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The Nature and Nurture of Intuition

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Mathematics

- Is mathematical ability innate
 or developed
  or both?

- While ability is likely both innate and developed, education needs to focus on the development.
- How do we help students improve their mathematical intuition and reasoning?

Definitions and Counter-Examples

- A circle is …

- Definitions in mathematics must be formal so they can be used in proofs. However, they also embody intuition.

A *circle* is the set of points a constant distance (the radius) from a fixed point (the center).
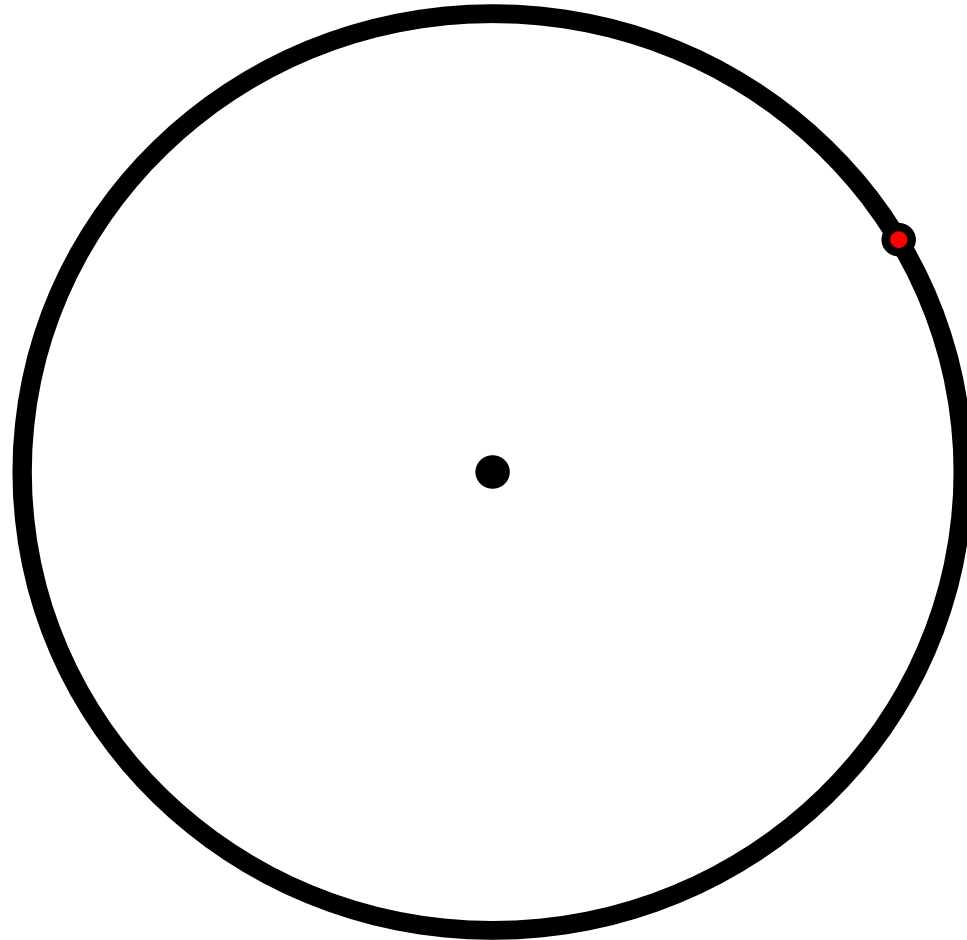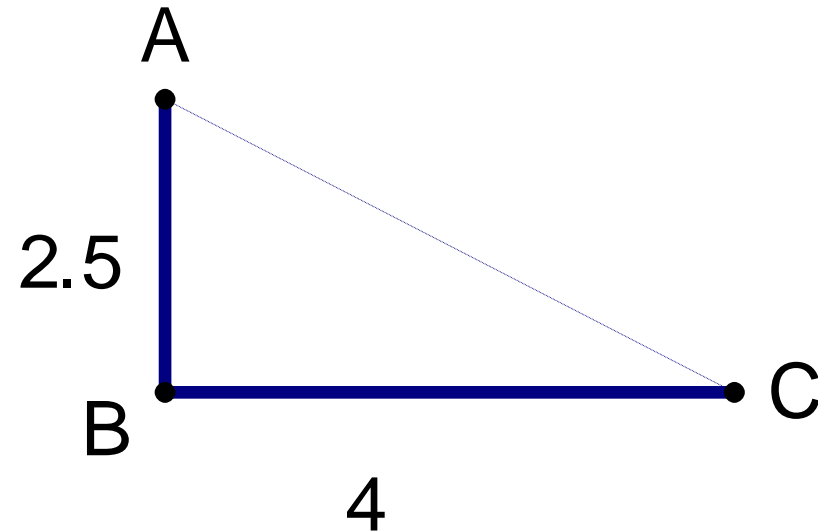
- "Intuition is not something that is given. I've trained my intuition to accept as obvious things which were initially rejected as absurd and I find everyone can do the same." – Benoit Mandelbrot
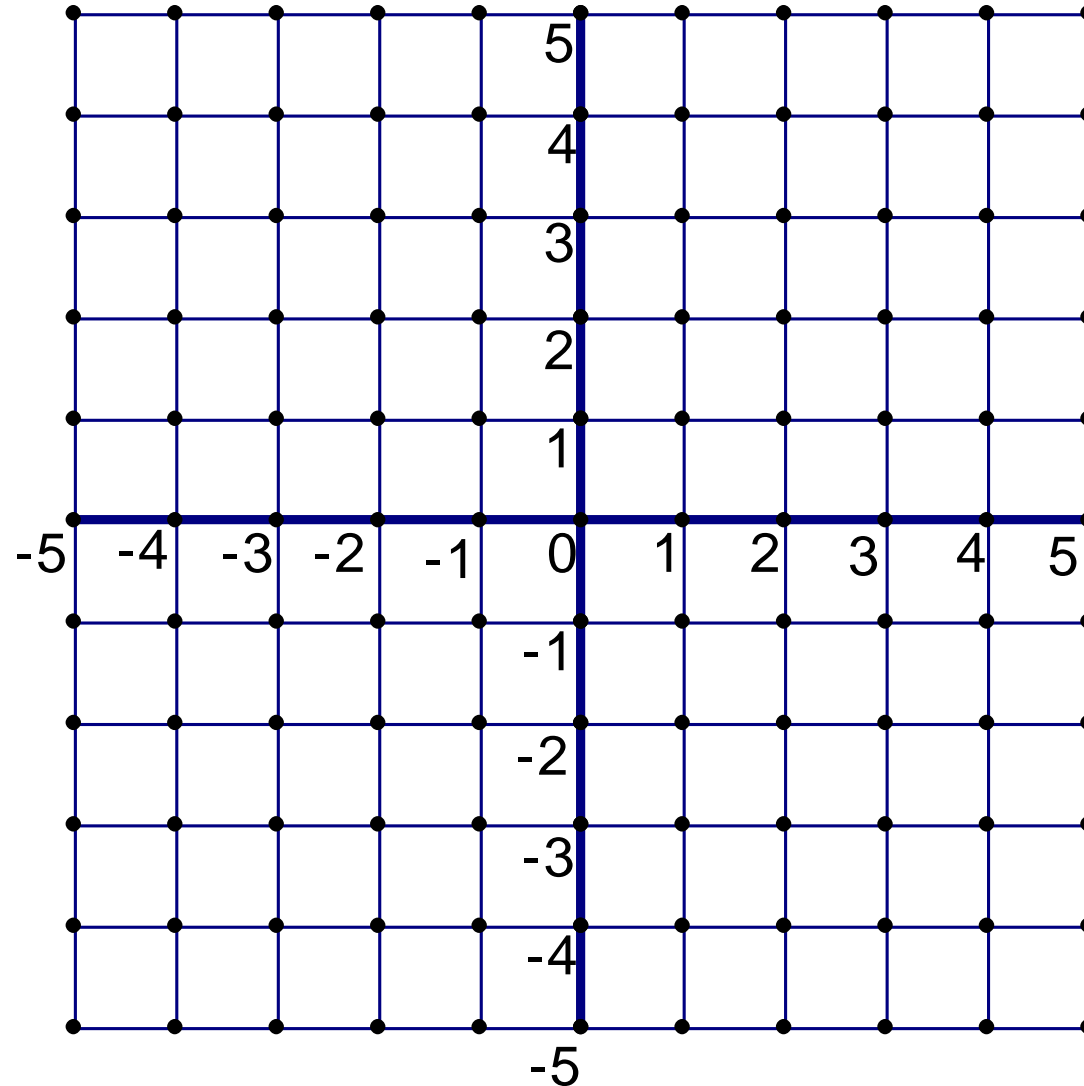
Taxicab Geometry

