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Insights from Actual Day-Ahead Bidding of Hydropower

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ABSTRACT

We analyse bidding behavior in the Nordic electricity market, where producers submit supply schedules for tomorrow's generation in a day-ahead auction. We use the two-stage stochastic mixed-integer linear program of Fleten and Kristoffersen (2007) [9] to generate efficient bids to assist in the analysis. These bids are compared to those submitted by three Nordic reservoir hydropower producers over four two week periods in 2011. Being price takers, the producers maximize their profits by bidding their marginal cost. We find that the hydropower producers often come close to the model-optimal result. However, not all marginal costs are taken into account, possibly leading to overproduction at low prices. Marginal costs seem in some cases to be overestimated at high production levels, leading to underproduction in those situations.

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1. Introduction

This article analyses a set of bids submitted to the electricity spot market by three Norwegian reservoir hydro producers in four two week periods in 2011. Our objective is to provide insight into actual bidding, and our findings indicate that there is scope for improved efficiency. Not all marginal costs are taken into account, possibly leading to overproduction at low prices. Marginal costs seem in some cases to be overestimated at high price levels, leading to planned underproduction in scarcity situations. In summary, we document examples of bidding that deviate from rational benchmarks.

As indicated, our contribution lies in analysing bidding behavior empirically. This has been done for electricity markets before [27, 25, 13]. However, their purpose is to detect abuse of market power, whereas we are looking for sub-optimal behavior for individual producers. On the other hand, normative (optimization) approaches to this problem abound, and here we cite a few. Refs. [6, 2] develop stochastic mixed integer optimization models for production bidding given uncertain prices. We extend [9] who develop piecewise linear bidding curves for Nordic hydropower producers. Also building on [9], [17] formulate the bidding problem as an intraday problem considering bidding into the day-ahead market, whereas the longer-term interday problem is modeled as a Markov decision process managing storage operations over time. Further, [7, 5] optimize the day-ahead bidding for a generic set of power plants in the same hydrological system. Finally, [1] compares a stochastic bidding approach with current best practice in the Nordic market. We refer to [14, 16, 15] for wider reviews on bidding strategies, including thermal generation.

The analysis is of interest to hydropower producers who want to improve their bidding process. However, it is also relevant for regulators and market analysts who are doing market surveillance and day-ahead price

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analysis. Although our data is from one country only, the day-ahead structure is common in many liberalised electricity markets and so our results are of interest in a wider setting, providing an in-depth picture of how day-ahead bids are formed, and thus of the market micro-structure of day-ahead markets.

The outline of the article is as follows. In Sections 2 and 3 we present the premises that flexible hydro producers have to deal with when bidding day-ahead. Section 4 presents the bids and gives the main empirical analysis, structured around the producers' decisions of deciding the bid volumes, setting the price points and deciding what type of bids to use, respectively. In Section 5 we report on results from implementing a stochastic optimization model for bidding, where optimized bids are compared with actual ones. Finally, we conclude in Section 6.

2. Physical Electricity Markets

Power producers have to choose where to sell their physical power. However, besides bilateral agreements with large industrial power consumers, the major marketplace to trade large volumes of power is the day-ahead auction. In total 334 TWh, or 77%, of the electricity produced in the Nordic market was traded through the day-ahead auction in 2012 [20]. The corresponding numbers for EEX are; 321 TWh turnover at the day-ahead market, i.e. about 28% of consumption. At 12:00 noon both retailers and producers submit bids to these two European day-ahead markets for buying or selling electricity for the coming day, that is the next 12–36 hours. The participants can use several combinations of prices and volumes for each hour, thus creating a piecewise linear bid function, in addition to other types of bids.

After receiving all bids, the market operators sets the uniform system and zonal spot prices. At around 12:30–12:45 the prices are made public and a producer will learn how large a volume he is committed to produce for every hour the next day. All power producers and suppliers have balancing responsibility, overseen by the respective transmission system operator in each country. If a producer for some reason cannot or does not want to comply with the committed volume, he must make an adjustment trade. For this purpose the market operators also organizes intraday trading markets.

There still might be outages or other incidents that cause deviations from the day- ahead (and intraday) schedule. Therefore, there are balancing markets, where the system operators accepts bids for upwards or downwards ramping of production. In addition to the balancing markets, system operators also coordinate markets for primary reserves. The extent to which bidding into one of these markets must be planned together with bidding into other markets is an active research question [23, 8,4].

Since producers can not be guaranteed any production in the closer to real time markets, producers looking to sell in an efficient market must necessarily bid much of their power into the day-ahead. The problem faced by the producers, hereby referred to as the bidding problem, thus consists of how much power to offer for tomorrow, at what prices and for which hours through what type of bids. This problem is further complicated by technical requirements and constraints in physical production, variable feed-in fees to the grid owner, as well as start up costs and variable efficiency curves for the generating units.

As a price taker in a competitive market you achieve your optimal outcome by offering your good to marginal cost [12]. However, where thermal power plants can relate their marginal costs to the cost of fuel, hydropower producers get their water for free. For flexible hydro producers, the marginal costs translates into the opportunity cost of not being able to sell power from this water at a later stage [22]. And determining the latter part is far from easy, as value of an additional unit of water in the reservoirs, the marginal water value, depends on more than just future price expectations. It is also dependent on the current reservoir level, local inflow expectations and the size of reservoir compared to it's average inflow and production capacity [24, 19].

Next we outline some factors that affect how the bids are formed in the context of a Norwegian hydropower producer, namely transmission tariffs, license power, marginal water value, efficiency curves of generating units, and start up costs.

3. Internal Premises for Bidding

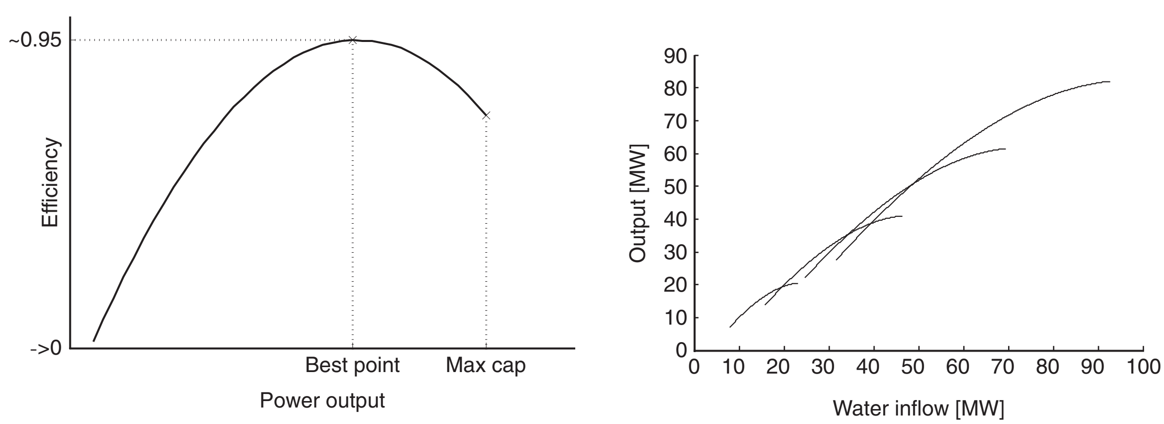
Power producers in Norway pay a fee when delivering power to the electricity grid, from here on referred to as the feed-in fee. This fee consists of a fixed part of 1 EUR/MWh paid to the system operator, Statnett, and a variable part paid to the grid owner. The fixed part of the fee is set for several years at a time to cover costs for Statnett and is equal for all power stations and all hours of the week. The variable part equals the marginal loss rate multiplied with the day-ahead price for every hour. The marginal loss rate is set by the system operator on a weekly basis to account for grid losses. If your production is closer to the power drain, you might in fact improve the grid situation by supplying the grid. Thus the marginal loss rate can be both positive and negative. The marginal loss rate is given as two different values over the span of a week, one for weekdays (07:00–22:00) and one for weekends and nights (22:00–06:00). With regards to bidding, the fixed part should not have any effect, whereas the variable part should be added or subtracted in determining the price points for every hour. Note that even though the rates are given in advance, the price for the next day is still unknown and day-ahead price variations will add uncertainty to the fee payment.

Hydropower producers in Norway are required to deliver up to 15% of the electricity production to the local and county councils and to the state at an estimated price set by the government [21]. This obligation is known as license power. The arrangement ensures that the local community benefits from the economic surplus generated by the electricity production and trade. For 2011 the license power price is set at 13.35 EUR/MWh. For certain producers this license power may be evident in the bidding through buying power at very low prices, simply for the possibility to cover the obligation through purchases at a favourable price level compared to producing it themselves.

The most important parameter when bidding hydropower is the value of an additional unit of water in a reservoir, the marginal water value, or simply the water value. As a fully correct water value calculation is complex, most producers use specialized software to perform an approximate calculation [26]. Some do their own simplified calculations in customized computer programs or as simple functions of the reservoir levels. We do not have exact water values for all three producers, nor will we create our own models for estimating them. However, through deduction it is possible to infer the water values from the bids.

