Analysis of Achievement for Understanding Geometry

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Abstract

The purpose of this study was to investigate the effectiveness of a mathematics professional development course. More specifically, in this study we examine whether geometric experiences have an impact on level of performance in mathematics. The van Hiele (Fuys, D., Geddes, D., & Tischler, R., 1988) model of geometric understanding provided a research framework from which to view geometric understanding. This model suggests five levels of understanding that should be taken into consideration when examining levels of geometric thinking: Visual, Descriptive/Analytic, Abstract/Relational, Formal Deduction/Proof, and Rigor.

The sample under study was three cohorts of practicing elementary teachers and mathematics coaches engaged in a 50-hour P-5 Mathematics Endorsement course entitled *Understanding Geometry*. Data collected through pre- and post-tests provided evidence that participants made significant improvement in geometric content knowledge and levels of understanding, thus verifying the effectiveness of their professional development experience. Also, this study points toward the importance of participants' entering level of understanding for achieving the course objectives.

Analysis of Achievement for Understanding Geometry

Given the commonly held belief that a causal relationship exists between teacher content knowledge and student achievement, it is essential that Georgia mathematics teacher educators contribute to strengthening the mathematics content knowledge of teachers. It is especially important to provide teachers with the knowledge and skills needed to effectively teach the new Georgia Performance Standards (GPS.) The GPS depend on building knowledge over time; therefore a strong foundation in P-5 is crucial.

The GPS promote a shift toward applying mathematical concepts and skills in the context of authentic problems. Students should understand concepts rather than merely follow a sequence of procedures. Like the National Council of Teachers of Mathematics (NCTM) standards, the GPS place a greater emphasis on problem solving, reasoning and proof, representation, communication, and connections. As students progress through the elementary years, learning opportunities that actively engage students through the use of manipulatives and various representations should be provided to allow them to grow in their geometric skills and understanding through grade-level appropriate activities such as:

- K – describe and sort objects
- 1st grade – observe, create, and decompose geometric shapes and solve simple problems including those involving spatial relationships
- 2nd grade – classify shapes and see relationships among them by recognizing their geometric attributes
- 3rd grade – broaden understanding of characteristics of previously studied geometric figures
- 4th grade – develop understanding of measuring angles with appropriate units and tools; understand the characteristics of geometric plane and solid figures
- 5th grade – expand understanding of computing area and volume of simple geometric figures; understand the meaning of congruent geometric shapes and the relationship of the circumference of a circle to its diameter.

Many recent professional development activities have been designed to improve teachers' knowledge of mathematics. In Georgia, it is possible to earn a P-5 Mathematics Endorsement by completing a sequence of four rigorous mathematics courses. One of these courses is *Understanding Geometry*. The Metropolitan Regional Educational Service Agency (MRESA) has received approval by the Professional Standards Commission to offer the Endorsement. To date, three cohorts have completed the sequence of courses under the same instructor and a fourth cohort is in progress. In this paper we will showcase the content and methods used to broaden understanding of fundamental concepts in geometry, construct and justify arguments, and interpret solutions, with a reference to the van Hiele theory of geometric understanding.

Statement of the Problem

Through this study, we sought to examine the effectiveness of the *Understanding Geometry* P-5 Mathematics Endorsement course on the performance of the participants. Specifically, we investigated the following question: *How do geometric experiences* *encountered in the* Understanding Geometry *course of the P-5 Mathematics Endorsement impact increased performances at higher van Hiele Levels?*