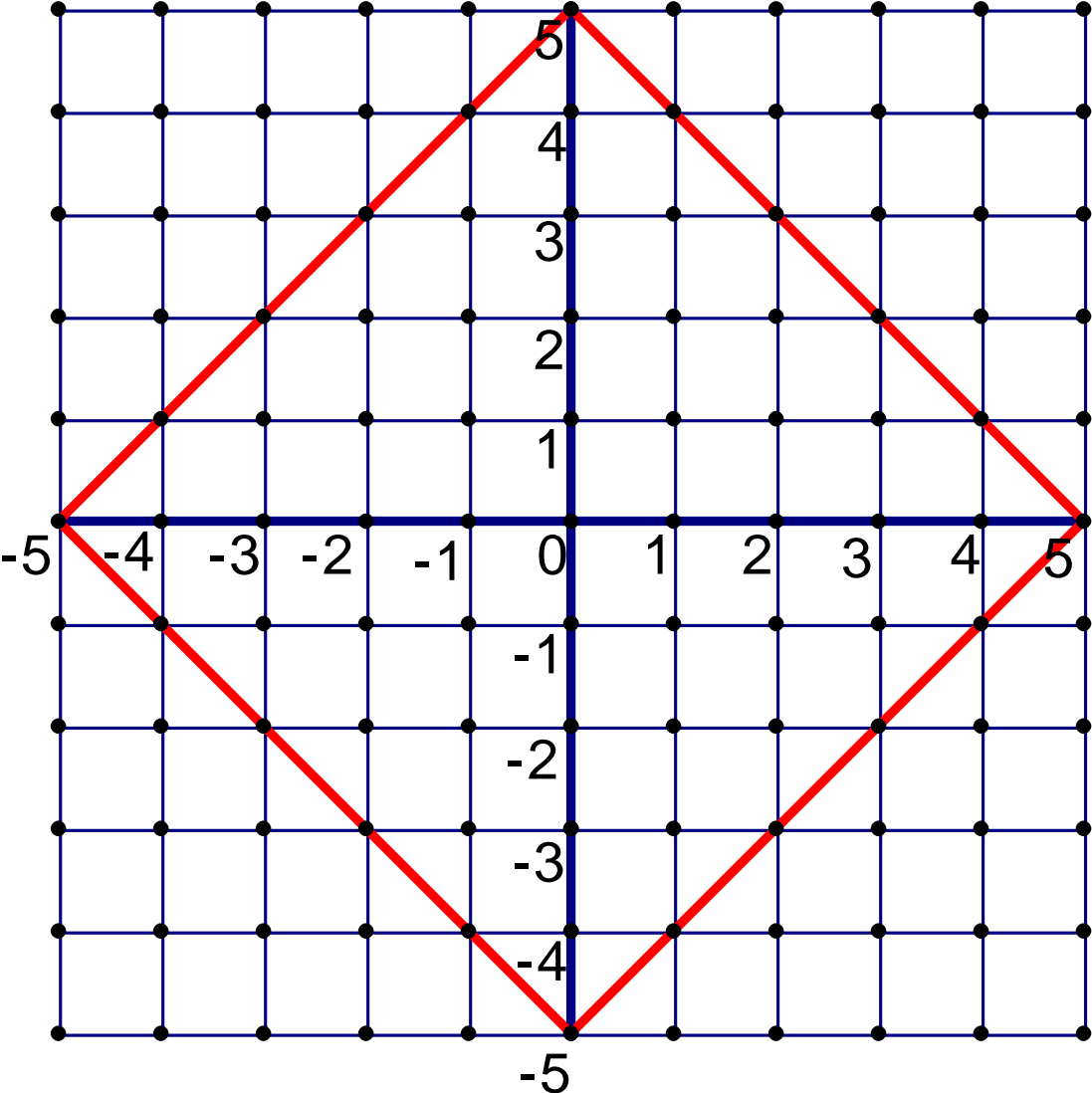- Distance is measured along vertical and horizontal directions only.

