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HIGH TEMPERATURE SYMMETRY NON-RESTORATION AND INVERSE SYMMETRY BREAKING IN THE $Z_2 \times Z_2$ MODEL

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Abstract. *The patterns of high temperature symmetry non-restoration (SNR) and inverse symmetry breaking (ISB) in the $Z_2 \times Z_2$ model are investigated in detail for a specified parameters.*

I. INTRODUCTION

At present it is well known that all physical systems can be classified into several categories:

- a):* The first one corresponds to those, in which the symmetry broken at $T = 0$ is restored at high temperature [1-3]. In addition, there is another alternative phenomenon, the behavior of which associates with more broken symmetry as temperature is increased. This is the so-called inverse symmetry breaking (ISB). Here high temperature means that $T/M \gg 1$ for mass scale M of the system in question.
- b):* The second category deals with those cases which exhibit symmetry non-restoration (SNR) at high temperature. This phenomenon emerges in a lot of systems and materials [4]. In the context of quantum field theory, the high temperature SNR has been considered in [5-9] and recently developed in many papers in connection with various important cosmological applications [10-22].

In this respect, there remains growing interest on studying in [23], basing on the CJT effective action at finite temperature [24], we considered the $Z_2 \times Z_2$ model, which was used in [10, 17] for the domain wall problem and in other Refs. [25, 26]. This paper concerns a detailed investigation of phase transitions, which correspond to high temperature SNR/ISB of the $Z_2 \times Z_2$ model for a specified set of the model parameters. In Section II, the main results of [23] are resumed. Section III is devoted for phase transition study. The conclusion and discussion are given in Section IV.

II. CONDITIONS FOR SNR/ISB

Let us start from the system described by the simple Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{\mu_1^2}{2}\phi^2 + \frac{\lambda_1}{4!}\phi^4 + \frac{1}{2}(\partial_\mu\psi)^2 + \frac{\mu_2^2}{2}\psi^2 + \frac{\lambda_2}{4!}\psi^4 + \frac{\lambda}{4}\phi^2\psi^2 + \Delta\mathcal{L}. \quad (1)$$

The counter-terms are chosen as

$$\Delta L = \frac{\delta\mu_1^2}{2}\phi^2 + \frac{\delta\lambda_1}{4!}\phi^4 + \frac{\delta\mu_2^2}{2}\psi^2 + \frac{\delta\lambda_2}{4!}\psi^4 + \frac{\delta\lambda}{4}\phi^2\psi^2.$$

The boundedness of the potential appearing in (1) requires

$$\lambda_1 > 0, \quad \lambda_2 > 0 \quad \text{and} \quad \lambda_1\lambda_2 > 9\lambda^2. \quad (2)$$

Shifting $\{\phi, \psi\} \rightarrow \{\phi + \phi_0, \psi + \psi_0\}$ leads to the interaction Lagrangian

$$\begin{aligned} \mathcal{L}_{int} &= \frac{\lambda_1 + \delta\lambda_1}{24}\phi^4 + \frac{\lambda_1 + \delta\lambda_1}{6}\phi_0\phi^3 + \frac{\lambda_2 + \delta\lambda_2}{24}\psi^4 + \frac{\lambda_2 + \delta\lambda_2}{6}\psi_0\psi^3 + \frac{\lambda + \delta\lambda}{4}\phi^2\psi^2 \\ &+ \frac{\lambda + \delta\lambda}{2}\phi_0\phi\psi^2 + \frac{\lambda + \delta\lambda}{2}\psi_0\psi\phi^2 \end{aligned}$$

and the tree-level propagators

$$\begin{aligned} D_0^{-1}(k; \phi_0, \psi_0) &= k^2 + \mu_1^2 + \delta\mu_1^2 + \frac{\lambda_1 + \delta\lambda_1}{2}\phi_0^2 + \frac{\lambda + \delta\lambda}{2}\psi_0^2, \\ G_0^{-1}(k; \phi_0, \psi_0) &= k^2 + \mu_2^2 + \delta\mu_2^2 + \frac{\lambda_2 + \delta\lambda_2}{2}\psi_0^2 + \frac{\lambda + \delta\lambda}{2}\phi_0^2. \end{aligned}$$

