IFM and Its Dual Form for Eigen Value Analysis of Plate Bending Problems

G. S. Doiphode,V. A. Patel and S. C. Patodi

***Abstract*—Integrated Force Method (IFM) is now well accepted method for the analysis of framed and continuum structure problems under static and dynamic loading. The methodology proposed in the present paper attempts to calculate the frequency using the force based eigen value analysis, while the present literature emphasizes on displacement based eigen value analysis. The suggested formulation is based on the Cauchy's equilibrium operator, Saint Venant's compatibility operator and Hooke's material matrix operator. Element equilibrium and flexibility matrices are derived by discretizing the expression of potential and complimentary strain energies respectively. The displacement field is decided using Hermits interpolation function, while the stress field is approximated using the traditional polynomial of approximate order. Formulation developed earlier for static analysis using rectangular element having nine force degree of freedom and twelve displacement degree of freedom (RECT_9F_12D) is extended. Lumped mass and consistent mass matrices are also derived. A modified formulation of IFM which is named as Dual Integrated Force Method (DIFM) is also explored. Plate bending problems with two different boundary conditions are attempted. Various discretization patterns are used to check the convergence of frequency values towards the analytical solution. Results obtained for natural frequencies, force mode shapes for each frequency value and corresponding nodal displacements are presented. Results obtained for natural frequency are compared with the exact solution; a good agreement is found.**

***Index Terms*—Eigen value, Equilibrium matrix, Flexibility matrix, IFM and DIFM.**

I. INTRODUCTION

MANY practical problems of dynamic analysis can not be solved effectively by using classical methods. Approximate numerical methods using digital computer give fast and acceptable solutions. Therefore, Finite Element Method (FEM) has become quite popular in structural dynamics field for tackling the problems involving complex material properties, loading and boundary conditions. Using engineering judgment on the view of safety, serviceability and economy with all required safety factors, a link between real physical problem and idealized mathematical model is developed in FEM, which gives feasible solution including all the assumptions imposed on the real practical problem. Integrated Force Method (IFM) has been successfully applied to thin square plate subjected to static loading under different boundary conditions [1]. It has also been successfully applied to small scale structures subjected to lumped mass systems at important nodes, which gives natural frequencies [2]. In the present paper, a force based integrated force method and a displacement based dual integrated force method are developed for the dynamic analysis of rectangular plates subjected to lumped or consistent masses as per the discretization pattern. Matrices necessary for the solution of the problem such as Global Equilibrium Matrix [S], Global Flexibility Matrix [G], Global Lumped Mass Matrix [ML] and Global Consistent Mass Matrix [MC] are derived using basic theory. Results obtained using proposed formulations are compared with the available exact solution based on the classical theory of plates [3, 4].

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II. GOVERNING EQUATIONS OF IFM

The IFM equations, for a continuum discretized in finite number of elements with 'n' and 'm' force and displacement degrees of freedom respectively, are obtained by coupling the 'm' number of equilibrium equations and r = n - m compatibility conditions. The m equilibrium equations are written as

$$[B]\{F\} = \{P\} \tag{1}$$

and the' r' compatibility conditions are written as

$$[C][G]\{F\} = \{\delta R\} \tag{2}$$

A displacement based approach uses the following equation of equilibrium for calculating the natural frequency of vibration of structure.

These conditions are combined to obtain the IFM governing equations for static analysis as follows [1]:

$$\left[\frac{[B]}{[C][G]}\right]\{F\} = \frac{\{P\}}{\{\delta_r\}} \tag{3}$$

or [S]{F} = {P}

A displacement based approach uses the following equation of equilibrium for calculating the natural frequency of vibration of structure.

$$[M]\{\omega^2\}\{X\} - i\{\omega\}[C]\{X\} = [K]\{X\} = \{P\} \tag{4}$$

Where [M], [K] and [C] are the mass, stiffness and damping coefficient matrices respectively. The basic frequency equation can be obtained by eliminating X and P between equations (1) and (4) as follows:

$$\left[\frac{[B]}{[C][G]}\right]\{F\} = \{\omega^2\}[M][J][G]\{F\} - i\{\omega\}[C^\star]\{F\} \tag{5}$$

where $[C^\star] = [C][G][F]$

Above equation is the eigen value equation of integrated force method, where F is the force mode shape and ? is the circular frequency of the structure. Using Eqn. (3) the displacement mode shape x can be calculated.

