First Order Space Time Autoregressive Stationary Model on Petroleum Data

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Abstract. First order Space-Time Autoregressive model is one of the models which involves location and time. STAR(1;1) model stationary can be used to forecast future observation at a location based on one previous time of its own location and the spatial neighborhood. STAR(1;1) model on petroleum productivity data in Balongan, Indramayu, West Java with eigenvalue less than 1. It indicates that STAR (1;1) model on petroleum productivity data in Balongan, Indramayu, West Java meets the stationary requirement.
Keywords : autoregressive, petroleum data, STAR, first order

1. Introduction

Space-Time Autoregressive (STAR) model is the development of a univariate time series model into a multivariate time series model[1-4]. The STAR(1;1) model states that the current time observation at a particular location is influenced by the observation of previous time at the location and its spatial neighborhood which are a part of the same group [13]. For the simplicity of the model, the study will be focused on time lag 1 and spatial lag 1. A simple, stationary time series model is first order Autoregressive model [5-11] : AR(1). AR(1) model developed into Vector Autoregressive(1) model: VAR(1), first order Space Time Autoregressive(1) model: STAR(1;1)[2]. Petroleum production in Balongan, Indramayu, West Java is a time series problem, which can be modeled by AR(1) model, VAR(1) model, and STAR(1:1) model. [12] Providing procedures in the form of; identification, parameter estimation, and diagnostic examination.

2. Methods used

Applying STAR (1;1) stationary model on petroleum data in Balongan, Indramayu, West Java. The STAR(1;1) model states that the current time observation at a particular location is influenced by the observation of previous time at the location and its spatial neighborhood which are a part of the same research group, thus STAR(1;1) model is applied on petroleum data, specifically on well-1 and well-3 petroleum drilling data in Balongan, Indramayu, West Java. For the simplicity of the model, the study will be focused on time lag 1 and spatial lag 1 on STAR(1;1) model at several lag locations. And the STAR(1;1) model is examined or assessed. After the assessment phase, model validation is conducted through *error*. If the model determined as adequate, then it can be used to forecast future observation on petroleum data in Balongan, Indramayu, Jawa Barat.

3. Results and Discussion

Bivariate VAR(1) model is said to be first order stationary, if the mean and covariance are not dependent on *t*:

$$E[z(t)] = \mu$$
$$Cov[z(t), z(s)] = E[z(t)\, z(s)'] = E[z(0)z(s-t)']$$
$$= C(0,s-t)=C(t-s)= C(s-t) \quad (1)$$

From equation (1) covariance matrix C only depends on time shift or time lag (*t*-*s*) [5] in [3]

The theorem of VAR(*p*) model stationary requirement with $u = 1,2, \ldots N$ variate, presented [5] in [3] as follows:

If x_u equation solution

$$\left| x_u \mathbf{I} - \sum \Phi(j)\, x_u \right| = 0$$

located in a unit circle($|x_u|$<1), then the VARMA(p,q) process is stationary.

Next [6] provides Hannan stationary requirement equivalence above for VAR(1) model, as follows: If $\forall B \in C$, $|B|>1$ applies $|\mathbf{I} - \Phi B| = 0$, then VAR(1) is stationary. In other words, the B roots of $|\mathbf{I} - \Phi B| = 0$ are outside the unit circle.

STAR(1;1) model by[4] in[3] is stated:

$$\mathbf{z}(t)=\mathbf{z}(t-1)+\mathbf{W}\mathbf{z}(t)+\boldsymbol{e}(t) \quad (2)$$

$\mathbf{W} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ equal weight

STAR(1;1) model equation for 2 locations can be presented as follows:

$$\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \phi_{01} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \phi_{11} \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \quad (3)$$

Or

$$\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \phi_{01} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \phi_{11} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}$$