Next the expressions for the renormalized CJT effective potential $V_\beta^{CJT}[\phi_0, \psi_0, D, G]$ and the gap equations at finite temperature are derived

$$\begin{aligned} V_\beta[\phi_0, \psi_0] &= \frac{\mu_{1R}^2}{2}\phi_0^2 + \frac{\lambda_{1R}}{24}\phi_0^4 + \frac{\mu_{2R}^2}{2}\psi_0^2 + \frac{\lambda_{2R}}{24}\psi_0^4 + \frac{\lambda_R}{4}\phi_0^2\psi_0^2 + Q_f(M_{1R}) + Q_f(M_{2R}) \\ &- \frac{\lambda_{1R}}{8}[P_f(M_{1R})]^2 - \frac{\lambda_{2R}}{8}[P_f(M_{2R})]^2 - \frac{\lambda_R}{4}P_f(M_{1R})P_f(M_{2R}), \end{aligned} \quad (3)$$

$$\begin{aligned} \left[\mu_{1R}^2 + \frac{\lambda_{1R}}{6}\phi_0^2 + \frac{\lambda_R}{2}\psi_0^2 + \frac{\lambda_{1R}}{2}P_f(M_{1R}) + \frac{\lambda_R}{2}P_f(M_{2R}) \right] \phi_0 &= 0, \\ \left[\mu_{2R}^2 + \frac{\lambda_{2R}}{6}\psi_0^2 + \frac{\lambda_R}{2}\phi_0^2 + \frac{\lambda_{2R}}{2}P_f(M_{2R}) + \frac{\lambda_R}{2}P_f(M_{1R}) \right] \psi_0 &= 0. \end{aligned} \quad (4)$$

and

$$\begin{aligned} M_{1R}^2 &= \mu_{1R}^2 + \frac{\lambda_{1R}}{2}[\phi_0^2 + P_f(M_{1R})] + \frac{\lambda_R}{2}[\psi_0^2 + P_f(M_{2R})], \\ M_{2R}^2 &= \mu_{2R}^2 + \frac{\lambda_{2R}}{2}[\psi_0^2 + P_f(M_{2R})] + \frac{\lambda_R}{2}[\phi_0^2 + P_f(M_{1R})]. \end{aligned} \quad (5)$$

where

$$\begin{aligned} P_f(M) &= \frac{M^2}{16\pi^2} \ln \frac{M^2}{\mu^2} - \int \frac{d^3k}{(2\pi)^3} \left[E(\vec{k}) \left(1 - e^{-\frac{E(\vec{k})}{T}} \right) \right]^{-1}, \\ Q_f(M) &= \frac{M^4}{64\pi^2} \left(\ln \frac{M^2}{\mu^2} - \frac{1}{2} \right) + T \int \frac{d^3k}{(2\pi)^3} \ln \left(1 - e^{-\frac{E(\vec{k})}{T}} \right). \end{aligned}$$

Considering high temperature SNR/ISB let us assume that $\mu_1^2 < 0$ and $\mu_2^2 > 0$. As a consequence,

$$\phi_0 \neq 0 \quad \text{and} \quad \psi_0 = 0,$$

which means that at $T = 0$ symmetry of the system is spontaneously broken in ϕ sector and unbroken in ψ sector.

It is easily obtained from (5) that the parameters are constrained by

$$\begin{aligned} \lambda_1 > 0, \lambda_2 > 0, \mu_1^2 < 0, \mu_2^2 > 0, \\ \lambda_1 \lambda_2 > 9\lambda^2, \lambda < 0, |\lambda| > \lambda_1, \lambda_2, \end{aligned} \quad (6)$$

for the present model, in which both SNR/ISB simultaneously take place at high temperatures in corresponding sector.

