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An Elementary Development of Mass-energy Equivalence

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advanced impulse, due to the next crossing of the wall, which raises its velocity to $\frac{1}{2}v$ towards the origin. After passing the wall it receives the final retarded impulse which raises its velocity to v towards the origin. So the cycle continues, with an increased period of $4t_2 > 4a/v$. The symmetry between advanced and retarded impulses is beautiful to behold. In the limit of infinite propagation speed, $s \to \infty$, the results reduce to the usual instantaneous square well. As $\frac{1}{2}v$ approaches s, the particle can penetrate farther and farther behind the wall before the retarded impulse has

time to catch up. As $\frac{1}{2}v \to s$, $t_2 \to \infty$, and $x(t_2) \to \infty$, and the particle escapes.

ACKNOWLEDGMENT

[1]C. H. Blanchard, Am. J. Phys. **49**, 987 (1981).
[2]P. Havas, in *Causality and Physical Theories*, edited by W. B. Rolnick (AIP, New York, 1974).

An elementary development of mass–energy equivalence

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In this note we describe a simple derivation of the mass–energy equivalence equation that we have not seen previously in the literature. The derivation is of the type which Einstein used[1,2] and employs a "thought experiment" in which an elementary photon emission event is examined by two sets of inertial observers. A comparison of the analyses of these observers brings the validity of the principle of momentum conservation into question. However, since the conservation of momentum is the cornerstone of dynamics it is preserved at all costs. This is accomplished by allowing the concepts of the mass and energy to undergo redefintion.

A block which is stationary with respect to observers A^0 and B^0 emits two photons of equal frequency in opposite directions as shown in Fig. 1. Using the principle of the conservation of momentum, and the Maxwell $(|p| = E/c)$ and Planck–Einstein $(E = h\nu)$ relationships for electromagnetic radiation, A^0 and B^0 infer that the change in momentum of the block is zero. In other words,

$$\Delta p^0_{\text{block}} = -\Delta p^0_{\text{photon}} = -(\nu^0_B - \nu^0_A)\,h/c = 0 , \qquad (1)$$

since $\nu^0_A = \nu^0_B = \nu^0$. In addition these observers find that the energy change of the block is

$$\Delta E^0 = -2h\nu^0 . \qquad (2)$$

Relative to observers A and B, shown in Fig. 2, the block is moving with velocity v. Their statement of the conservation of momentum is

$$\Delta p_{\text{block}} + (\nu_B - \nu_A)\,h/c = 0 , \qquad (3)$$

where for these observers $\nu_A \neq \nu_B \neq \nu^0$.

Owing to the Doppler effect, B finds that photon B, emitted by a source approaching with veloicty v, has a frequency given by

$$\nu_B = [(1 + v/c)/(1 - v/c)]^{1/2}\, \nu^0 . \qquad (4)$$

Observer A, on the other hand, finds that photon A, emit-

ted by a source receding with velocity v, has a frequency given by

$$\nu_A = [(1 - v/c)/(1 + v/c)]^{1/2}\, \nu^0 . \qquad (5)$$

Substitution of Eqs. (4) and (5) into Eq. (3) yields after rearrangement

$$\Delta p_{\text{block}} + [2h\nu^0/(1 - v^2/c^2)^{1/2}](v/c^2) = 0 . \qquad (6)$$

These observers find that the energy change of the block to be

$$\Delta E = -h\nu_A - h\nu_B$$
$$= -2h\nu^0/(1 - v^2/c^2)^{1/2}$$
$$= \Delta E^0/(1 - v^2/c^2)^{1/2} . \qquad (7)$$

Substitution of Eq. (7) into Eq. (6) gives

$$\Delta p_{\text{block}} - \Delta E\,(v/c^2) = 0 . \qquad (8)$$

The classical definition of momentum is maintained by requiring that $p_{\text{block}} = mv$. Since no recoil is observed in the rest frame, the relativity principle requires that $\Delta v = 0$ in the moving frame also. This means that the principle of the conservation of linear momentum can only be preserved for A and B by postulating a mass change for the block upon emission of the photons, or that $\Delta p_{\text{block}} = \Delta(mv) = v\Delta m$. In light of these considerations, Eq. (8) can be written as

$$\Delta E = \Delta mc^2 . \qquad (9)$$

We note that an equivalent equation $(\Delta E^0 = \Delta m^0 c^2)$ can be written for the rest frame by combining Eqs. (7) and (9) and interpreting $\Delta m(1 - v^2/c^2)^{1/2}$ as the change in rest mass of the block resulting from the emission of the two photons:

$$\Delta m^0 = \Delta m(1 - v^2/c^2)^{1/2} . \qquad (10)$$

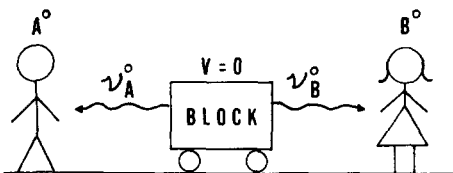Generalizing on the basis of Eqs. (9) and (10) suggests that

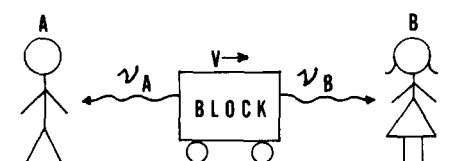Fig. 1. Photon emission as viewed by two observers at rest relative to the block.