A

2.5

B

4

C

Distance from A to C
is 2.5 + 4 = 6.5.

On graph paper draw a taxicab "circle" with center at (0,0) and "radius" 5.

Taxicab Circle

- Aquinas (and later Descartes) used the example of a "square circle" as a contradiction in terms.

- Counter-examples force us to re-examine our naïve mathematical intuitions and enable us to mature.

Finding and Proving Properties

# Mathematical proof needs both careful reasoning and insight (intuition).

- Kant considered mathematics "synthetic *a priori*"—genuinely new knowledge, but independent of experience.

- Modern psychology and Mandelbrot beg to differ. Math may be neither "Platonic" nor "empirical."

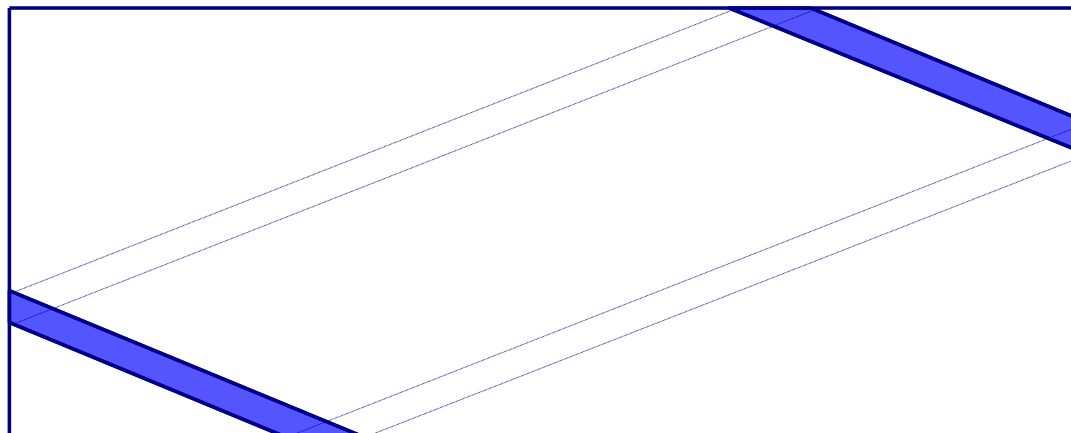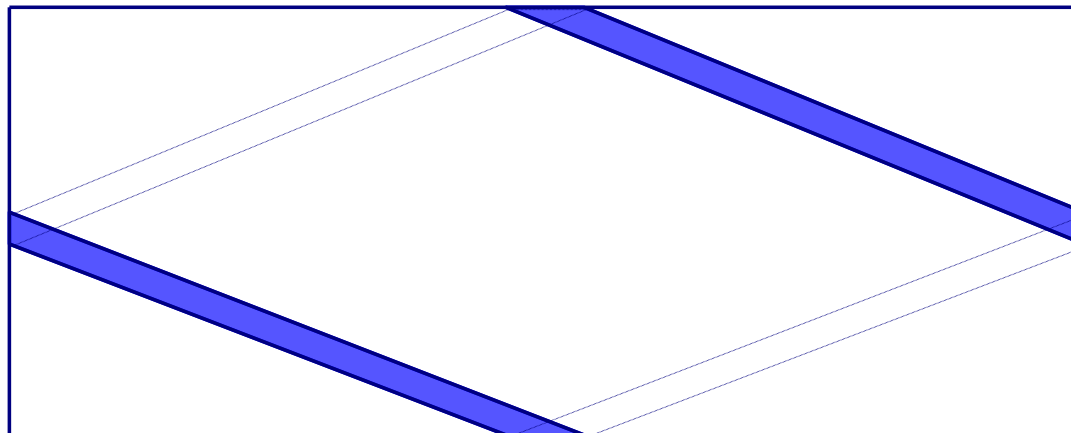