and resulting stress and displacement in underground opening in frozen ground. In the present study, the comprehensive finite element program ABAQUS was used to analyze the temperature distributions around a deep, uninsulated airway in Permafrost. This program was developed by Hibbitt, Karlsson and Sorensen, Inc., (1992). Detailed description of ABAQUS can be found in ABAQUS user manuals.

Analysis of the roof thawing depths (Wu, 1996), for the deep, uninsulated airways (Table 1) indicated that a two-meter thawing after ten years' ventilation can be reasonably expected (Figures 2, 3, and 4). To reduce the transfer of heat flux from the ventilating air, one possible approach is to insulate the airway. In this situation, the heat is transferred from the ventilation air to the insulation material rather than to the frozen ground. If it is assumed that the contact between the airway walls and the insulation is complete, then only the conductive heat transfer would exist on the interface of the insulation and the frozen soil. The mathematical model of the thermal system is the same as that of an uninsulated airway as shown below:

$$k[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}] = [\rho C_p + L\delta(T - T_0)]\frac{\partial T}{\partial t} \qquad (10)$$

In an insulated mine airway, the ventilation air is not on direct contact with the frozen ground. In this situation, the heat transfer process can be viewed as heat flowing through different mediums with different thicknesses and varying thermal-physical properties.

When two different materials are in contact, an interface exists between them. If the contact between the different materials is considered as a complete one, then convective and radiative heat transfers can be totally neglected. In such cases, the equations representing the heat transfers in the composite materials are the same as that in a single medium. At each interface, however, the interfacial conditions (additional boundary conditions) must reflect two facts:

1. In the absence of thermal contact resistance, the temperature of each body at the contact interfaces will be the same;
2. Heat flow from the body of higher temperature must be equal to the heat flow into the body of lower temperature.

Since the governing equations for an insulated mine airway is the same as that of an uninsulated airway, the Equations (1-9) are not reproduced here. Only the additional boundary conditions are presented and discussed here.

In an insulated mine airway, the general interfacial boundary conditions between the insulation and airway walls expressed mathematically are:

$$T_1 = T_2 \quad t>0 \qquad (11)$$

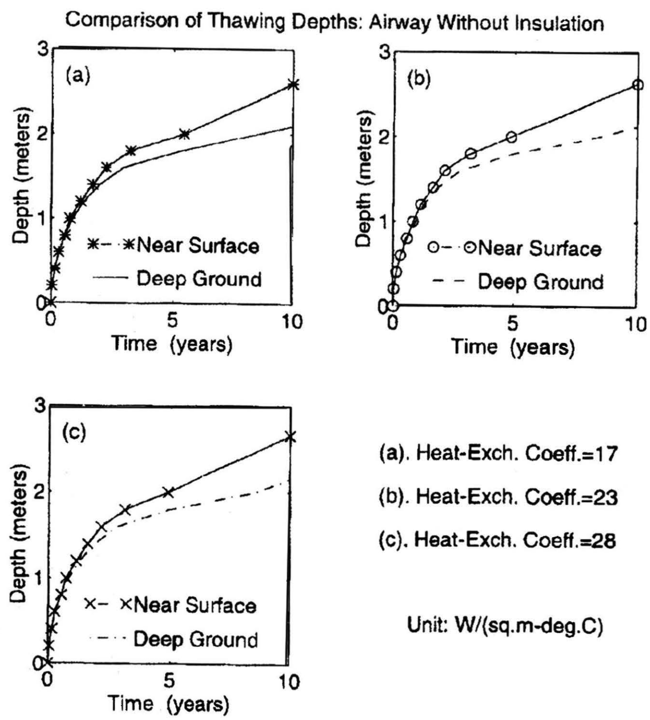

Figure 2. Thawing depth around an uninsulated, deep mine airway (silt).