Fig. 2. Photon emission as viewed by viewed two observers relative to which the block has velocity v.

we can write

$$E = mc^2 ,\tag{11}$$

where

$$m = m^0(1 - v^2/c^2)^{-1/2} .\tag{12}$$

[1] A. Einstein, Ann. Phys. **18**, 639 (1905). An English translation can be found in *The Principle of Relativity*, edited by H. A. Lorentz *et al.* (Dover, New York, 1952), p. 69–71.

[2] A. Einstein, in *Out of My Later Years* (Philosophical Library, New York, 1950), p. 116.

Comment on "The concept of temperature and its dependence on the laws of thermodynamics"

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Ehrlich[1] has recently given an extended analysis of the concept of temperature, based on the laws of thermodynamics. In that paper he stressed the concept of temperature.

Some year ago I published a restatement of the zeroth law of thermodynamics,[2] viz.:

There exists a *scalar quantity* called temperature, which is a property of all thermodynamic systems (in equilibrium states), such that temperature equality is a *necessary and sufficient* condition for thermal equilibrium.

The terms "scalar quantity" and "necessary and sufficient" appear to establish a temperature order and overcome some of the gaps which Ehrlich finds in other treatments. Of course, my formulation achieves this by postulating something beyond the conventional form of the zeroth law, which then follows as a corollary.

The above form also explicitly includes the requirement for equilibrium states. In Ehrlich's approach his domain D_t is presumably restricted to systems which are either in equilibrium states or very close to equilibrium.

There is one minor problem in Ehrlich's interesting paper, i.e., the definition of heat reservoir in the beginning of Sec. VI. He states:

By a heat reservoir we mean an idealized element of D_t, whose state undergoes no change as a result of partaking in a heat interaction.

Actually, whenever the reservoir either gains or loses heat in an interaction, there must be a change in its internal energy and hence, by the first law, a change in state. In principle one can make the *change* in any *intensive* property (such as temperature) as small as desired by letting the mass of the reservoir increase appropriately. However, the corresponding *change* in an *extensive* property will not in general approach zero. Thus Ehrlich's definition is not an idealization based on a limiting process and does not seem to be a permissible thermodynamic concept.

However, the difficulty can be easily remedied simply by requiring that after any heat interaction the reservoir R remains "as hot as" it was before. This might be operationally confirmed by dividing R into two subsystems, R_1 and R_2. At the time of division obviously R_1 is as hot as R_2. If after any permissible heat interaction involving R_1 it still remains as hot as R_2, then R_1 qualifies as a heat reservoir.

[1] P. Ehrlich, Am. J. Phys. **49**, 622 (1981).

[2] J. S. Thomsen, Am. J. Phys. **30**, 294 (1962).

Further support for a "physical" interpretation of the primes

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In March 1982, H. Gutfreund and W. A. Little published a "physicist's proof of Fermat's theorem of primes."[1] In their note they suggest an analogy between the primes and lack of symmetry of certain Ising-spin systems. This physical interpretation of primes can be further supported by a proof of Wilson's theorem on primes: if p is a prime, then $(p - 1)! = -1 \pmod{p}$. This proof will be presented here.

Let us consider a ring of p sites and ascribe to each site i an Ising-spin variable s_i. Each Ising-spin variable s_i assumes one of the $p = 2j + 1$ possible spin projections $-j$, $-j + 1,...,0, 1, ..., j$, in such a way that the whole ring contains a permutation of the values $-j,...,j$, and always $s_1 = 0$.

Clearly, there are $(p - 1)!$ possible configurations. They will be divided into classes according to the following: Two configurations α and β belong to the same class, iff β can be obtained from α by adding a fixed number K to each of the s_i of α (where increasing $s_i = j$ by one means switching over to $s_i = -j$), and, after that, performing a rotation of the new ring so that the new s_1 equals 0 (notation $\beta = \alpha + K$).

For all configurations α we have $\alpha = \alpha + p$, so any class cannot contain more than p different configurations. In fact, if a class contains less than p different configurations, it contains only one, for if $\alpha + K = \alpha + L$, then $\alpha = \alpha + (L - K)$, so $\alpha = \alpha + t$ $(L - K)$ (multiplication modulo p); $t = 0, 1,..., p - 1$. Because of the fact that p is a prime, we can conclude $\alpha = \alpha + n$; $n = 0, 1, 2, ..., p - 1$.