The bidding problem also relates to the efficiency of the power plants and each separate turbine. The efficiency ε at which a turbine runs can be given by its power output, w, in MW divided by the flow of water, q, also in MW: $\eta = w\ (q)/q$ (at nominal water height and flow). Modern turbines and generators are able to convert up to 95% of the kinetic energy to electric power. Both above and below best point the efficiency usually drops a few percentage points, illustrated with the concave curve in Figure 1a. Naturally, producers want to run their turbines at best point for much of the time. However, as the spot price rises so should the producer's willingness to produce above best point. On the other hand, due to high start up costs or minimum flow constraints, a producer might also end up producing below best point.

When bidding for power stations with multiple turbines, producers also have to consider the combined efficiencies of two or more turbines. Depending on the efficiency curves of the turbines, it might be better to run three turbines at a certain point of production than two, or vice versa. Figure 1b shows actual combined output



(a) Efficiency curve for a single turbine related to power output.

(b) Combined efficiency curves for one to four identical turbines.

Figure 1: Efficiency and output curves for one and several combined turbines. The single efficiency curve shows the actual efficiency relative to the power output while the multiple turbine curve shows output relative to the water inflow.

of one to four identical turbines relative to water inflow. At a given flow of water, producers naturally want to get the highest possible power output in return. In this case we see a slight overlap when using one and two turbines, and an increasing overlap when including more turbines. Where the curves intersect, the producer is indifferent to using one more (or one less) turbine. We will use the term efficiency curve for both types of curves shown in Figure 1 as they both are able to display the efficiency of turbines.

If a producer starts a turbine from a stand-still, both direct and indirect costs will occur. The direct costs come in the form of inefficient use of water (for a few seconds) and potential extra use of manpower, while the indirect costs are wear and tear on the equipment and a risk of a failure in the start up procedure. Although very low compared to other power production technologies, start up costs still have to be accounted for when running a hydro plant.

Large hydropower producers have many power stations, some with multiple turbines. After Nord Pool Spot clears the market, producers are given a total production commitment according to the spot price and their submitted bids. Producers then need to plan their allocation of the realized volume commitment across their power plants and turbines in a way that minimizes the total costs. At this point they usually run a deterministic short term optimization, for instance SHOP [11]. A stochastic approach is suggested by [10] and developed further by [3]. Thus, the possibility of post commitment production allocation favours big producers with many power plants. In a sense, it releaves some of the pressure on bidding optimally as, with luck, even bidding somewhat poorly might in the end result in a production scheme where every turbine runs at a highly efficient output level. A further remedy for unfortunate bidding and commitment is the intraday market. Finally, penalties for imbalances in the balancing market, although always positive, may not always be substantial.

4. Bid Analysis

In the following section we present the empirical analysis of the bids. First we give a summary of the bids and other information we have received in 4.1. In 4.2 we analyze the bids based on the underlying factors found in Section 3. Finally, in Section 4.3 we analyse the relative performance of the bidding for each producer, over the whole data period.

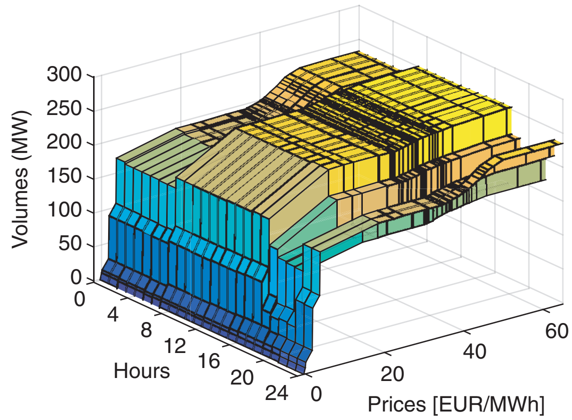
4.1. Presentation of bid data

We have gathered a unique data set from three Norwegian hydropower producers. This includes all variations of bids submitted by the producers to Nord Pool Spot for four two week periods representing four seasons in 2011. Additionally, Producer A in particular has provided us with highly detailed data regarding all their cascades and power stations and thus enabled a more extensive analysis of their bidding. The annual electricity generation is less than 10 TWh for all of the producers, placing them below top 10 among Nordic producers, and below top 3 in Norway, regarding their relative size. Table 1 gives a brief overview of the data we have received from the three producers.

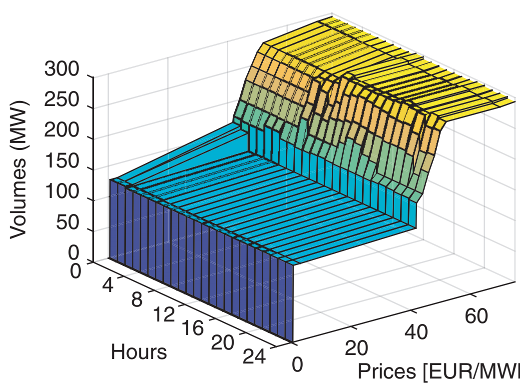
The hourly bid is the most common type of bidding product used by Norwegian hydropower producers, and is submitted to the market operator in matrices. The hourly bid matrix can be quite large, spanning 24 rows and up to 64 columns representing respectively the hours and the price points. Actual bids are presented in cut-outs of full matrices in Section 4.2 to illustrate and support the analyses. However, to exemplify the dimensions, the matrices are plotted along three axes in Figure 2 below. We present three plots of these matrices, one for each company, and each matrix with different characteristics. For Producer A we also include a block bid, i.e. a bid valid for several hours (here 18:00–22:00, for 40 MW), that will be accepted if and only if the average spot price for those hours is above 18.75 EUR/MWh for this case.

Table 1: Overview of the data received from producers A, B and C. A cascade is two or more sequential reservoirs and power stations in the same hydrological system. H = regular hourly bids, B = block bids, L = linked blocks.

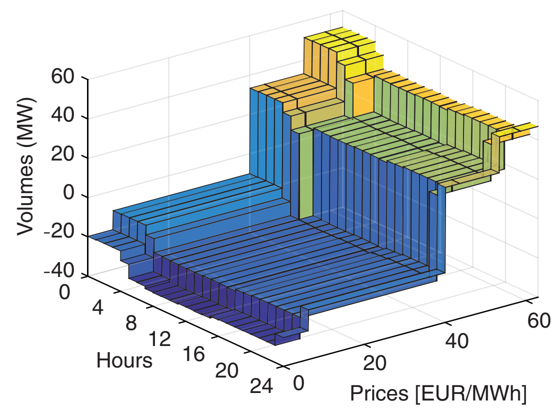
Producer	Scope	Indiv. power station bids	Bid types in use	Water values	Efficiency curves
A	All bids	Yes	H + B + L	Yes	Yes
B	Cascade	No	H + B	No	No
C	All bids	No	H + B	No	No



(a) Bids a winter day for Producer A, including ablock of 40 MW from18:00 to 22:00 at price 18.75 EUR/MWh. The bid uses 37 price points in total. Most of the volume is bid at low price levels, after which the bid flattens out. We also see how the drop in hourly bid volume corresponds with the entrance of the block.



(b) Bids a weekend summerday for Producer B. The bids are quite stable throughout the day. Volumes at the max. and min. prices are constant over the day.



(c) Both demand and supply bids a spring day for Producer C. At 40 EUR/MWh the bids shift from buying to selling. We also see distinct volume shifts at hour 6 and hour 21.

Figure 2: Representative bids for Producer A, B and C, respectively.

4.2. Patterns in the bids

We find that there are basically three decisions that a bidder faces in the day-ahead auction. These are deciding the volumes to bid, setting the exact price points you want to bid in, and finally figuring out what type of bids to use. We analyze the first two of these. Naturally, all these decision are strongly interconnected. A producer would likely never set a price point without having an idea of which volume to connect it to. To clarify the presentation they are still analyzed separately.

4.2.1. Deciding the volumes

The production volumes found in a bid matrix or a block bid are naturally connected to the technical specifications of the turbines a producer controls. Depending on the price level and price expectations, a producer usually wants to run his turbines at the minimum level, at best point, at maximum capacity or somewhere in between the latter two. These levels of production are fixed and do not change unless the producer decides to physically alter the design of the power station. Submitting sensible bids thus consists mainly of setting a limited set of volumes at strategic price points. However, a number of other elements come into play when setting the volume points. These include bilateral contracts, pumping and an assessment of the maximum available capacity.

