Neutron Star Heating Constraints on Wave-Function Collapse Models

Antoine Tilloy[1,*] and Thomas M. Stace[2]
[1]Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany
[2]ARC Centre for Engineered Quantum Systems, School of Mathematics and Physics, The University of Queensland, Brisbane, Queensland 4072, Australia

(Received 25 February 2019; published 21 August 2019)

Spontaneous wave-function collapse models, like continuous spontaneous localization, are designed to suppress macroscopic superpositions while preserving microscopic quantum phenomena. An observable consequence of collapse models is spontaneous heating of massive objects. We calculate the collapse-induced heating rate of astrophysical objects, and the corresponding equilibrium temperature. We apply these results to neutron stars, the densest phase of baryonic matter in the Universe. Stronger collapse model parameters imply greater heating, allowing us to derive competitive bounds on model parameters using neutron star observational data, and to propose speculative bounds based on the capabilities of current and future astronomical surveys.

DOI: 10.1103/PhysRevLett.123.080402

Collapse models, like the continuous spontaneous localization (CSL) model [1,2], aim at solving the measurement problem of quantum mechanics through a stochastic nonlinear modification of the Schrödinger equation [3,4]. Such modifications have sometimes been conjectured to be caused by gravity, with the most famous example being the Diósi-Penrose (DP) model [5,6]. In general, collapse models posit an intrinsic (possibly gravitational) noise, which endogenously collapses superpositions of sufficiently macroscopic systems (in a particular basis) while preserving the predictions of quantum mechanics at small scales. One notable consequence of these models is the spontaneous heating of massive objects. Neutron stars, which are extremely dense, macroscopic quantum-limited objects, offer a unique system on which to test this prediction. Here, we estimate the equilibrium temperature of a neutron star radiating heat generated from spontaneous collapse models. We find that neutron stars are competitive for constraining the parameter diagram of collapse models. Theoretically or observationally improving upper bounds for neutron star equilibrium temperatures could in principle eliminate historically proposed CSL parameter values.

Collapse models.—Continuous Markovian collapse models modify the Schrödinger equation with a nonlinear noise term,

$$\partial_t|\psi_t\rangle = -\frac{i}{\hbar}H|\psi_t\rangle + F(\eta_t, |\psi_t\rangle), \tag{1}$$

where η_t is a white noise process and F some function which is partially constrained by consistency conditions [7,8], and it is chosen to yield a spontaneous collapse in the position basis.

Although this stochastic description (1) of the state vector is required to understand why collapse models actually achieve their purpose and solve the measurement problem, their empirical content is fully contained in the master equation obeyed by $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$. For *most* Markovian nondissipative collapse models proposed thus far [4], it takes the form $\partial_t\rho_t = -(i/\hbar)[H,\rho_t] + \mathcal{D}[\hat{M}]\rho_t$, with

$$\mathcal{D}[\hat{M}]\rho = -\int d\mathbf{x}d\mathbf{y}f(\mathbf{x}-\mathbf{y})[\hat{M}_{r_c}(\mathbf{x}),[\hat{M}_{r_c}(\mathbf{y}),\rho]], \tag{2}$$

where f is a positive definite function and $\hat{M}_{r_c}(\mathbf{x})$ is a regularized mass density operator:

$$\hat{M}_{r_c}(\mathbf{x}) = g_{r_c}\circ\hat{M}(\mathbf{x}) = g_{r_c}\circ m a^\dagger(\mathbf{x})a(\mathbf{x}). \tag{3}$$

In this expression, m is the mass of the particle considered (we will consider neutrons), $a_k^\dagger(\mathbf{x})$, $a_k(\mathbf{x})$ denote the usual (here, fermionic) creation and annihilation operators, g_{r_c} is a regulator which smooths the mass density over a length scale r_c, and "$\circ$" denotes the convolution product. Typically, the regulator function is taken to be Gaussian:

$$g_{r_c}(\mathbf{x}) = e^{-\mathbf{x}^2/(2r_c^2)}\Big/\left(\sqrt{2\pi r_c^2}\right)^3. \tag{4}$$

The regulator length scale has to be much larger than the Planck length and even the nucleon Compton wavelength, with the usual choice being $r_c \simeq 10^{-7}$ m [9].

