01 Jan 1994

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David Newton Richardson
Missouri University of Science and Technology, richardd@mst.edu

Recommended Citation

D. N. Richardson, "Estimation of Resilient Modulus of Fine-Grained Subgrade Soils," *Proceedings of the 37th Annual Asphalt Conference (1994, Rolla, MO)*, University of Missouri--Rolla, Jan 1994.

ESTIMATION OF RESILIENT MODULUS FOR FINE-GRAINED SUBGRADE SOILS

Presented at the 37th Annual UMR Asphalt Conference
November 9-10, 1994
University of Missouri-Rolla

David Richardson, PE
Area Coordinator for Construction Materials,
Construction Engineering, and Transportation

WHAT IS THE RESILIENT MODULUS?

The resilient modulus is the stiffness of the material under cyclic load conditions, such as the on-off loading from traffic.

HOW IS IT MEASURED?

The resilient modulus of soil or granular base material is nothing more than the elastic modulus that is obtained from cyclic loading of the material specimen while recording load and deformation. Knowing the cross-sectional area of the specimen, load (lbs) is converted to stress (psi). And knowing the length of the specimen (or gage length), deformation (in.) is converted to strain (in./in.). From the plot of stress vs strain, the slope is obtained. This is the resilient (elastic) modulus. The test is run at about 1 cycle per sec.; usually the slopes of the last five load cycle stress-strain plots are averaged to give the resilient modulus (M_R). Because M_R is a function of stress state, the above process is repeated at many different levels of load and confining pressure.

WHY DO WE NEED M_R?

Resilient modulus of soil is the representation of soil support that is required input for such pavement design methods as those from AASHTO and the Asphalt Institute, and for pavement analysis programs such as KENLAYER.

ALTERNATE METHODS OF DETERMINING M_R

As shown above, M_R testing of soil is time-consuming, equipment-intensive, and not operator-friendly. An alternate way, based on estimation of M_R by use of soil index properties, is presented next.

ESTIMATION OF FINE-GRAINED SUBGRADE RESILIENT MODULUS

The resilient modulus of fine-grained soils is a function of soil type, degree of saturation, compacted density, and state of stress within the pavement structure. In Fig. 1 are shown *examples* of four soils of varying consistency or stiffness: very soft, soft, medium, and stiff. As can be seen, the M_R-σ_d curves of each of these four soils has the same general shape and equal line slopes (K_3 and K_4). The parameters that distinguish one soil's consistency from another are the maximum and minimum moduli (boundary conditions on a possible spectrum of stiffness) and K_1, except for the very soft material which has a K_4 of zero.

The purpose of this design guide is to enable the user to:

1. **Establish the M_R-σ_d curve for the soil in question, at the prevalent conditions of compaction and in-service moisture content.**
2. **Determine the appropriate value of σ_d (deviator stress) with which to enter the figure.**
3. **M_R is thus determined by coming up vertically with σ_d, striking the curve, and moving horizontally to read the corresponding M_R value on the y-axis.**
4. **Because subgrade moisture content changes seasonally, and because M_R changes with moisture content, several M_R-σ_d curve positions will have to be established through a design year.**
5. **Finally, the AASHTO method of finding the overall weighted average M_R for the design year (E_{sg}) will be used.**

ESTABLISHING THE M_R-σ_d CURVE POSITION

Input values to describe the four soils are shown in Table 1. Note that E_{sg} is synonymous with M_R for fine-grained subgrade soils.

Table 1.Typical Input Soil Constants for KENLAYER Analysis.

Soil Consistency	K_1 (psi)	K_2 (psi)	K_3	K_4	E_{sg} (max) (psi)	E_{sg} (min) (psi)
very soft	1000	6.2	1110	0	5662	1000
soft	3020	6.2	1110	178	7682	1827
medium	7680	6.2	1110	178	12,342	4716
stiff	12,340	6.2	1110	178	17,002	7605

From Fig. 1 it is seen that E_{sg} is also a function of stress state. Thus, there is an interaction between the stress transmitted to the base and the subgrade, with the modulus of both materials fluctuating with stress state. Elastic layer solution computer programs such as KENLAYER perform numerous iterations to reconcile the base, subbase, and subgrade moduli with stress states. KENLAYER is the PC-based software that comes with the book Pavement Analysis and Design by Huang, available through Prentice-Hall publishers (1). It is not necessary to use an elastic layer solution program - later in this paper a method is presented so that the designer can still calculate resilient modulus by hand-solution. However, the reader should continue to read this section prior to attempting hand-solutions.

Table 1 is based on work by Thompson and Robnett (2). Note that with the exception of very soft soils (K_1 = 1000 psi or less), the slopes of the lines in Fig. 1 are all the same, thus the most significant variable is K_1. If KENLAYER is being used, K_1 is input into KENLAYER; this sets the curve position. KENLAYER computes deviator stress (σ_d); E_{sg} is thus determined by moving along the curve in accordance with the point where σ_d intersects. K_1 can be determined by test (resilient modulus testing of subgrade soil) or by approximation from the following equation:

$$K_1 = 3.63 + 0.1239(P_{CLAY}) + 0.4792(PI) + 0.0031(P_{SILT}) - 0.3361(GI) \quad \textbf{(1)}$$

where: K_1 = resilient modulus of soil at σ_d = 6.2 psi, ksi
P_{CLAY} = material finer than O.002 mm, %
PI = plasticity index
P_{SILT} = material between 0.05 and 0.002 mm, %
GI = group index, "new" method (infinite scale)

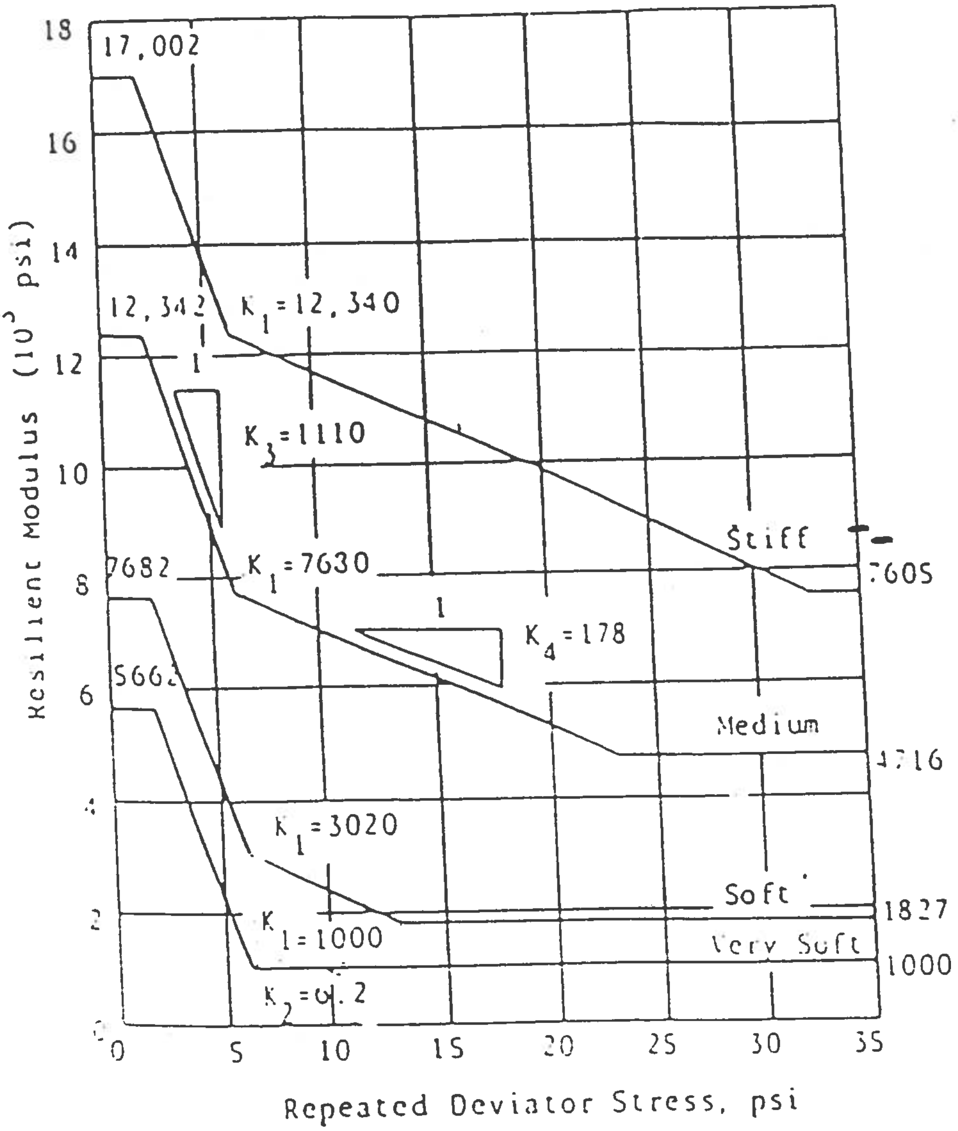


Fig. 1. Resilient Modulus – Deviator Stress Relationship.