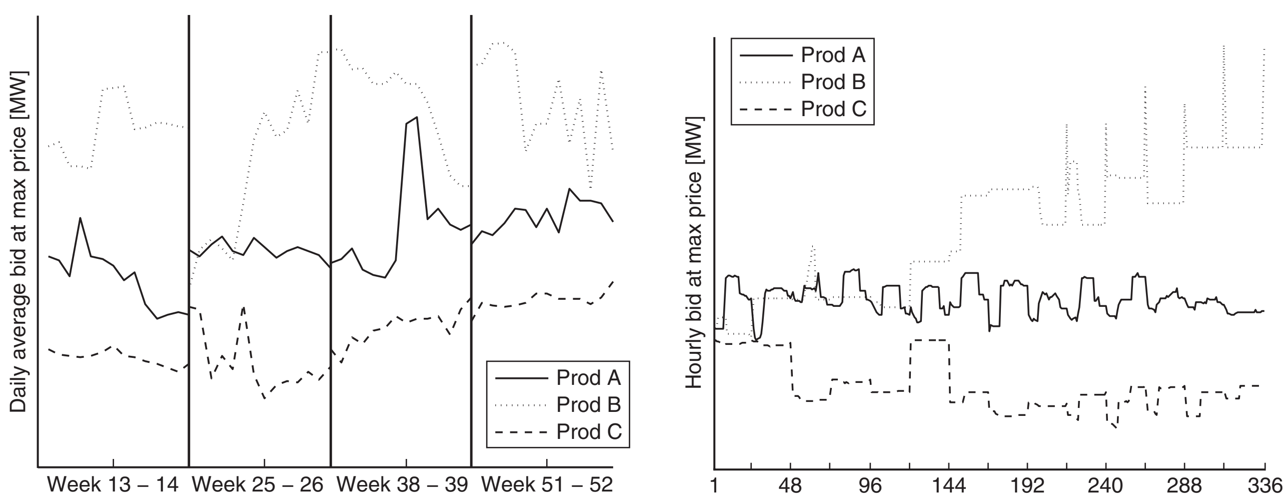
Bilateral agreements include the obligation to deliver concession power, as well as directly to industrial consumers. We can therefore find bids in the bid matrices where producers bid to purchase power at low price levels. Further, purchase bids can also represent bids to run pumping stations. Analysis of these two issues is available upon request.

It might seem natural for a producer to bid the combined technical capacity of all its turbines to Elspot at the highest allowable price point of 2000 EUR/MWh. However, this is not exactly the case. Assuming that the producers want to produce as much power as they can at the maximum price of 2000 EUR/MWh, the maximum production capacity becomes the sum of the hourly bid at maximum price and the submitted block bids in an hour. A surprising finding is that the maximum bid volume varies greatly, even in shorter periods of time such as a week, or even a day. Figure 3 shows variations in maximum capacities, with Figure 3a showing daily average maximum output over the weeks. Variations on hourly intervals for weeks 25–26 are shown in Figure 3b. There are a number of reasons for the observed variation. Producers commit production in other markets besides day-ahead spot as they see it best to reduce risk and maximize profits, selling for example bilaterally to power intensive industry, or setting aside capacity for ancillary services markets. Additionally, hydropower producers in Norway must deliver concession power at at a varying level. In certain periods a producer can experience reservoirs that are empty, or nearly empty, to further disrupt the total output capacity. Maintenance is another reason. Finally, the maximum output also depends on the head of water which varies with the reservoir level. All in all, these factors strongly affect producers' ability to deliver to Elspot. As we can see the deviations for all producers are quite high.

4.2.2. Setting the right price points

Once a producer has established which volumes are sensible to bid, he must figure out the right prices to connect them to. The marginal water value is a typical anchor at which the producers would want to bid their best point volume. However this is necessarily not sufficient. The producers also have to account for the fact the Nord Pool Spot will interpolate their bids between consecutive price points as well as the fact that costs for feeding power onto the grid often are not included in the water values. Additionally, at sufficiently high price levels, producers will want to produce above best point, and thus have to consider the drop in efficiency relative to the increase in price points.

Most often, the spot price will not equal any of the price points chosen by the producer. The exact hourly bid commitment will then be an interpolated value between volumes of the two neighboring price points. Interpolation between bid points can be unfavorable for the producers, because of the risk of committing to produce in an infeasible or inefficient range. Table 2 illustrates how Producer C bids to avoid volume interpolation. The price points in italics show his marginal water values for four separate reservoirs when all marginal costs are accounted for. If the spot



(a) Daily avg. max. cap., all weeks

(b) Hourly max. cap. week 25–26

Figure 3: Maximum production capacity bid to Elspot for producers A, B and C, showing high variations both within fortnightly and daily time perspectives.

price exceeds these marginal water values, Producer C will produce at exactly best point for the respective turbines, unless the spot price happens to land between the 0.125 EUR price gaps, thus interpolating the bids. This is accomplished by setting two price points at the smallest allowed interval apart combined with a sharp increase in volume, leading effectively to almost piecewise constant bidding curves, as opposed to piecewise linear. Notice also how the producer bids best point volume at the technical maximum price, implying that the turbine best point is calibrated to lie at maximum production.

Producers also purposely bid so as to control the interpolation between price points. Bids with noticeable gaps between price points and volume points can represent a linear approximation of the falling efficiency above best point. Table 3 below gives a real example of a bid matrix generated by Producer A for a single power station. The strategy in the bid is to allow for interpolation whilst letting higher price levels balance the loss of efficiency. Figure 4 illustrates this graphically. The efficiency η in Table 3 is relative to best point efficiency set at 1, and have been used to calculate the necessary price levels to compensate for respective efficiency loss of running above best point.

In Table 3 Producer A seems to want a disproportionally high premium to produce at maximum production. This sort of behavior is found in most of the bid matrices for Producer A. This finding complies with evidence for smaller bidders in Texas electricity market [13]. The particular reservoir in this example had a filling level of more than 90%, along with the other reservoirs in the hydrological system. At this storage

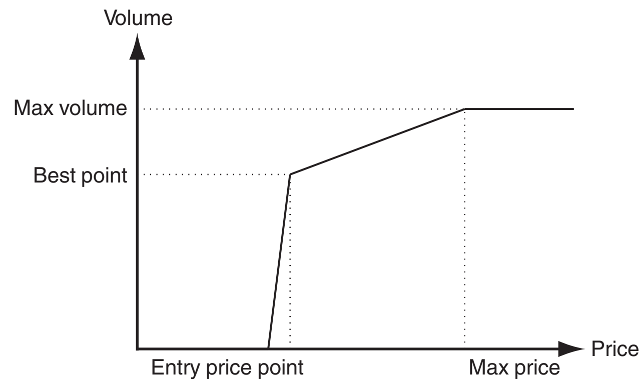


Figure 4: Graphical display of an hourly bid for a single turbine. The bid enters at the best point production level whereas the next price point hits at maximum production. The linearly increasing price between the two points should approximately balance the loss of efficiency from best point to maximum production.

Table 2: Producer C uses neighboring price points to minimize the risk of interpolation of his hourly bids. Top row shows bid prices in EUR/MWh, and table entries are in MW. Positive entries are purchases, and negative are sales. Water values in italics.

Hour\Price	–263	41.875	*42*	46.625	*43.75*	56.25	*56.375*	58.625	*58.75*	2625
10	27.9	27.9	27.9	27.9	–16.6	–16.6	–16.6	–16.6	–34.8	–34.8
11	28.0	28.0	28.0	28.0	–16.5	–16.5	–16.5	–16.5	–34.7	–34.7
12	27.7	27.7	27.7	27.7	–16.8	–16.8	–16.8	–16.8	–35.0	–35.0
13	27.8	27.8	27.8	27.8	–16.7	–16.7	–16.7	–16.7	–34.9	–34.9
14	27.8	27.8	27.8	27.8	–16.7	–16.7	–16.7	–16.7	–34.9	–34.9
15	27.4	27.4	27.4	27.4	–17.1	–17.1	–17.1	–17.1	–35.3	–35.3
16	26.8	26.8	26.8	26.8	–17.7	–17.7	–17.7	–17.7	–35.9	–35.9

Table 3: Producer A setting strategic price points to allow and control interpolation, compensating for loss of efficiency above best point by using higher price points, single turbine. Bid prices in top row are in EUR/MWh, and table entries are in MW, where negative numbers indicate sales.

Hour\Price	–263	32.25	46.38	52.50	2625
1	0	–28	–31	–35	–36
2	0	–28	–31	–35	–36
.	.	.	.	.	.
24	0	–28	–31	–35	–36
Efficiencies relative to best point, η	–	1	0.98	0.97	0.967
Price pts to compensate for η (32.25/η)	–	32.25	32.88	33.25	33.38

level, the turbines could run at full capacity for over 1700 hours without emptying the reservoir. Thus we find this bidding pattern even more peculiar. Even if the spot price should reach 1250 EUR/MWh the producer will be allocated closer to 35 than 36 MWh, missing out on large profits, however improbable. Producer A explains this by their high price expectations regarding the frequency controlled normal-run reserves market, closer to real time. Thus Producer A wants a large premium to dedicate all his production capacity to the spot market.

We find similar bidding patterns for Producer B, yet he charges a lesser premium to run at maximum capacity. For Producer C we find that he does not charge a premium over its best point production, simply because the maximum production level is at, or very close to, the best production for all its turbines (Table 2). This simplifies the bidding process and allows the producer to run at best point more frequently.

The lowest price point at which the producer starts bidding to supply electricity is from here on referred to as the *entry price point*. Below this point the producer will be offering zero supply and perhaps submit demand bids to cover other commitments cheaply. The entry price point should consist of what the producer sees as his marginal cost of production, including the marginal water value, the feed-in fees, etc. Producers sometimes let portions of the water in their reservoirs run through their power stations no matter what the price level; this is rational if the water cannot be stored. The producers set the marginal value of this water to zero. Thus if they can produce electricity from it at a price above their power stations' direct marginal cost, they will. If not, they simply let water run through while disconnecting the generators. The lowest price at which the producers should bid for 'non-storable' production is henceforth known as the *break-even price point*. This should also include all marginal costs except the marginal water value. Any time a producer for some reason employs the break-even price point, this should also be the entry price point, as a producer should never bid to produce below the break-even price point.