III. GOVERNING EQUATIONS OF DIFM

IThe modified form of Integrated Force Method i.e., Dual IFM (DIFM) is formulated by mapping the forces into displacements. The basic equations of the dual formulation of the IFM are given below.

The nodal displacement - external load relation is written as

$$[D]\{X\} = \{P\} \tag{6}$$

where

$$[D] = [B][G]^{-1}[B]^T$$

is a symmetric matrix of size m x m, which is assembled at element level. The nodal displacement - internal force relation is written as

$$\{F\} = [G]^{-1}[B]^T[X] \tag{7}$$

Displacement based approach yields the basic equation of equilibrium for calculating the natural frequency of vibration of structure as

$$[M]\{\omega^2\}Xi\{\omega\}[C]\{X\} = [K]\{X\} = \{P\} \tag{8}$$

Equating Eqns. (6) and (8), one can write the modified form as follows.

$$[D]_{mxm}X_{mx1} = \{\omega^2\}\{X\}_{mx1} - i\{\omega\}_{1xm}[C]_{mxm}\{X\}_{mx1} \tag{9}$$

Dynamic analysis is carried out by taking into consideration the inertial effects by using either lumped or consistent mass matrix. In lumped mass approach, the distributed mass is assigned to nodal points which contribute towards the translational degree of freedom along diagonal terms of mass matrix only whereas in consistent mass approach, the complete mass is distributed corresponding to each degree of freedom at each node of the considered element.

IV. DEVELOPMENT OF VARIOUS MATRICES

A. Element Equilibrium Matrix [Be]

The elemental equilibrium matrix written in terms of forces at grid points represents the vectorial summation of 'n' internal forces F and 'm' external loads P. The nodal EE in matrix notation can be stored as rectangular matrix [Be] of size m x n. The variational functional is evaluated as a portion of IFM functional which yields the basic elemental equilibrium matrix [Be] in explicit form as follows:

$$[U_p = \int_D \left\{ M_x \frac{\partial^2 w}{\partial x^2} + M_y \frac{\partial^2 w}{\partial y^2} + M_x y \frac{\partial^2 w}{\partial x \partial y} \right\} d_x d_y \tag{10}$$

$$= \{M\}^T\{\epsilon\}ds \tag{11}$$

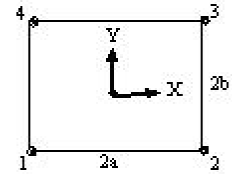

Fig. 1. A rectangular plate element

where,

$$\{M\} = (M_x, M_y, M_{xy})$$

are the in-plane internal moments and

$$\{\epsilon\}^T = \left(\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, \frac{\partial^2 w}{\partial x \partial y} \right)$$

represents the curvatures corresponding to each internal moment.

Consider four-noded, 12 ddof (X1 to X12) rectangular element of thickness t with dimensions as 2a x 2b along the x and y axes as shown in Fig. 1.

The force field is chosen in terms of four independent forces as;

$$\{F\} = (F_1, F_2, ...F_9) \tag{12}$$

Relations between internal moments and independent forces are written as.

$$M_x = F_1 + F_{2x} + F_{3y} + F_{4xy} \tag{13}$$

$$M_y = F_5 + F_{6x} + F_{7y} + F_{8xy} \tag{14}$$

$$M_{xy} = F_9 \tag{15}$$

Arranging Eqns. (13), (14) and (15) in matrix form

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} 1 & x & y & xy & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & x & y & xy & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \{F_e\} \tag{16}$$

or

$\{M\} = [y]\{F_e\}$ where,

$$\{F_e\} = [F_1, F_2, F_3, ...F_9]^9 \tag{17}$$

The variation of above forces is considered bilinear along both directions. The displacement fields satisfy the continuity condition and the selected forces satisfy the mandatory requirement.