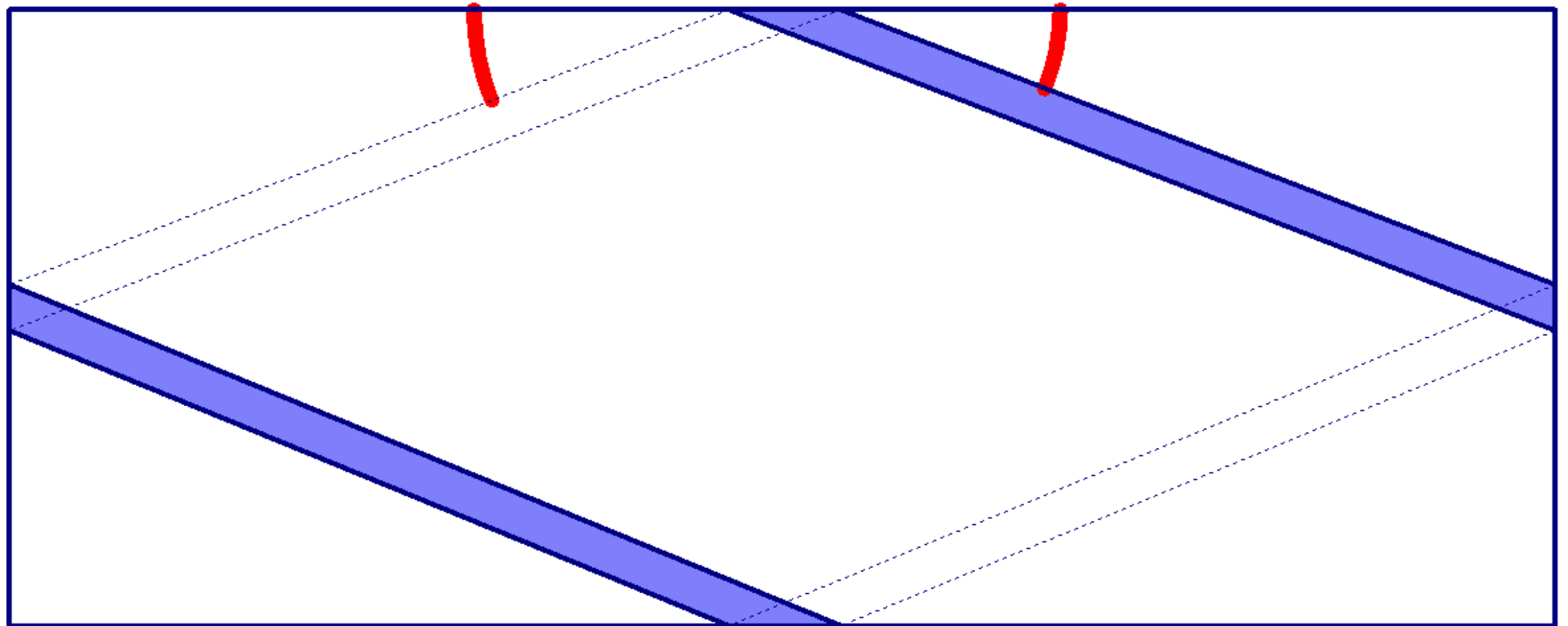Remove ribbon without cutting or untying
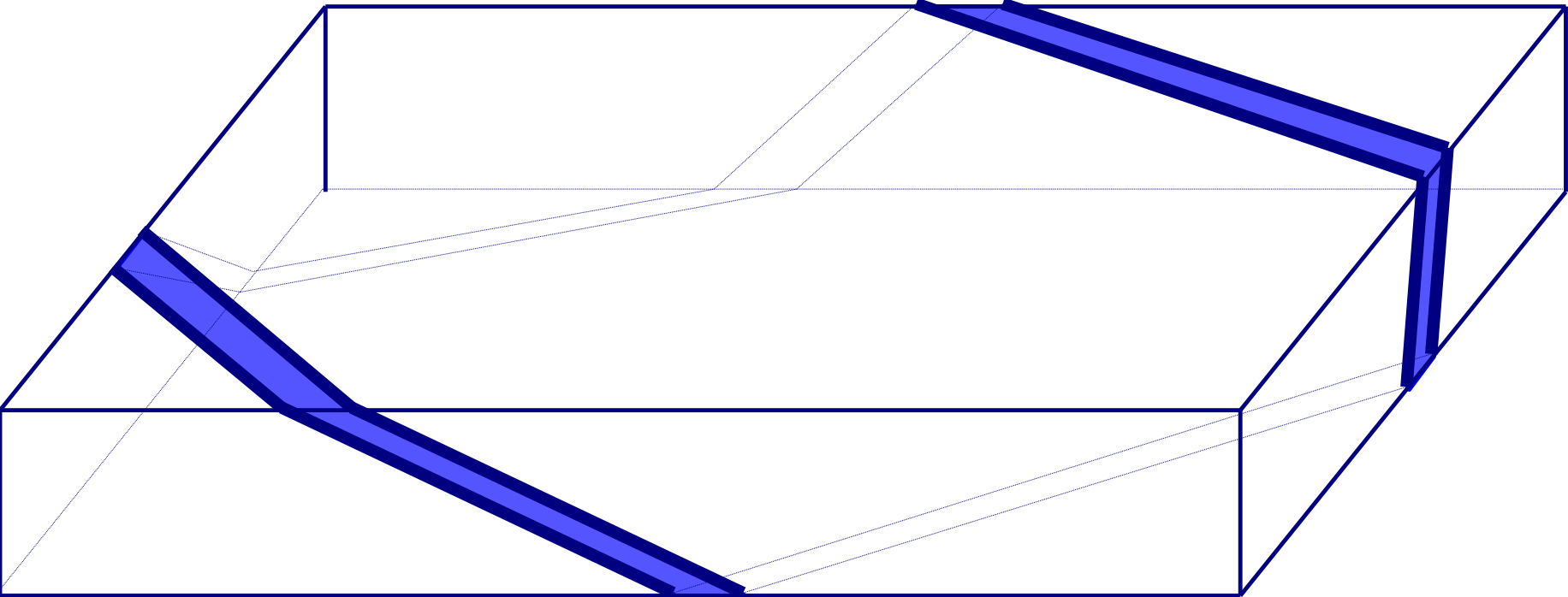
Why can we remove the ribbon? Simplification: Think about this problem in two dimensions.
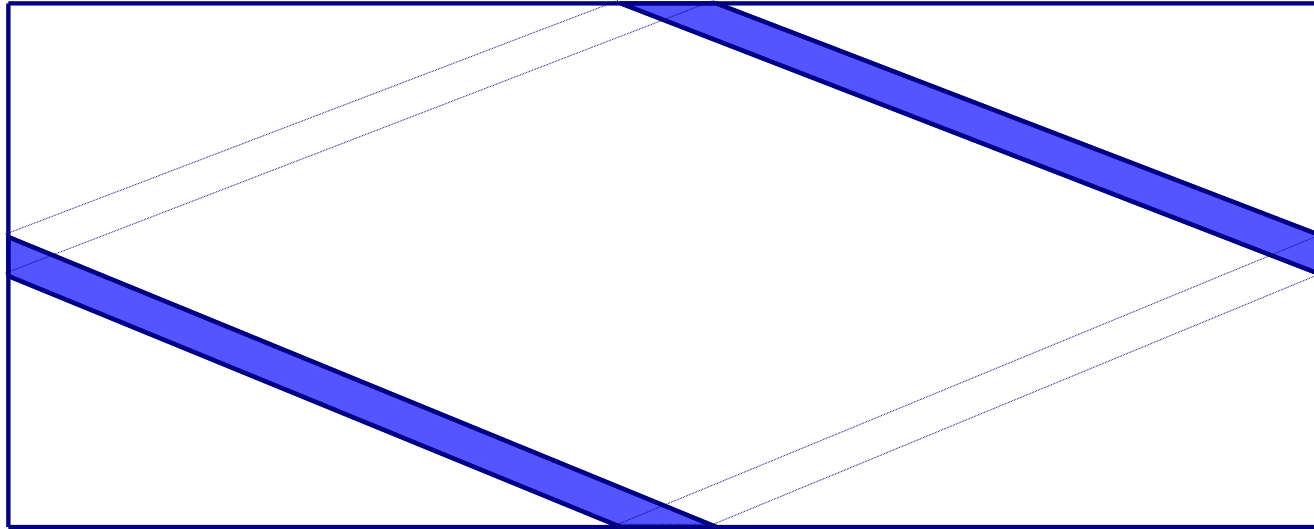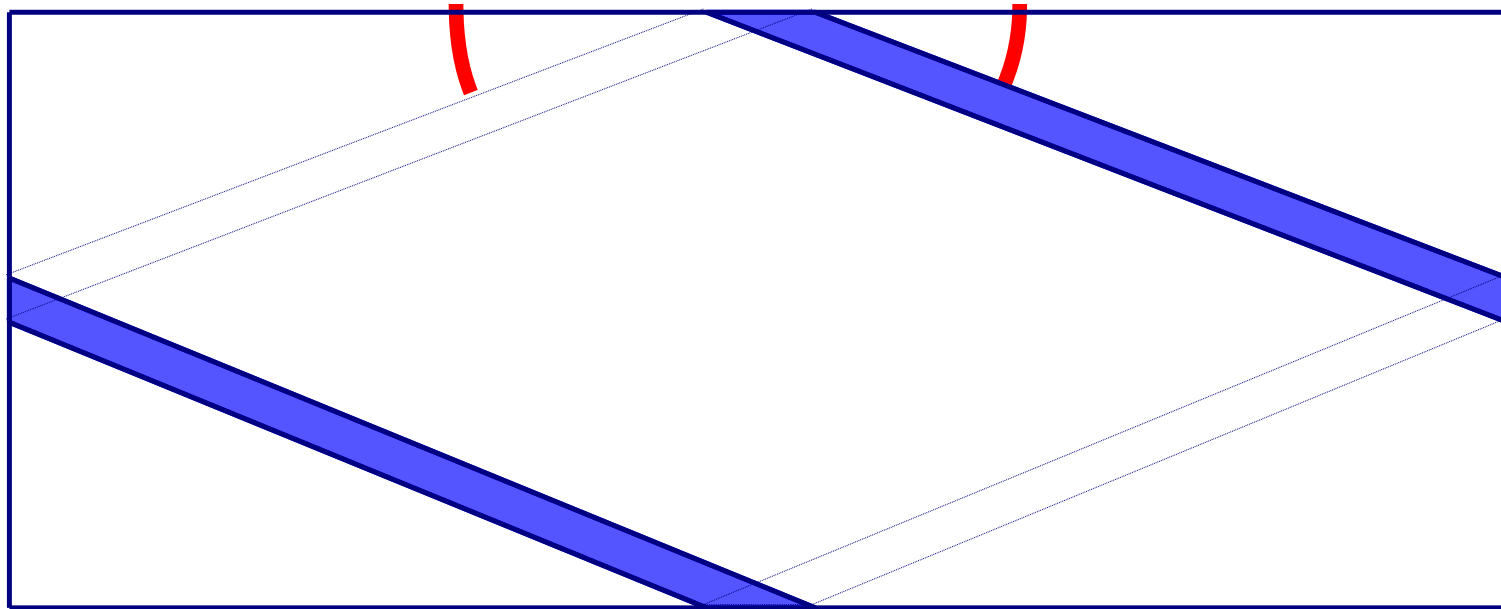
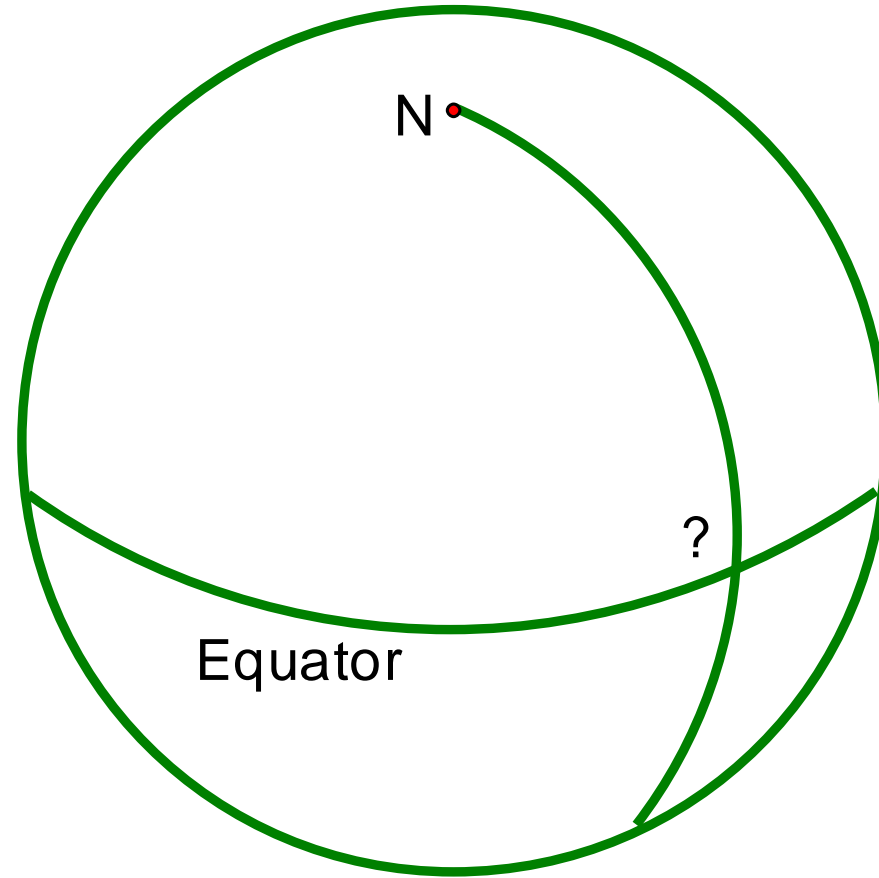# Key: Similar Triangles