Surprisingly, the producers often bid at the break-even price point for water in flexible reservoirs. Our study gives no conclusive answers to why they do this, but in interviews they state it is most often due to the reservoir situation in combination with the weather forecast. Strangely, we find there is plenty of available reservoir storage capacity in some of the periods when the producer bids power at the break-even price point. This implies that they see that a certain amount of water must under all circumstances be released from the reservoir. Yet, they value the remainder of the water in the reservoir at a higher price. Thus they can be seen as having two marginal water values, one at zero and one usually in the range of 10's of EUR/MWh. Table 4 shows an example of Producer A's bids for an individual power station where there seems to be two water values in play. As in Table 3, Producer A charges a high premium for higher volumes. This particular power station is situated at the bottom of a cascade, so there should be no incentive to release water simply to be able to produce further downstream.

Some producers are more exposed to high feed-in fees and have to take the fee into consideration more than others when bidding. In certain areas during the winter season the variable part of the fee can be as high as 20% of the day-ahead price. Others experience the variable part being close to 0% year around. The variable part of the feed-in fee can thus comprise a significant part of the direct marginal cost of production, and have a great impact on the entry- and break-even price points. Hence, we should be able to observe the changing feed-in fee reflected in changing entry price points. However, even though the marginal loss rate used to calculate the feed-in fee is known, it is also

Table 4: Producer A seemingly operating with two marginal water values for a single flexible reservoir, indicated from the use of the break-even price point as well as much higher price points. Producer A bids to let water through at the break-even price point of 3.75 EUR/MWh while bidding to produce more at much higher prices. The high premiums above 12.5 EUR/MWh are in line with Table 3. Top row shows bid prices in EUR/MWh, while table entries are in MW, with negative numbers indicating sale.

Hour\Price	−263	3.625	3.75	19.875	20	28.75	28.875	50	2625
1	0	0	−74	−74	−110	−110	−150	−165	−170
·	0	0	−74	−74	−110	−110	−150	−165	−170
24	0	0	−74	−74	−110	−110	−150	−165	−170

dependent on the unknown price the next day. Producers can therefore at best use their price forecasts for the next day to predict the variable feed-in fee. We use the same price forecast as Producer A had in hand when setting price points for the next day. The marginal loss rates are multiplied with the average price forecast, respectively between 06:00–22:00 and 22:00–06:00, to obtain forecasted variable feed-in fees for daytime and for the night. In this example, the change in the forecasted variable feed-in fee over a day is well reflected in Producer A's bidding for an individual power station, as seen in Table 5 below.

This example illustrates how a producer ideally should match the variable feed-in fee with the entry price point. Yet, this not often the case for the producers in this study. The marginal loss rate can vary significantly from week to week and within any weekday, thus we should see a corresponding change in the entry price points the producers employ in their bid matrices. An example of where the change in the variable feed-in fee is not taken into account is presented below in Tables 6 and 7. In the example, Producer A bids to produce at 0.125 EUR/MWh, below the feed-in fee alone and thus below his break-even point. Neither does he change the entry price point according to change in the marginal loss rate from hour 6 to 7. This will cause a direct loss to the producer if the spot price should land below 2.75 EUR/MWh. Most likely is the feed-in fee not taken well into account in the higher price points either, thus causing a loss taking the marginal water value into account. The bid can therefore be said to be irrational. To improve the bid, the producer can simply bid at lower price points at night and at higher price points during the day according to the changing feed-in fees.

4.2.3. Choosing the bid type

The choice between bidding hourly bids or block bids, with or without links, is perhaps the most complex decision the bidding responsible has to deal with. Linked block bids are block bids that are conditional on another block being accepted, i.e. the linked block bid only becomes valid if the parent block is accepted. The idea of allowing block bidding is to give producers with startup costs and other inflexibilities a predictable production schedule. An advantage stated by our set of producers is being able to set the exact best point production directly at the marginal water value. Simply put, an average spot price above the marginal cost gives the most efficient production, and a price below means no production. Such practise can be seen as rational and is a common way to submit block bids.

A common strategy is to combine hourly bids with block bids, submitting the block bid at best point and the marginal water value, while the hourly bids cover the production from best point to maximum production at higher price points. Often the producers find that spot prices hover around the marginal water value, so that hourly bidding will result in many startups, which is not desirable.

Table 6: Average feed-in fees seen by Producer A in relation to Table 7.

	Day	Night
Approx. marginal loss rate	3%	−3%
Average variable feed-in fee	14.1	−12.9
Average total feed-in fee	22.1	−4.9

Table 7: Bid matrix showing irrational bidding as the entry price point (in italics) is below the feed-in fee alone, shown in Table 6. First row numbers are in EUR/MWh, while table entries are in MW, with negative numbers indicating sales.

Hour	−263	0	0.125	75	87.5	2625
1	0	0	−8	−8	−11	−11
.	0	0	−8	−8	−11	−11
24	0	0	−8	−8	−11	−11

Table 5: Change in variable feed-in fee reflected well in Producer A's entry price points. The average forecasted change in the variable feed-in fee is very near the change in Producer A's use of entry price point.

	Day	Night	Difference
Approx. marginal loss Rate	−5%	20%	25%
Entry price points [EUR/MWh]	35	26.88	8.125
Average variable feed-fee [EUR/MWh]	6.94	−1.64	8.49

Nord Pool Spot gives participants the possibility of submitting up to 100 block bids per day, where the blocks can be any consecutive combination of minimum three hours. The latter implies there are $\sum_{n=1}^{22} n = 253$ possible combinations to choose from. However the producers rather use a few combinations of hours that give practical meaning with regards to peak and off-peak price hours, working shifts and feed-in costs, and stick to simple rules when making block bids. Blocks are usually submitted sequentially in time, with the overlap often taking place at the shift of the marginal loss rate at 06:00, or according to work shifts around 07:00–09:00.

The application of bid types varies quite a lot between the three producers. Table 8 shows the volume share of the two main bidtypes submitted and realized over 8 weeks by the three producers. The producers each submit a sizeable share of block bids, which drop quite a bit once the bids are actually realized in Elspot. Producer B submits 27% of the total volume in block bids, but only 6% are actually realized. Producer C is the most noteworthy in terms of the volume of block bids used, submitting 71% of his capacity in the spot market in blocks. A total of 60% was still realized and produced as block bids. Producer C gives no other reasons for submitting block bids than what we have already mentioned, mainly bidding blocks to avoid starts and stops. However, Producer C's largest power station comprises a sizeable portion of his total capacity and is almost exclusively bid in using blocks.

4.3. Performance analysis

To find the potential increase in income for the producers, we perform an analysis of a case where the producers are able to predict every price peak within each two week period. In other words, we simulate having perfect foresight for the entire period, and producing at maximum bided capacity in the hours with the highest spot prices. Given the limited technical and hydrological data on two of the producers some simplifications were necessary. We assume the producers' capacity to deliver to the spot market is given by the sum of its hourly and block bids submitted for each hour. We aggregate the total volume of flexible electricity produced over each two week period, meaning we have subtracted bid volumes we consider less than fully flexible, i.e. bids at breakeven price points. Thus the remaining volume should be 100% flexible. This energy is then reallocated to the highest priced hours until the total amount is allocated, where we assume we achieve the same average efficiency of the turbines. This is of course a somewhat unfair analysis as it disregards both the start up costs, the reservoir levels and the potentially lower efficiency achieved from always running at maximum capacity. Yet it does paint a picture, and to a certain degree, the analysis can give an indication of the producers' bidding performance. This is displayed in Table 9 below.

Table 8: Volume share of hourly bids and block bids submitted by and realized for Producers A, B and C over 8 weeks.

Producer	A	B	C
Hourly bids submitted	76%	70%	29%
Block bids submitted	24%	30%	71%
Hourly bids realized	87%	94%	40%
Block bids realized	13%	6%	60%

We see that the producers perform quite well, competing against *perfect* price information. The highest potential increase in income over the six periods is 11.1%. On average over the 8 weeks, the potential increase for all producers was 5.4%. Producer A displayed a more stable performance than B and C. For Producer C in week 38–39 there was very low net trade due to low price levels, making this analysis less interesting. We observe that all producers on average performed best in week 13–14. Producers A and B had their weakest performance in week 38–39, and all had their second poorest results in week 25–26. Clearly, there is some correlation between producers' performance. It appears to be easier to perform closer to the upper bound in the spring and winter weeks, than in the summer and autumn weeks. In Table 10 we have calculated the standard deviation of the zonal price relative to the average price in the respective periods.

Table 9: Potential increase in actual realized income in each two week period given complete knowledge of future price levels.

	Week 13–14	Week 25–26	Week 38–39	Week 51–52
Producer A	5.2%	5.8%	9.4%	5.8%
Producer B	1.2%	7.3%	11.1%	3.2%
Producer C	0.4%	6.2%	–	3.7%
Average	5.4%			

Table 10: Standard deviation of zonal price relative to average price in period, showing a certain correlation to the producers potential to increase income shown in Table 9.

