Review Article: 96-120

# **Review Article:**

Jean-Claude Martzloff, Le calendrier chinois: Structure et calculs (104 av. J.-C.-1644). Indétermination céleste et réforme permanente. La construction chinoise officielle du temps quotidien discret à partir d'un temps mathématique caché, linéaire et continu. Paris: Honoré Champion, 2009, 464 pp.

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"Omnem movere lapidem."

This book opens with a Latin motto from Erasmus: "Omnem movere lapidem" – leave no stone unturned. Like Copernicus warning his readers on the title page of the De Revolutionibus that it was written for mathematicians, not for the faint of heart, the words of Erasmus serve here to alert the reader to the thoroughness and detail that await, and indeed, the author has been indefatigable in his search to understand not only the history of Chinese calendars, but the very essence of Chinese calendar making. Martzloff is interested in both the structure of the calendars and their calculation – the various mathematical techniques that were devised by the Chinese over several millennia to set their calendars. This was not simply a means of almanac making to aid farmers with agricultural seasons, but was of grave political and social significance, a matter of state that explains why emperors devoted substantial resources to employ a retinue of expert astronomers/astrologers skilled in making the instruments and applying the mathematics that would predict phenomena like solar and lunar eclipses, the first appearance of a new moon, solstices, equinoxes, the exact timing of noon and midnight, and planetary conjunctions, among other important matters.

For many readers, this book will divide naturally into two parts, the first 100 pages or so constituting a very readable, detailed overview of the basic features of the Chinese official calendars and their construction; the

rest of the book is devoted to the technical details of how these calendars were actually constructed and used. This is by no means a matter of simply understanding one calendar and then applying the results across the board, for each calendar has its own peculiarities, from the data one may champion over another, to the number systems they might apply (often with the same words applied to the divisions of time but with wholly different numerical significance), so that unraveling all the various skeins that serve to make up any single calendar is indeed an imposing task, but one that Martzloff, nevertheless, has managed to accomplish and for which anyone wanting to know the mathematical details will be grateful.

In Part I, devoted to "Le calendrier chinois," Martzloff characterizes the Chinese calendar as an "objet paradoxal," on the one hand a "powerful conservator of ancient traditions," loathe to abandon ancient elements and resistant to innovations. Martzloff quotes Michael Loewe about the esoteric signs and expressions of Qin and Han calendars that persist in the calendars to be found today in Taiwan and Hong Kong. And yet, on the other hand, between 104 BC and 1644 (the timeframe of calendars Martzloff considers), some 90 proposals were made to reform the calendar, an average of a new calendar every 35 years. Although some lasted for centuries, others were abandoned after only a few decades or less. Martzloff likens this paradoxical situation to the optical illusions of M. C. Escher, notably of stairways that seem to go up and down at the same time, one seeming just as possible as the other.

In the case of the Chinese calendar, however, Martzloff offers an explanation for the seeming paradox of its static reflection of tradition while giving constant attention to its reform. He suggests that the calendar should not be regarded in two different ways, in terms of its tangible sense of continuity and then in terms of its ceaseless reforms; instead, these should be regarded as essentially connected, each an aspect of the same thing, each a reflection of the other.

Martzloff explains it this way. If the calendar for a given number of years is considered as a structure (A, B) where A represents the calendar from the point of view of the computational techniques used to establish it, and B is the manifest structure of the annual calendar, i.e. a list of days and months organized in various ways and related to all sorts of other elements, then A determines the content of B in a unique way, although as Martzloff says, the inverse is not true. Knowing B, i.e. the calendar for a given year, it is not possible to reproduce the computations that were used to construct it without already knowing them.

Now, to appreciate how the calendars changed over the course of time, it is important not to confuse the two. To appreciate how changes in A

might affect B, it is also necessary to distinguish two aspects of the structure of B, one invariable, the other not. The invariable part of B is the fact that between 104 BC and 1644, the days of the Chinese calendar were always numbered according to a sexagesimal cycle, to which there was no exception. Likewise, the Chinese calendar never departed from months that were either 29 or 30 days long, and the number of solar periods into which the year was divided was always 24 (denoted by 24 solar "airs," about which more below). Related to these invariable aspects of the Chinese calendar are a number of other permanent elements that contribute to a sense of its essentially unchanging character.

The variable part of B, however, is not so readily apparent, but relates to the different types of successions of 29- and 30-day months. Usually a year consisted of a regular alteration of 29- and 30-day months, but not always, and without going into the details here, it is the exceptions and the rules for them that affect the computations of A.