It was proved [23] that the constraints (6) for there being SNR/ISB is very stable in a large temperature interval due to the T logarithmic dependence of coupling constants.

III. PHASE TRANSITION PATTERNS FOR SPECIFIED VALUES OF PARAMETERS

In order to gain an insight into the model it is very interesting to consider the phase transitions for specified values of the model parameters. As is easily seen, there is no value of λ which fulfils both conditions

$$\lambda_1 \lambda_2 > 9\lambda^2, \quad |\lambda| > \lambda_1, \lambda_2.$$

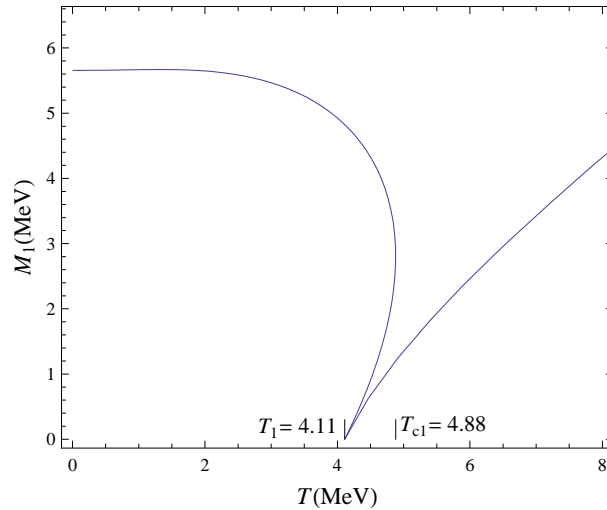


Fig. 1. The T dependence of M_1 , corresponding to the region that the broken symmetry in ϕ -sector is restored (see Fig. 3). The phase transition happens in the interval $[T_1, T_{c1}]$.

In this respect, let us proceed to the phase transitions study for the case, in which broken symmetry gets restored in ϕ sector and ISB takes place in ψ sector. Accordingly,

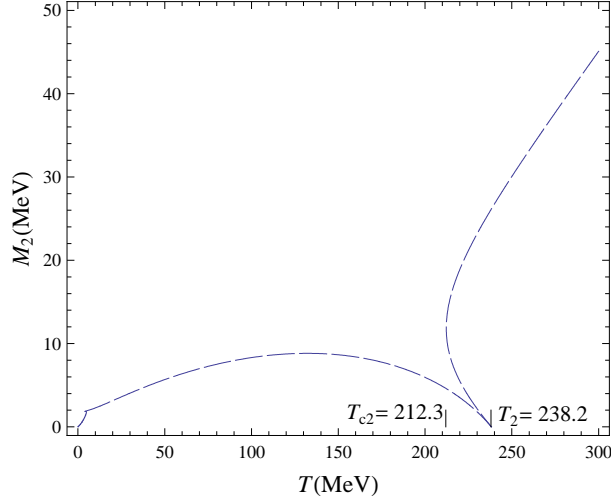


Fig. 2. The T dependence of M_2 , corresponding to the region that the symmetry in ψ -sector is broken (see Fig. 5). The phase transition happens in the interval $[T_{c2}, T_2]$.

the parameters are constrained as follows.

$$\begin{aligned} \lambda_1 > 0, \lambda_2 > 0, \mu_1^2 < 0, \mu_2^2 > 0, \\ \lambda < 0, \lambda_1 > |\lambda| > \lambda_2, \lambda_1 \lambda_2 > 9\lambda^2. \end{aligned} \quad (7)$$