	Week 13–14	Week 25–26	Week 38–39	Week 51–52
Producer A	8%	23%	41%	28%
Producer B	7%	22%	39%	15%
Producer C	6%	23%	41%	28%

The price deviations indicate that the higher the volatility in prices, the more difficult it is to bid optimally and take advantage of the high prices in a period.

5. Comparison with Optimization-Based Bids

We implement the two-stage stochastic programming model in [9] for Producer A, in order to compare actual bids with optimized bids. The next subsection explains assumptions used in the model setup, while Section 5.2 discusses the results. See the appendix for details.

5.1. Case description

The system has four reservoirs and five power stations. Inflow is assumed deterministic, while price scenarios were generated based on data on forecast errors, where forecasts were gathered from SKM Market Predictor, a market analysis company. In particular, price scenarios are constructed as a normal distribution around these forecasts with a standard deviation equal to that of the area spot price for the respective hour 40 weekdays or 16 weekend days back in time. An example of 500 generated price scenarios for day-ahead is displayed in Figure 5.

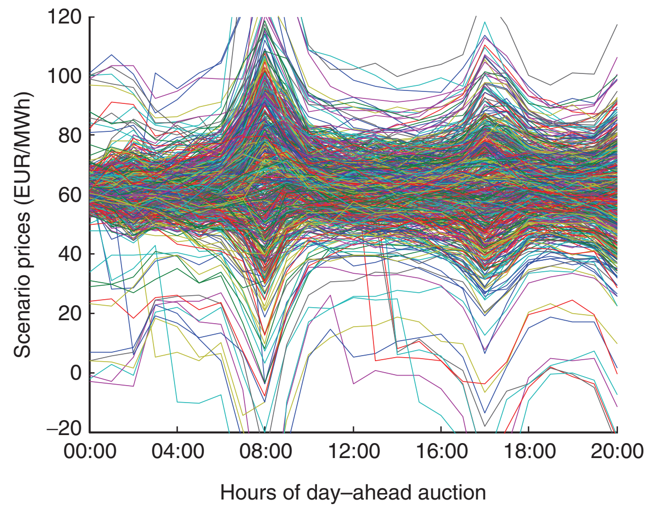


Figure 5: Plot of 500 generated price scenarios for day-ahead NO3 spot prices, 05.04.2011.

The efficiency curves are based on the producer's records of measured water flow versus power output. The need for a linear formulation is taken care of by linearly approximating the efficiency curves for each turbine. The efficiency curve is usually concave within the turbine's operating range. The slope of the line stretching from the origin represents the best point conversion rate. This is displayed in Figure 6. Each turbine is also modeled with a minimum output, which in the figure is where the thick unbroken curve begins.

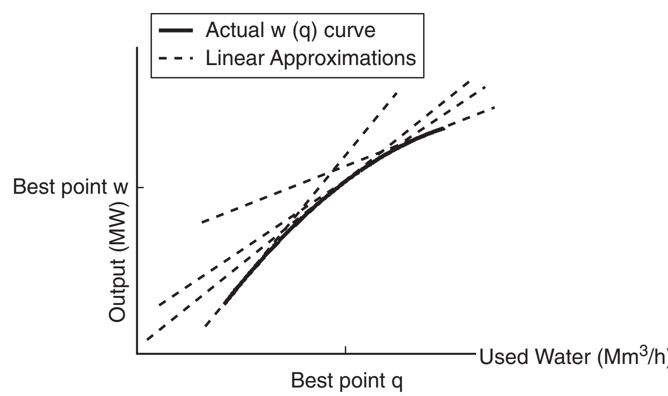


Figure 6: Linear approximations of efficiency curve formulated as conversion from water flow to output power. Minimum and maximum production is indicated by the location of the solid line.

The model is run with 300 scenarios over a two week horizon. We focus on day-ahead bidding for two particular days, 25 and 26 September 2011. In the optimization we bound the end-of-horizon reservoir level to match the realized one, making sure the optimization uses the same amount of water as was used in actual operation. We use only hourly bids and block bids; linked block bids are excluded to make the results more interpretable. For the same reason we exclude participation in intraday, balancing or ancillary services markets.

Feed-in fees are included in order to reflect time-varying transmission costs, and bid price points (parameters set in advance) are set using efficiency curves and actual water values when such are available, besides making sure that bid points are located closely where the price scenarios are dense.

In summary, we set up and solve a two-stage stochastic programming model for the bidding problem of Producer A.

5.2. Implementation and results

The model is implemented in XPRESS 7.2.1 on a PC with 4x3.4GHz i7 processors and 16 GB RAM. Typical solution time is 1000 seconds for 300 scenarios for a problem having a two week horizon.

5.2.1. Actual Bids and Model-Generated Bids for 25 Sep

The aggregated actual power station bids and the model-generated bids are displayed below in Figure 7, respectively. The bids are roughly equal for all hours. The price axes have been cut from 60 EUR/MWh to make the figures readable. Worth noting first and foremost is the difference in entry price points. Whereas the actual bids place the first volume of 14MW already at 1.4 EUR, the model generates its first bid volume of 40MW at 7.20 EUR. The marginal loss rate for the likely power station with a capacity of 15 MW, is 9.4% this weekend. This implies a total feed-in cost of $C^{feed} = \pi \times C^{m1r} + C^{fixed} = 1.4 \times 0.094 + 1.1 = 1.23$ EUR/MWh produced if the spot price turns out to be 1.4 EUR. Most likely this reservoir has a marginal water value above 0.17 EUR/MWh, and thus the bid is irrational. The generated bids are not that easily split up into power station bids, and thus the same analysis can not be done. However through searching the model for a scenario price neighboring 7.20 EUR, we see that the total feed-in costs are 67.04 EUR, implying an average average realized feed-in cost of 1.68 EUR/MWh. This implies a water value of 5.52 EUR/MWh. And with all reservoirs being far from full, and an average price forecast of 24.9 EUR, this too is irrational bidding. However, if a water flow constraint is binding, this sort of instance might occur.

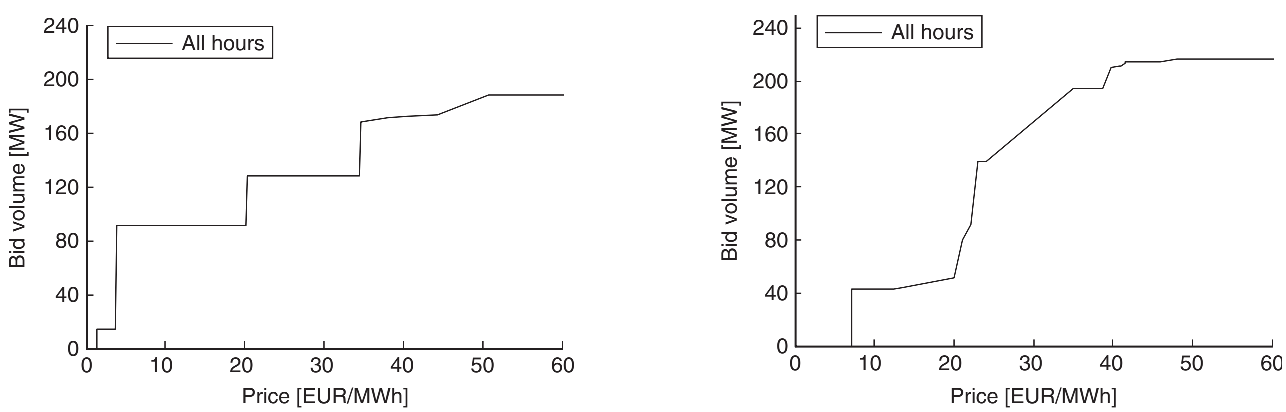
The figures also show that the model generates a higher hourly bid at maximum price. This comes from the fact that in addition to the hourly bids, the producer has also submitted block bids for this day, given in Table 11. The model has not used any block bids, so adding in block volumes, the two bids equal approximately at a maximum volume of 217 MW. However the actual bids only max to 217 MW for the first 9 hours of the day. We can not find any good reasons why the bid volumes should drop to 215 MW for the remainder of the day, and thus see it as irrational bidding behavior.

5.2.2. Out of Sample Comparison

We run the actual bids through the same model instance that generated the comparing bids, but with new price scenarios, to see how they perform under uncertainty. The

Table 11: Actual block bids of 25.09.2011 for the hydropower system in question. Notice how the three block bids do not equal in volume, even though the hourly bids are constant throughout the day. Volumes are in MW and prices are in EUR/MWh.

Start	Stop	Volume	Price	Block
00:00	09:00	−23	33.5	B0009
09:00	20:00	−21	35.5	B0920
20:00	24:00	−21	35.5	B2024



(a) Actual hourly bids, 25.09.2011

(b) Generated hourly bids, 25.09.2011

Figure 7: Bids for Sunday 25.09.2011 from Producer A and the bidding model, respectively. The price axes have been cut from 2000 EUR/MWh to make the figures readable.