GI = $(P_{200}-35)(0.2 + 0.005\ (LL-40)) + 0.01\ (P_{200}-15)(PI-10)$ in accordance with AASHTO M145

P_{200} = material finer than #200 sieve, %

LL = liquid limit

Thus, by performing Atterberg limits and sieve and hydrometer analyses, K_1 can be estimated. Again, K_2, K_3, and K_4 are as shown in Table 1 for any fine-grained soil.

The K_1 equation is based on a dry density equal to 95% standard proctor (T-99) maximum and at optimum moisture content (OMC). For an increase in density to 100%, an increase of about 1.4 ksi is suggested (2). For densities between 95 and 100%, Eq. 2 can be used:

$$Denscor = 1.4 * \left[\frac{(P_{comp} - 95)}{5}\right] \quad \textbf{(2)}$$

where:

Denscor = density correction to K_1, ksi

P_{COMP} = in-service compaction, %

Thus, K_1 is corrected by adding "Denscor" to it. This is only done when increasing density from 95% to 100% T-99 maximum density on the dry side of optimum moisture content (OMC). If the in-service moisture content (MC_{IS}) will be greater than OMC, it is recommended that the use of the Denscor should be omitted. However, judgement should be exercised to keep the K_1 values from becoming unrealistically low.

More significantly, K_1 must be corrected for in-service moisture content. Thus, the in-service moisture content must be estimated. An increase in in-service moisture content above OMC will reduce the K_1 to $K_{1(corr)}$ by adding the moisture correction (Satcorr) as follows:

$$\text{Satcorr} = 0.334\left(\text{Sat}_{\text{OMC}} - \text{Sat}_{\text{SVC}}\right) \quad \textbf{(3)}$$

Where:

Satcorr = correction to K_1 for increase in moisture content above OMC, ksi

Sat_{SVC} = in-service degree of saturation, %, at the in-service dry density(SDD):

$$Sat_{SVC} = \frac{MC_{IS}}{\frac{62.4}{SDD} - \frac{1}{sp.grav.}} \tag{4}$$

Sat_{OMC} = degree of saturation at T-99 OMC, %, at 95% standard proctor maximum dry density (MDD) in pcf.

$$Sat_{OMC} = \frac{OMC}{\frac{62.4}{(0.95)(MDD)} - \frac{1}{sp.grav.}} \tag{5}$$

$$SDD = \frac{CDD}{1 + \frac{\%swell}{100}} \tag{6}$$

$CDD=(P_{comp}/100)(MDD)$

Where CDD is the compacted dry density and moisture is in "%".

Note that "dry density" will be different in Sat_{SVC} and Sat_{OMC} because 1) the subgrade will probably not be compacted at MDD, and 2) the soil may swell if it becomes wetter than OMC over time.

Thus:

$$K_1(corr) = K_1 + \left(1.4\left(P_{COMP} - 95\right)/5\right) + \left(0.334\left(Sat_{OMC} - Sat_{SVC}\right)\right) \tag{7}$$

Use of Satcorr is made for in-service moisture contents above OMC, but on the dry side of OMC the application of the correction is limited down to only, say, 2% moisture content below OMC. Also note that if the in-service degree of saturation increases above the compacted moisture content, Satcorr becomes negative, thus K_1 (corrected) is lower than K_1. Note that correction of K_1 will change its position on the E_{sg} - σ_d plot, as shown in Fig. 2. So, the curve will move down as the season (and subgrade) becomes wetter.

Thus, to calculate Satcorr, the in-service moisture content must be determined. Kersten (3) suggests that in pavement structures the moisture content of clays generally exceeds their plastic limits, silty soils are equal to or just under their plastic limits, and sandy loams are less than their plastic limits. Plastic limits generally are higher than OMC values. A large proportion of all fine-grained soils exhibit in-service moisture contents in excess of their optimum moisture contents. So, a lower bound on estimated in-service moisture contents would be between OMC and plastic limit (PL). An upper bound would be above the plastic limit but less than 100% saturation. **Judgement must be exercised so that the Sat_{svc} is not estimated as being excessively high; this could lead to negative values for K_1 (corr), an impossible situation.** Unfortunately, dry density often changes when moisture content changes, thus the degree of saturation is a function of both factors. Also, for a given moisture content, the greater the compacted dry density, the closer the soil is to 100% saturation. Proximity to a certain moisture content, like the plastic limit, becomes meaningless. One must consider both the moisture content and the dry density when estimating the in-service degree of saturation. Field dry density can be estimated knowing CDD and %swell, as shown in Eq. 6.

A review of the literature indicates that the highest degree of saturation a fine-grained subgrade will attain (disregarding spring thaw conditions) averages about 8% above the saturation at OMC at MDD. See Figs. 1b and 1c.

Thus, by knowing LL, PL, and OMC, K_1 can be corrected (usually downward): If using KENLAYER, K_1(corr) would then be input into KENLAYER. From the pavement

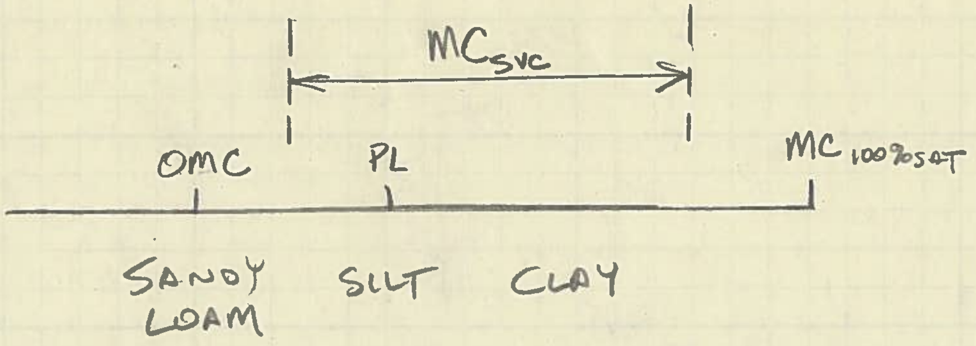


FIG. 1 b: MOISTURE CONTENT SPECTRUM

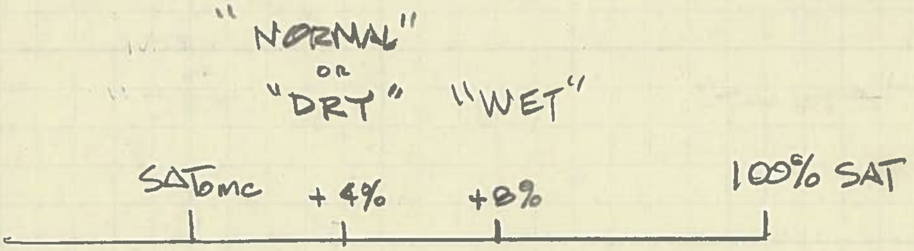


FIG. 1c: DEGREE OF SATURATION SPECTRUM

cross-section and material moduli, KENLAYER will compute the deviator stress in the subgrade, and knowing K_1(corr), it will compute the resilient modulus of the subgrade.

RESILIENT MODULUS BY USE OF ESTIMATED DEVIATOR STRESS

At some point during routine design, the E_{sg} must be determined, but the use of KENLAYER may not be possible or appropriate. In this case, E_{sg} can be estimated by calculation of K_1 (corrected) as shown above, followed by estimation of σ_d. Several options for estimation of σ_d are open. To determine what these options would be, 237 runs of KENLAYER were performed (Richardson et al 1994) which represented pavement cross-sections of a range from 2 in asphalt over 4 in base to 15 in asphalt over 18 in base. The following σ_d values were noted: 2.1 psi minimum, 12.2 psi maximum, and 5.1 psi average. The 5.1 psi average σ_d is less than 6.2 (the "knee" in Fig. 1), thus the E_{sg} is situated on the steep-sloped portion of the curve (Fig. 1) and E_{sg} would be greater than K_1. On the other hand, by use of $\sigma_{d\,max}$ (12.2 psi), E_{sg} would be less than K_1.

