Spectrum Sensing Based on Censored Observations in Time-Varying Channels using AR-1 Model

D. K. Patel and Y. N. Trivedi

***Abstract*—Non-parametric sensing algorithms are preferred in cognitive radio. In this paper, spectrum sensing method based on censored observations is proposed. We evaluate the performance of Censored Anderson-Darling (CAD) sensing method in time-varying and flat-fading channel using Monte Carlo simulations. We have shown the performance of the CAD sensing in terms of receiver operating characteristic (ROC). The considered channel is modeled by Gaussian variables and characterized by a first ordered autoregressive process ($AR1$). It is shown that the proposed method outperforms prevailing techniques such as the Energy detection (ED) sensing and Order-statistic (OS) based sensing in time-varying channel at lower signal to noise ratio.**

***Keywords*—Spectrum sensing, goodness of fit test, type-2 right censoring, time-varying channel.**

I. Introduction

CURRENT demand of higher data rate in the limited frequency spectrum can be achieved using Cognitive Radio (CR) [1]. One of the most important components in CR is spectrum sensing. The main task in spectrum sensing is to detect the licensed users, known as primary users (PU). This task is performed by unlicensed users, known as secondary users (SU), which can use the spectrum of PU such that they do not cause interference to PU.

In recent past, many spectrum sensing algorithms have been proposed with an interest to provide spectrum access in an opportunistic way. The spectrum sensing methods are broadly categorized in two ways; Parametric sensing and Non-parametric sensing. In parametric sensing, some information about PU is available at SU. Some of the parametric algorithms are cyclostationary based sensing, waveform-based sensing, matched filtering etc [2], [3]. In case of non-parametric sensing, no information about PU is available at SU. Some of the non-parametric algorithms are energy detection (ED) [4] and Goodness of Fit (GoF) tests [5]. The ED is the most common method for spectrum sensing due to its low complexity. However, the performance of ED degrades at low signal-to-noise ratio (SNR). In this scenario of low SNR, GoF tests such as Anderson-Darling (AD) test, Kolmogorov-Smirnov (KS) test and Student t-test are preferred for identifying the presence of PU at SU [6].

In GoF tests, detection is made based on testing of null hypothesis. From the received observations, the Empirical Cumulative Distribution Function (ECDF) is determined. This ECDF is compared with the known CDF under the null hypothesis. The distance of the ECDF from the CDF decides whether PU is present or not. Furthermore, to give more emphasis to the tails of the CDF, a special weight function has been used in Anderson Darling (AD) sensing [7].

In AD sensing, all the received observations are used to determine ECDF. However, the distance of the CDF and ECDF is higher especially at the right tail due to less number of observations. This incomplete information of CDF on the right tail introduces an error in determining statistics in AD sensing, especially at low SNR. To overcome this, the concept of censored data is proposed which has been used in survival analysis. The censoring is applied on total individuals, when incomplete information about the survival time of some individuals is available [8]. In view to this, we drop some observations in the right tail, which carry incomplete information for the CDF.

In the literature of GoF based sensing, the detection performance of spectrum sensing algorithms has been shown assuming Additive White Gaussian (AWGN) or quasi-static channel. In this paper, we consider time-varying Gaussian channel and modified AD test. The underlying time varying channel is modeled by first order AR process as was used in [9]. The modify AD test is used by introducing censoring of observations and call it as Censored Anderson Darling (CAD) test. In this test, the observations from right tail are removed. For this CAD test, modified statistic of the AD test is used as derived in [10]. This statistic has been obtained by modifying the upper limit of the integration for lower number of observations. Using Monte Carlo simulations, the receiver operating characteristics (ROC) is presented for different time-varying channel conditions. We have also compared our results of the CAD test with ED sensing and Order-Statistics (OS) based sensing method [11]. We have found that CAD sensing outperforms these methods at lower values of SNR in time-varying channels.