Although the goal of the civil calendar was to mirror as closely as possible the appearances of the heavens, there were nevertheless "dérèglements astronomiques contingents" that reflected deviations from the calculations, due to what were taken to be arbitrary motions of the planets or the occurrences of heavenly phenomena the calendar may have failed to predict, such as eclipses, or predicted eclipses that failed to occur. As Martzloff puts it for cases of apparent anomalies in the motion of the Sun: "ce type d'analyse qui attribue au soleil une sorte de liberté cinématique, inaccessible à l'analyse rationelle, ruine à la fois la possibilité d'accorder du crédit à la possibilité d'existence de lois de nature, au moins dans le cas du soleil." The important question of how Martzloff's study of the Chinese calendar serves to illuminate Chinese understanding of natural phenomena in general, and how it differed in essential ways from western assumptions about nature and the universe, will be considered at the end of this review.

As for the substance of Martzloff's book, Part I of *Le calendrier chinois* distinguishes between the civil calendar that was printed and circulated for use throughout the empire, and the official astronomical calendars, that were only to be known within court circles as a state secret of the most important rank. Concerning the civil calendars, these generally sought to conform with heavenly phenomena, but from time to time the heavens might depart from previously computed positions due to anomalies that did not necessarily mean the computations that had determined the calendar were themselves wrong.

Thus constructing the civil calendars was at times an ad hoc matter of keeping in mind various constraints that had to be observed in addition to the direct mathematical calculations that had to be made. There were

an abundant number of rules that were to be followed in general but might be violated from time to time, and several of the more important of these are discussed in greater detail below. One important question is the extent to which these extra-computational constraints might override what the mathematical assumptions and calculations might otherwise determine as a date that for pragmatic or other reasons might be changed. Martzloff discusses violations to the basic rules through the book, but his primary interest is first to understand the mathematics of the computations, the rules that were followed, and then to explain the reasons why anomalies nevertheless occurred. It turns out that in fact Chinese astronomers expected anomalies to occur as a matter of course, and this in turn added an element of arbitrariness from time to time in determining their calendars despite the accuracy of their computations.

Among the details Martzloff carefully explains are the basic fundamentals of the Chinese calendar, determination of the day, the solar year, the 24 "solar airs" (*souffles solaires*), the 72 seasonal indicators (*hou*), the 5 agents (*wu xing*), the lunar year, ordinary and intercalary months, the structure of the lunar year, the relative occurrences of "full" and "empty" months, origins of the lunar year, the numeration of the lunar years, etc. Martzloff devotes as entire section to pseudo-cycles, which include decimal cycles, duodecimal and sexagesimal cycles, the system of 9 palacecolours, the planetary week, the 28 mansions, the *jianchu* and *nayin* cycles. A final discussion of holidays, special days, and irregular years ends the first part of the book and sets the stage for the rigorously detailed discussion of the calendars and their calculations that follows in parts II and III of the book.

Before turning to more technical matters, however, it is worth considering that the Chinese concept of li 曆 (calendar) has a long and complex history of its own. As Martzloff explains, as early as the *Shujing* it refers not only to lunisolar phenomena, but includes the planets as well, although the Chinese civil calendars (both official and nonofficial calendars) do not mention eclipses or the planets. While the notion of the civil calendar is more limited than that of *li*, the earliest such calendars from the second-century BC provide lists of daily sexagesimal dates indicating new moons and some solar phenomena, but the recording of this information is not systematic. By the Tang dynasty (618-907), and with the greater availability of paper, the civil calendars became more detailed in their listing of auspicious days, or days to avoid doing certain things, like visiting friends, undertaking construction projects, practicing acupuncture, etc. New day counts were introduced, including planetary weeks, and later the 28-*xiu* 宿 enumeration of days that followed the reform. No doubt these were innovations prompted by the numerous divination systems of Medieval China.

In contrast to civil calendars, *nong li* 農 曆 or agricultural calendars are not so ancient and according to Martzloff, are most likely a product of the nineteenth century. The most ancient calendars before and during the Han were variously called *rili* 日 曆, *liri* 農 日, or *rishu* 日 書. Those from Dunhuang are called *juzhuli* 具 注 曆 ("annotated calendars," a reference to the fact that in addition to the lunisolar data of the civil calendars, these added information about auspicious and inauspicious days, divination, etc.). More recent calendars are simply called *li* 曆, and references to *huangli* 黃 曆 (the "yellow" or "imperial calendar") are again fairly recent, probably a term from the Qing dynasty (1644-1911).