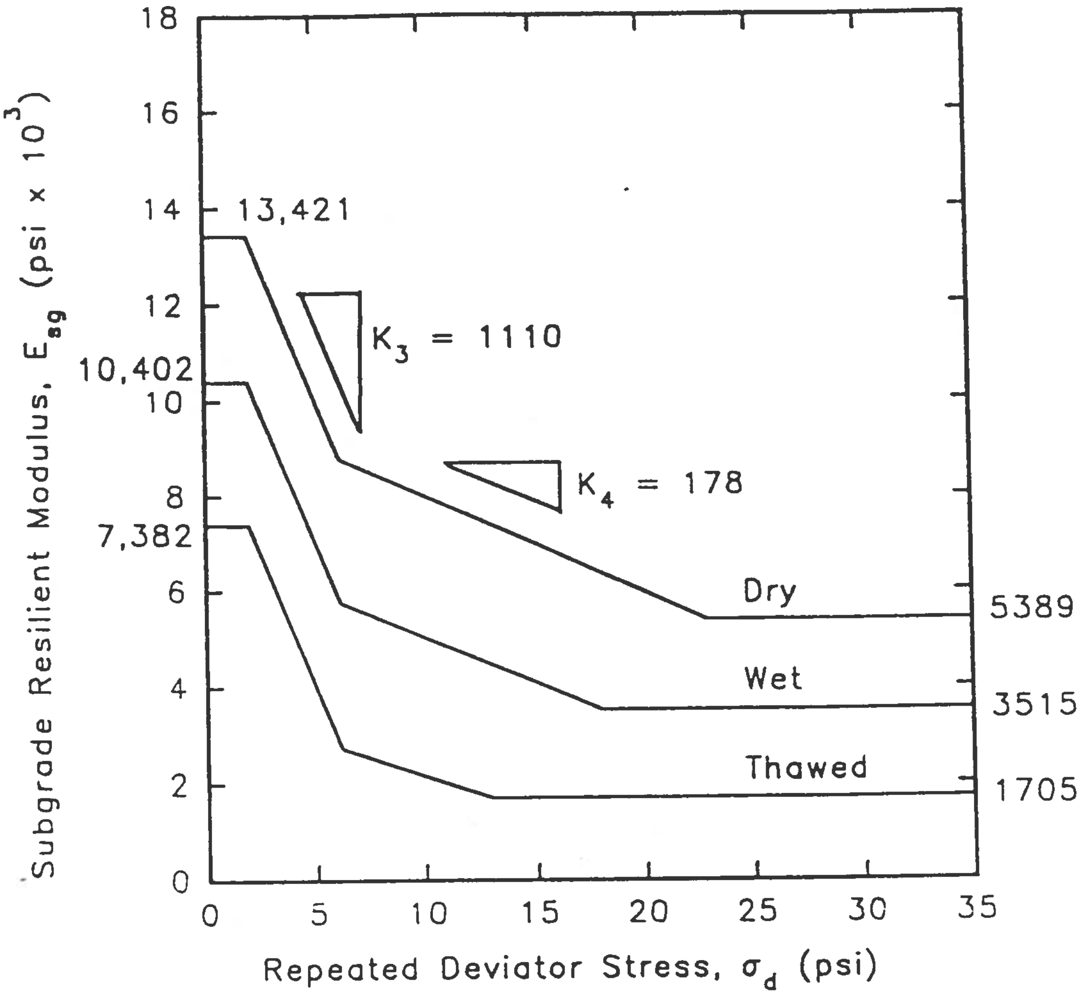


Fig. 2 . Relationship of Road Test Subgrade Resilient Modulus and Deviator Stress for Three States of Moisture Content.

When determining resilient modulus on a given curve, the following equations are useful:

$E_{sg} = K_1(corr) + K_3 (K_2 - \sigma_d)$ when $\sigma_d < K_2$ .... **(8)**

$E_{sg} = K_1(corr) - K_4 (\sigma_d - K_2)$ when $\sigma_d > K_2$ .... **(9)**

where:

$K_2 = \sigma_d$ at the knee of the curve; 6.2 psi is used in KENLAYER

K_3 = upper slope of curves, 1.110 is used in KENLAYER

K_4 = lower slope of curve, 0.178 is used in KENLAYER

Note that E_{sg} (minimum) = K_1 when K_1 = 1.0 ksi or less. The selection of 5.1 psi is the least conservative option, and Elliot (5) suggests it is not even appropriate. Conversely, use of $\sigma_{d\,max}$ = 12.2 psi may be unduly conservative. Thus, it is suggested here that a value of 6.2 psi be used and thus E_{sg} should be set at the K_1 (corrected) value. This would be considered the "normal" condition.

SEASONAL VARIATION OF E_{SG} FOR USE IN THE AASHTO DESIGN METHOD

The 1986 AASHTO Guide (5) recommends that a further correction should be performed to account for seasonal moisture changes, freezing, and length of each season. The final weighted average E_{sg} ("effective resilient modulus") should be the value used in the AASHTO design equation. This modulus is found in the following manner.

In essence, the year is divided into equal periods, say 12 or 24. The E_{sg} is determined for each period, as in the example shown in Fig. 3. These 12 or 24 moduli are then converted to a single "Effective Roadbed Soil Resilient Modulus" in accordance to the procedure given in the AASHTO Guide, as discussed later. Actually, only four values are required to determine the 12 or 24 modulus values: $E_{sg(ns)}$, $E_{sg\,(wet)}$, $E_{sg\,(frozen)}$, and $E_{sg\,(thaw)}$. These are found by first determining the seasonal change in moisture content.

Seasonal Variation of Moisture Content

The first task is to determine the seasonal variation in subgrade moisture contents. Lacking real data, the following is a suggested method for estimation of these moisture levels. First, periods of wet and dry soil conditions can be found by running the program MODAMP. Use of MODAMP is explained in the MODAMP Users Guide originated at UMR (7). An example of the output from MODAMP is shown as Fig. 4 which utilized Columbia, Missouri weather data. Each of the 12 months in a year are described as having a moisture surplus ("1.00") or deficit ("0.00"), as shown on the line "Month Sat". Additionally, the monthly air temperatures are given (line "TEMP C"), which can be useful in estimation of frozen subgrade conditions.

If it is not feasible to use MODAMP, an alternate procedure is available as shown below, which is based on the determination of the local "Climate Condition." A rough estimation of Climate Condition is shown in Table 2. Note that t_{wet} should be subdivided into spring and fall; if no local data are available, divide t_{wet} into two equal intervals.

Table 2. Climate Condition Season Lengths.

Climate Condition	Season (Months)			
	Roadbed Frozen (t_{froz})	Roadbed Thawing (t_{thaw})	Roadbed Wet (t_{wet})	Roadbed Dry (t_{ns})
A	0.0	0.0	2.0	10.0
B	0.0	0.0	5.5	6.5
C	3.0	1.5	1.0	6.5
D	0.5	0.5	1.5	9.5
E	3.0	1.5	2.0	5.5
F	0.5	0.5	5.0	6.0