Equation (1) can be stated in a form:

$$\boldsymbol{z}(t) = [\phi_{01}\boldsymbol{I} + \phi_{11}\boldsymbol{W}]\boldsymbol{z}(t-1) + \boldsymbol{e}(t)$$
$$\mathbf{z}(t) = \mathbf{z}(t-1) + \boldsymbol{e}(t) \quad (4)$$
$$\boldsymbol{z}(t) = [\phi_{01}\boldsymbol{I} + \phi_{11}\boldsymbol{W}]\mathbf{z}(t-1) + \boldsymbol{e}(t)$$

with

$$\boldsymbol{\Phi} = [\phi_{01}\mathbf{I} + \phi_{11}\mathbf{W}]$$

Next, validation model is conducted through error. If the model determined as adequate, then it can be used to forecast future observation by using linear equation model [17]:

$$\mathbf{y} = X\,\boldsymbol{\beta} + \boldsymbol{e}(t),\ \boldsymbol{e}(t) \overset{iid}{\sim} N(\mathbf{0},\sigma^2) \quad (5)$$

STAR(1;1) model can be written

$$\boldsymbol{z}(t) = [\boldsymbol{z}(t-1) \quad \mathbf{W}\boldsymbol{z}(t-1)] \begin{bmatrix} \phi_{01} \\ \phi_{11} \end{bmatrix} + \boldsymbol{e}(t) \quad (6)$$

and

$$\mathbf{y} = X\,\boldsymbol{\beta} + \mathbf{e}(t)$$

Thus, STAR (1;1) model parameter estimation is obtained by using the smallest square method

$$\hat{\beta} = \begin{pmatrix} \hat{\phi}_{01} \\ \hat{\phi}_{11} \end{pmatrix} = (X' \quad X)^{-1} X' \hat{\boldsymbol{y}}. \qquad (7)$$

with

$$X = [\boldsymbol{z}(t-1) \quad \mathbf{W}\boldsymbol{z}(t-1)]$$

Matrix X above is a 2x2 size matrix contains:

$$X = \left[\begin{matrix} z_1(t-1) \\ z_2(t-1) \end{matrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} \right] \qquad (8)$$

Theorem: STAR(1;1) is said to be stationary, if it meets the following condition;

$$|\phi_{01}| + |\phi_{11}| < 1$$

(9)

[4] in [2] has proven the stationary requirement as follows:

$$|x_u \boldsymbol{I} - \quad \phi_{10} \boldsymbol{I} - \quad \phi_{11} \boldsymbol{W}| = 0$$

(10)

or

$$|-\phi_{11} \mathbf{W} \quad -(\phi_{10} - x_u)\mathbf{I}| = 0$$

with $\lambda(x_u) = \phi_{10} - x_u$
and $\lambda(x_u)$ is Eigenvalue A= $-\phi_{11} \mathbf{W}$

Each Eigenvalue A located in one of closed unit circles, stated by

$$|x - a_{ij}| \leq \sum_{j \neq 1} |a_{ij}|$$

For STAR (1;1) model case, the application of calculus theorem conducted by taking A= $-\phi\mathbf{W}$ and

$$\sum_{j=1} |a_{ij}| = \sum_{j=1} w_{ij} \, |\phi_{11}| x \, \mathbf{I} = \, |\phi_{11}|$$

$$\boldsymbol{w_{ij}} = 0$$

$$\sum_{j=1}^{N} w_{ij} = 1$$

thus

$$a_{ij} = \phi_{11}$$

can be written as :

$$|\lambda - 0| = |\phi_{11}|$$

And because all circles are the same, thus all Eigenvalues need to meet:

$$|\lambda| \leq |\phi_{11}|$$

For STAR(1;1) model $\lambda(x_u) = \phi_{10} - x_u$

therefore

$$|\phi_{10} - x_u| \leq |\phi_{11}| \quad (11)$$

equation (2) is the same as follows

$$|\phi_{10} - x_u| \leq |\phi_{11}|$$

and

$$x_u - \phi_{10} \leq |\phi_{11}|$$

thus

$$\phi_{10} - |\phi_{11}| \leq x_u \leq \phi_{10} + |\phi_{11}|$$

because

$$|x_u| < 1 \text{ or } -1 < u < 1,$$

thus

$$-1 < \phi_{10} - |\phi_{11}| \leq x_u \leq \phi_{10} + |\phi_{11}| < 1$$

or

$$\phi_{10} - |\phi_{11}| > -1$$

and

$$\phi_{10} - |\phi_{11}| < 1$$

Both conditions combined, so that the STAR (1;1) model will be stationary with (ϕ_{10}, ϕ_{11}) parameter meets