The two most common continuous collapse models are the CSL model and the Diósi-Penrose model (the latter having a heuristic link with gravity): (1) The CSL model is obtained for

$$f^{\rm CSL}(\mathbf{x}-\mathbf{y}) = \frac{\gamma}{2m_N^2} \times \delta(\mathbf{x}-\mathbf{y}), \tag{5}$$

where m_N is the mass of a nucleon and γ is the collapse "strength." It is a rate $\times$distance^3; the corresponding rate is $\lambda_{\rm CSL} \equiv \gamma/(4\pi r_c^2)^{3/2}$, historically fixed at $\lambda_{\rm CSL} \simeq 10^{-16}$ s^{-1} [the so-called Ghirardi-Rimini-Weber (GRW) value]. (ii) The DP model is obtained for

$$f^{\rm DP}(\mathbf{x}-\mathbf{y}) = \frac{G}{4\hbar} \times \frac{1}{|\mathbf{x}-\mathbf{y}|}. \tag{6}$$

Because the collapse strength is fixed by the gravitational constant, there is one parameter fewer [10]. Modern motivation for Eq. (6) is given by attempts at constructing models of fundamentally semiclassical gravity [11,12]. We note that, at least at the master equation level, the regulator applied on the mass density operator can equivalently be applied on the kernel f:

$$\mathcal{D}[\hat{M}]\rho = -\int d\mathbf{x}d\mathbf{y} f_{r_c}(\mathbf{x}-\mathbf{y})[\hat{M}(\mathbf{x}),[\hat{M}(\mathbf{y}),\rho]] \tag{7}$$

with $f_{r_c} = g_{r_c} \circ f \circ g_{r_c}$.

We also note in passing that the two models we consider here are nonrelativistic. Efforts towards developing relativistic collapse models for quantum fields have shown that their construction is possible (albeit challenging; see, e.g., Refs. [13–19]). Here, we simply assume that such relativistic extensions can be constructed, and that, in the limit where relativistic effects are not dominant, their predictions would be similar to those of the nonrelativistic CSL or DP models.

Spontaneous heating.—The additional decoherence term, Eq. (2) in the master equation, does not commute with the kinetic part of the Hamiltonian; hence, the expectation of the energy $\langle H\rangle_t \equiv {\rm tr}[H\rho_t]$ is no longer conserved. This spontaneous heating provides a natural test of collapse models [20–22].

Recent proposals to test these models have, e.g., been built around ultracold atoms [23], which may provide good platforms to obtain bounds on the parameters in the theory, as the heating effect should be significant in relative terms. An alternative which has been overlooked so far is to consider instead maximally dense systems, exploiting the mass density dependence of the heating for all collapse models. In this respect, neutron stars are ideal candidates.

Neutron star cooling has been studied theoretically and observationally. At early stages when $T_{\rm star} \sim 10^9$ K, neutral stars cool by various baryonic emission processes, but at later stages, when $T_{\rm star} \sim 10^6$ K or colder, the cooling is radiation dominated [24–26]. Thus, the equilibrium temperature is attained by the balance of the spontaneous collapse induced heating with Stefan-Boltzmann radiation, so it is determined by the heat balance condition $P_{\rm heat} = P_{\rm rad}$, where

$$P_{\rm heat} = \partial_t \langle H\rangle_t = {\rm tr}[H\mathcal{D}[\hat{M}]\rho_t] \tag{8}$$

and

$$P_{\rm rad} = S\sigma T^4, \tag{9}$$

where S is the neutron star surface area and $\sigma = 5.6 \times 10^{-8}$ W m^{-2} K^{-4} is Stefan's constant. It follows that at equilibrium $T_{\rm star} = [P_{\rm heat}/(S\sigma)]^{1/4}$.