Professional Development Literature

Geometric Content Knowledge

Clearly, a critical component of mathematics teacher education is the acquisition of appropriate content knowledge. We agree with Tapan and Arslan's statement that, –the successful teaching of geometry at the elementary school depends critically on the subject knowledge of teachers.‖ (2009, p. 1) In this context, however, it is important to clarify that the term –subject or content knowledge‖ means much more than the mastery of mathematical terms and procedures. Because –content knowledge‖ is a broad term with different levels of meaning, we would like to specify that we equate *geometric content knowledge* with *conceptual knowledge/understanding*. Although definitions of *conceptual knowledge* also differ, we will adopt the statement from Hiebert and Lefevre (1996, pp. 3-4) that conceptual knowledge is ―knowledge that is rich in relationships… Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network.‖ Educational experiences that include cooperative learning and reflective discussion enhance the construction of relationships, the depth of understanding and the likelihood of retention (Daniels et al., 1993; Garrity, 1998). Mikusa notes further that –[E]xploring geometry using manipulatives or computers, creating conjectures, and then arguing about those conjectures with classmates is essential in helping students develop the use of propositional knowledge with visual knowledge… [H]aving students try to convince others of their mathematical ideas not only forces them to reflect on their own ideas, but to elaborate these ideas, making them more mathematically explicit‖ (1995, p. 7).

Pedagogical Content Knowledge

Chamberlain and Powers (2007) note that the knowledge of mathematics for teaching is more than simply mathematics content knowledge. They add that it also includes a specialized knowledge regarding teaching, such as the ability to analyze students' mathematical thinking. This ―specialized knowledge for teaching‖ is more commonly called *pedagogical content knowledge.* Reporting the results of her work with preservice and inservice elementary teachers, Fuller (1996) adopts Schulman's description of *pedagogical content knowledge*: –[PCK] includes… the most useful forms of representation of … ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that makes it comprehensible to others… [It] also includes an understanding of what makes the learning of specific concepts easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning‖ (p. 9).

Battista and Clements (1995) report the results of their project in which 3^{rd}- to 5^{th}-grade students worked in pairs to determine the number of cubes in 3-D arrays. The results of this study convinced the researchers that reflection and cognitive conflict were essential components of learning. –[The students'] work illustrates that, like scientists, students are theory builders… The difference between the scientist and the student is that the student interacts with a teacher, who can guide his or her construction of knowledge‖ (pp. 8-9). Teacher education, whether for preservice or inservice teachers, must equip teachers to be the guides their students will need.

In her book, *The Middle Path in Math Instruction: Solutions for Improving Math Education* (2004), Shuhua An's perception of *pedagogical content knowledge,* as described by Jeremy Kilpatrick's review, includes a mathematics teacher's ability for –addressing and correcting students' misconceptions‖ (2005, p. 256). A very useful implication of An's findings is the importance of having teachers build their own conceptual understanding to enable them to identify and correct their students' misconceptions. In other words, for teachers to develop into the –guides‖ their students need, professional development courses must provide participants with frequent constructivist learning experiences.

Implications for Teacher Education

While a majority of the research studies related to teacher education involved preservice teachers, those involving inservice teachers or a combination of teachers used similar approaches. Those that we investigated focused on the importance of content and pedagogy. Olkun and Toluk (2004) described their success in a math methods course for preservice elementary teachers. The researchers focused on the development of both content and pedagogical content knowledge, moving their students toward more formal use of the concepts, as well as to a higher van Hiele level. Their method, which had three components: manipulatives, guided questioning, and collective argumentation, also resulted in an increase in concept retention. Fuller (1996), who worked with a combination of 26 preservice and 28 experienced elementary teachers, used a similar approach. She describes her method as –the synthesis or integration of teachers' subject matter knowledge into an understanding of how particular topics are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction… [It] is that form of knowledge that makes teachers teachers…‖ Fuller further noted that research into the pedagogical content knowledge of both preservice and experienced teachers has shown that –teachers who themselves are tied to a procedural knowledge of mathematics are not equipped to represent mathematical ideas to students in ways that will connect their prior and current knowledge and the mathematics they are to learn, a critical dimension of *pedagogical content knowledge."* (p. 12, italics added)