This paper is organized as follows. The system under consideration and channel model are introduced in Section II. The problem of spectrum sensing as GoF testing for censored observations is formulated as null hypothesis testing problem in Section III. In Section IV, the detection performance of the CAD sensing algorithm is presented and compared with OS and ED sensing methods. Finally, the paper is concluded in Section V.

II. System Model

Let us consider a communication link in a time varying and flat fading channels, characterized by a first ordered

D. K. Patel is with Information and Communication Technology Dept., Institute of Engineering and Technology, Ahmedabad University, Ahmedabad, Gujarat, India Email: dhaval.patel@ahduni.edu.in

Y. N. Trivedi is with Department of Electrical Engineering, Institute of Technology, Nirma University, Ahmedabad, Gujarat, India Email: yogesh.trivedi@nirmauni.ac.in

autoregressive ($AR1$) model [9]

$$h_i = ah_{i-1} + \sqrt{1-a^2}v_i \quad (1)$$

where $h_i \sim \mathcal{N}(0,1)$ and v_i denotes independent and identically distributed ($i.i.d$) as Gaussian with mean zero and variance one. In (1), a indicates correlation coefficient where $0 \leq a \leq 1$. Here $a = 1$ and $a = 0$ result in independent or constant channel. The value of a will be determined using Jake's autocorrelation function [9]. At the secondary user (SU), received observations x_i, for $1 \leq i \leq n$, are real valued and represented as,

$$x_i = \sqrt{\rho}mh_i + w_i, \quad i = 1, 2, 3, \cdots\cdots n, \quad (2)$$

where $m \in \{0, 1\}$, ρ is the received SNR and additive noise w_i, for $1 \leq i \leq n$, are the samples from any arbitrary continuous probability distribution function. In (2), $m = 1$ and 0 denote presence and absence of PU respectively.

III. Summary of CAD Sensing Algorithm

Without loss of generality, we assume that all n observations are in ascending order. It means $x_1 \leq x_2 \leq \cdot\cdot, x_n$. Now, we retain first r observations and drop or censor the last $n - r$ observations as shown in Fig. 1. Hence, x_r is the highest valued observation. This method of censoring $n - r$ highest valued observations is known as right censoring with Type-2 [8].

In this scenario, the problem of spectrum sensing as null hypothesis testing problem as GoF testing is defined as [7],

$$\begin{aligned} H_0 &: F_X(x) = F_0(x) \\ H_1 &: F_X(x) \neq F_0(x) \end{aligned} \quad (3)$$

where $F_0(x)$ denotes the CDF of w_i.

The CAD sensing for AWGN and quasi-static channel is proposed by us in [12]. In brief, the steps involved in CAD sensing algorithm is summarized below:

Step:1 Find the threshold λ for a given probability of false alarm P_f using,

$$P_f = \mathbb{P}\{\ _pA_n^2 > \lambda | H_0\} \quad (4)$$

As given in [5], the value of λ is determined for a specific value of P_f and censoring ratio p. For example, when $P_f = 0.05$ and $p = 0.4$, the value of λ is 1.133.

Step:2 Sorting all the observations in ascending order, we get

$$x_1 \leq x_2 \leq \cdot\cdot \leq x_r \leq x_{r+1} \leq \cdot\cdot \leq x_n,$$

where $x_{r+1} \leq x_{r+2} \cdot\cdot \leq x_n$ observations are censored.

Step:3 Calculate the required test statistic $_pA_n^2$ for the observations $x_1 \leq x_2 \leq \cdot\cdot \leq x_r$ as defined in (5).

$$\begin{aligned} _pA_n^2 = &-\frac{1}{n}\sum_{i=1}^{r}(2i-1)(lnz_i - ln(1-z_i)) - 2\sum_{i=1}^{r} ln(1-z_i) \\ &- \frac{1}{n}[(r-n)^2 ln(1-z_r) - r^2 lnz_r + n^2 z_r], \end{aligned} \quad (5)$$

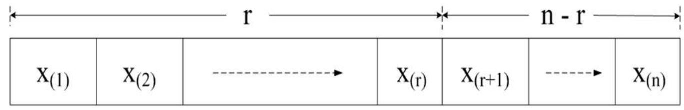

Fig. 1: Number of received (n) and censored $(n - r)$ observations

where $z_i = F_0(x_i)$.