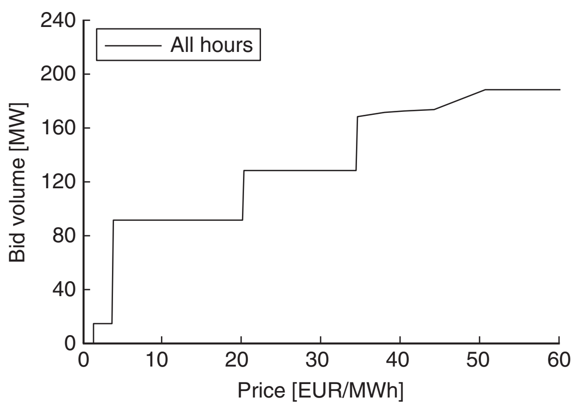
results are displayed in 12. The actual bids end up with a little less generated output, but not enough to justify the loss in spot revenues. The average price achieved is 24.95 EUR/MWh, compared to the generated bids' 26.79 EUR/MWh. We also see that the start-up costs are greater for the actual bids than the generated ones, even though the former use block bids. This can attributed to the fact that the model optimizes bidding for all five stations combined, whereas the actual bids have been constructed as bids for the individual stations. We conclude there is room for optimization-based improvements in bidding.

5.2.3. Backtest

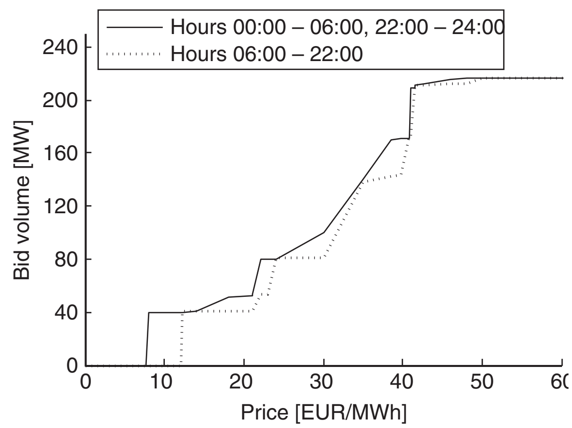
For reference we also include a similar table with a one-scenario run-through of actual spot prices, Table 13. We see that the generated bids still outperform the actual bids. The realized spot prices for 25 Sep turned out to be quite a lot higher than the forecast and thus the resulting numbers go up. The start-up costs are assumed to be equal 400 EUR/start-up for all turbines, thus the number of start-ups are easily recognized. Still, even though the empirical block bid is accepted, the generated bids achieve less startups through using hourly bids only.

5.2.4. Actual and Model-Generated Bids from 26 Sep

The resulting reservoir levels from 25.09.2011 was input as starting reservoirs for 26.09.2011. The aggregated actual power station bids as well as the bids generated by the model, are displayed below in Figure 8. The actual hourly bid from Producer A is identical to the bid from 25.09.2011. The bids generated on the other hand are split in two periods, displayed as the dashed line plot for 06:00-22:00 and the continuous line for the other 8 hours. Now notice how the generated bids clearly differ in entry price points for the two plots. This reflects perfectly the fact that 26.09.2011 was a Monday and the



(a) Actual hourly bids, 26.09.2011



(b) Generated hourly bids, 26.09.2011

Figure 8: Bids for 26.09.2011 from from Producer A and the bidding model, respectively. The price axes have been cut from 2000 EUR/MWh to make the figures readable.

Table 12: Results from fixing bids as parameters to the model with 300 price scenarios. Note that the price scenarios are not the same that generated the bottom row bids. All numbers are averaged from the scenario realizations. Output is given in MW, and the other numbers in EUR.

Bids	Output	Spot income	Start-up costs	Feed-in costs	Profit
Actual	3 803	94 882	1 036	3 768	90 078
Generated	3 832	102 654	706	3 974	97 973

Table 13: Running results from fixing bids as parameters to the model with actual spot prices for 25.09.2011. Output is given in MW, and the other numbers in EUR.

Bids	Output	Spot income	Start-up costs	Feed-in costs	Profit
Actual	3 895	118 775	1 200	4 567	113 008
Generated	3 944	120 557	800	4 637	115 120

marginal loss rates now vary intraday. The continuous line consequently enters at an earlier price than the dashed line which implies a lower feed-in cost. This makes perfect sense, seeing that all the power stations in question had a lower marginal loss rate for the night hours than the day hours.

The figures show that the model still generates a higher hourly bid at maximum price. The difference in volumes are made up for through the first of the block bids given in Table 14. However, the irrationality continues also here as the volumes for the other blocks do not equal the first. The change from 09:00 to 07:00 in the bidded blocks reflect the change from weekend to weekday, which again can relate both to peak hours for prices and the actual work shifts of the power stations.

5.2.5. Out of Sample Comparison

Once more, we run the actual bids through the same model input that generated the comparing bids, but with new price scenarios. The results on how the bids deal with new stochasticity are displayed in Table 15. Again the generated bids do better than the actual ones. It should be this way though, seeing that the optimal bids were generated using the same model and a price scenario set with the same properties. The average price achieved is 27.55 EUR/MWh, compared to the generated bids' 28.09 EUR/MWh. The start-up costs however, are in fact higher for the received bids than for the generated ones. The only conlusion we draw from this is that the model does not weigh start-up costs very heavily, neither should it, seeing that the estimated and used cost per start-up of 400 EUR is less than 0.4% of the spot revenues.

Table 14: Block bids for 26.09.2011 for the hydropower system in question. The volumes blocked are not equal for all hours. Volumes are in MW and prices are in EUR/MWh.

Start	Stop	Volume	Price	Block
00:00	07:00	−23	33.5	B0009
07:00	20:00	−22	35.5	B0920
20:00	24:00	−21	35.5	B0920

5.2.6. Backtest

A similar table with a one-scenario run-through of actual spot prices is shown in Table 16. Now the actual bids gives way more output than the generated ones. What happens is that the actual bids naturally hits the exact water usage values per station as they were set based on the bids and efficency curves. The generated bids on the other hand now used a too high price forecast in generating new price forecasts, so that when the spot realizes way below forecast the commitments become way too low. The model hits the water usage per station, as it must, but does it through spilling whatever water it cannot produce. The average realized spot prices for 26 Sep are 36.7 EUR/MWh and 37.0 EUR/MWh, respectively, for actual and generated bids. The generated bids now do one more start-up, and we conclude that comparing actual bids to the model bids and getting unambiguous results is easier said than done.

5.2.7. Simulation: Stochastic Versus Deterministic

In Section 4.3 we showed that Producer A could have increased their week 13–14 period income by 5.2% given complete knowledge of future price levels.

Table 15: Running results from inputting bids as parameters to the model with 300 price scenarios. Note that the price scenarios are not the same that generated the bottom row bids. All numbers are averaged from the scenario realizations. Output is given in MW, and the other numbers in EUR.

Bids	Output	Spot revenue	Start-up costs	Feed-in costs	Profit
Actual	3 762	103 633	1 744	4 116	97 773
Generated	3 799	106 731	2 001	4 239	100 491

Table 16: Running results from inputting bids as parameters to the model with actual spot prices for 26.09.2011. Output is given in MW, and the other numbers in EUR

Bids	Output	Spot revenue	Start-up costs	Feed-in costs	Profit
Actual	4 113	146 738	1 200	5 827	139 711
Generated	3 270	121 021	1 600	4 657	116 367

Running a similar test for the single five-station cascade shows a 1.6% potential in increased income. In the following, we test to see how much profit the model can realize through the same period, testing both with a 300–scenario model and a 1-scenario deterministic model with price forecasts. The model is run iteratively for each day, first with price forecasts and free bid variables, then with actual prices and fixed bid parameters. The first day of running actual reservoir levels are used as initial levels, while the end-of period reservoir levels is fixed at the real end level. This initial run will generate bids that are put back into the model again, but now with actual prices for the first day. Now, the reservoir levels at $h = 24$ from this fixed-bid real-prices run are given as input to the next day's model run, where all other parameters are updated as the time span H is reduced by 24 hours. This way we run through the model a total of 14 times, every other time being with fixed bids and real prices. 64 price points are used and 40 different blocks are possible to bid at every day, and every stochastic run uses 300 price scenarios based around the day-ahead forecast.

The results in Table 17 show that total profit over 14 days is 1.9% greater for the generated bids. Such a small number and with only one run-through is not enough to draw any conclusions. Yet we would like to point out our suspicion that the bidding design is such that we see the need to model short term price uncertainty to that great an extent. The stochastic model ends up realizing higher total start-up costs than the deterministic model. A possible explanation for this is again the fact that the stochastic model allocates a lot of total bid volumes to block bids, which may not be accepted in the deterministic real price run. In fact 23.6% of the block volumes are rejected. Thus the model may need to shut down turbines as their volume was set aside for block production. The huge variance in daily feed-in costs can be explained to a large extent by the two weekends present. Excluding the weekends gives standard deviations of 15% and 13% for the stochastic and deterministic model, respectively.

Also worth noticing from Table 17 is the relatively higher standard deviation in the deterministic model run. The actual revenue increase compared to reality was minuscule 0.4% and -0.2%, for the stochastic and deterministic run-throughs respectively. We believe the improvement would have been higher if the prices and price forecasts had not been so stable.

As of the 1.6% potential improvement shown in Section 4.3, a deterministic run of the model with actual spot prices gives an increase in income of 0.9%, which confirms that the value of perfect information cannot be very high for this particular case. We have not run tests through the entire 14 days with actual bids to find a profit for comparison, however find it likely that this potential increase would be higher.