The Hermits Interpolation function for lateral displacement for rectangular element is as follows:

$$w(x,y) = N_1(x,y)X_1 + N_1(x',y)X_2 + N_1(x,y')X_3 + N_4(x,y')X_{12} \tag{18}$$

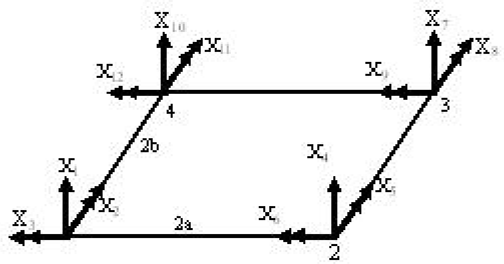

Fig. 2. Nodal displacements.

Where,

$$w(x,y) = N_{1(x)}N_{1(y)}, N_1[x',y] = N'_{1(x)}N_{1(y)}$$
$$N_1(x,y') = N_{1(x)}N'_{1(y)}$$

and so on.Here,

$N_{1(x)} = \frac{x^3-3a^2x+2a^3}{4a^3}$ $N_{2(x)} = \frac{-x^3+3a^2x+2a^3}{4a^3}$

$N_{2(y)} = \frac{y^3+3b^2y+2b^3}{4b^3}$ $N'_{1(x)} = \frac{x^3-ax^2-a^2x+a^3}{4a^2}$

$N_{1(y)} = \frac{y^3-by^2-b^2y+b^3}{4b^2}$ $N'_{2(y)} = \frac{y^3+by^2-b^2y-b^3}{4b^2}$

are associated with nodal displacements X1, X2,...X12 as shown in Fig. 2.

By arranging all force and displacement functions properly, one can discretize the Eq. (10) to obtain elemental equilibrium matrix as follows.

$$U^e = \{X\}^T[B^e]\{F\} \tag{19}$$

Where

$$[B^e] = \int_s [Z]^T[Y]ds \tag{20}$$

Here,

$$[Z] = [L][N] \tag{21}$$

where [L] is the differential operator matrix, [N] is the displacement interpolation function matrix and [Y] is the matrix of force interpolation function. Substituting Eq. (17) and Eq. (18) in Eq. (20) and integrating yields the following non zero components of non-symmetrical equilibrium matrix B_e of size 12 x 9.

$B^e_{31} = b, B^e_{61} = -B^e - 31, B^e_{91} = -B^e_{31}$
$B^e_{121} = B^e_{31}, B^e_{12} = -B^e_{31}, B^e_{22} = 0.33b_2$
$B^e_{32} = -ab, B^e_{42} = B^e_{12}, B^e_{52} = -B^e_{22},$
$B^e_{62} = B^e_{32}, B^e_{72} = -B^e_{12}, B^e_{82} = B^e_{22}$
$B^e_{92} = -B^e_{62}, B^e_{102} = -B^e_{22}, B^e_{112} = -B^e_{22}$
$B^e_{122} = -B^e_{32}, B^e_{33} = -0.4b_2, B^e_{63} = B^e_{33}$
$B^e_{93} = B^e_{33}, B^e_{123} = B^e_{33}, B^e_{14} = B^e_{33}$
$B^e_{24} = -0.66b_3, B^e_{34} = 0.4ab_2, B^e_{44} = B^e_{14},$
$B^e_{54} = B^e_{24}, B^e_{64} = -B^e_{34}, B^e_{74} = B^e_{13}$
$B^e_{84} = 0.2b_3, B^e_{94} = B^e_{34}, B^e_{104} = -B^e_{13}$
$B^e_{114} = -B^e_{24}, B^e_{114} = B^e_{94}, B^e_{25} = -a,$
$B^e_{55} = a, B^e_{85} = a, B^e_{115} = -a,$
$B^e_{26} = 0.4a_2, B^e_{26} = -0.4a_2, B^e_{86} = 0.4a_2,$
$B^e_{116} = -0.4_{a2}, B^e_{17} = a, B^e_{27} = ab,$
$B^e_{37} = -0.3a_2, B^e_{47} = -a, B^e_{57} = ab, B^e_{67} = 0.3a_2,$
$B^e_{77} = -a, B^e_{87} = ab, B^e_{97} = -0.3a_2,$
$B^e_{107} = a, B^e_{71} = ab, B^e_{127} = 0.3a_2,$
$B^e_{18} = -0.4a_2, B^e_{28} = -0.4a2b, B^e_{38} = 0.6a_3$
$B^e_{48} = B^e_{18}, B^e_{58} = B^e_{28}, B^e_{68} = B^e_{38},$
$B^e_{78} = B^e_{18}, B^e_{88} = B^e_{28}, B^e_{98} = -B^e_{18}$
$B^e_{108} = -B^e_{18}, B^e_{118} = -B^e_{28}, B^e_{128} = B^e_{38}$
$B^e_{128} = B^e_{38}, B^e_{19} = -2, B^e_{49} = 2,$
$B^e_{79} = -2, B^e_{19} = -2, B^e_{49} = 2, B^e_{79} = -2, B^e_{109} = 2. \quad (22)$

