A variational approach to denoising problem

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Date and location of PhD thesis defense: 29 June 2016, Tula State University, Tula city, Russia.

Abstract

In many problems of image processing, digital images are always offered in the form of mathematical function. We call this function as a brightness or an intensity function. Such functions can be considered in the continuous or discrete forms. A digital image can be created by different digital devices, such as digital cameras, X-ray scanners, etc. In practice, such devices can give unexpected defects, for example, noise.

One of important problems of image processing is the denoising problem [1-3]: it needs to reconstruct the original version of a corrupted initial image given. This problem has many applications in many fields. The main goal of reconstructing is an image quality improvement based on image processing by specially developed denoising algorithms.

Many different types of noises can arise during the process of image formation in the form of signal-dependent or signal-independent noises [4]. The signal-dependent noise can be usually approximated by Poisson distribution (we call it Poisson noise). The signal-independent noise can be approximated by Gaussian distribution (we call it Gaussian noise). The Gaussian noise appears in almost types of digital images, and the Poisson noise, for example, in X-ray images. The problem of removing these noises is actual today.

When we remove noise in digital images, we usually prefer the Gaussian noise and omit other types of noise. Many methods have been developed in order to remove the Gaussian noise, such as: median filter [5], Wiener filter [6], ROF-model [7], etc. However, the Poisson noise is important too. Its variance is not the constant, but it depends on the intensity value of pixels. Many methods have been developed in order to remove this noise too, such as non-local PCA [8], ICA domain [9], Anscombe transform [10], modified ROF-model [11], etc.

Real noises can be effectively represented by the mixed Poisson-Gaussian noise [12]. This mixed noise usually appears in electronic microscopic images, in aerospace images, etc.

Hence, the problem of the mixed noise removing is the actual problem too. Theoretical models to solve this problem are based on the mixed Poisson-Gaussian noise distribution. In order to remove this noise, there are many methods, such as: multiscale gradient (Benvenuto) [13], alternative minimization (Gil-Rodrigo) [14], PURE-LET [4], generalized Anscombe transform based method [15], etc.

Theoretical models to remove the mixed noise usually are complicated ones. Such complexity is usually caused by multiple parameters of such models. Therefore, the performance of processing procedures is reduced, if the estimations of these parameters are not sufficiently optimal. Therefore, the problem to develop the mixed denoising

model with the small number of parameters to simplify a processing procedure and to increase the processing performance is actual right now.

In this dissertation we solve the problem of the mixed noise removal by using total variation, where the components of the mixed noise are given. Then we have to define the proportion of every component in the mixed noise. We call this noise linear combination of noises. Therefore, we consider the problem of linear combination of Gaussian and Poisson noises removal. Besides, we also consider this problem for the Gaussian noise and Poisson noise separately.

The variational approach is well-known and very promising. This concept was pioneered by Rudin. He proposed the ROF model [7]. The ROF model is used to remove only Gaussian noise. However, another important type like Poisson noise is usually presented in medical X-ray images. In order to remove this noise, Le T. developed so called modified ROF model [11]. Nevertheless, ROF and modified ROF models ineffectively treat this combination. ROF model gives priority to Gaussian noise, but modified ROF model gives it to Poisson noise.

Our goal is to combine ROF model (for Gaussian noise) and modified ROF model (for Poisson noise) to create new model that can treat this combination effectively. Our model will treat this combination with considering proportion of noise between them.

The difference of proposed approach from other known approaches is: the noise that we consider is not superposition, but linear combination of components of noises. This reason allows us obtain the simple model with small number of parameters and also obtain the simple algorithm to guaranty the high quality processing results.
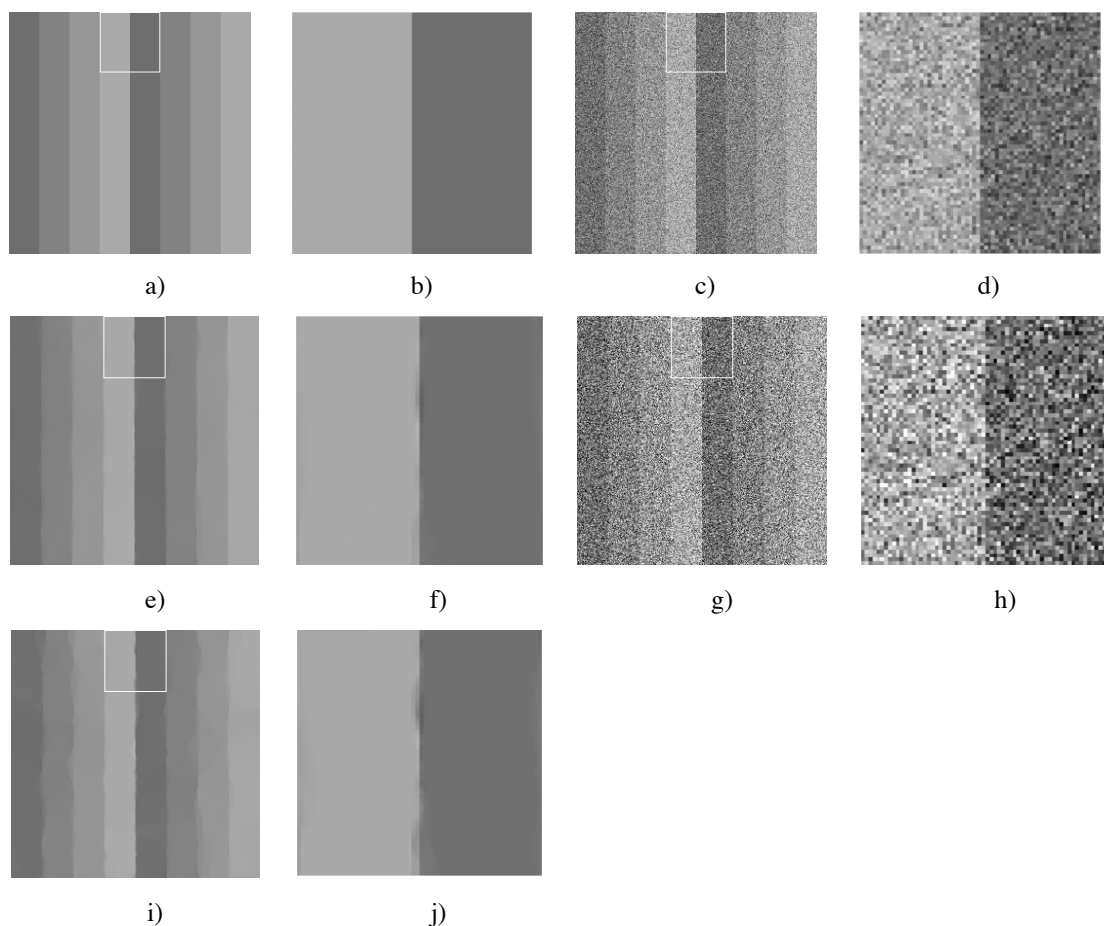
Experiment

Figure 1: Denoising of the artificial image: a)-b) original image, c)-d) noisy image with linear combination of noises, e)-f) denoised image (c), g)-h) noisy image with superposition of noises, i)-j) denoised image (g).

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