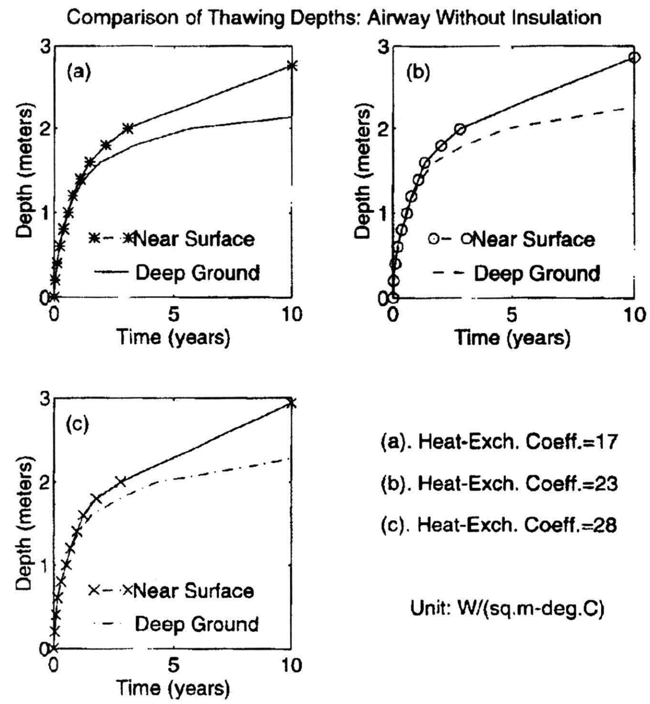

Figure 3. Thawing depth around an uninsulated, deep mine airway (gravel).

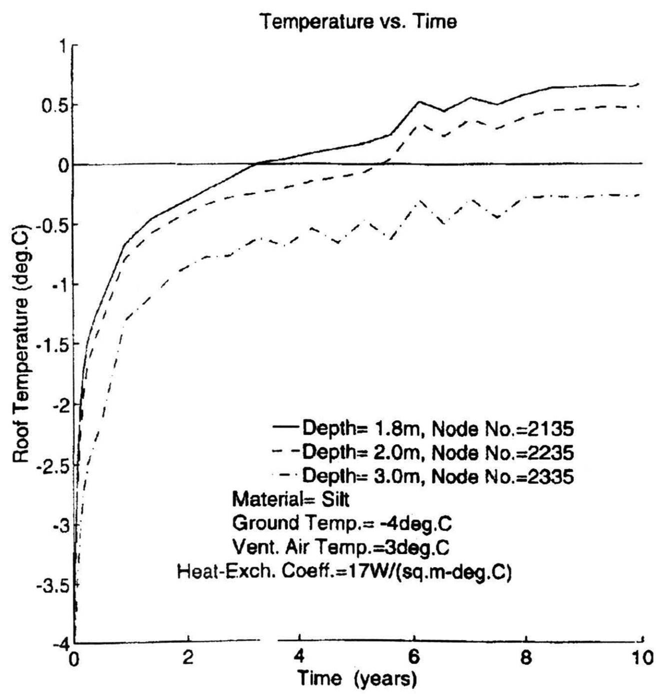

Figure 4. Temperature variations at selected roof points in an uninsulated, deep mine airway.

and

$$k_1 \frac{\partial T_1}{\partial n} = k \frac{\partial T_2}{\partial n} \qquad (12)$$

where k_1 is the heat conductivity of the insulation material.

For a partially insulated mine airway (Figure 5), the boundary condition interfaces of airway roof "A-B" is:

$$k_1 \frac{\partial T_1}{\partial y} = k \frac{\partial T_2}{\partial y} \qquad (13)$$

the boundary condition at the contact interfaces of airway roof "B-C" is:

$$k_1 \frac{\partial T_1}{\partial x} = k \frac{\partial T_2}{\partial x} \qquad (14)$$

Equations (13) and (14) above define the additional interface conditions are for a partially insulated mine airway. With those additional boundary conditions, the temperature distribution around a deep partially insulated airway can now be analyzed in conjunction with the mathematical model developed for an uninsulated airway.

A PARTIALLY INSULATED DEEP AIRWAY

In practice, in an active mine, it is difficult to insulate effectively the floor of an airway. Also thawing of the floor may have less influence on the stability of a mine airway when the airway is horizontal or it has a gentle slope.

Based on approach taken to analyze a completely uninsulated deep airway, it is not difficult to examine the behavior of heat transfer around a partially insulated deep airway. The additional work needed is to modify the ABAQUS code (Hibbitt, Karlsson and Sorensen, Inc., 1992), in such a way that the floor of the airway has no insulation layer elements and impose a convective boundary condition directly on the mine floor surface.

Since the thickness of insulation as well as thermal properties vary widely, to examine the influence of insulations with different thicknesses, a total of twelve variants were analyzed for a completely insulated airway. Three types of insulation materials (Polyurethane foam, Fiber glass, and Cement fiber) and four insulation thicknesses (30 mm, 50 mm, 80 mm, and 100 mm) were considered in this study. A partially insulated deep airway domain is illustrated in Figure 5 and the finite element meshes for a partially insulated airway are shown in Figure 6. Triangle elements were used to mesh the airway wall-interface layer.

