30

HEAT TRANSFER ANALYSIS OF HEAT GENERATING WIRE USING FINITE ELEMENT METHOD

Dipak J Parmar and Bhargav M. Chavada

Abstract—This paper describes the numerical results of the heat transfer from heat generating wire at different conditions by finite element method. The parametric effects on heat transfer were investigated. The varied parameters included ambient conditions, as well as the shape of the cross-section. The numerical results show that the type of the medium where the heat generating wire immerges has strong effects on the heatdissipation rate. As the size of the diameter the heat dissipation to the ambient is decreased. The effects of free convection conditions on the wire surface are also significant. However, the effect of the shape of the cross-section is not very strong. The results presented in this paper provide useful information for the application of heat generating wire.

Index Terms-Heat transfer, finite element, heat dissipation

I. INTRODUCTION

H EAT conduction issue is one of the hot topics in the research area. It has wide applications in industry and our daily life, such as electrical furnaces, electrical heating appliances, and ventilation of computer chips. Among those heat conduction problems, the heating elements are the core topic of the heat conduction problems. Different heating elements demonstrate different characteristics in heat conduction. Among many different structures of the heating elements, the wire has been used in many areas because of its compact structure and large thermal expansion. This study has analyzed the heat transfer of the heat generating wire under different conditions by using the finite element method. It provides some useful information on designs and applications of heat generating wire.

II. FINITE ELEMENT METHOD FOR HEATCONDUCTION

In general, heat conduction problems can be studied by experimental, analytical, and numerical methods. In the experimental method, measurements may be very difficult in some applications. In analytical method, it is impossible to get accurate solutions of differential equations in most problems. With the advancement of computational technologies, the numerical simulation is now widely used to help resolving the difficulty. Various numerical methods can be used to solve heat conduction problems, such as finite differential method, variation perturbation. Boundary element method and finite element method. In this study, the finite element method is used. Heat conduction problems could be divided into two categories: the steady state and the unsteady state. In a .three dimensional heat conduction with internal heat generation problem, the unsteady state temperature distribution T(x, y, z,t) in a Cartesian coordinator system in the domain can be solved by the following differential equation'':

$$pc\frac{\partial \mathbf{T}}{\partial \tau} - \frac{\partial}{dx}\left(k\frac{\partial \mathbf{T}}{dx}\right) - \frac{\partial}{dy}\left(k\frac{\partial \mathbf{T}}{dy}\right) - \frac{\partial}{dz}\left(k\frac{\partial \mathbf{T}}{dz}\right) - Q = 0;$$
(1)

The equation is a heat balance equation, where t is time, k is heat conductivity, p is the density, c is the heat capacity, and \mathbf{Q} is the rate of the heat generated per volume from the heat source of the material. The boundary conditions for solving this equation should meet the following conditions in the Ω domain:

$$\Gamma_1 + \Gamma_2 + \Gamma_3 = \Gamma$$

where, $\Gamma_1 + \Gamma_2 + \Gamma_3 = \Gamma$ are 1^{st} , 2^{nd} and 3^{rd} types of boundary conditions, respectively. If T and Q are unchanged with the time, i.e. $\frac{\partial T}{\partial \tau}$ -o, the Equation (1) becomes the steady state heat conduction equation. This study considers the steady state heat conduction problems with the heat generation. In the finite element method, Galerkin's approach can be used to convert the heat conduction equation (1) to the finite element equation". The corresponding finite element matrix of the unsteady state heat conduction is given as follows:

$$CT + KT = F \tag{2}$$

Where, C is the matrix of the heat capacity, K is the matrix of the heat conduction, T is the matrix of temperature, and F is the matrix of the temperature loading. In Equation (2) if the first term becomes zero, the equation becomes the steady state heat conduction finite element equation. For a 2-Dimentional heat conduction problem, the elements of the matrix could be expressed as

$$K_v = \sum_e K_v^e + \sum_e H_v^e \tag{3}$$

$$F_i = \sum_e F_{Qt}^e + \sum_e F_{qt}^e + \sum_e F_m^e \tag{4}$$

Each term in Equations 3 and 4 can be expressed as followings: The contribution of each cell to the matrix of heat conduction:

$$K_{ij}^{e} = \int_{Q^{e}} \left(k \frac{\partial N_{i}}{\partial x} \frac{\partial N_{i}}{\partial x} + k \frac{\partial N_{i}}{\partial y} \frac{\partial N_{i}}{\partial y} \right) \partial \Omega$$

Dipak Parmar is with Department of Mechanical Engineering Institute of technology Nirma University Ahmedabad-382481,India and Bhargav M. Chavada is with G.H.Patel College of Engineering,V.V.Nagar 38xxxx, India dipakjparmar@nirmauni.ac.in

The correction of heat transfer boundary condition of each cell to the matrix of heat conduction:

$$H_{ij}^e = \int_{\Gamma_a^e} h N_i N_j \partial \mathbf{I}$$

The temperature loading generated by heat souse in each cell:

$$F_{Qi}^e = \int_{\Omega^e} \rho \ \mathbf{Q} N_i \ N_j \ \partial \ \Omega$$

The temperature loading from constant surface heat flux:

$$F_{qi}^e = \int_{\Gamma_2^e} \mathbf{q} N_i \ \partial \ \Gamma$$

The temperature loading from the convection on the surface:

$$F_{Hi}^e = \int_{\Gamma_a^e} HT_a N_i \partial \Omega$$

Where, Ni, Nj are the weighing functions. This approach has converted the steady state heat conduction differential equation in a space domain ? into the initial value problem of an ordinary integration equation of temperature T at a number of nodes. For a 3-Dimentional heat conduction problem, a similar method can be used for deriving the corresponding finite element equation of heat conduction.