$$|\phi_{10}| + |\phi_{11}| < 1 \quad (12)$$

Table Petroleum Data

	S1	S3		S1	S3		S1	S3
1.	635	737	**21.**	262	506	**41.**	527	486
2.	534	672	**22.**	333	454	**42.**	478	426
3.	598	717	**23.**	288	405	**43.**	496	462
4.	541	673	**24.**	321	567	**44.**	610	513
5.	489	890	**25.**	539	518	**45.**	620	457
6.	455	751	**26.**	446	395	**46.**	657	476
7.	503	793	**27.**	374	361	**47.**	573	432
8.	573	662	**28.**	420	395	**48.**	611	337
9.	571	518	**29.**	501	353	**49.**	528	383
10.	534	606	**30.**	520	318	**50.**	484	398
11.	414	557	**31.**	512	352	**51.**	606	387
12.	575	551	**32.**	471	342	**52.**	586	403
13.	635	518	**33.**	464	327	**53.**	520	421
14.	443	694	**34.**	505	328	**54.**	568	330
15.	445	654	**35.**	577	376	**55.**	592	347
16.	448	504	**36.**	428	306	**56.**	610	402
17.	450	629	**37.**	404	332	**57.**	646	409
18.	468	508	**38.**	392	386	**58.**	702	432
19.	501	380	**39.**	494	503	**59.**	714	388
20.	443	444	**40.**	533	542	**60.**	849	306

> Summary

S1	S3
Min.:262.0	Min.:306.0
1st Qu.:449.5	1st Qu.:382.2
Median:516.0	Median:438.0
Mean:516.9	Mean:478.6
3rd Qu.:579.2	3rd Qu.:544.2
Max.:849.0	Max.:890.0