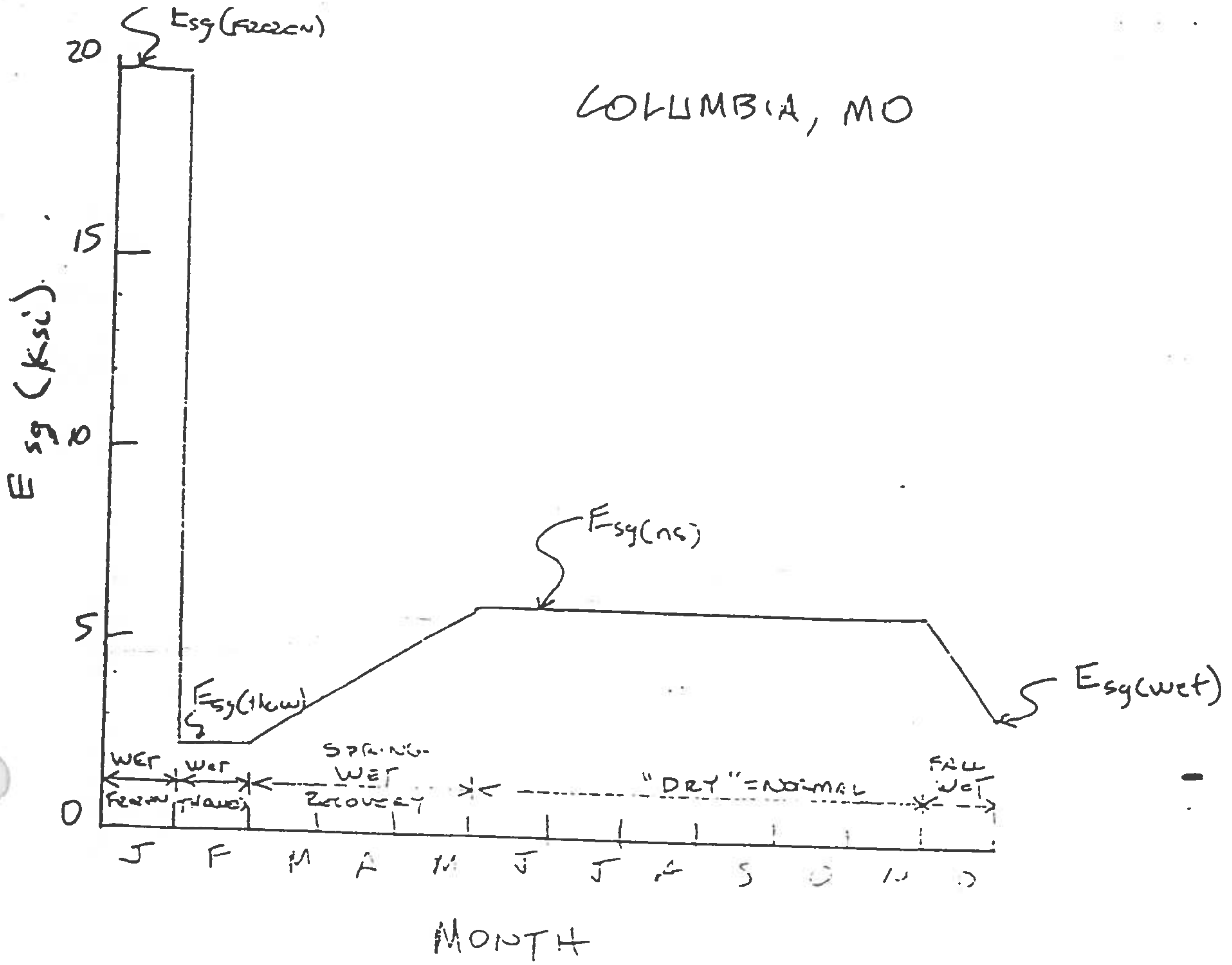
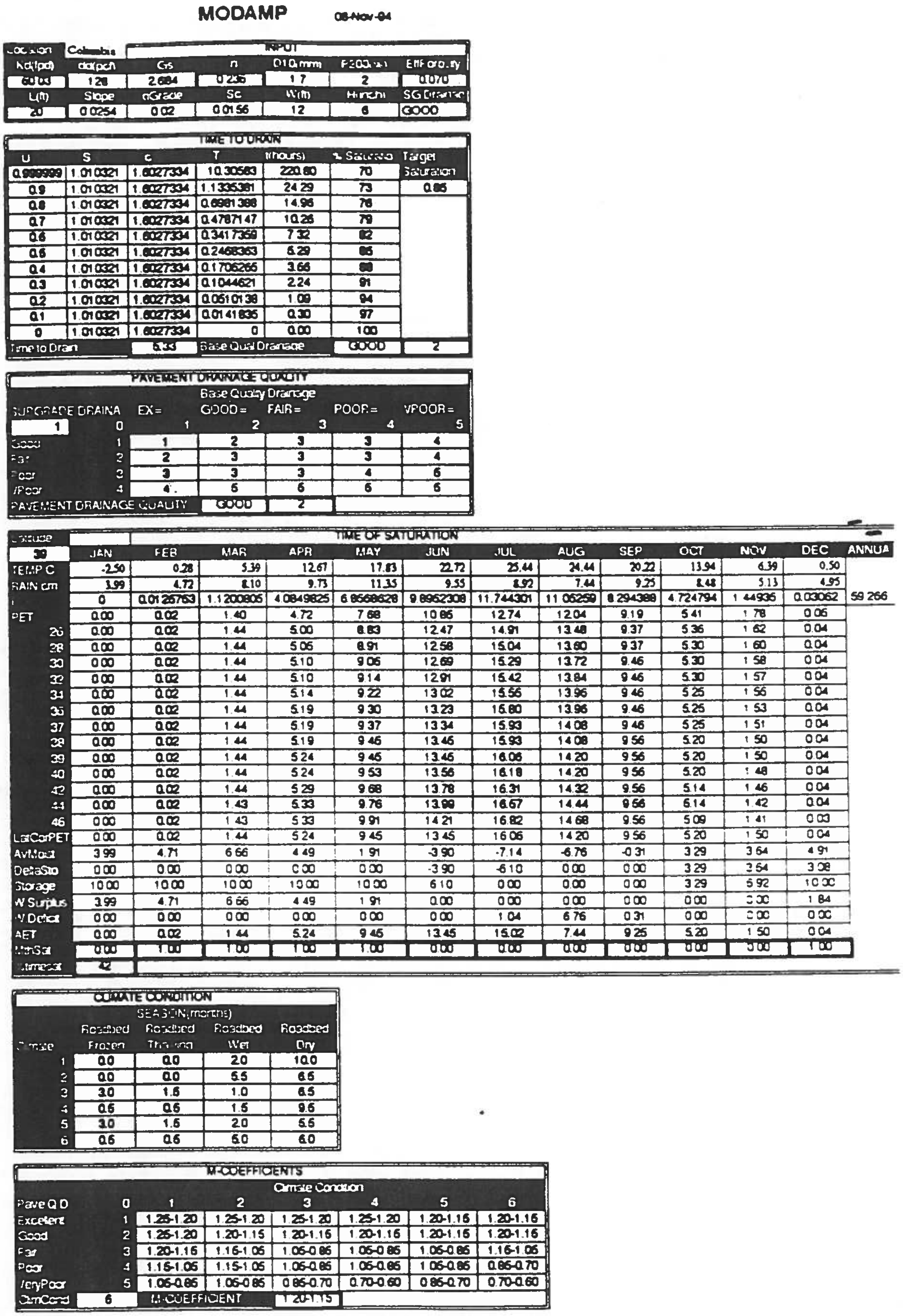


Fig. 3. Variation of Subgrade Resilient Modulus Through the Year.



MODAMP

08-Nov-94

Location	Columbia	INPUT				
Kd(fpd)	dd(pcf)	Gs	n	D10,mm	P200(%)	Eff Porosity
60.03	128	2.684	0.236	1.7	2	0.070
L(ft)	Slope	nGrade	Sc	W(ft)	H(inch)	SG Drainage
20	0.0254	0.02	0.0156	12	6	GOOD

TIME TO DRAIN

U	S	c	T	t(hours)	% Saturation	Target Saturation
0.999999	1.010321	1.6027334	10.30583	220.80	70	
0.9	1.010321	1.6027334	1.1335381	24.29	73	0.85
0.8	1.010321	1.6027334	0.6981388	14.96	76	
0.7	1.010321	1.6027334	0.4787147	10.26	79	
0.6	1.010321	1.6027334	0.3417359	7.32	82	
0.5	1.010321	1.6027334	0.2468363	5.29	85	
0.4	1.010321	1.6027334	0.1706265	3.66	88	
0.3	1.010321	1.6027334	0.1044621	2.24	91	
0.2	1.010321	1.6027334	0.0510138	1.09	94	
0.1	1.010321	1.6027334	0.0141836	0.30	97	
0	1.010321	1.6027334	0	0.00	100	
Time to Drain		6.33	Base Qual Drainage		GOOD	2

PAVEMENT DRAINAGE QUALITY

		Base Quality Drainage				
SUBGRADE DRAINA		EX=	GOOD=	FAIR=	POOR=	VPOOR=
1	0	1	2	3	4	5
Good	1	1	2	3	3	4
Fair	2	2	3	3	3	4
Poor	3	3	3	3	4	6
VPoor	4	4	6	6	6	6
PAVEMENT DRAINAGE QUALITY			GOOD	2		