To provide a better focus, the analysis was limited to airways in frozen silt. Numerical simulations

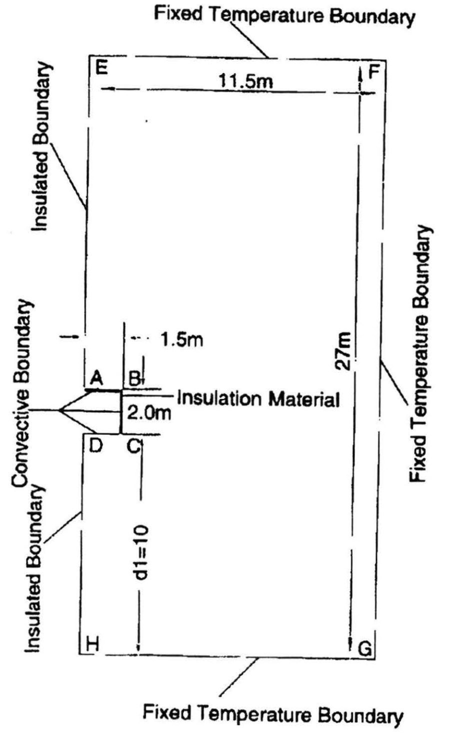

Figure 5. Domain of heat transfer around a partially insulated, deep mine airway.

were conducted for a time interval of ten years and twelve cases presented in Table 2 were analyzed with partial insulation on the airway.

SIMULATION RESULTS

Relationship between roof thawing depth and the ventilation time are presented in Figures 7 through 9. Also, there are seven cases out of twelve where the roof temperatures are above the freezing points. They are the polyurethane foam of 30 mm, the fiber glass of 30 mm and 50 mm, and the cement fiber of 30 mm, 50 mm, 80 mm and 100 mm. The depth of thawing and the time needed to reach these thawing depths are summarized in Table 2.

From these figures, it can be seen that the thermal regime surrounding an airway can be controlled by insulating the airway. The roof temperature can be kept at -0.7°C by applying a 50 mm thickness of polyurethane foam or an 80 mm thickness of fiber glass insulation.

If the roof temperature is required to be kept at -1.6°C, a 80 mm polyurethane foam may, however, be required. Similarly, for keeping the roof temperature at -1.3°C a 100 mm thickness of fiber glass, and at -1.8°C by 100 mm of polyurethane foam would be needed (Figure 10).

CONCLUSIONS

In this study, the finite element program ABAQUS was used to analyze thermal regimes surrounding a deep partially insulated airway in permafrost.

Results of the analyses show that without any thermal control, there would be a stable change in temperature around the mine airway. With different insulations on the walls of the airway, roof thawing can be reduced and in certain cases, can be completely controlled. Roof thawing decreases with increasing thickness of insulation, and for the same insulation thickness, insulation with smaller conductivity results in smaller thawing depth.

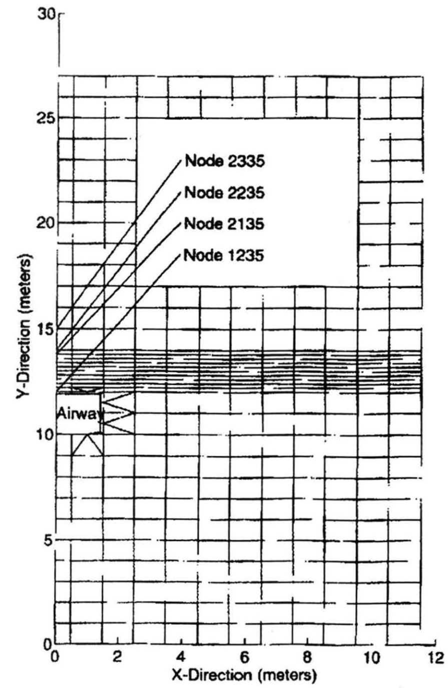

Figure 6. Finite element mesh of a partially insulated, deep mine airway.

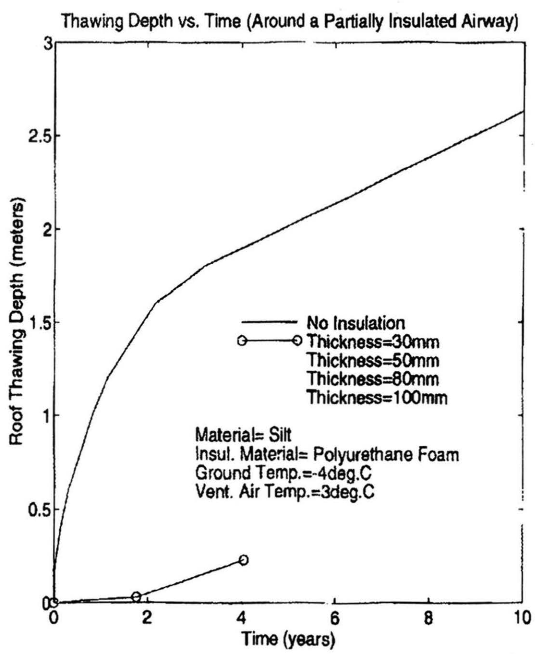

Figure 7. Relationship between insulation thickness (polyurethane) and thawing depth.