Step:4 For detection at secondary user, based on censored observations, null hypothesis is rejected when $_pA_n^2 > \lambda$, where λ is the value of threshold.

Step:5 Compute performance metric as probability of detection (P_d) with a given value of P_f. The analytical P_d can be computed as,

$$\begin{aligned} P_f &= \mathbb{P}\{\ _pA_n^2 > \lambda | H_1\} \\ &= 1 - F_{_pA_n^2, H_1}(\lambda) \end{aligned} \quad (6)$$

IV. Simulation Results

In this section, the performance of the proposed CAD sensing algorithm is shown in terms of ROC and P_d vs SNR using Monte Carlo simulations, where ROC is a curve between probability of detection (P_d) versus probability of false alarm (P_f). The values of SNR are in dB. In the considered CAD test, n and $n - r$ denote number of observations received and number of observations censored based on the censoring ratio $\left(p = \frac{r}{n}\right)$. The performance has been shown for different values of correlation coefficient a. Furthermore, the ROC of the proposed one is compared with prevailing schemes like AD, ED and OS based sensing.

Fig. 2 shows ROC for CAD sensing for different values of p such as 0.2, 0.4, 0.6 and 0.8, and fixed value of n as 20 with an SNR of -4dB. It can be seen that P_d increases with p for a fixed value of P_f. It is expected because higher number of observations improves the detection probability.

Fig. 3 shows the impact of time varying nature of the channel on ROC of the proposed scheme at -2dB of SNR using different values of correlation coefficient (a) such as 0, 0.9, 0.95, 0.99, 1 taking n = 20 and r = 12. It means 12 observations are used for the detection of PU to identify its presence or absence. It can be seen that P_d is improved as the value of a increases towards unity.

Fig.4, shows P_d versus SNR for $P_f = 0.05$ for the same values of n, r and a. As SNR increases, P_d increases as per expectation. From the results shown in Fig. 3 and Fig. 4, we can say that CAD sensing improves P_d, when the channel is quasi-static ($a = 1$). However, as the value of a decreases, the performance degrades. It should be noted that in the considered CAD sensing, test statistic and threshold are dependent upon variance of noise only, not on the signal or channel component.

The ROC of the proposed CAD sensing is compared with the existing GoF sensing schemes as shown in Fig. 5. We

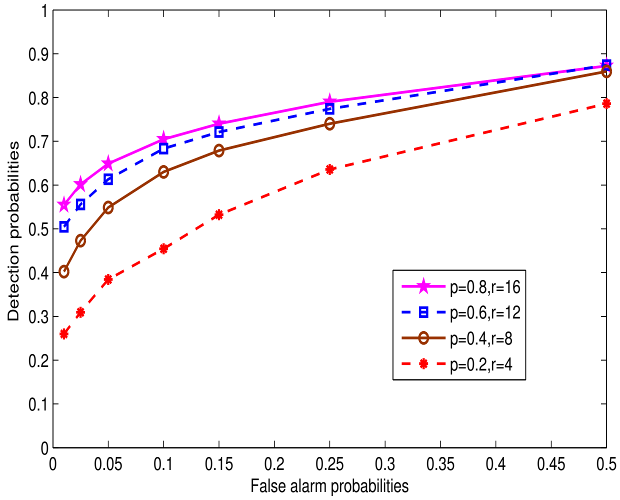

Fig. 2: ROC for CAD sensing with $n = 20$ at SNR = -4dB.