In summary, four results are noted; first, optimized and actual bid curves are qualitatively similar, lending

Table 17: Results from running an iterative 14-day comparison between the stochastic and deterministic model. The standard deviations are given as a percentage of average. Prices are given in Euros, while costs, incomes and profits are given in 1000 Euros. Finally, volumes are in MWs.

	Stochastic			Deterministic		
	Average	***Total***	***St.dev***	***Average***	***Total***	***St.dev***
Price forecast	60.55	–	6.3%	60.55	–	6.3%
Actual spot price	60.52	–	6.2%	60.52	–	6.2%
Spot revenue	234.2	3 280	11%	230.2	3 222	24%
Feed-in cost	1.6	22.8	34%	2.0	28.6	127%
Start-up cost	1.3	18.1	8.3%	1.0	14.0	60%
Profit	231.3	3 239	13%	227.1	3 180	22%
# unique volumes	97	–	35%	15	–	22%
# unique price	49	–	14%	11	–	33%
# block bids	24	–	36%	0	–	–
# blocks used	15	–	28%	0	–	–
Avg. bids per block	2	–	17%	0	–	–
Avg. vol. per block	18	–	66%	0	–	–
Tot. vol. bid as block	1 832	25 648	31%	0	–	–
Block commitment	1 399	19 592	38%	0	–	–

empirical support to the model of [9]. Second, the entry price point for the actual bids are too low and do not reflect reasonable marginal water values including transmission tariffs. This is consistent with the empirical analysis in the previous section. Third, at high volumes, the actual bids require higher prices than the optimized bids. One may ask whether the widespread use of such a practice may lead to higher prices than necessary in scarcity situations (and explain why economists tend to find empirical evidence of 'non-competitive' markups of prices over marginal costs [18].). Further, the actual bids imply a maximum production that decreases after the first seven hours of the day. Finally, using out-of-sample scenarios, the optimized bids give a revenue that is higher by 1.9%. It outperforms the actual bidding also in terms of start up costs, even though the actual bids contain block bids, while the optimization model do not. It seems that the optimization model is able to exploit the capability of the joint system of power stations; the actual bids are made from adding bids from individual power stations.

6. Conclusion

This analysis gives insight into how day-ahead bidding is done in practice, and as such provides a basis for improved system operation. The most decisive factors when bidding are the marginal water value, feed-in fees, technical and hydrological characteristics, and bilateral agreements outside the spot market. We find that the producers take some of these factors well into account, and others not always so well. Among the things they consider well are the efficiency curves of the turbines and that they must choose to strategically interpolate or avoid it, as to hit suitable points of the efficiency curve. The feed-in fees are not taken as well into account. Producers' entry price points do not always adapt to changes in the variable feed-in fee over the course of a weekday, and some bids are submitted at price points below the feed-in fee alone. We also find that the producers do not fully utilize the range of price points allowed.

The producers perform quite well in their bidding, as indicated by the analysis of the maximum potential increase in spot revenues. On average over the 8 weeks, the potential increase for all producers was 5.4%, competing against perfect price information. This comes both from the fact that price forecasts are generally good, and that the system design with several price-volume bids and uniform spot prices is well-functioning. We still find that their performance correlates with the standard deviation of the price levels, meaning it is more difficult to bid optimally when price variations are high.

The overall conclusion is that bidding is not always rational, but that the consequences of this are often limited. There is room for improved bidding, e.g. through optimization approaches, however, the potential gains in average profit over time are likely to be modest.

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A Model for Optimal Bidding of Hydropower

This section presents a version of the [9] model to optimize day-ahead bidding given price scenarios for a given period forward in time as well as reservoir levels at the period's start and end. The mathematical formulation of the model, with parameters, variables, constraints and objectives is given in Section A.1.

Notational conventions To help the readability of the model, indices are always defined by small single latin letters. Sets have capital letters in a calligraphic font, with potential controlling subscripted index and/or superscripted letters to point out that it is a subset of special characteristics. Parameters are defined similar to sets, except with a regular font. All decision variables are lower case single latin letters, with lower case controlling subscripts. For reference the units used on parameters and decision variables in the implementation are included in the descriptions that follow.

A.1 Mathematical formulation

Sets and indices The model has a generic formulation with regard to reservoirs, power stations and turbines. The following sets and indices are used consequently in the modeling.

- $\mathcal{S}$: Set of scenarios, indexed by s, sized to S
- $\mathcal{H}$: Set of all hours the model spans over, indexed by h, sized to H
- $\mathcal{I}$: Set of bidpoints, indexed by i
- $\mathcal{B}$: Set of possible blocks, indexed by b
- $\mathcal{R}$: Set of reservoirs, indexed by r
- $\mathcal{K}$: Set of power stations, indexed by k
- $\mathcal{T}$: Set of turbines, indexed by t
- $\mathcal{E}_t$: Set of efficiency segments for turbine t, indexed by e

Subsets The model is split up into one day-ahead part that is directly related to the bidding and one part for the coming days, that will not need bidding. Thus the hour resolution is also split up into two subsets corresponding to the two model types. The other subsets relate to the topography and positioning of reservoirs and turbines.

- $\mathcal{H}^{\mathcal{D}} \subseteq \mathcal{H}$: Set of hours for the day-ahead bidding
- $\mathcal{H}^{\mathcal{L}} \subseteq \mathcal{H}$: Set of hours for long term part
- $\mathcal{K}_r \subseteq \mathcal{K}$: Subset of power stations that tap water from reservoir r
- $\mathcal{K}_r^+ \subseteq \mathcal{K}$: Subset of power stations directly above reservoir r
- $\mathcal{T}_k \subseteq \mathcal{T}$: Subset of turbines in power station k

Parameters The constants and coefficients given as parameters in the model are stated below. Notice that the price points for hourly bids and block bids are the same. Also note that the capacity of a turbine is time dependent.

- π_{sh}: Area spot price for scenario s in hour h [EUR/MWh]
- ρ_s: Probability of scenario s
- P_i: Price at bidpoint i [EUR/MWh]
- P_{bi}: Price for block b at bidpoint i [EUR/MWh]
- B_b^{start}: The first hour of block b
- B_b^{end}: The last hour of block b
- B_{sb}^{ave}: Average spot price for block b in scenario s [EUR/MWh]
- W_{ht}^{cap}: Maximum output from turbine t in hour h [MW]
- W_t^{min}: Minimum output from turbine t [MW]
- E_{te}: Efficiency for turbine t and segment e [MW]
- E_{te}^{0}: Efficiency constant for turbine t and segment e [Wh/m^3]
- Q_{hk}^{min}: Minimum flow for power station k in hour h [Mm^3/h]
- C_{shk}^{feed}: Feed-in fee for station k in scenario s and hour h [EUR/MWh]
- C_t^{start}: Start-up cost for turbine t [EUR]
- R_r^0: Initial reservoir level of r [Mm^3]
- R_r^{max}: Maximum reservoir level in r [Mm^3]
- R_r^{min}: Minimum reservoir level in r [Mm^3]
- R_r^{end}: Final end-of-period reservoir level in r [Mm^3]
- F_{hr}: Inflow to reservoir r in hour h [Mm3/h]

Decision variables The variables can be split up into two groups, the ones related to the bids and the ones related to the actual water flow. The x-variables are the sole variables not dependent on scenario, and thus the only first-stage variables. All other variables are second-stage recourse variables.

- x_{hi} : Volume bid in hour h at bidpoint i [MW]
- $\hat{x}bi$: Volume bid for block b and bidpoint i [MW]
- y_{sh} : Commitment for hourly bids in scenario s and hour h [MW]
- $\hat{y}_{sb}$: Commitment from block bid b in scenario s [MW]
- w_{sht} : Power output from turbine t in scenario s and hour h [MW]

q_{sht} : Flow through turbine t in scenario s and hour h [Mm^3/h]
$\hat{q}_{shk}$: Flow through power station k in scenario s and hour h [Mm^3/h]
l_{shr} : Reservoir level of r in scenario s and hour h [Mm^3]

$$\gamma_{sht} = \begin{cases} 1, \text{ if turbine } t \text{ is running in scenaro } s \text{ and hour } h \\ 0, \text{ otherwise} \end{cases}$$
$$\delta_{sht} = \begin{cases} 1 \text{ if turbine } t \text{ starts up in scenaro } s \text{ and hour } h \\ 0, \text{ otherwise} \end{cases}$$

Objective function The objective maximizes the total revenues from day-ahead and the period to come, less costs associated with start-ups and feed-in fees.

$$\mathcal{Z}^{dt} = \sum_{s \in S} \rho s \left(\sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}_k} \left(\sum_{h \in \mathcal{H}^D} w_{sht} \left(\pi_{sh} - C^{feed}_{shk} \right) - \delta_{sht} C^{start}_{t} \right) \right) \quad (1)$$

$$\mathcal{Z}^{long} = \sum_{s \in S} \rho s \left(\sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}_k} \sum_{h \in \mathcal{H}^L} w_{sht} \left(\pi_{sh} - C^{feed}_{shk} \right) \right) \quad (2)$$

$$\max \mathcal{Z} = \mathcal{Z}^{dt} + \beta \times \mathcal{Z}^{long} \quad (3)$$