For a system of N fermions with a nonrelativistic Hamiltonian, one can show that the spontaneous collapse induced heating $P_{\rm heat}$ is independent of the potential (which commutes with the mass density) and more surprisingly does not depend even on the quantum state. For the CSL model, it reads [27]

$$P_{\rm heat}^{\rm CSL} = {\rm tr}[H\mathcal{D}[\hat{M}]\rho_t] = \frac{3\lambda\hbar^2}{4r_c^2 m} N, \tag{10}$$

where N is the number of neutrons in the star. Similarly, for the DP model it reads [27]

$$P_{\rm heat}^{\rm DP} = \frac{G\hbar m}{8\sqrt{\pi} r_c^3} N. \tag{11}$$

The CSL model.—We take the typical neutron star radius $L \sim 10$ km and mass $M_{\rm star} \sim M_\odot \simeq 2.0 \times 10^{30}$ kg, and hence $N = M_{\rm star}/m_N \simeq 10^{57}$ neutrons. For the values historically proposed for the CSL model, $\lambda = 10^{-16}$ s and $r_c = 10^{-7}$ m, one finds $P_{\rm heat} \sim 10^{14}$ W. On the other hand, the lowest observed temperature of an astronomical neutron star is $T^{\rm (obs)} = 0.28$ MK for the object PSR J 840-1419 [28]. This observed temperature corresponds to a radiative dissipation rate of $P_{\rm rad}^{\rm (obs)} \sim 10^{26}$ W, well above the power that would be radiated by the CSL model. Hence, the neutron stars we can currently observe are not cold enough to straightforwardly falsify the CSL model.

Naturally, neutron stars are expected to cool down to much lower temperatures than the ones we currently manage to see directly [26], and the bound from PSR J 840-1419 is thus an excessively conservative one. We discuss this further below.

The DP model.—Following the same reasoning as with the CSL model, we can constrain the only free parameter, the regularization length r_c, of the DP model using Eq. (11). The most conservative bound, given by PSR J 840-1419, yields $r_c \gtrsim 10^{-13}$ m, which excludes a regulator of the order of the neutron radius which was historically conjectured to be a possible cutoff. This lower bound is of the same order of magnitude as the current best one of 4×10^{-14} m yielded by constraints from gravitational wave detector data [29]. The bound improves with decreasing temperatures $r_c \propto T^{-4/3}$.

Discussion.—The analysis presented in this Letter makes "lumped-element" approximations that provide robust bounds on the radiated power. For example, we have assumed that the emissivity of a neutron star is unity and that the thermal conductivity throughout the core is large enough that the star temperature is approximately uniform. If these assumptions are relaxed, then the core temperature may be substantially higher than the observed surface temperature. Neutron superfluidity [25] has been hypothesized in the core of neutron stars. This phase will have a corresponding critical temperature T_c, which may provide a sensitive thermometric bound on tolerable heat generation rates in the star core: superfluidity will be suppressed if the internal temperature is too high. More generally, heat transfer models that include realistic constitutive models for the neutron star body may thus be able to provide even more stringent bounds on collapse model parameters than the lumped-element approximations that we have adopted here.

The positive bounds established above are based on observed temperatures of young, hot, bright neutron stars. There is a possibility for improvement in the bounds if colder neutron stars are observed, or if a large population of cold remnants can be excluded due to lack of observation, so we now speculate on the near-term prospects for wide survey observations.

The Dark Energy Survey (DES) has classified a significant fraction of astronomical objects down to apparent magnitude $m = 23$ [30]. The separation of neutron stars in the vicinity of the Sun is estimated to be around 10 pc [31], so the nearest neutron star is expected to be $d \approx 5$ pc away from Earth. At that range, $m = 23$ objects seen by DES correspond to a luminosity of 5×10^{18} W, and a neutron star surface temperature of 22×10^3 K (assuming a neutron star radius of 10 km). Thus, the DES should be able to see nearby, cool neutron stars. This would put a constraint on CSL models which is roughly comparable to the constraints from spontaneous x-ray emission studies [32].

In the future, the Large Synoptic Survey Telescope (LSST) will be able to image apparent magnitude $m = 28$ objects [33]; at 5 pc, such objects have a luminosity of 5×10^{16} W, and a surface temperature of 7×10^3 K $\approx T^{(\mathrm{Sun})}$. Such an observation which would improve bounds on the CSL model, as shown in Fig. 1.

In the event that either DES or LSST *fails* to observe such objects, it would suggest either a (surprisingly) low local density of neutron stars or that nearby neutron stars are unobservably cold (i.e., $T < T^{(\mathrm{Sun})}$). The latter inference would further rule out parts of the CSL parameter diagram, also shown in Fig. 1.