A Geometric Understanding Theoretical Framework

A component of our research is the van Hiele theory regarding how students learn geometry. Van de Walle (2001) describes the van Hiele theory as –the most influential factor in the American geometry curriculum‖ (p. 309). Developed in the mid-to late-20th century by Dutch educators Pierre van Hiele and Dina van Hiele-Geldorf, the theory defines five levels (0 – 4) of geometric thought development. At Level 0 (Visualization), students think in terms of the shapes of objects and what they look like. They are able to group those that are –alike.‖ Level 1 (Analysis) students are able to think in terms of classes of shapes rather than individual ones and to focus on properties. At Level 2 (Informal Deduction) students are able to use the relationships among the properties to classify shapes. Level 3 (Deduction) students are able to use the relationships among properties of shapes to formulate deductive proofs. Level 4 (Rigor) students have the ability to compare and contrast different axiomatic systems. The levels are sequential and movement from one to the next relies on geometric experiences, not maturation. (Fuys et al, 1998)

Olkun and Toluk (2004) state that they expect their pre-service elementary school teachers to be at van Hiele Level 3, given that they have all completed the secondary school geometry program. However, when Halat (2008) compared preservice elementary and secondary mathematics teachers, he found that the elementary group's average was below Level 2 and the secondary group's average was below Level 3. Attempting to teach geometry at a level which students have not reached does not work. So it is crucial that a diagnosis of van Hiele levels be done prior to and incorporated into the lesson planning.

Methodology

The Participants

The composition of the cohorts included in this study has varied due to increased communication of the expectations and refinement of the admission policy. The first cohort was comprised of classroom teachers from six different elementary schools in one of the 12 school systems in the MRESA service area. While twenty candidates were accepted into the P-5 Mathematics Endorsement, several self-selected out when they learned the rigorous demands of the courses. Of the fifteen who began *Understanding Numbers and Operations*, the first course, nine completed the requirements to earn the endorsement. Two candidates were not allowed to continue in the endorsement beyond the first course since they did not acceptably complete the unit requirement.

After the completion of the endorsement by the first cohort, instructors for each of the content courses, serving as the P-5 Mathematics Advisory Council, made some recommendations. The first recommendation was to provide an information session regarding the rigor and length of the endorsement. The second recommendation was that a placement test be given since the intent of the endorsement is to take good elementary math teachers and make them coach material. A placement test was created and administered to the other two cohorts included in this study. Individuals completing eight of twelve items correctly dealing with number and operations were admitted into the program. Those who completed fewer than eight items correctly were encouraged to take another course prior to re-applying for the admission to the Mathematics Endorsement. Retention of cohort members improved after implementation of these recommendations.

The second cohort consisted of teachers from a school cluster in another system at the request of their area superintendent. Seventeen of the twenty-two who took the placement test did well enough to be admitted. Ten of those seventeen began the endorsement, and seven completed it.

The third cohort consisted of nine individuals from different school clusters in the same system as the second cohort. The endorsement was open to any elementary teacher in the system. This cohort has completed three of the four content courses to date. Seven are progressing to the last course.

Means of Assessment

All participants from each cohort completed both a pre- and post-test. The test was compiled by two of the researchers using a collection of items from various sources. At the beginning of this study each item was categorized by van Hiele level. Some test items were not appropriate to classify at van Hiele levels; however two items were classified as Level 1 (Analysis - students are able to think in terms of classes of shapes rather than individual ones and to focus on properties) and five items were classified as Level 2 (Informal Deduction - students are able to use the relationships among the properties to classify shapes). One of the Level 2 test items examined participants' ability to classify geometric shapes by examining their properties – regular, irregular, and concave polygons for the first problem; and triangles, regular polygons and polygons with symmetry for the second problem. This item, Venn Diagram –Labels for Polygon Sort (see Figure 1), was a preliminary focus of this study.

> Polygons can be grouped in many different ways in addition to being grouped according to the number of sides. Two other ways included regular polygons and concave polygons. Regular polygons have sides that are all the same length and angles that are all the same size. Concave polygons look like they are collapsed or have one or more angles dented in. Any polygon that has an angle measuring more than 180 is concave. How should the Venn diagram be most specifically labeled?