TIME OF SATURATION

Latitude 39	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	ANNUA
TEMP C	-2.50	0.28	5.39	12.67	17.83	22.72	25.44	24.44	20.22	13.94	6.39	0.50	
RAIN cm	3.99	4.72	8.10	9.73	11.35	9.55	8.92	7.44	9.25	8.48	5.13	4.95	
i	0	0.0126763	1.1200806	4.0849825	6.8568628	9.8962308	11.744301	11.06259	8.294388	4.724794	1.44936	0.03062	59.266
PET	0.00	0.02	1.40	4.72	7.68	10.86	12.74	12.04	9.19	5.41	1.78	0.06	
26	0.00	0.02	1.44	5.00	8.83	12.47	14.91	13.48	9.37	5.36	1.62	0.04	
28	0.00	0.02	1.44	5.06	8.91	12.58	15.04	13.60	9.37	5.30	1.60	0.04	
30	0.00	0.02	1.44	5.10	9.06	12.69	15.29	13.72	9.46	5.30	1.58	0.04	
32	0.00	0.02	1.44	5.10	9.14	12.91	15.42	13.84	9.46	5.30	1.57	0.04	
34	0.00	0.02	1.44	5.14	9.22	13.02	15.56	13.96	9.46	5.25	1.56	0.04	
35	0.00	0.02	1.44	5.19	9.30	13.23	15.80	13.96	9.46	5.25	1.53	0.04	
37	0.00	0.02	1.44	5.19	9.37	13.34	15.93	14.08	9.46	5.25	1.51	0.04	
38	0.00	0.02	1.44	5.19	9.46	13.46	15.93	14.08	9.56	5.20	1.50	0.04	
39	0.00	0.02	1.44	5.24	9.46	13.46	16.06	14.20	9.56	5.20	1.50	0.04	
40	0.00	0.02	1.44	5.24	9.53	13.56	16.18	14.20	9.56	5.20	1.48	0.04	
42	0.00	0.02	1.44	5.29	9.68	13.78	16.31	14.32	9.56	5.14	1.46	0.04	
44	0.00	0.02	1.43	5.33	9.76	13.99	16.67	14.44	9.66	6.14	1.42	0.04	
46	0.00	0.02	1.43	5.33	9.91	14.21	16.82	14.68	9.56	5.09	1.41	0.03	
LatCorPET	0.00	0.02	1.44	5.24	9.45	13.45	16.06	14.20	9.56	5.20	1.50	0.04	
AvMoist	3.99	4.71	6.66	4.49	1.91	-3.90	-7.14	-6.76	-0.31	3.29	3.64	4.91	
DeltaSto	0.00	0.00	0.00	0.00	0.00	-3.90	-6.10	0.00	0.00	3.29	3.64	3.08	
Storage	10.00	10.00	10.00	10.00	10.00	6.10	0.00	0.00	0.00	3.29	6.92	10.00	
W Surplus	3.99	4.71	6.66	4.49	1.91	0.00	0.00	0.00	0.00	0.00	0.00	1.84	
W Deficit	0.00	0.00	0.00	0.00	0.00	0.00	1.04	6.76	0.31	0.00	0.00	0.00	
AET	0.00	0.02	1.44	5.24	9.45	13.45	15.02	7.44	9.25	5.20	1.50	0.04	
MthSat	0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	
timesat	42												

CLIMATE CONDITION

	SEASON(months)			
Climate	Roadbed Frozen	Roadbed Thawing	Roadbed Wet	Roadbed Dry
1	0.0	0.0	2.0	10.0
2	0.0	0.0	5.5	6.5
3	3.0	1.5	1.0	6.5
4	0.5	0.5	1.5	9.5
5	3.0	1.5	2.0	5.5
6	0.5	0.5	5.0	6.0

M-COEFFICIENTS

		Climate Condition					
Pave Q D	0	1	2	3	4	5	6
Excellent	1	1.25-1.20	1.25-1.20	1.25-1.20	1.25-1.20	1.20-1.15	1.20-1.15
Good	2	1.25-1.20	1.20-1.15	1.20-1.15	1.20-1.15	1.20-1.15	1.20-1.15
Fair	3	1.20-1.15	1.15-1.05	1.05-0.85	1.05-0.85	1.05-0.85	1.15-1.05
Poor	4	1.15-1.05	1.15-1.05	1.05-0.85	1.05-0.85	1.05-0.85	0.85-0.70
VeryPoor	5	1.05-0.85	1.05-0.85	0.85-0.70	0.70-0.60	0.85-0.70	0.70-0.60
ClimCond	6	M-COEFFICIENT		1.20-1.15			

Fig. 4. MODAMP Output.

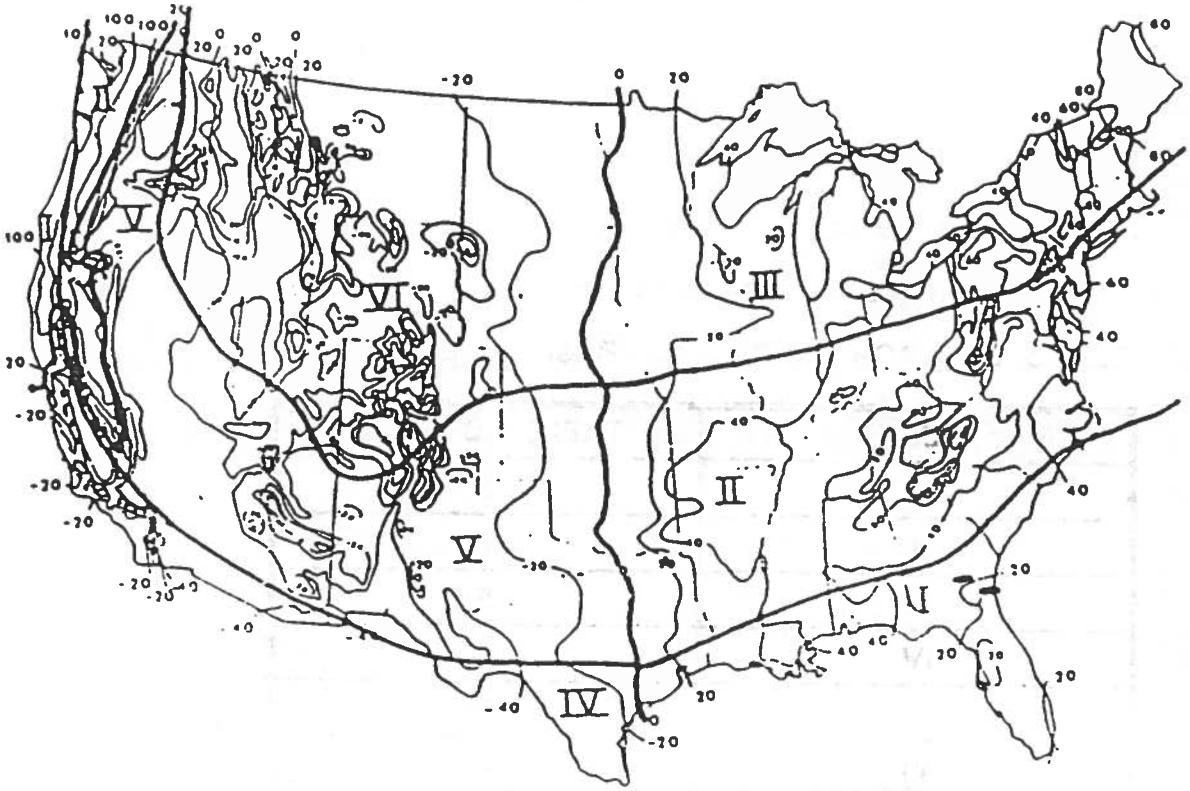
Lacking weather data, the following can be used to determine in which Climate Condition (A through F) the project resides. The AASHTO Guide divides the USA into six zones in regard to climate, as shown in Fig. 5. Examination of weather data indicates that the following relationships shown in Table 3 can be used to convert AASHTO zones to Climate Conditions:

Table 3. Zone ʙ Climate Condition Relationships.

ZONE	TABLE 1 COLUMN
I	B
II	F
III	E
IV	A
V	D
VI	C

Note that even though Zone V is quite dry, it is recommended that the designer consider downgrading the column choice from D to F if freeze/thaw conditions exist at the particular project site.

Going back to our Columbia example (using MODAMP), in Fig. 3 is shown an example of monthly moisture variation. The concept is that in the winter the moisture content remains "constant" because the soil is frozen at whatever moisture content the soil was at when it froze. Technically this is not correct because water will be drawn to the ice lenses, but in terms of subgrade <u>behavior</u>, the modulus will be unaffected because the soil is frozen. During spring thaw, the soil becomes extremely wet because of the melting of ice lenses which renders a much higher moisture condition than would be achieved



REGION	CHARACTERISTICS
I	Wet, no freeze
II	Wet, freeze – thaw cycling
III	Wet, hard-freeze, spring thaw
IV	Dry, no freeze
V	Dry, freeze – thaw cycling
VI	Dry, hard freeze, spring thaw

Fig. 5. Six Climatic Regions in the U.S.A. According to AASHTO.

through normal capillary action. In this example, the thaw period lasts one month. Then the excess water from the melting of ice is slowly removed by drainage during the "wet" months until the equilibrium or normal moisture content is reached. It may be desirable to divide the year into 24 half-month periods to accommodate the values given in Table 2.