More speculatively, we might hope to one day be able to eliminate the possibility of an equilibrium temperature like that of our own planet, $T^{(\mathrm{Earth})} \sim 3 \times 10^2$ K, which would falsify the historical GRW values by 2 orders of magnitude.

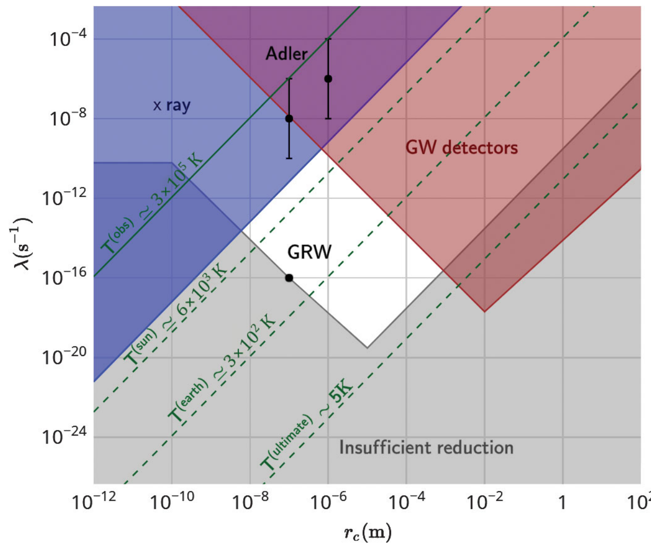

FIG. 1. CSL parameter diagram. (Top) Zones formerly excluded by gravitational wave detectors (red) [29,34], spontaneous x-ray emission (blue) [32], and insufficient macroscopic localization [35]. The value historically proposed by GRW [9] and the range put forward by Adler [36] are shown with black dots. The green line delineates the upper left regions that are excluded by currently observed neutron stars (solid line). More speculative bounds, obtained assuming various equilibrium temperatures for neutron stars, are showed as dashed green lines.

What might be the ultimate observable limit, even in principle? Neutron stars would be net thermal sources indefinitely if their minimum equilibrium temperature were to exceed the cosmic microwave background (CMB) temperature. Though this would be difficult to observe terrestrially, it does offer an intriguing limit. Below $T^{(\mathrm{ultimate})} = 5$ K $\gtrsim T^{(\mathrm{CMB})}$, we find that the CSL parameter bounds are too low for collapse models to be effective, as shown in Fig. 1.

For the DP model, upper bounds on r_c can be obtained if the model is required to provide a consistent theory of fundamental semiclassical gravity [12]. In this context, the regulator g_{r_c} affects the Newtonian potential, and the $1/r^2$ law of the gravitational force breaks down for $r \sim r_c$. The Newtonian force is well measured for distances as short as 100 μm [37], which provides a conservative upper bound, $r_c \lesssim 10^{-4}$ m. Even supposing cold neutron stars of a few kelvin, we find that $r_c \gtrsim 10^{-7}$ m. Hence, the range of values allowed for the DP model could not (even in principle) be closed by the temperature of neutron stars alone, and gravitational upper bounds would need to be improved in parallel.

On the other hand, refinements and extensions of the CSL model with colored noise [38,39], dissipation [40], or both [41] containing additional parameters (such as a high frequency cutoff or a temperature) are known to yield weaker heating effects. Consequently, the constraints we put forward here would be weaker for these models.

In summary, with a conservative estimate of neutron star cooling based on the currently observed coldest neutron stars, one obtains constraints on the CSL model (albeit weaker than from spontaneous x-ray emission studies) and on the DP model ($r_c \gtrsim 10^{-13}$ m, competitive with state-of-the-art gravitational wave interferometer data). Improving the observational upper bound on neutron star equilibrium temperatures would yield substantial improvements. If we could measure an old cold neutron star, one could test more of the CSL parameter diagram. This provides motivation for a systematic survey of nearby cold neutron stars.

We thank Philip Pearle and Tamara Davis for the helpful discussions during the development of this work. A. T. was supported by the Alexander von Humboldt foundation. T. M. S. was supported by Australian Research Council Centre of Excellence for Engineered Quantum Systems Grant No. CE170100039.