> Cor

	S1	**S3**
S1	1.00000000	-0.06468568
S3	-0.06468568	1.00000000

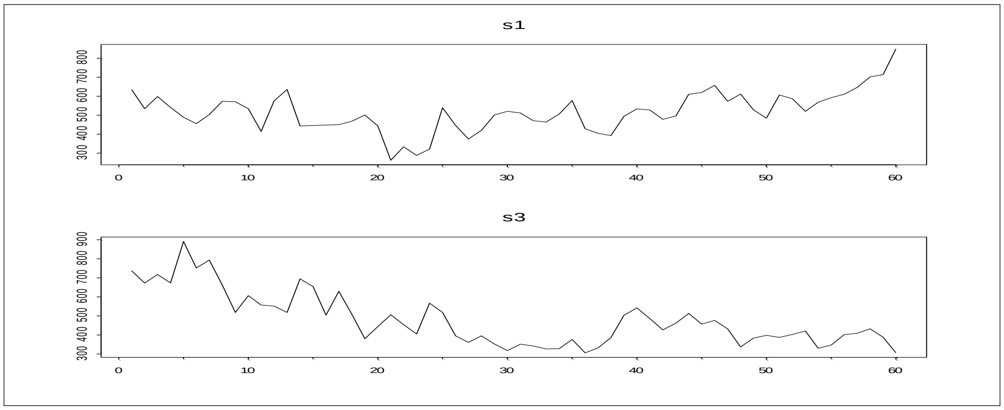

Image 1: graphs graphs of Petroleum Wells Production data

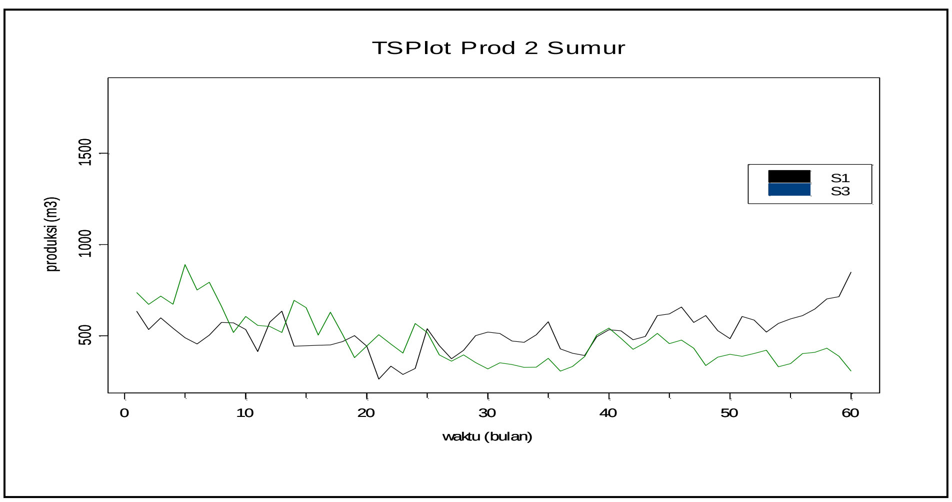

Image 2: Combined graphs of 2 Petroleum Wells Production data

The stationary requirement for STAR(1;1) model is fulfilled, because STAR (1;1) model parameter estimation which is ϕ_{01} and ϕ_{11} values as follows:

ϕ_{01} = 0,91697

ϕ_{11} = 0,07295

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Implying stationary requirement is fulfilled as follows

$$|\phi_{01}| + |\phi_{11}| < 1$$

This value indicates that STAR (1;1) model on petroleum production data is stationary. Those values obtained by using Microsoft Excel.
STAR model (*Time Series*) (Excel 2003)
matrix phi

phi 01 = 0,91697
Phi11 = 0,072946

Stationary: 0,989916

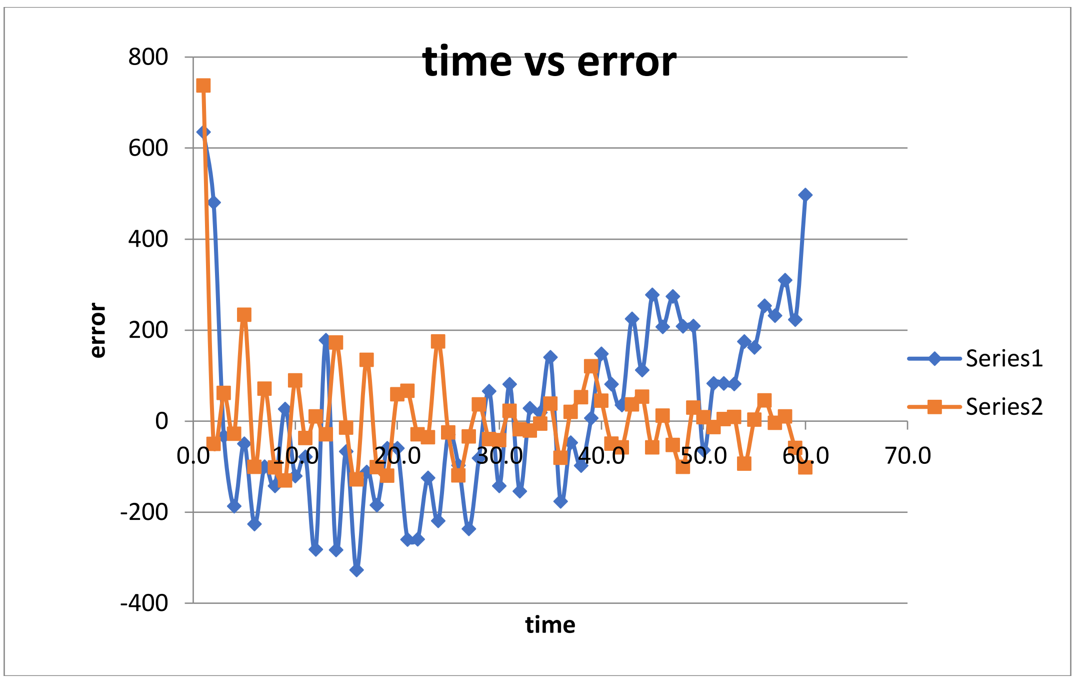

Image 3: Error graph of 2 petroleum wells

4. Conclusion

STAR(1;1) model on two petroleum wells data is stationary and meets the requirement.

$$|\phi_{01}| + |\phi_{11}| < 1.$$

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