$E_{sg(ns)}$ and $E_{sg(wet)}$

Here is where we are headed with this. We want E_{sg} through the year, so we must calculate $K_{1(corr)}$ through the year. According to Eq. 7, to calculate $K_{1(corr)}$, we need to calculate the degree of saturation (Sat_{svc}) as it changes, which is governed by Eq. 4. Looking at Eq. 4, the variable we need to track is MC_{IS}, the in-service moisture content, which is changing with the seasons.

The "normal" moisture content is the moisture content as discussed earlier which is relative to the plastic limit, as per Kersten (see Eqs. 4,8,9). The modulus at this moisture content is termed $E_{sg(ns)}$. The moisture content remains in this condition through the "dry" period until a "wet" period supplies a surplus of water and the soil approaches an upper moisture limit through capillary action, wet weather springs, and so forth. As a guideline to determine the upper limit of in-service soil moisture, use of the following is suggested:

$$MC_{wet} = S_{wet}\left(\frac{62.4}{\gamma_d} - \frac{1}{G_s}\right) \quad \textbf{(10)}$$

where:

S_{wet}= percent saturation

γ_d = in-service dry density, pcf

G_s = specific gravity

Note that the MC_{wet} is higher than the normal in-service content, but lower than the moisture content during spring thaw. If in-situ data is not available for subgrade moisture contents, assume a degree of saturation ($Sat_{wet} = Sat_{svc}$ in this case) of 8% above the saturation point at OMC and 100% MDD.

So, K_1 (corr) is calculated by use of Eq. 7 for two in-service moisture levels: the normal (say, at 4% above saturation at OMC), and near-100% saturated at about 8% above saturation at OMC.

$$SAT_{normal} = \frac{OMC}{\frac{62.4}{MDD} - \frac{1}{G_s}} + 4 \qquad \textbf{(11)}$$

(Note: choosing an excessively high "S" in Eq. 10 may render an MC_{wet} that is so high that upon substitution into Eq. 4, the calculated K_1 (corr) becomes negative. In this case, a lower "S" should be chosen). **Also note that calculation of MC_{wet} is not necessary for calculation of $K_{1(corr)}$, rather, Sat_{wet} is used directly in Eq. 7.** See Fig. 6.

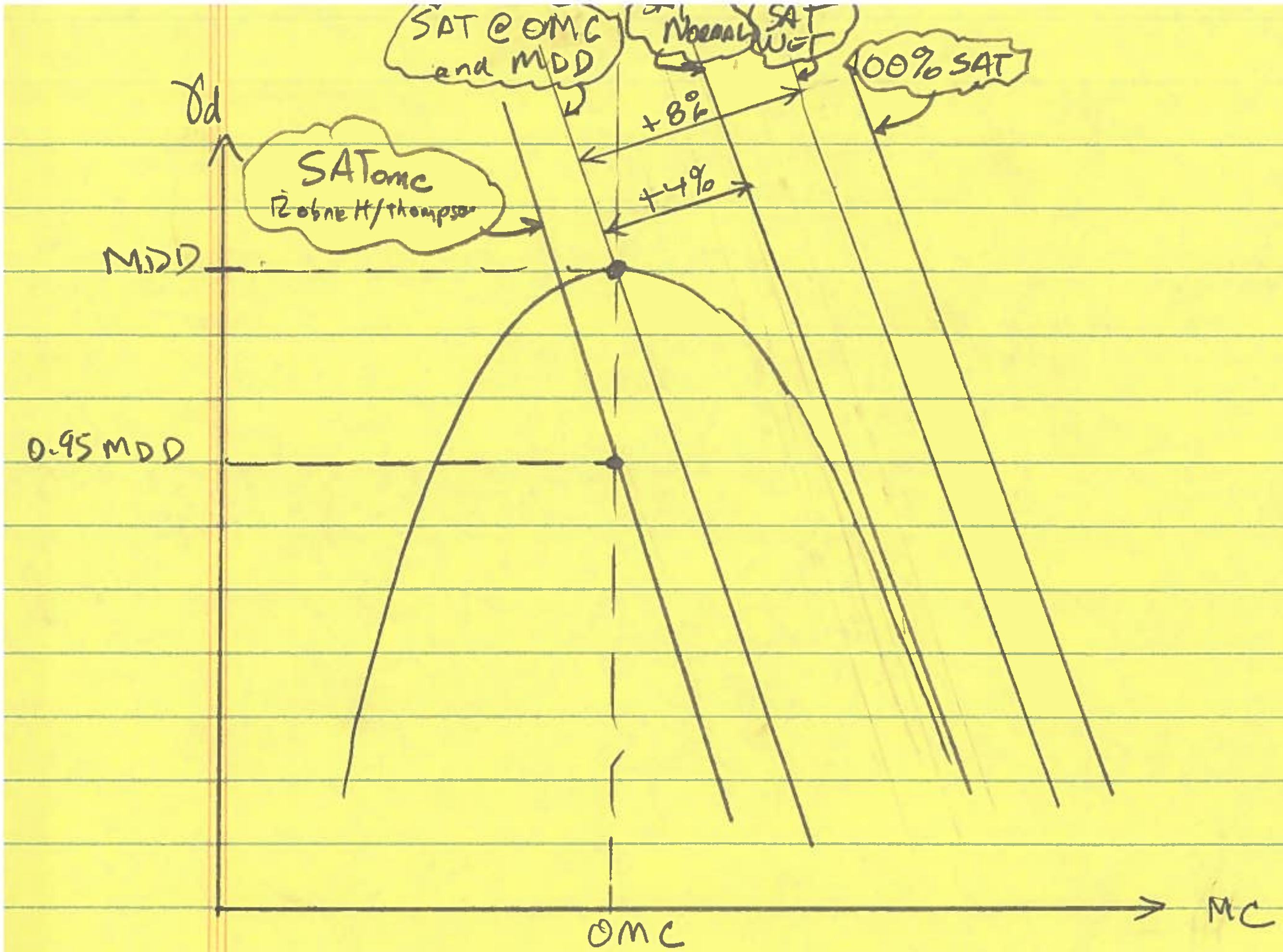
$$SAT_{wet} = \frac{OMC}{\frac{62.4}{MDD} - \frac{1}{G_s}} + 8$$

Next, to obtain the position of the E_{sg}-σ_d curves, values for E_{min} and E_{max} are assigned, relative to the K_1 (corr) values. E_{min} and E_{max} can be calculated by use of Eqns. 8 and 9:

$$E_{max} = K_{1(corr)} + K_3 (K_2 - \sigma_d)$$

$$E_{min} = K_{1(corr)} - K_4 (\sigma_d - K_2)$$

Assuming: $K_3 = 1.110$
$K_4 = 0.178$
$K_2 = 6.2$ psi
σ_d @ $E_{max} = 2$ psi
σ_d @ $E_{min} = q_u$ (unconfined compressive strength, psi) [probably for "normal" condition]



$$SAT_{omc} = \frac{OMC}{\frac{62.4}{0.95\,MDD} - \frac{1}{G_s}}$$

$$SAT @ OMC = \frac{OMC}{\frac{62.4}{MDD} - \frac{1}{G_s}}$$

$$SAT_{normal} = \frac{OMC}{\frac{62.4}{MDD} - \frac{1}{G_s}} + 4$$

$$SAT_{WET} = \frac{OMC}{\frac{62.4}{MDD} - \frac{1}{G_s}} + 8$$

Fig. 6. IN-SERVICE DEGREES OF SATURATION

For the "wet" condition, E_{max} can be calculated as shown above. E_{min} can be estimated graphically by sketching the E_{sg} - σ_d curve in a manner similar to the curves in Fig. 7. The slope of the dashed line is about 0.818 ksi/psi. Then, KENLAYER can be used to compute the σ_d for each of the two sets of [K_1 (corr)/E_{min}/E_{max}] data, and the E_{sg} for each will be output. Or, more simply, $E_{sg(ns)}$ can be estimated as equal to $K_{1(corr)(ns)}$, and $E_{sg(wet)}$ can be estimated as equal to $K_{1(corr)(wet)}$.

In summary, to determine the values in Fig. 3, the following are required: length of seasonal periods of wet, dry, thawed, and frozen conditions (from MODAMP or Table 2), normal in-service moisture content, and the wettest in-service moisture content . Neither the spring thaw moisture content nor the frozen moisture content are required, as discussed next. At this point, on the E_{sg} vs time curve, similar to Fig. 3, you have established the frozen, thawed, wet recovery, dry (normal), and fall-wet intervals on the x-axis. Also, you have established $E_{sg(ns)}$ and $E_{sg(wet)}$. By connecting the points, you would now have the E_{sg}-time curve for the summer and fall (in this example: June through December).