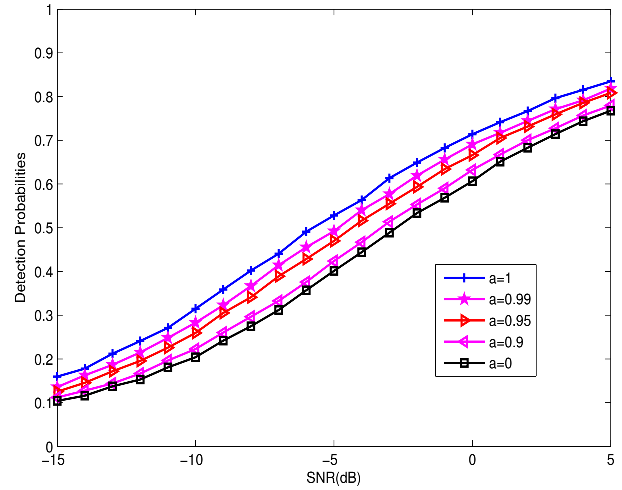

Fig. 4: P_d vs SNR for $P_f = 0.05$, $n = 20$.

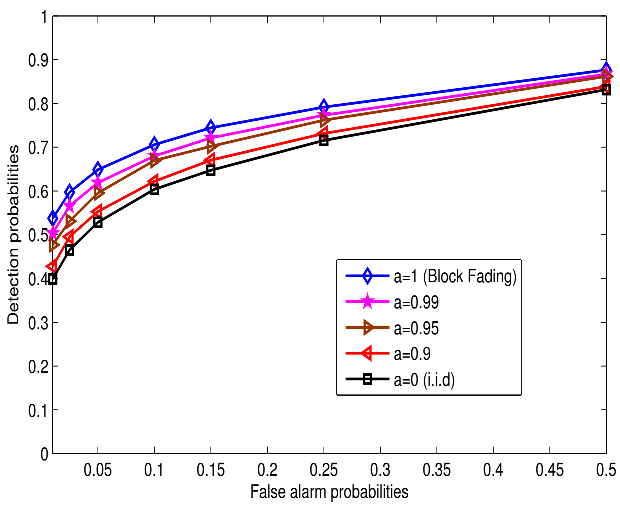

Fig. 3: ROC for CAD sensing with $n = 20$ and $r = 12$.

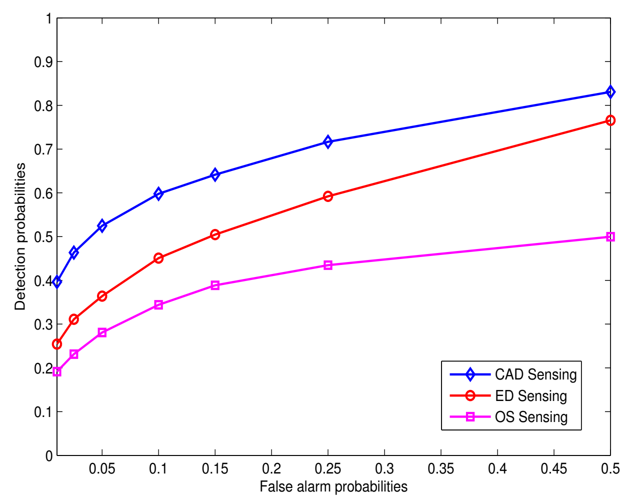

Fig. 5: ROC for $a = 0.99$, $n = 20$ at SNR = -5dB.

have considered the time-varying channel which is modeled using AR-1 model. We take $a = 0.99$, $n = 20$, SNR = -5dB and $r = 12$ (for CAD sensing only). It can be seen from the graph that CAD sensing outperforms the other two methods in the whole range of P_f. For $P_f = 0.05$, P_d in CAD sensing is 0.5247 whereas for ED and OS sensing it is 0.3641 and 0.2809 respectively.