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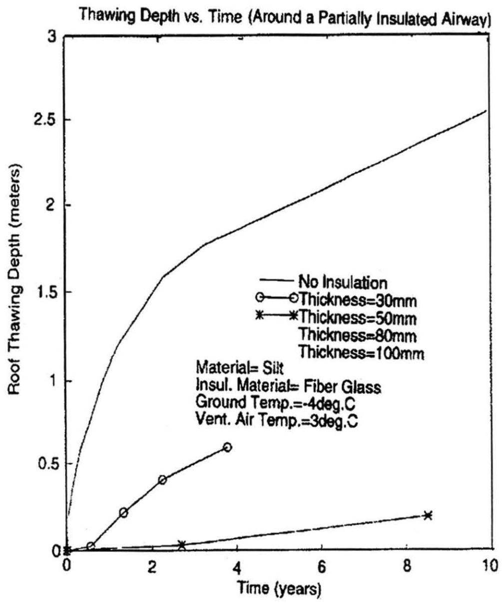

Figure 8. Relationship between insulation thickness (fiber-glass) and thawing depth.

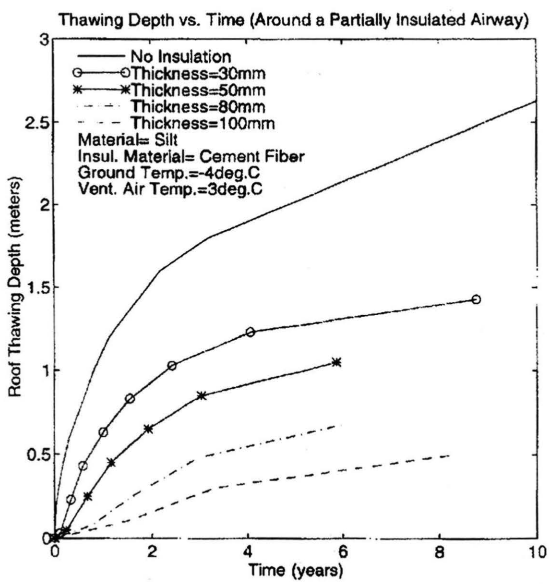

Figure 9. Relationship between insulation thickness (cement fiber) and thawing depth.

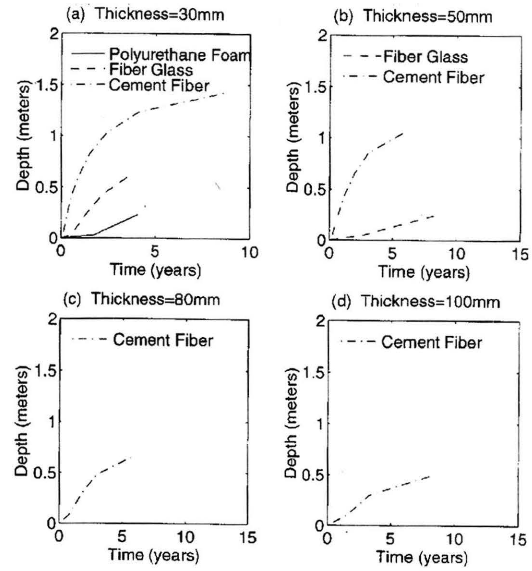

Figure 10. Comparisons of thawing depths in partially insulated airway.

Table 2. Roof thawing in partially insulated deep airways.

Number of case	Insulation	Soil	Thickness	Partially Insulated Deep Airway		
				Roof Thawing	Thawing Time	Roof Temp
			mm	m	year	°C
	No Insulation	silt	0	2	8.44	2.7
1	1. Polyurethane Foam	silt	30	0.23	4.15	0.4
2		silt	50			-0.4
3		silt	80			-1.2
4		silt	100			-1.5
5	2. Glass Fiber	silt	30	0.63	3.56	1.2
6		silt	50	0.25	8.57	0.3
7		silt	80			-0.7
8		silt	100			-0.9
9	3. Cement Fiber	silt	30	1.43	8.9	2.08
10		silt	50	1.05	5.92	1.6
11		silt	80	0.68	6.03	1.05
12		silt	100	0.5	8.25	0.35