$E_{sg\ (frozen)}$ and $E_{sg\ (thaw)}$

Although a thawed moisture content can be back-calculated from a 100% saturated condition, the actual soil density probably will be lower due to frost heave. Thus, the K_1 value (which is based on in-service density) would be erroneous. Also, the question arises as to how to calculate K_1 under frozen conditions. It turns out that calculation of K_1 is not needed for determination of E_{sg} in the above two cases because the E_{sg} values will be determined in a different manner. Quite simply, the frozen E_{sg} is arbitrarily taken as between 20 and 50 ksi in accordance with the AASHTO Guide, based on actual tests by others. KENLAYER requires input for E_{max}, E_{min}, and K_1. If we assume E_{max} = 20 ksi, then by use of Eqns. 8 and 9:

$$E_{max} = K_{1(corr)} + K_3\ (K_2 - \sigma_d)$$

$$20 = K_{1(corr)} + 1.110\ (6.2 - 2)$$
$$K_{1(corr)} = 15.338 \text{ ksi}$$

then, $E_{min} = K_{1(corr)} - K_4 (\sigma_d - K_2) = 15.338 = 0.178 (q_u - 6.2)$
To obtain q_u in the frozen state, extrapolate as with $q_{u,wet}$ (see Fig. 7).

The thawed E_{sg} can be taken as a percent of the $E_{sg(ns)}$ in accordance with Witczak (8). As a guideline, Witczak suggests that the percent retained (r_t) modulus under thawed conditions is a function of soil type and climate. Suggested values are shown in Table 4. Interpolation will be necessary for other climate-E_{sg} combinations, as shown in Fig. 8. Thus, $E_{sg(thaw)} = (r_t)(E)_{ns}$. In Table 4, MMAT is equal to the Mean Monthly Air Temperature from local/regional weather station data. KENLAYER requires input for E_{max}, E_{min}, and $K_{1(corrected)}$. Again, by use of Eqns. 8 and 9:

$$E_{max} = K_{1(corr)} + K_3 (K_2 - \sigma_d)$$
$$=(r_t)(K_{1(corr)ns}) + 1.110\ (6.2 - 2)$$

and $$E_{min} = K_{1(corr)} - K_4 (\sigma_d - K_2)$$
$$=(r_t)(K_{1(corr)ns}) - 0.178\ (q_u - 6.2)$$

Again, in the absence of q_u data in the thawed state, refer to Fig. 7.

Now that $E_{sg\ (frozen)}$, $E_{sg\ (ns)}$, $E_{sg\ (wet)}$, and $E_{sg\ (thaw)}$ are known, the rest of the E_{sg} values through the year are found by interpolation along the sloped lines, as shown in Fig. 3.

Table 4. Suggested Values of Percent of Retained Resilient Modulus During Periods of Spring Thaw.

MMAT (F)	$E_{sg(ns)}$ (psi)	r_t (%)
	4500	20
45	12,000	50
	22,000	70
	4500	30
60	12,000	60
	22,500	80

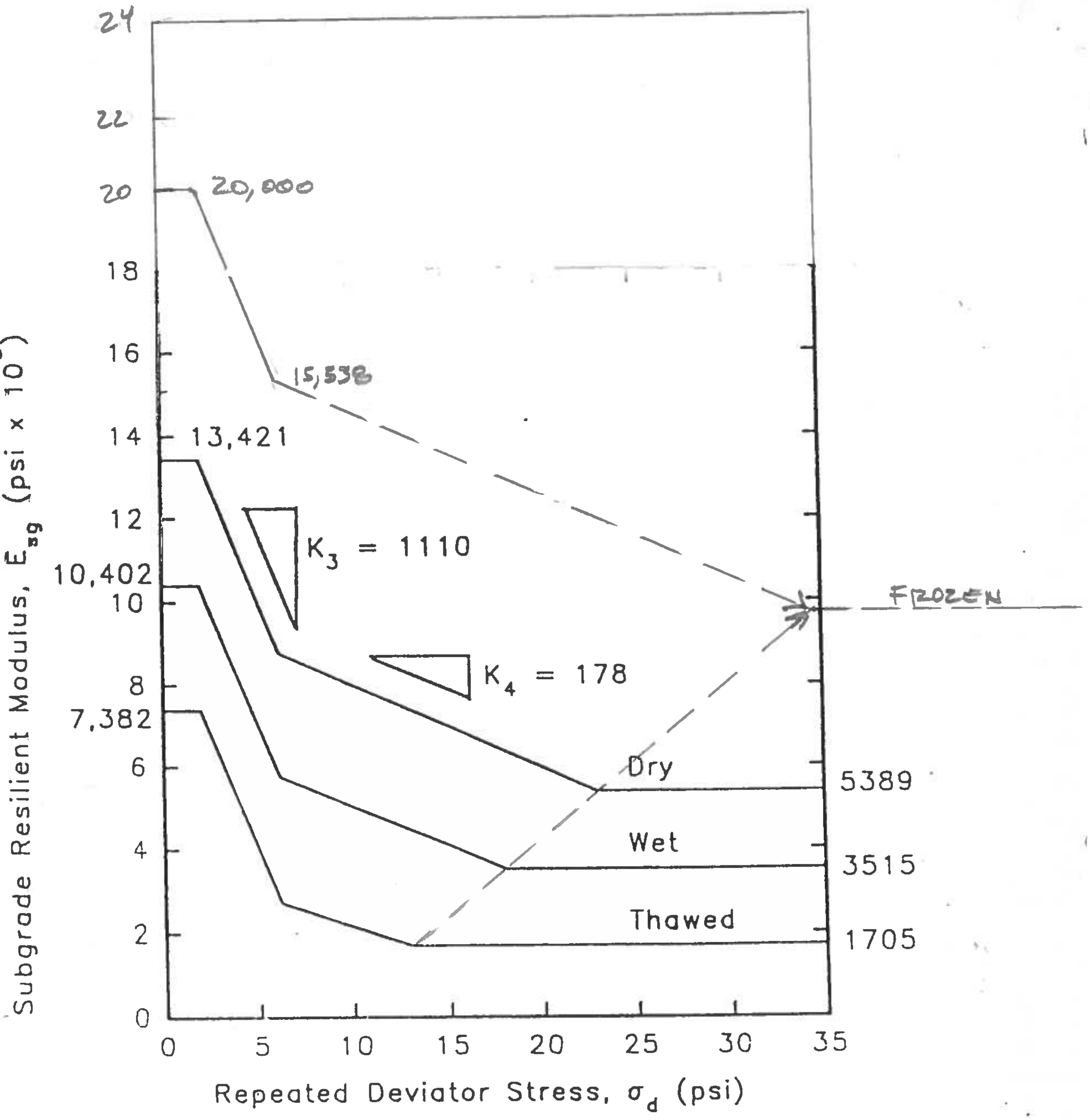


Fig. 7 . Relationship of Road Test Subgrade Resilient Modulus and Deviator Stress for Three States of Moisture Content.