B.Element Flexibility Matrix [Ge]

The elemental flexibility matrix is obtained by discretizing the complementary strain energy.

$[G^e] = \int [Y]^T[D][Y]dxdy$

where, [Y] is the force interpolation function matrix and [D] is the material property matrix. Substituting values in Eq. (11) and integrating yields the symmetrical flexibility matrix [Ge] as follows with a common multiplication term as $48ab/Et^3$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -\upsilon & 0 & 0 & 0 & 0 \\ 0 & \frac{a^2}{3} & 0 & 0 & 0 & \frac{-\upsilon a^2}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{b^2}{3} & 0 & 0 & 0 & \frac{-\upsilon b^2}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{a^2b^2}{9} & 0 & 0 & 0 & \frac{-\upsilon a^2b^2}{9} & 0 \\ -\upsilon & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-\upsilon a^2}{3} & 0 & 0 & 0 & \frac{a^2}{3} & 0 & 0 & 0 \\ 0 & 0 & \frac{-\upsilon b^2}{3} & 0 & 0 & 0 & \frac{b^2}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{-\upsilon a^2b^2}{9} & 0 & 0 & 0 & \frac{a^2b^2}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2(1+\upsilon) \end{bmatrix} \tag{24}$$

Where , E and t are the Poisson's ratio, modulus of elasticity of material and thickness of plate respectively.

C.Global Compatibility Matrix [C]

The compatibility matrix is obtained from the deformation displacement relation ($\beta = [B]^TX$). In DDR all the deformations are expressed in terms of all possible nodal displacements and the 'r' compatibility conditions are developed in terms of internal forces i.e., $F_1, ...F_{2n}$, where '2n' is the total number of internal forces in a given problem. The concatenating or global compatibility matrix [C] can be evaluated by multiplying the compatibility matrix [C] by global flexibility matrix [G].

V. Experimental Results and Discussion

The novel region based image fusion algorithm described in previous section has been implemented using Matlab 7. The proposed algorithm are applied and evaluated using large number of dataset images which contain broad range of multifocus and multimodality images of various categories like multifocus with only object, object plus text, only text images and multi modality IR (Infrared) and MMW (Millimeter Wave) images to verify the robustness of an algorithm and simulation results are shown in Fig. 5 to 10.

D.Global Mass Matrix [ML]

The global lumped mass matrix for corner quadrant discretized in 2 x 2 mesh is a diagonal matrix which is obtained by substituting nodal masses in the transverse displacement directions.

$$[M_L] = \frac{\rho abt}{4}\begin{bmatrix} 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&2&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&2&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&4 \end{bmatrix} \quad (25)$$

With ρ being the material density, the global consistent mass matrix is obtained for each element using,

$[M_c^1] = [M_c^2] = [M_c^3] = [M_c^4] = \rho \int_a^{+a} I \int_b^{+b} [N]^T [N] t d_x dy (26)$

where, [N] is the displacement interpolation matrix of size 3 x 12 in which non zero components are as follows:

$N_{11} = N_{1(x)} N_{1(y)}, N_{14} = N_{2(x)} N_{2(y)}, N_{17} = N_{2(x)} N_{2(y)},$
$N_{1,10} = N_{4(x)} N_{4(y)}, N_{22} = N'_{1(x)} N_{1(y)}, N_{25} = N'_{2(x)} N_{2(y)},$
$N_{28} = N'_{3(x)} N_{3(y)}, N_{2,11} = N'_{4(x)} N_{4(y)}, N_{22} = N_{1(x)} N'_{1(y)}$
$N_{36} = N_{2(x)} N'_{2(y)}, N_{39} = N_{3(x)} N'_{3(y)}, N_{3,12} = N_{4(x)} N'_{4(y)}$

After substituting all interpolation components in Eq. (26), consistent mass matrix is calculated for individual element. The global consistent mass matrix is obtained as per the global boundary conditions available in the unrestrained direction in assembled matrix (n x n). This matrix is modified by adding 'r' rows of zeroes at bottom, which is used for frequency calculations.

VI. V.PLATE BENDING EXAMPLES

Simply supported and fixed square plates of size 4000 x 4000 mm as shown in Fig. 3 are analyzed considering plate thickness 't' as 100 mm. The plate is made of steel having modulus of elasticity E as 2.069 x 105 N/mm2 and Poisson's ratio ? as 0.3. Dynamic analysis is carried by using a two- way symmetry. Lower left corner is discretizing into 1x1, 2x2, 3x3 and 4x4 grids by considering lumped and consistent masses at respective nodes.

Steps required for the solution, using IFM and DIFM, are illustrated here with the help of an example of a simply supported plate which is discretized into 2 x 2 mesh.

Step 1: A four-noded rectangular element (2a x 2b) with 12 ddof and 9 fdof is used for discretizing the problem into four elements. The elemental [Be] matrix is obtained by substituting a = 500 mm, b = 500 mm in Eq. (22) and assembled to have a matrix of size 16 x 36.

Step 2: The compatibility matrix for the four elements is obtained from the displacement deformation relations (DDR) i.e. ? = [B]TX. In the DDR, 36 deformations which correspond to 36 force variables are expressed in terms of sixteen displacements (X1, X2...X16). The problem requires 20 compatibility conditions [C] that are obtained by eliminating the sixteen displacements from the 20 DDR's.

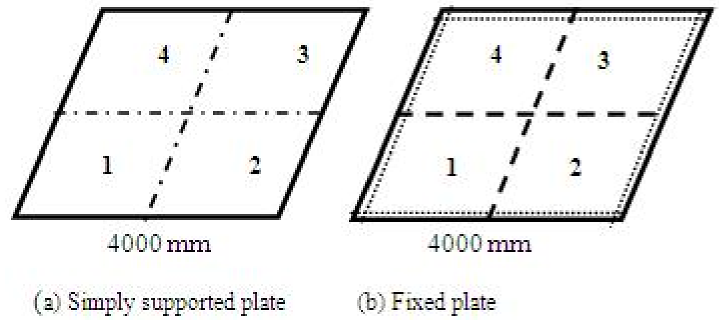

Fig. 3. Plate Bending Option

Step 3: The Flexibility matrix for the problem is obtained by diagonal concatenation of the four elemental flexibility matrices as;

$$[G] = \begin{pmatrix} Ge^1 & & & \\ & Ge^2 & & \\ & & Ge^3 & \\ & & & Ge^4 \end{pmatrix} (28)$$

where sub matrices are calculated using equation (23).

Step 4: The required compatibility matrix of size (20 x 36) is calculated by multiplying the global compatibility matrix [C] of size (20 x 36) by the global flexibility matrix [G] of size (36 x 36).