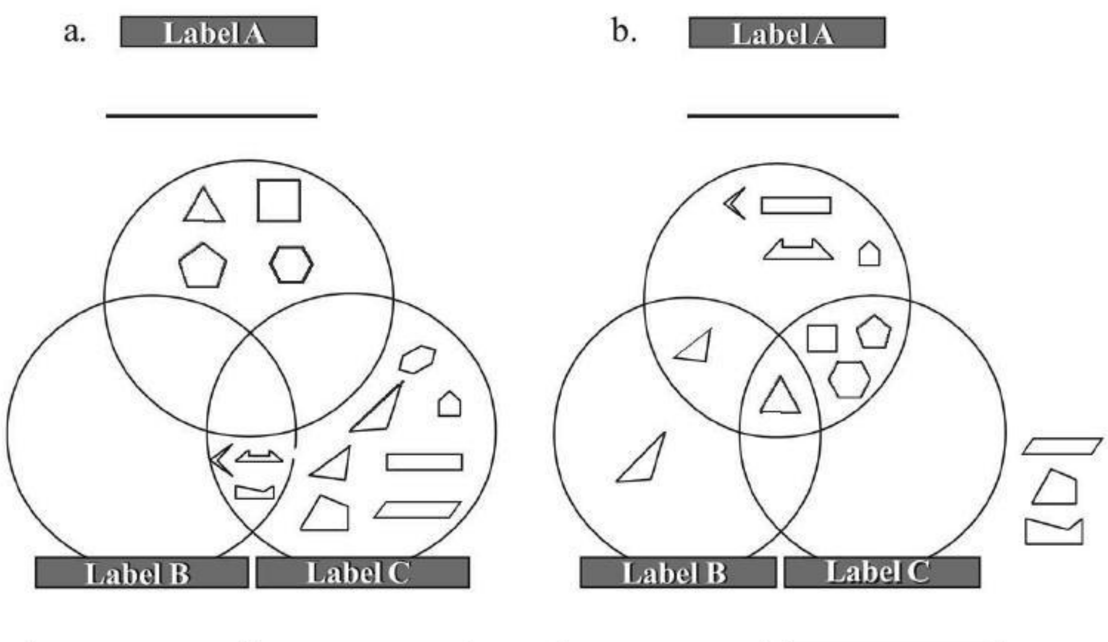

Figure 1: Venn Diagram – Labels for Polygon Sort

During the *Understanding Geometry* course, participants experienced a variety of activities sorting polygons such as Roping in Quadrilaterals (Gavin et al., 2001). In this activity participants placed sixteen given quadrilaterals into Venn diagrams they created from yarn. Once the quadrilaterals were sorted, appropriate labels were placed on the Venn diagram. Another activity was Mystery Rings (Gavin et al., 2001). In this activity participants progressed through six different tasks of increasing complexity, sorting quadrilaterals into Venn diagrams according to specified labels. For example, in Task 1 the labels are ―At least one right angle‖ and ―No right angles.‖ In Task 6 (using three rings) the labels are ―At least two pairs of adjacent sides equal,‖ ―All pairs of opposite angles equal,‖ and ―All adjacent angles equal.‖ Participants also experienced an interactive online sorting activity, Sorting Polygons (www.learner.org).

Each of the three experiences occurred on Day 3 of the 9-day course. In each cohort group, participants expressed their value of and appreciation for these sorting activities. Typical comments were ―This is such a rich activity‖ and ―I love the way it gets harder and harder. I couldn't do the last one if it had been first, but doing them in order I can get them all right.‖ Even though most participants found these activities challenging, they believed the experiences were appropriate for their grade 3-5 students.

Results

In examining the effectiveness of the *Understanding Geometry* course on the performance of the participants, we assessed increase in content knowledge and progress through the van Hiele levels. We began by analyzing aggregate differences in pre- and post-test scores by cohort for a particular problem. The results of this evaluation, plus consideration of changes in the entrance criteria for the course, led us to focus on Cohorts 2 and 3 as they were deemed more representative of future cohorts.

Specifically, aggregate data was compiled from all three cohorts and responses were examined on both parts of the pre/post-test item Venn Diagram –Labels for Polygon Sort. The data organization yielded four sets of data for each participant – problem A and problem B for both pre- and post-tests. Correct or incorrect responses on each Venn diagram label were compared from pre-test to post-test for each participant.