For illustration let us choose at random some specified values for $\mu_1^2, \mu_2^2, \lambda_1, \lambda_2$ and λ , which obey the above mentioned inequalities:

$$\begin{aligned} \mu_1^2 &= -(4 \text{ MeV})^2, & \mu_2^2 &= (2 \text{ MeV})^2, \\ \lambda_1 &= 24, & \lambda_2 &= 1, 8 \quad \text{and} \quad \lambda = -2. \end{aligned}$$

They are the inputs for numerical computations. We first remark that, in addition to the model parameters, the renormalization introduced another parameter μ , which is the renormalization scale. Then we must determine a suitable value μ_0^2 of μ^2 , which is defined as the real root of the following equation

$$\phi_0(\mu^2, 0)|_{\mu^2=\mu_0^2} = 2 \text{ MeV},$$

where $\phi_0(\mu^2, 0)$ is a solution of the system of Eqs. (4) and (5) at $T = 0$. The numerical computation gives $\mu_0 = 5.657 \text{ MeV}$.

In ϕ -sector, eliminating ϕ_0 from (4) and (5) leads to

$$\begin{aligned} M_1^2(T) &= -2\mu_1^2 - \lambda_1 P_f(M_1) - \lambda P_f(M_2), \\ M_2^2(T) &= \mu_2^2 + \frac{3\lambda}{\lambda_1} \mu_1^2 - \lambda P_f(M_1) + \left(\frac{\lambda_2}{2} - \frac{6\lambda^2}{4\lambda_1} \right) P_f(M_2). \end{aligned} \quad (8a)$$

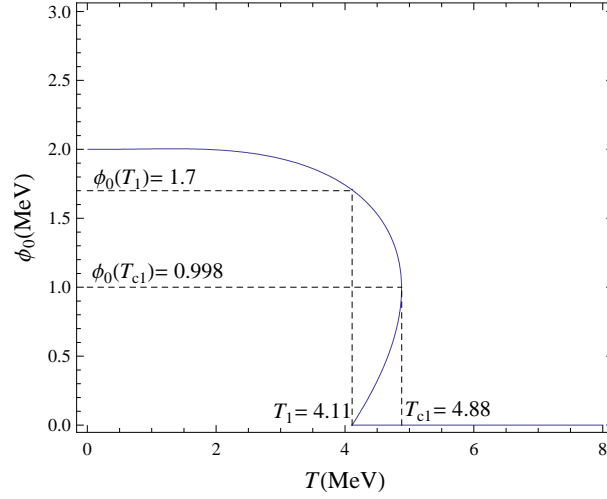


Fig. 3. The T evolution of the order parameter ϕ , in which the broken symmetry in ϕ -sector is restored. The phase transition happens in the interval $[T_1, T_{c1}]$. At T_1 , the value $\phi_0 = 0$ is in maximum of $V(\phi_0, T)$, while the value $\phi_0 = 1.7$ MeV is in minimum. In the interval $T_1 < T < T_{c1}$, the value $\phi_0 = 0$ is in minimum at $V(\phi_0, T) = 0$, value ϕ_{02} is in maximum, and ϕ_{01} in minimum. At T_{c1} , there is an inflexion point of $V(\phi_0, T)$ at $\phi_0 = 0.988$ MeV. (see Fig. 4).