Subject to:

$$x^{h}_{h(i-1)} \leq x^{h}_{hi}, s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I} \setminus \{1\} \quad (4)$$

$$y_{sh} = x^{h}_{h(i-1)} + \left(\pi_{sh} - P_{i-1} \right) \times \frac{x^{h}_{ih} - x^{h}_{h(i-1)}}{P_i - P_{i-1}}, \quad s \in \mathcal{S}, h \in \mathcal{H}^D, i \in \mathcal{I} \setminus \{1\} \mid P_{(i-1)h} \leq \pi_{sh} \leq P_{ih} \quad (5)$$

$$\hat{y}_{sb} = \sum_{i \in \mathcal{I} | P_{bi} \leq B^{ave}_{sb}} \hat{x}_{bi}, \qquad s \in \mathcal{S}, b \in \mathcal{B} \quad (6)$$

$$\sum_{b \in \mathcal{B} | B^{start}_{b} \leq h \leq B^{end}_{b}} \sum_{i \in \mathcal{I}} \hat{x}_{bi} + x_{hI} \leq \sum_{i \in \mathcal{I}} W^{cap}_{ht}, \qquad h \in \mathcal{H}^D \quad (7)$$

$$\sum_{t \in \mathcal{T}} w_{sht} = y_{sh} + \sum_{b \in \mathcal{B} | B^{start}_{b} \leq h \leq B^{end}_{b}} \hat{y}_{sb}, \qquad s \in \mathcal{S}, h \in \mathcal{H}^D \quad (8)$$

$$w_{sht} \leq \gamma_{sht} W^{cap}_{ht}, \qquad s \in \mathcal{S}, h \in \mathcal{H}^d, t \in \mathcal{T} \quad (9)$$

$$w_{sht} \geq \gamma_{sht} W^{min}_{t}, \qquad s \in \mathcal{S}, h \in \mathcal{H}^d, t \in \mathcal{T} \quad (10)$$

$$\delta_{sht} \geq \gamma_{sht} - \gamma_{s(h-1)t}, \qquad s \in \mathcal{S}, h \in \mathcal{H}^d \setminus \{1\}, t \in \mathcal{T} \quad (11)$$

$$\delta_{s(1)t} \geq \gamma_{s(1)t} - \gamma_{s(24)t}, \qquad s \in \mathcal{S}, t \in \mathcal{T} \quad (12)$$

$$w_{sht} \leq W^{cap}_{ht}, \qquad s \in \mathcal{S}, h \in \mathcal{H}^l, t \in \mathcal{T} \quad (13)$$

$$w_{sht} \leq E^{0}_{te} + E_{teqsht}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, t \in \mathcal{T}, e \in \varepsilon_t \quad (14)$$

$$\hat{q}_{shk} = \sum_{t \in \mathcal{T}_k} q_{sht}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, k \in \mathcal{K} \quad (15)$$

$$\hat{q}_{shk} \geq Q^{min}_{hk}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, k \in \mathcal{K} \quad (16)$$

$$l_{s1r} = R^{0}_{r} - \sum_{k \in \mathcal{K}_r} \hat{q}_{s1k} + F_{1r} + \sum_{k \in \mathcal{K}^{+}_{r}} \hat{q}_{s1k,} s \in \mathcal{S}, r \in \mathcal{R} \quad (17)$$

$$l_{shr} = l_{s(h-1)r} - \sum_{k \in \mathcal{K}_r} \hat{q}_{shk} + F_{hr} + \sum_{k \in \mathcal{K}^{+}_{r}} \hat{q}_{shk}, s \in \mathcal{S}, h \in \mathcal{H} \setminus \{1\}, r \in \mathcal{R} \quad (18)$$

$$l_{shr} \leq R^{max}_{r}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \quad (19)$$

$$l_{shr} \leq R^{min}_{r}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \quad (20)$$

$$l_{s(H)r} = R^{end}_{r} \qquad s \in \mathcal{S}, r \in \mathcal{R} \quad (21)$$

$$x_{hi}, \hat{x}_{bi}, y_{sh}, \hat{y}_{sb}, \overline{y}_{sh} \geq 0, \qquad s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I}, b \in \mathcal{B} \quad (22)$$

$$\begin{aligned} & w_{sht}, q_{sht}, \hat{q}_{shk}, l_{shr} \geq 0, \\ & s \in \mathcal{S}, h \in \mathcal{H}, k \in \mathcal{K}, t \in \mathcal{T}, r \in \mathcal{R} \end{aligned} \quad (23)$$

$$\gamma_{sht}, \delta_{sht} \in \{0,1\}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, t \in \mathcal{T} \quad (24)$$

The objective function, (1) to (3), is the probability-weighted sum of profit in all scenarios, both for the day-ahead part and for the long term part. It includes the costs associated with feed-in fees and start-up costs for the turbines. The latter would naturally push production away from day-ahead towards the rest the period, such as to avoid the costs of binary start-up variables. To make up for this shift we include a factor β, calibrated to compensate for this through assuring an equal output for all days, relative to the price forecast.

Constraint (4) reflects a rule given by the market operator making their problem easier to solve. It states that all bids have to be strictly non-decreasing, thus making sure a producer cannot bid totally stepwise constant bids, as discussed in Section 4.2.2. It also prohibits a decreasing hourly bid volume with rising prices, which otherwise might occur if block bids are bid in at a certain price or if participants are in possess of market power. Due to the previous constraint (4) the interpolation to the correct committed volumes is done as easily as in (5). Setting the commitment for each scenario based on scenario-independent bid variables also functions as the non-anticipativity constraint of the stochastic model.

Eq. (6) commits production from block bids if the price is below the average realized spot price. Constraint (7) makes sure the model never bids such that total volumes from hourly bids and block bids are greater than the combined turbine capacity in any hour. The sum of production in all turbines have to equal total commitment from hourly bids and block bids, expressed through (8).

Constraints (9) to (11) set the binary variables, while (12) says that hour 24 is related to hour 1. The latter states that if the turbine is not running in hour 24, then it needs to start to be able to run in hour 1. This is included to discourage the model from doing more start-ups in the earlier hours of the day than the later hours. If a turbine is running one night, it is not unlikely that it will run the next night as well.

A turbine cannot deliver more power than its capacity, (13). The conversion from output power to water flow through the turbine is simplified through the linearizations of efficiency in equation (14). Approximations et of the conversion rate from water flow to output power are given through the Y-axis intercept at E^0_{te} and a slope of E_{te}. Constraints (15) and (16) sum the flow through all turbines in power station k and bounds it to be equal to or higher than an hourly dependent minimum flow.

Eq. (17)-(20) control the reservoir levels and (21) states how the reservoir levels in the final hour have to equal the input end-of-period reservoir levels. Notice how there is no explicit modeling of potential spill over reservoirs. As we will not analyze spill any further, it would only enter the model as an increase in the upwards unbounded q variable.

A.2 Input parameters

This subsection will elaborate on the generation of input parameters for the model runs that are included in the results.

Price scenarios We have received historical day-ahead price forecasts from SKM Market Predictor AS for all the days in question, denoted by π_h $h \in \mathcal{H}^d$. Price scenarios have been constructed as a normal distribution based around these forecasts with a standard deviation σ_h equal to that of the area spot price for the respective hour 40 weekdays or 16 weekend days back in time. For the first day-ahead hour all scenarios will be normally distributed around the price forecast. Given that in a scenario s the price π_{sh} misses the forecast for hour h by

$$\Delta_{sh} = \pi_{sh} - \pi_h, \qquad s \in \mathcal{S}, h \in \mathcal{H}, \quad (25)$$

then the expectation for hour $h+1$ in same scenario s will be

$$\mathbb{E}\left[\pi_{s(h+1)}\right] = \pi_{h+1} + \frac{\Delta_{sh}}{\sigma_h}\sigma_{h+1}, \qquad s \in \mathcal{S}, h \in \mathcal{H} \setminus \{H\} \quad (26)$$

Thus we have

$$\pi_{s1} = \pi_h + \phi^{-1}\left(z_{s(1)}\right) \times \sigma_h, \qquad s \in \mathcal{S}, z_{s(1)} \in \mathcal{U}(0,1) \quad (27)$$

$$\begin{aligned} & \pi_{sh} = \mathbb{E}\left[\pi_{sh}\right] + \phi^{-1}\left(z_{sh}\right) \times \sigma_h, \\ & s \in \mathcal{S}, h \in \mathcal{H} \setminus \{1\}, z_{sh} \in \mathcal{U}(0,1) \end{aligned} \quad (28)$$

where the cumulative distribution function is

$$\phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp^{-x^2/2} \, dx \tag{29}$$

Since we only possess day-ahead forecasts, the forecast for days to come, $\pi_h \; h \in \mathcal{H}^L$ have been assumed to equal the day-ahead forecast, $\pi_h \; h \in \mathcal{H}^D$, for the remainder of the period, adjusting to weekends and weekdays according to average weekend versus weekday ratios for 2010. When going several days forward, we have also used a steadily increasing weight towards the forecasts to make sure the scenarios do not go way out of hand. An example of 500 generated price scenarios for day-ahead is displayed in Figure 5.