*antoine.tilloy@mpq.mpg.de

[1] P. Pearle, Phys. Rev. A **39**, 2277 (1989).
[2] G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A **42**, 78 (1990).
[3] A. Bassi and G. Ghirardi, Phys. Rep. **379**, 257 (2003).
[4] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, Rev. Mod. Phys. **85**, 471 (2013).
[5] L. Diósi, Phys. Lett. A **120**, 377 (1987).
[6] R. Penrose, Gen. Relativ. Gravit. **28**, 581 (1996).
[7] N. Gisin, Helv. Phys. Acta **62**, 363 (1989).
[8] H. Wiseman and L. Diósi, Chem. Phys. **268**, 91 (2001).
[9] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).
[10] Note that the factor 1/4 in Eq. (6) has also been fixed at 1/8 in the literature.
[11] A. Tilloy and L. Diósi, Phys. Rev. D **93**, 024026 (2016).
[12] A. Tilloy and L. Diósi, Phys. Rev. D **96**, 104045 (2017).
[13] L. Diósi, Phys. Rev. A **42**, 5086 (1990).
[14] P. Pearle, Phys. Rev. A **59**, 80 (1999).
[15] R. Tumulka, J. Stat. Phys. **125**, 821 (2006).
[16] D. J. Bedingham, Found. Phys. **41**, 686 (2011).
[17] P. Pearle, Phys. Rev. D **91**, 105012 (2015).
[18] A. Tilloy, arXiv:1702.06325.
[19] B. A. Juárez-Aubry, B. S. Kay, and D. Sudarsky, Phys. Rev. D **97**, 025010 (2018).
[20] P. Pearle and E. Squires, Phys. Rev. Lett. **73**, 1 (1994).
[21] M. Bahrami, Phys. Rev. A **97**, 052118 (2018).
[22] S. L. Adler and A. Vinante, Phys. Rev. A **97**, 052119 (2018).
[23] F. Laloë, W. J. Mullin, and P. Pearle, Phys. Rev. A **90**, 052119 (2014).
[24] J. M. Lattimer, K. A. van Riper, M. Prakash, and M. Prakash, Astrophys. J. **425**, 802 (1994).
[25] J. M. Lattimer and M. Prakash, Astrophys. J. **550**, 426 (2001).
[26] D. Yakovlev and C. Pethick, Annu. Rev. Astron. Astrophys. **42**, 169 (2004).
[27] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.123.080402 for a derivation of the energy increase in the CSL and DP models.
[28] E. F. Keane, M. A. McLaughlin, M. Kramer, B. W. Stappers, C. G. Bassa, M. B. Purver, and P. Weltevrede, Astrophys. J. **764**, 180 (2013).
[29] B. Helou, B. J. J. Slagmolen, D. E. McClelland, and Y. Chen, Phys. Rev. D **95**, 084054 (2017).
[30] A. Drlica-Wagner *et al.*, Astrophys. J. Suppl. Ser. **235**, 33 (2018).
[31] N. Sartore, E. Ripamonti, A. Treves, and R. Turolla, Astron. Astrophys. **510**, A23 (2010).
[32] K. Piscicchia, A. Bassi, C. Curceanu, R. D. Grande, S. Donadi, B. C. Hiesmayr, and A. Pichler, Entropy **19**, 319 (2017).
[33] P. A. Abell *et al.* (LSST Science and LSST Project Collaborations), arXiv:0912.0201.
[34] M. Carlesso, A. Bassi, P. Falferi, and A. Vinante, Phys. Rev. D **94**, 124036 (2016).
[35] M. Toros, G. Gasbarri, and A. Bassi, Phys. Lett. A **381**, 3921 (2017).
[36] S. L. Adler, J. Phys. A **40**, 2935 (2007).
[37] C. A. Hagedorn, Ph.D. thesis, University of Washington, 2015.
[38] A. Bassi and G. C. Ghirardi, Phys. Rev. A **65**, 042114 (2002).
[39] S. L. Adler and A. Bassi, J. Phys. A **40**, 15083 (2007).
[40] A. Smirne, B. Vacchini, and A. Bassi, Phys. Rev. A **90**, 062135 (2014).
[41] L. Ferialdi and A. Bassi, Phys. Rev. A **86**, 022108 (2012).