Fig. 6, shows P_d versus SNR for AD, ED and OS based sensing in the considered time-varying channel and compared them with the proposed CAD sensing for $P_f = 0.05$, $n = 20$, $a = 0.99$ and $p = 0.6$. To present fair comparison with the CAD sensing, the AD sensing is considered without censoring. The $P_d = 0.88,\ 0.867,\ 0.81$ and 0.48 are achieved for AD, CAD, ED and OS sensing respectively at SNR of 8dB. Thus, similar trend in P_d can be seen over here for a wider range of SNR from -15dB to 10dB. However, we can observe that the detection performance of CAD sensing degraded after 10dB of SNR which shows that the CAD sensing performs better at the lower value of SNR. From Fig. 6, it can be seen that the AD sensing has improved detection than CAD sensing. However, CAD sensing uses lower number of observations $(r = 12)$ for achieving almost same detection performance as obtained for AD sensing at $n = 20$. So, the CAD sensing helps for saving the processing energy of secondary user and

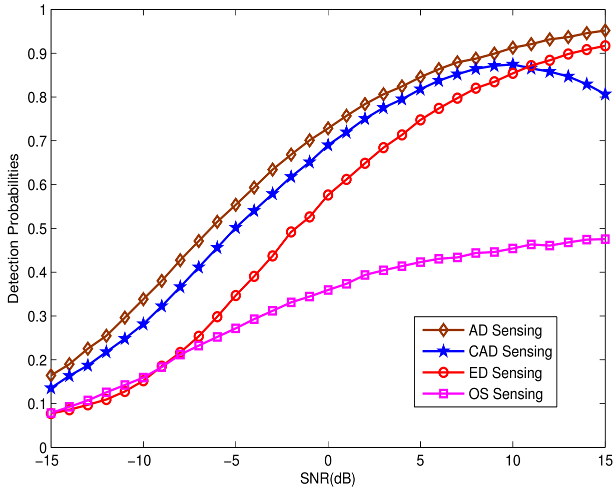

Fig. 6: P_d vs SNR for $a = 0.99$, $n = 20$.

reducing the computational complexity too. The OS based sensing performs poorly in comparison with CAD and ED sensing methods. However, OS sensing outperforms under the AWGN channel only [11].

V. Conclusion

We considered CAD spectrum sensing with censored observations in time varying channel. The underlying channel is characterized by $AR1$ model. The ROC is presented for the CAD sensing for different values of correlation coefficient a. It is observed that the performance degrades with reducing the value of a. The detection performance of CAD sensing is also compared with the ED and OS based sensing schemes. We found that CAD sensing outperforms the ED and OS based sensing in time varying channel only at lower value of SNR.

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D K Patel received his B.E degree in Electronics and Communication engineering from North Gujarat University, India in 2003. He obtained his M.E degree in Communication Systems from Gujarat University, Ahmedabad, India in 2010. Since 2011, he has been working as research scholar toward the PhD degree in PG lab, Institute of Technology, Nirma University, Ahmedabad, India, where he is working on nonparametric detection in cognitive radio. His area of interest includes statistical signal processing, Cognitive Radio and MIMO systems.

Y.N Trivedi is an Associate Professor with the Dept of Electrical Engineering at Institute of Technology-Nirma University, Ahmedabad, India. His current research interests are in performance analysis and simulation of wireless communication systems and cognitive radio. He has published several papers in national/international conferences and international journals. He received his B.E degree in Electronics from Sardar Patel University, Vallabh Vidya Nagar, India in 1993 and M.E degree in Industrial Electronics from M.S University of Baroda, India in 1995. He received Ph.D degree in Electrical Engineering from IIT Kanpur, India in 2011. From 1995 to 1996, he worked as a lecturer in Faculty of Technology and Engineering, M.S University of Baroda. He has been guiding students for their M.Tech and Ph.D degrees in Nirma University.