Step 5: The set of governing equations is as follows.

$$\left[[S] - \omega^2 \left[\frac{[M_L|J|G]}{0}\right]\right] \{F\} = 0 (29)$$

where, $[M_L]$ is the lumped mass matrix, [J] consists of 'm' rows of $[S^{-1}]^T$ matrix and [G] is the global flexibility matrix. Solving above set of equations, using Eigen solver of MatLab, the solution for natural frequency vector ω_{Lx1}, which has L (ddof per node x number of nodes loaded with lumped masses) entries is obtained. Also, by replacing lumped mass matrix $[M_L]$ by consistent mass matrix $[M_c]$ and using the same equation (29), eigen vector ω is calculated

The same problem when solved by the dual integrated force method requires modification in different matrices. The final equation for frequency analysis using dual integrated force method is as follows:

$$\lfloor \lfloor D_{ifmd_{global}} \rfloor - \omega^2 [M_L] \rfloor \{X\} = 0$$

with

$[D_{ifmd_{global}}] = [D_{ifmd(e)^1} : D_{ifmd(e)^2} : D_{ifmd(e)^3} : D_{ifmd(e)^4}] (30)$

where $[M_L]$ is global lumped mass matrix, $[Be^1]$ $[Be^2]$ $[Be^3]$ and $[Be^4]$ are the basic elemental equilibrium matrices and $[Ge^1]$, $[[Ge^2]$, $[Ge^3]$ and $[Ge^4]$ are the equilibrium and flexibility matrices of the four elements. Using eigen solver of MatLab, one can get the solution for natural frequency, force mode and displacement mode shapes for both the problems.

Step 6: Substituting value of 'ω' in Eq. (29), the values of internal forces can be worked out using direct elimination procedure with F_1 = 1.

Step7: Substituting values of all internal moments in equation X = m rows of $[S^{-1}]^T[G]F$, the nodal displacements can be calculated.

Results obtained for frequencies for simply supported and fixed plate are shown in Table I whereas force mode and displacement mode shapes obtained by considering lumped mass criteria are shown in Tables II. Results of convergence study carried out for simply supported and fixed plates are depicted in Figs. 4 and 5 respectively.

TABLE I
RESULTS FOR NATURAL FREQUENCIES

Mesh	Simply Supported Plate Lumped Mass IFM/DIFM	Consistent Mass IFM/DIFM	Exact	Fixed Plate Lumped Mass IFM/DIFM	Consistent Mass IFM/DIFM	Exact
1 x 1	27.04	35.63	34.01	--	--	--
2 x 2	30.76	35.43	34.01	23.19	24.79	18.75
3 x 3	32.64	35.32	34.01	19.46	21.73	18.75
4 x 4	33.90	34.75	34.01	19.03	19.97	18.75