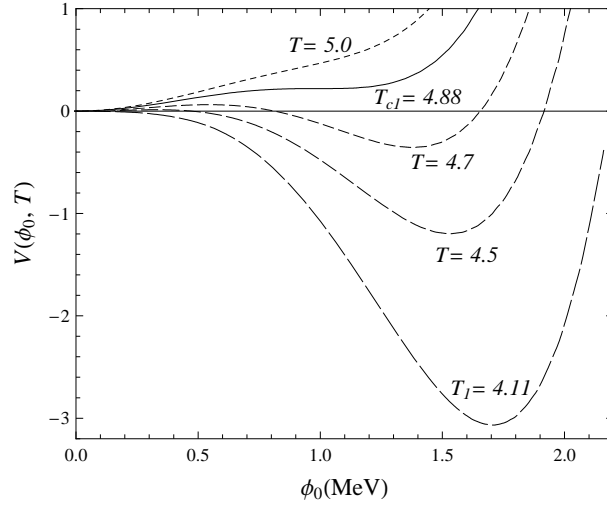


Fig. 4. The evolution of the $V(\phi_0, T)$ as a function of the order parameter ϕ_0 for several temperature steps: $T = 4.11, 4.5, 4.7, 4.878, 5$. MeV from bottom to top. At T_1 , the value $\phi_0 = 0$ is in maximum of $V(\phi_0, T)$, while the value $\phi_0 = 1.7$ MeV is in minimum. In the interval $T_1 < T < T_{c1}$, the value $\phi_0 = 0$ is in minimum at $V(\phi_0, T) = 0$, value ϕ_{02} is in maximum, and ϕ_{01} in minimum (see Fig. 4). At T_{c1} , there is an inflexion point of $V(\phi_0, T)$ at $\phi_0 = 0.988$ MeV.

for $\phi_0 \neq 0$, and

$$\begin{aligned} M_1^2(T) &= \mu_1^2 + \frac{\lambda_1}{2} P_f(M_1) + \frac{\lambda}{2} P_f(M_2), \\ M_2^2(T) &= \mu_2^2 + \frac{\lambda_2}{2} P_f(M_2) + \frac{\lambda}{2} P_f(M_1). \end{aligned} \quad (8b)$$

for $\phi_0 = 0$.

Inserting $\mu^2 = \mu_0^2$ into (8) and then solving numerically this system of equations we obtain the solutions M_1 in Fig. 1, and similar to ψ -sector we have M_2 presented in Fig. 2.

The T dependence of the order parameter ϕ_0 is given in Fig. 3. It is observed in these figures that for $0 < T < T_1 \approx 4.18$ MeV a first order phase transition persists. When $T = T_1$ a second order phase transition emerges and in the interval $T_1 \leq T \leq T_{c1}$ both phase transitions coexist up to $T_{c1} \approx 4.878$ MeV, at which

$$\left. \frac{d\phi_0(T)}{dT} \right|_{T=T_{c1}} = \infty.$$

T_{c1} is exactly the critical temperature, where the system transform from first order phase transition to second order one. This phenomenon is highlighted by means of the numerical computation performed for $V_\beta[\phi_0, \psi_0 = 0]$, as function of ϕ_0 at several values of T . It is easily proved that the curve, corresponding to $T = T_{c1} = 4.878$ MeV in Fig. 4, has an inflexion point at $\phi_0(T_{c1}) = 0.998$ MeV and $V[\phi_0(T_{c1})] = 0.227$ MeV. The broken symmetry is then restored at T_{c1} .

In order to consider the high temperature ISB in ψ sector the T dependence of $\psi_0(T)$ for large T are plotted in Fig. 5.