But why are they similar?

Taut ribbon implies "angle of incidence equals angle of reflection." — Fermat and Menelaus

Spheres

- What is the angle made by the equator with an arc from the North pole to the South pole?

What is the angle made by the equator an arc from the North pole to the South pole?

- Euclid (300 BCE): If two intersecting lines make adjacent angles equal, both angles are right angles (90°).

- Euclid (Book I, Proposition 32) The angle sum of a triangle is 180°.

- (Menalaus, 100 CE) In spherical geometry the amount by which the angle sum of a triangle exceeds 180° is proportional to the area of the triangle.

- Gauss (1777 — 1855) proved a generalization of Euclid's and Menelaus' results for triangles formed by *geodesics* (roughly, shortest paths) on curved surfaces.
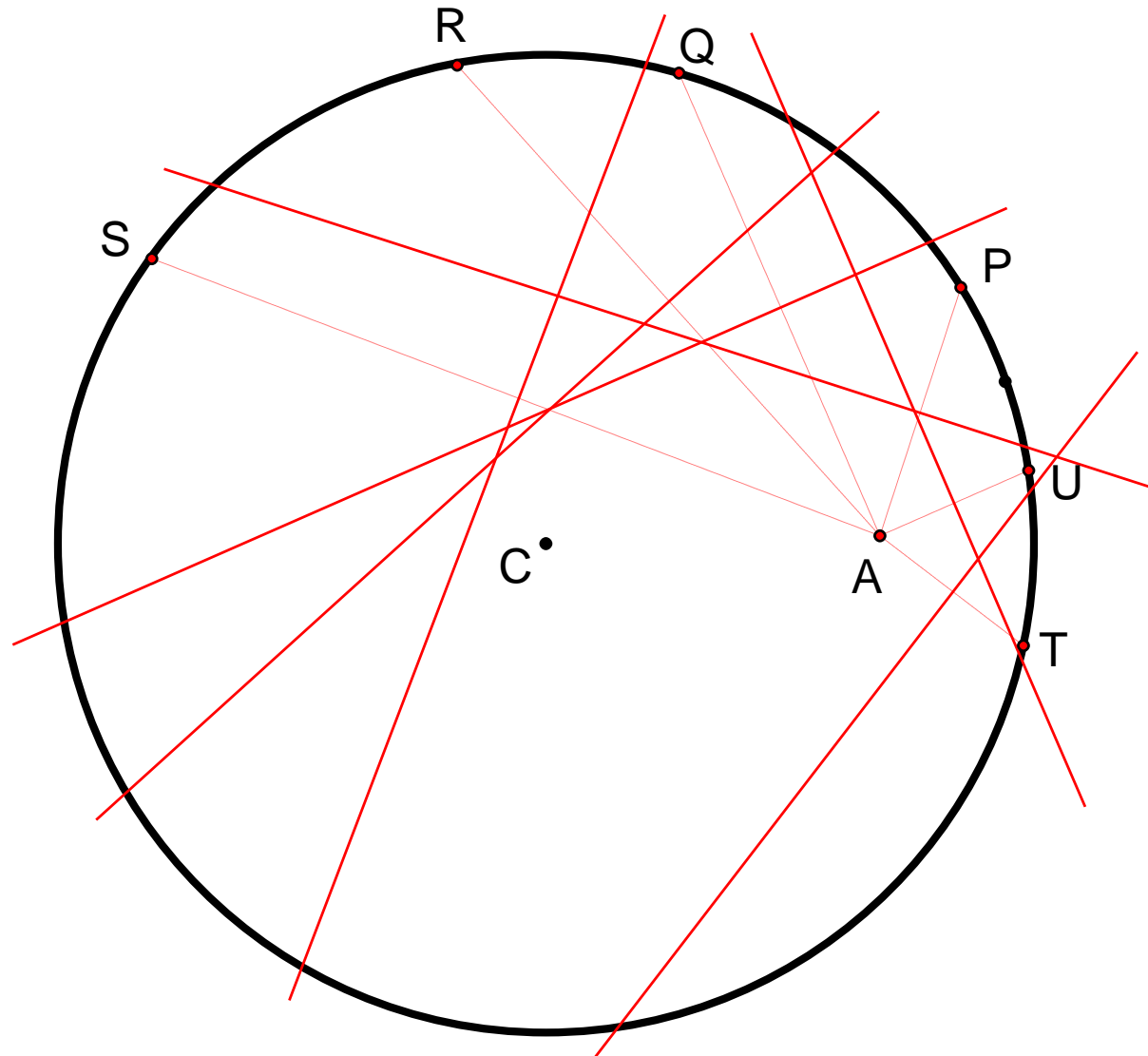
- Riemann generalized Gauss' result to any number of dimensions, providing language and intuition essential for Einstein's general theory of relativity.