TABLE II
RESULTS FOR INTERNAL FORCES AND NODAL DISPLACEMENTS

Simply Supported Plate Forces {F}		Displacement{X}		Fixed Plate Forces {F}		Displacement{X}	
F_1	1.0	X_1	1.0	F_1	1.0	X_1	1.0
F_2	-3.87	X_2	0.38	F_2	-2.00	X_2	0.78
F_3	1.0	X_3	1.76	F_3	1.00	X_3	0.98
F_4	3.87	X_4	1.0	F_4	2.00	X_4	1.12
F_5	1.0	X_5	-0.87	F_5	1.00	X_5	1.98
F_6	-5.47	X_6	-2.57	F_6	3.7	X_6	1.92
F_7	5.47	X_7	1.0	F_7	1.0	X_7	0.99
F_8	2.87	X_8	1.66	F_8	4.8	X_8	0.78
F_9	1.0	X_9	2.99	F_9	5.7	X_9	0.96
F_{10}	1.0	X_{10}	1.0	F_{10}	1.0	X_{10}	0.89
F_{11}	-1.44	X_{11}	-1.76	F_{11}	-1.6	X_{11}	0.89
F_{12}	1.44	X_{12}	-1.69	F_{12}	1.0	X_{12}	1.0
F_{13}	18.30	X_{13}	1.0	F_{13}	1.6	X_{13}	0.00
F_{14}	-18.30	X_{14}	0.44	F_{14}	-1.0	X_{14}	0.00
F_{15}	7.6	X_{15}	0.80	F_{15}	1.6	X_{15}	0.00
F_{16}	-5.8	X_{16}	1.0	F_{16}	7.8	X_{16}	0.00
F_{17}	5.8	X_{17}	0.0	F_{17}	8.7	X_{17}	0.00
F_{18}	1.0	X_{18}	0.0	F_{18}	-1.8	X_{18}	0.00
F_{19}	-3.3	X_{19}	0.0	F_{19}	0.9	X_{19}	0.00
F_{20}	3.3	X_{20}	0.0	F_{20}	1.0	X_{20}	0.00
F_{21}	4.8	X_{21}	0.0	F_{21}	16.7	X_{21}	0.00
F_{22}	8.9	X_{22}	0.0	F_{22}	1.0	X_{22}	0.00
F_{23}	-8.9	X_{23}	0.0	F_{23}	-15.9	X_{23}	0.00
F_{24}	4.1	X_{24}	0.0	F_{24}	-1.87	X_{24}	0.00
F_{25}	2.9	X_{25}	0.0	F_{25}	-1.87	X_{25}	0.00
F_{26}	1.0	X_{26}	0.0	F_{26}	1.0	X_{26}	0.00
F_{27}	1.0	X_{27}	0.0	F_{27}	1.0	X_{27}	0.00
F_{28}	15.44	X_{28}	0.0	F_{28}	4.9	X_{28}	0.00
F_{29}	-15.44	X_{29}	0.0	F_{29}	-5.3	X_{29}	0.00
F_{30}	4.9	X_{30}	0.0	F_{30}	1.0	X_{30}	0.00
F_{31}	8.7	X_{31}	0.0	F_{31}	11.6	X_{31}	0.00
F_{32}	1.0	X_{32}	0.0	F_{32}	12.34	X_{32}	0.00
F_{33}	-12.9	X_{33}	0.0	F_{33}	-12.34	X_{33}	0.00
F_{34}	12.9	X_{34}	0.0	F_{34}	-11.6	X_{34}	0.00
F_{35}	1.0	X_{35}	0.0	F_{35}	1.0	X_{35}	0.00
F_{36}	1.0	X_{36}	0.0	F_{36}	12.6	X_{36}	0.00

VII. CONCLUSION

1) Forces are primary unknowns of IFM. Its dual form (DIFM) is developed by mapping forces in to displacements at the element level. The equations of IFM and DIFM are mathematically equivalent hence the natural frequencies, forces and displacements obtained by either of the methods are identical.

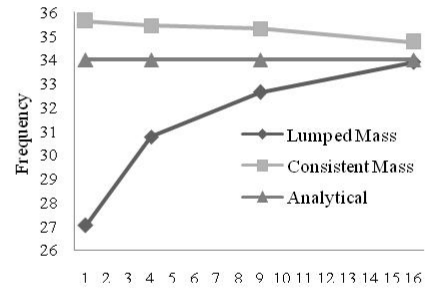

Fig. 4. Graph for simply supported plate.

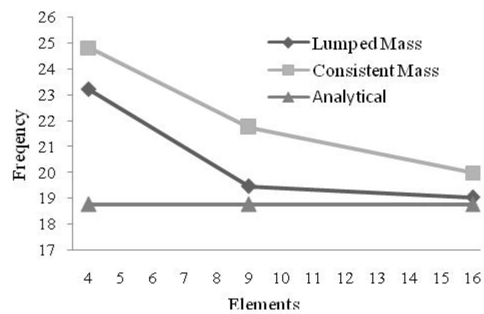

Fig. 5. Graph for fixed plate.

2) The values of natural frequencies for simply supported plate using lumped mass matrix with various discretization patterns are found lower bound to analytical solution, while using consistent mass matrix with the same discretization patterns gives upper bound solution. However, both are found to converge to the exact solution with increase in number of elements.
3) For the fixed plate example, both lumped and consistent mass matrix solutions for frequency are found to converge to the exact solution from the upper side.
4) DIFM is meant for the displacement based eigen value analysis which provides displacement mode animation, whereas dynamic analysis through IFM gives stress mode animation. The stress mode animation technique is helpful for the structures which are subjected to uncertain loading having number of critical failure zones.
5) Both Integrated and Dual Integrated Force Methods can be readily extended to steady state, transient and random vibration problems by making minor modifications in the formulation.

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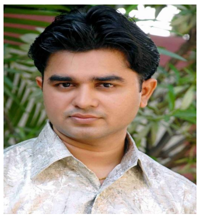

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