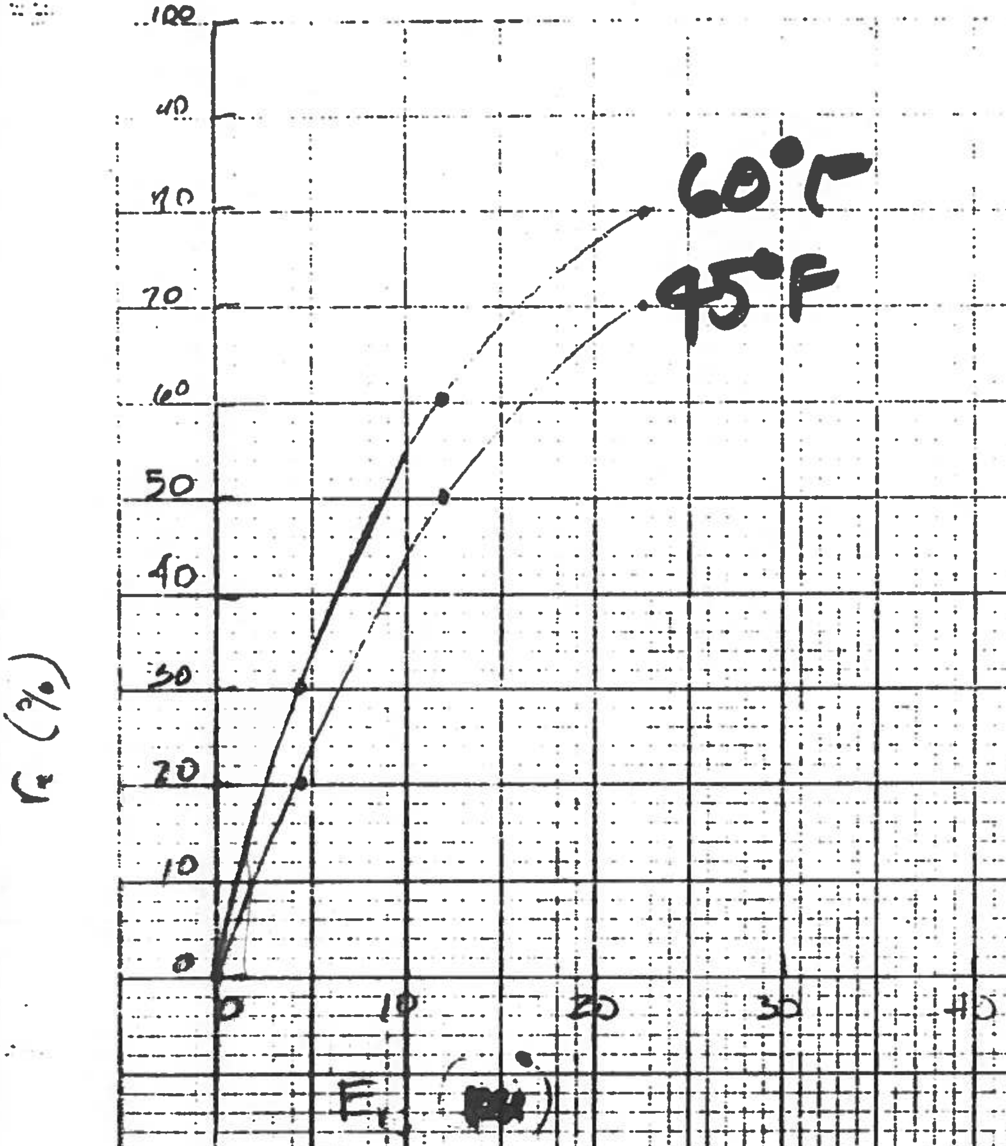


FIG. 8. THAWED CORRECTION FACTOR.

EFFECTIVE ROADBED MODULUS

Thus, E_{sg} is found for each period of the year as moisture and temperature vary. The final step in the calculation of an overall weighted average is to follow the method presented in the AASHTO Guide. Essentially, the effective (weighted average) modulus is tied to the damage that occurs in the pavement when a given E_{sg} is in effect. For instance, in the spring when E_{sg} drops significantly, a large amount of damage occurs compared to the rest of the year. Thus, this low E_{sg} should be given a large weight in the overall average E_{sg}. The way that weights are assigned to each seasonal E_{sg} is as follows.

$$u_f = (1.18x10^{8})(E_{sg})^{-2.32} \tag{13}$$

A serviceability damage factor (u_f) is calculated for each E_{sg}:

The results are tabulated in Table 5. The average for the 12 or 24 u_f values for the year is calculated: $\overline{u}_f$. Using this average u_f in Eq.13, the weighted average E_{sg} is back calculated $\left(\overline{E}_{sg} = \left(1.18x10^{8}/\overline{u}_f\right)^{1/2.32}\right)$. The E_{sg} is termed the "effective roadbed subgrade modulus" and is the input value necessary in the AASHTO design equation.

The u_f equation is based on serviceability criterion. The design should also be checked based on subgrade vertical compressive strain criteria(9). Calculate the damage factor $u_{rs,i}$ for each seasonal interval:

$$u_{rs,i} = 4.022x10^{7}(E_{sg})^{-1.962} \tag{14}$$

Then calculate the design E_{sg}

$$\overline{E_{sg}} = \frac{\Sigma(E_{sg}\ x\ u_{rs,i})}{\Sigma u_{rs,i}} \tag{15}$$

Use the lower of the two design values, E_{sg}

Table 5. Determination of Effective Roadbed Modulus

Period	Esg	Uf	Urs	Esg x Urs
1	15538	0.0223	0.2404	3735.3
2	15538	0.0223	0.2404	3735.3
3	2265	1.9414	10.515	23816
4	2265	1.9414	10.515	23816
5	2936	1.0634	6.3198	18555
6	3607	0.6596	4.2201	15222
7	4278	0.444	3.0196	12918
8	4949	0.3167	2.2688	11228
9	5620	0.2358	1.7679	9935.6
10	6291	0.1815	1.4169	8914
11	6291	0.1815	1.4169	8914
12	6291	0.1815	1.4169	8914
13	6291	0.1815	1.4169	8914
14	6291	0.1815	1.4169	8914
15	6291	0.1815	1.4169	8914
16	6291	0.1815	1.4169	8914
17	6291	0.1815	1.4169	8914
18	6291	0.1815	1.4169	8914
19	6291	0.1815	1.4169	8914
20	6291	0.1815	1.4169	8914
21	6291	0.1815	1.4169	8914
22	6291	0.1815	1.4169	8914
23	5020	0.3064	2.2063	11076
24	3748	0.6035	3.9142	14671
	avg Uf=	0.4132	63.647	264589
	Eeffsg	4413	Eeffsg	4157

Effm.xls

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SUMMARY OF STEPS TO FIND RESILIENT MODULUS FOR USE IN THE AASHTO PAVEMENT DESIGN NOMAGRAPH

(NON-KENLAYER METHOD)

STEP	ACTION
1	$GI = (P_{200} - 35)(0.2 + 0.005(LL - 40)) + 0.01(P_{200} - 15)(PI - 10)$ $K_1 = 3.63 + 0.1239(P_{clay}) + 0.4792(PI) + 0.0031(P_{silt}) - 0.3361(GI)$ in ksi
2	$K_1(corr) = K_1 + (1.4(P_{comp} - 95)/5) + (0.334(Sat_{omc} - Sat_{svc}))$ in ksi $Sat_{omc} = \dfrac{OMC}{\dfrac{62.4}{(0.95)(MDD)} - \dfrac{1}{sp.grav.}}$ Sat in % DD in pcf If field data is available: $Sat_{svc} = \dfrac{MC_{IS}}{\dfrac{62.4}{SDD} - \dfrac{1}{Sp.grav.}}$ SDD=CDD/[1+(%swell/100)] *MDD G* If field data is not available: $SAT_{svc} = \dfrac{OMC}{\dfrac{62.4}{MDD} - \dfrac{1}{G_s}} + 4 \text{ or } 8$ If necessary, $OMC = 1.83(PL)^{0.5}(\ln PL) - 7.1$ in % $MDD = 177.9 - 20.45(OMC)^{0.438}$ in pcf
3	Estimate $\sigma_d = 6.2$ psi, $E_{sg} = K_1$ (corr)
4a	$E_{sg(ns)} = K_1(corr)$ where Sat_{svc} is at OMC saturation S+ 4%
4b	$E_{sg(wet)} = K_1(corr)$ where Sat_{svc} is at an elevated amount, say, OMC saturation+8% or CBR soaked %MC
4c	$E_{sg\ (froz)} = 15.338$ ksi
4d	$E_{sg\ (thaw)} = (r_t)(E_{sg,ns})$ where $r_t = 0.2$ to 0.8

5	Determine t_{ns}, t_{wet}, t_{froz}, t_{thaw} from MODAMP or AASHTO Table (Table 2 in handout). Divide t_{wet} into spring and fall as per MODAMP.
6	• Choose number of subperiods in a year, n = 12 or 24 • Calculate E_{sg} for each subperiod (knowing $E_{sg(ns)}$, $E_{sg(wet)}$, $E_{sg(froz)}$, $E_{sg(thaw)}$ and t_{ns}, t_{wet}, t_{froz}, t_{thaw}, get slopes of recovery and wetting periods of the year).
7	Calculate u_f corresponding to each E_{sg}: $u_f = (1.18 \times 10^8)(E_{sg})^{-2.32}$
8	Calculate Average $\overline{u}_f = \frac{\Sigma u_f}{n}$
9	$\overline{E_{sg}} = (1.18 X 10^8 / \overline{u}_f)^{1/2.32}$ Calculate average E_{sg}: This is the value to use in the AASHTO nomagraph which solves for Structural Number, SN.
10	Check: $u_{rs,i} = 4.022 x 10^7 (E_{sg})^{-1.962}$ $\overline{E_{sg}} = \frac{\Sigma(E_{sg})(u_{rs,i})}{\Sigma u_{rs,i}}$

As can be seen $\overline{E}_{sg}$ is a function of:

- soil characteristics
- in-service compacted density and degree of saturation
- stress state
- climate (months frozen, thawing, wet-recovery, normal moisture, wetting)
- relative damage caused by variable level of support