Each data set was comprised of three labels for the Venn diagram; therefore there were three possible correct answers. The analysis score was computed as post-test score minus pre-test score. If there was no change between pre- and post-test responses, a "0" was recorded. If one label (out of 3) showed improvement, "+1" was recorded. The best possible analysis score was +3 (all wrong on pre-test and all correct on post-test). If a participant scored better on the pre-test than the post-test that was recorded as -1, -2 or -3. Overall the possible analysis scores were -3, -2, -1, 0, 1, 2, or 3 for each participant on each problem. To compare cohorts, the scores were added for each problem, A and B. These aggregate scores are listed below:

Problem A, Cohort 1 (n=9) : aggregate score –3
Problem A, Cohort 2 (n=6) : aggregate score +3
Problem A, Cohort 3 (n=8) : aggregate score +4

Total across Cohorts (n=23) on Problem A : +4
Problem B, Cohort 1 (n=9) : aggregate score –4
Problem B, Cohort 2 (n=6) : aggregate score +4
Problem B, Cohort 3 (n=8) : aggregate score +6
Total across Cohorts (n=23) : aggregate score +6

Preliminary analysis of this data confirmed our perception that Cohort 1 was markedly different from Cohorts 2 and 3. The aggregate scores indicate that overall, participants in Cohort 1 scored better on the pre-test than on the post-test. In fact, only one participant from Cohort 1 showed an improved score on the post-test. Of the remaining eight participants, four showed no difference on pre- and post-tests and four showed a decline in accomplishment. Several factors could have impacted performance. Participants in Cohort 1 did not take the mathematics placement test required of subsequent Cohorts; nor did they have the benefit of an information session to advise them of the rigor demanded in the Endorsement courses. In addition, Cohort 1 experienced a more time-compressed course than either of the other Cohorts – nine class days in two weeks as opposed to nine class days in six weeks. The condensed pace, combined with unrealistic course expectations, may have created debilitating anxiety during the post-test. Alternatively, it is possible that the post-test was not taken seriously by this inaugural group of participants.

Changes in the course entrance criteria and course pace resulted in Cohort 1 being distinctly different from the other two cohorts, therefore we chose to focus our analysis on data gathered from Cohorts 2 and 3 which were deemed more representative of future cohorts. The analysis focused on changes in two outcomes: (1) content knowledge and (2) van Hiele level of understanding.

To determine changes in content knowledge, a paired t-test (null hypotheses $\mu_2 - \mu_1 = 0$ and alternative hypothesis $\mu_2 - \mu_1 > 0$) was conducted on pre- and post-test scores. The results showed a significant improvement ($p = 0.00001$) in participants' geometric content knowledge by the end of the course.

To determine whether progress had been made in van Hiele levels of understanding, a subset of pre- and post-test problems were identified as Level 1 (Analysis - students are able to think in terms of classes of shapes rather than individual ones and to focus on properties) or Level 2 (Informal Deduction - students are able to use the relationships among the properties to classify shapes) assessments based on the type of understanding necessary for correct responses to the items. Again, results of a paired t-test showed notable gains. There was a significant difference in the percentage of correct responses at both Level 1 ($p = .003$) and Level 2 ($p = 0.00000002$). It makes sense that the Level 1 growth would be smaller because many of the participants began the course with a Level 1 understanding. It is very gratifying to find such a large increase in the percentage of teachers who had achieved Level 2 understanding by the end of the course.

In addition, pre- and post-test scores for the van Hiele Level 2 problem <u>Venn Diagram –Labels for Polygon Sort</u> were compared. The analysis found participants' progress from pre- to post-test was also significant (A, $p = 0.0143$; B, $p = 0.0093$).

Future Plans

We will continue to examine future cohorts of the Mathematics Endorsement course entitled *Understanding Geometry* to confirm whether geometric experiences in that course have an impact on level of performance in mathematics. In addition, we will examine the progress of cohorts in one or more of the other three courses of the P-5 Mathematics Endorsement: *Understanding Numbers and Operations, Understanding Algebra, and Understanding Data Analysis and Probability*.

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