III. RESULTS AND DISCUSSION

Based on the above heat conduction analysis using the finite element method, the heat transfer from a wire at different conditions was analyzed in this study. A wire is a typical structure used for generating heat. Figure 1(a) shows the schematic of a wire. The shape of the cross-section of the wire is circular. The finite element model of the wire is shown in figure 1(b).



Fig. 1. Model of solid wire

Table 1 lists a typical set of parameters conductivity, ambient temperature used for the baseline case in this study. In theoretical analysis, an infinite wire is assumed. In the numerical analysis the length of the wire is taken 5m and adiabatic condition is assumed at the two ends of wire.



Fig. 2. FEM of wire

Diameter of wire	0.5m
Length of wire	5m
Heat generation rate	30 kW/m3
Convection heat transfer coefficient	60 W/m2K
Conductivity of Pure Copper	401 W/m K
Ambient Temperature 30oC	
Table: - 1 Set of parameters for wire	

1 Set of parameters for wire.

Based on the above heat conduction analysis using the finite element method, the problem of heat generating wire is solved by ANSYS 9 analysis software for above set of parameters. The temperature distribution with in the heat generating wire is shown in figure 2.



Fig. 3. Temperature distribution in Heat Generating Wire

A. The Effects of Medium Type

Heat transfer between a heat generator and its surroundings is affected by the type of mediums. In this paper, calculations are performed for a pure copper wire with the circular crosssection in still water, air and a magnesia brick. Only heat transfer by conduction in the medium is considered. Figure 3 shows the surface temperature versus the heat generation at different types of mediums. At the same heat generation



Fig. 4. Temperature distribution in Heat Generating Wire

rate, the surface temperature is different at different medium. The air and water has small heat conductivity, the corresponding surface temperature is therefore higher. However, the Mg brick's heat conductivity is high, causing high heat transfer rate between the wire and the Mg brick and thus low surface temperature of the wire. In general the smaller the heat conductivity of the medium, the less heat transfers between the wire and the medium. It is also clear that the surface temperature increases linearly with the heat generation. However, the slope is different at different medium. The smaller the heat conductivity of the medium, the larger the slope, which indicates again less heat is transferred.



Fig. 5. Effect of Medium type

B. The Effects of Free Convection

When the free convection in the air is considered, the surface temperature decreases significantly as shown in Figure 4. This indicates significant impacts of the free convection to the heat transfer between the wire and the air. In the calculation, the free convection coefficient is specified based on experimental data. When heat generation increases, the surface temperature also increases, but with smaller increment because the free convection causes lower heat resistance. The above results indicate that the trend of the surface temperature versus the heat generation is very similar for different medium. In the following parametric studies, only air will be used as the



Fig. 6. Effect of Free Convection

medium, free convection is considered, and the constant heat generation is considered.

C. The Effects of Materials

Different material has different heat conductivity. Figure 5 shows the effects of material of wires. It plots the relationship between the surface temperature and the thermal conductivity for 6 different materials. The surface temperature decrease slightly with the increases of heat conductivity when the heat conductivity is less than 50 W/m.K and then show little effects afterwards.

D. The Effects of Diameter

Figure 6 shows the effect of the diameter of the wire on the surface temperature. For a given heat generation rate, when the diameter increases, initially the surface temperature decreases very rapidly and then the surface temperature decreases grad-ually.



Fig. 7. Effect of Materials

E. Effects of Heat Generation Rate

Figure 7 shows the effect of heat generation rate. As expected, the higher the heat generation rate, the higher the surface temperature.



Fig. 8. The Effects of Diameter

F. Effects of Ambient Temperature

This paper has also studied the effects ambient temperature on heat transfer. As shown in Figure 8 at low ambient temperature, there is little change of the surface temperature with the ambient temperature. At higher ambient temperature, the surface temperature increases with the increase of ambient temperature.



Fig. 9. Effects of Heat Generation Rate

IV. CONCLUSION

This paper has established the heat conduction model of the finite element method based on the basic theory of heat conduction. Numerical analysis was conducted for various heat generating wire. A number of computations were performed to investigate parametric effects on heat transfer. It is found that the type of medium and the free convection have significant impacts on heat transfer between the wire and the medium. The effects of the area and the diameter of the wire are relatively large. There is almost no effect of type of materials. In general, the increase of heat generation rate and ambient temperature causes higher surface temperature.



Fig. 10. Effects of Ambient Temperature

REFERENCES

- [1] J.P. Holman, 1976. "Heat Conduction", McGraw-Hill. pp.5.
- [2] T.M. Shih,1984, "Numerical Heat Conduction", Hemisphere Publishing Corporation. pp.225
- [3] F.P. Incropera, and D. P. Dewitt. 1981, "Fundamentals of Heat Conduction", John Wiley Sons, Inc.pp.519-527.
- [4] Kun Xiangqian, 1986, "the applications of the finite element method in heat conduction", science publisher, Beijing.
- [5] Bian Jiantao, 2002 "Numerical analysis of Heat transfer from heating coil"



Dipak Parmar is Assistant professor in the Department of Mechanical Engineering, Institute of Technology, Nirma University, he has obtained B.E. From S.P. University in 2003 and M.E. in Jet propulsion and gas Turbine in 2006 from the M.S. University. He is life member of ISTE.



Bhargav Chavda is Asst. professor in the Department of Mechanical Engineering, G. H. Patel college of Engineering at Vallabh Vidyanagar, he has obtained B.E. in mechanical engineering from L.E. college of Engineering in 2003 and M.E. in Thermal Science from M.S. University Baroda in 2006. His research interest are Heat transfer analysis, Fluid flow analysis and Finite Element method.