"It is by logic we prove, it is by intuition we invent." — Henri Poincaré
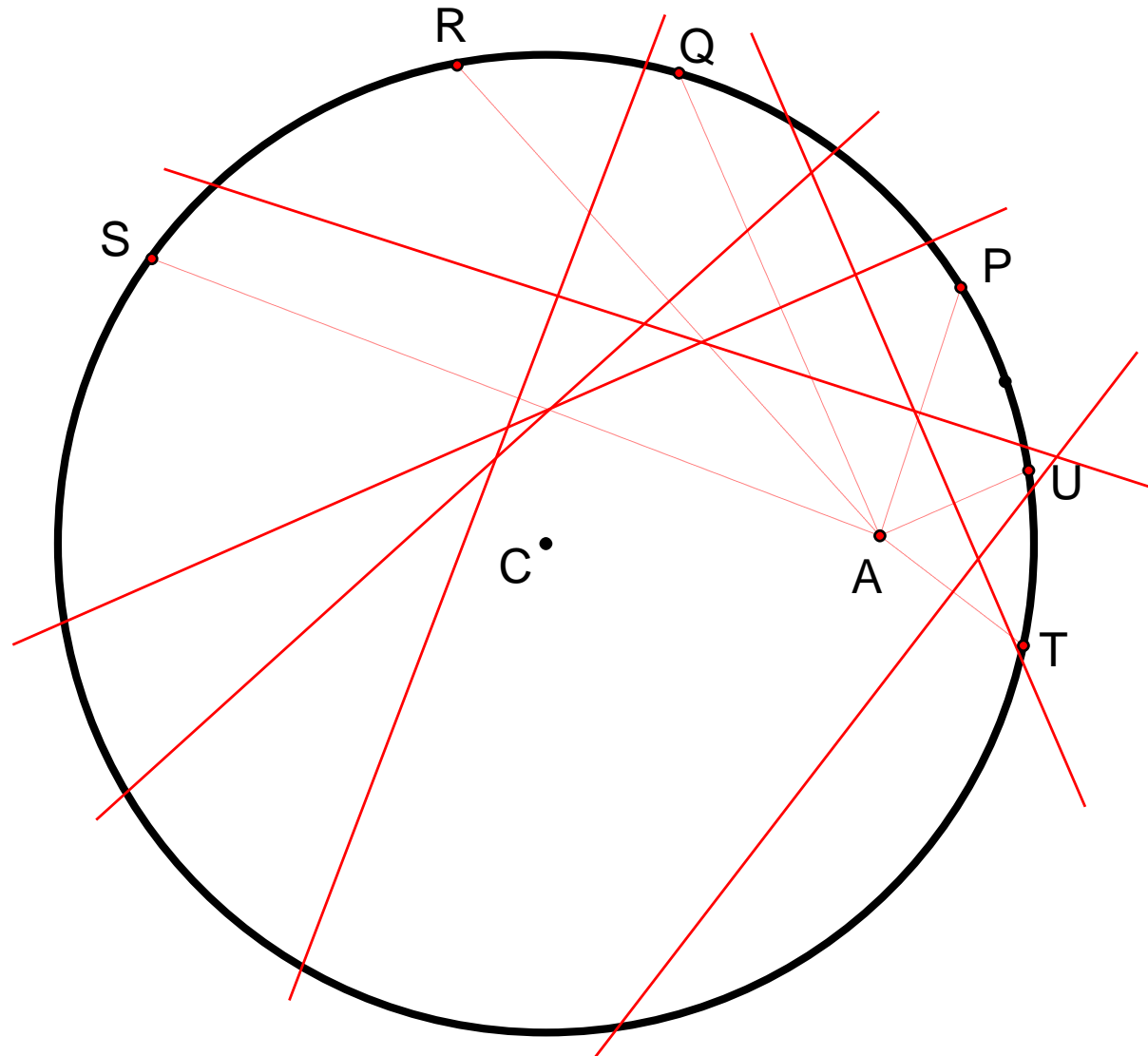
Paper Folding Curve
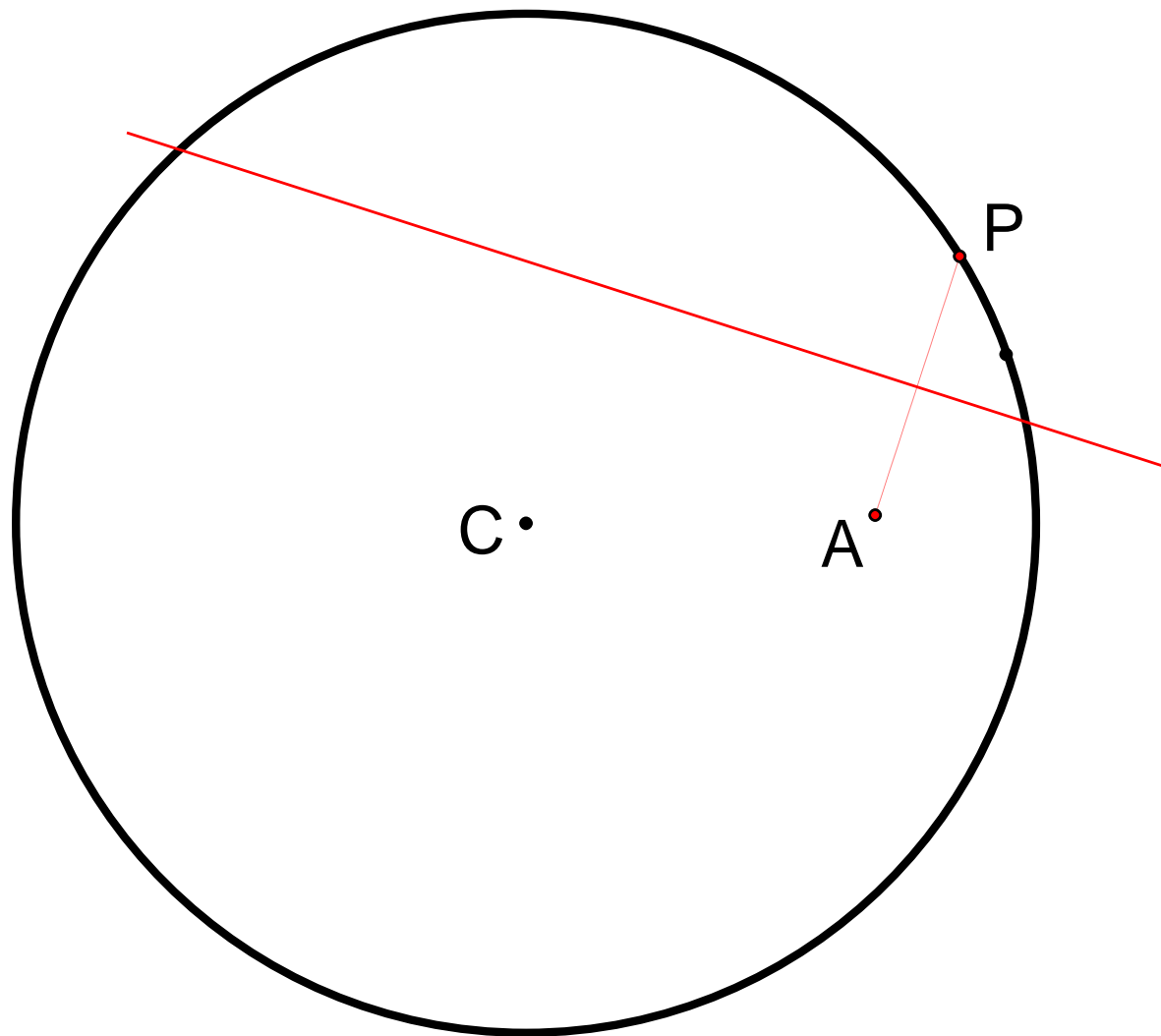
What curve is outlined? Why?

- An ***ellipse*** with *foci* F_1 and F_2 is the set of points P so that the sum of the distances PF_1 and PF_2 is a fixed value.

What are the two foci and the fixed distance?

Ellipse: CW + WA = CP = radius