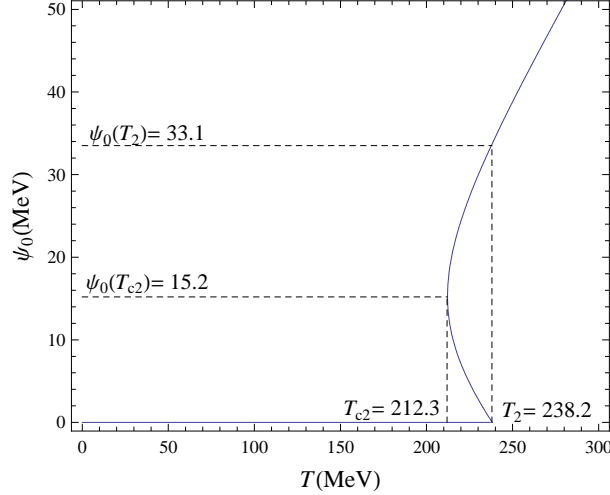


Fig. 5. The T evolution of the order parameter ψ , in which the symmetry in ψ -sector is broken. The phase transition happens in the interval $[T_{c2}, T_2]$. At T_2 , the value $\psi_0 = 0$ is in maximum of $V(\psi_0, T)$, while the value $\psi_0 = 33.1$ MeV is in minimum. In the interval $T_{c2} < T < T_2$, the value $\psi_0 = 0$ is in minimum at $V(\psi_0, T) = 0$, value ψ_{02} is in maximum, and ψ_{01} in minimum. At T_{c2} , there is an inflexion point of $V(\psi_0, T)$ at $\psi_0 = 15.2$ MeV. (see Fig. 6).

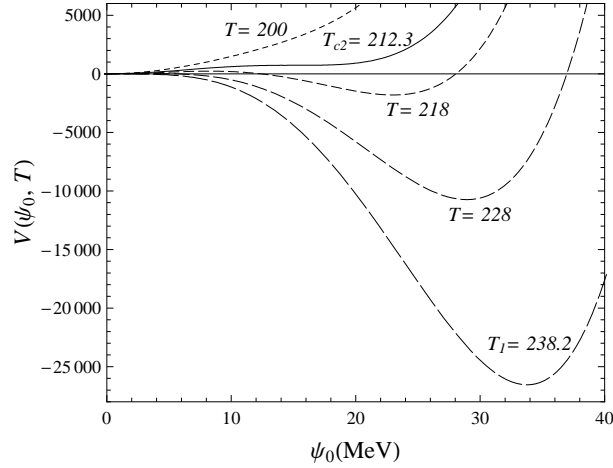


Fig. 6. The evolution of the $V(\psi_0, T)$ as a function of the order parameter ψ_0 for several temperature steps: $T = 200, 212.253, 218, 228, 238.232$ MeV from top to bottom. At T_2 , the value $\psi_0 = 0$ is in maximum of $V(\psi_0, T)$, while the value $\psi_0 = 33.1$ MeV is in minimum. In the interval $T_{c2} < T < T_2$, the value $\psi_0 = 0$ is in minimum at $V(\psi_0, T) = 0$, value ψ_{02} is in maximum, and ψ_{01} in minimum (see Fig. 5). At T_{c2} , there is an inflexion point of $V(\psi_0, T)$ at $\psi_0 = 15.2$ MeV.

It is evident that the symmetry is broken for $T = T_{c2} = 212.253$ MeV, at which

$$\left. \frac{d\psi_0(T)}{dT} \right|_{T=T_{c2}} = \infty.$$

T_{c2} is the critical temperature when the system exhibits simultaneously first and second order phase transition. It is the temperature for ISB to take place in ψ sector. The evolution of $V_\beta[\phi_0 = 0, \psi_0]$ against ψ_0 for different temperatures is shown in Fig. 6. It is properly asserted that the inflection point of the curve $T = T_{c2} = 212.253$ MeV possesses coordinates $\psi_0(T_{c2}) = 15.230$ MeV and $V_\beta[\psi_0(T_{c2})] = 789.02$ MeV.

IV. CONCLUSION AND DISCUSSION

In this paper the phase transitions were considered for $Z_2 \times Z_2$ model by means of the finite temperature CJT effective action. We investigated in detail phase transitions for a set of parameter chosen at random. The numerical solutions for the gap equations and the shape of effective potential, as function of order parameters at different temperatures, exhibit the coexistence of first and second order phase transitions for SNR in ϕ sector and ISB in ψ sector. Although the model studied earlier is too simple, but all those we observed in the preceding section are extremely interesting and their main feature does not depend on the chosen set of parameters, provided the latter obeys (7), of course. The generalization to $O(M) \times O(N)$ -model is straightforward and produces similar results. Our present study, in some sense, could be considered to be complementary to those obtained in [6, 13, 14, 25, 26].

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