At the beginning of the twentieth century, an immense collection of more than 40,000 manuscripts in the form of scrolls, books, isolated folios, separate sheets of paper, paintings, and various odd fragments dating from the Tang (618-907), the Five Dynasties (907-960), and the early Song (960-1279), written primarily in Chinese and Tibetan, but also in Khotanese, Pali, Sogdian, Tibetan, etc., were discovered in grotto 17 of the cave complex of the 1000 Buddhas at Mogaoku, 20 kilometers to the south-east of Dunhuang, in Gansu Province. Together with Marc Kalinowski and Alain Arrault, Martzloff has studied more than 50 or so calendars from those among the material from Dunhuang [Arrault and Martzloff, 2003]. Most of these calendars are non-official, some are quite fragmentary, and their dates often are not exactly the same as those listed in the official Chinese chronologies (their solar and lunar dates frequently differ by as much as one or two days). Possibly some of these represent exercises, never meant to be used as real calendars. Of the calendars, all are manuscripts with the exception of three that are printed. Among the printed almanacs is the British Library's S-P6 recto, which Martzloff describes at length. It is characterized by its detailed contents and complex layout, and lists the months and successive days throughout the year 877, not only with written information but also with diagrams and designs relative to all sorts of divinatory procedures.

Some calendars include planetary information and therefore might best be called "ephemerides," since they are the result of computations but without any clear astrological character, although as Martzloff has shown, those that include the so-called fictitious planets, *rahu* and *ketu*, are borrowed from Indian astrology and are associated with ascending and descending nodes of the moon. However, Chinese ephemerides are not associated as such with any astrological significance (although they were used at times for astrological purposes). According to Martzloff, only one of these ephemerides has survived from the Ming dynasty, and a very few from the Qing are also known, including one in the Biblio-

thèque nationale in Paris. But given their extreme rarity, Martzloff suggests they should be referred to as "secret" ephemerides, and he has expressly not included these in the present book, saying that to do so would have required reconstructing their underlying calculations, an enormous undertaking that would have taken him far afield from the rich material that this book focusing on Chinese civil calendars has now made available.

An important aspect of all the calendars that Martzloff addresses is the indeterminacy of astronomical phenomena that was a recurring problem for Chinese astronomers, who as already mentioned did not conceive of the heavens as controlled by some sort of inexorable system of gears and wheels and therefore susceptible of exact mathematical prediction. Nevertheless, the Chinese did devise complex algorithms to help them approximate the apparent regularity of astronomical phenomena, despite the anomalies they clearly understood to doom any particular calendar or system to eventual failure.

Unlike solar calendars, like the Julian and Gregorian, or the Islamic lunar calendar, the Chinese calendar is a lunisolar calendar based on both cycles of the sun and moon. Chinese calendrical calculations begin from an arbitrarily set Grand Origin, a time-zero from which the calendar would be calculated, but different calendars differed considerably as to what the Grand Origin should be. Traditionally, the calendar is said to have been invented by the mythical Emperor Huangdi in the sixty-first year of his reign (2637 BC). There was then a sixty-year cycle governed by a combination of what the Chinese call the 10 celestial stems and the 12 earthly branches that was used to identify the years elapsed since the Grand Origin.

It seems that the idea of referring the Grand Origin to the time of the Yellow Emperor was a device invented by historians wishing to introduce something in Chinese chronology similar to BC/AD. As far as calculations are concerned, the Grand Origin was meant to antedate all possible historical events. Although Martzloff defines the Grand Origin in terms of the lunisolar calendar, in general it is a concept that also implies an initial grand conjunction of the planets, both spatially and temporally, a complex problem because it involves planetary calculations in particular.

Concerning the overall concept of the Grand Origin in Chinese astronomy, it is important to distinguish between its significance from the point of view of calendrical calculations themselves, and secondly, in terms of the role the Grand Origin plays in year counts with respect to the civil calendars. This can all become very complex. Strictly speaking, the Grand Origin only concerns the official astronomical canons (OAC here, or C.A.O. = *canons astronomiques officiels* as Martzloff abbreviates them), and was never an issue for year-counts in the civil calendar. The civil calendars, in fact, never mention a Grand Origin, nor do they make reference to the sometimes extraordinarily large number of years at which some Grand Origins were fixed (the largest was that of the *Chunyou* 淳 祐 calendar: 129, 267, 647 years before this calendar was officially adopted in 1250 AD). Instead, in civil calendars time is referred to reign-periods of the emperors. This system of reference seems to have begun in 140 BC. Before then, dating systems referred to the reign-year of a given ruling lord (in the reign period system, the name of the Emperor is never used).

Thus there are two different systems of time reckoning that come into play, one for calculations and the other for civil calendars. Martzloff distinguishes the two by referring to "deep" structures of the OAC and "surface" structures of the civil calendars. What is important here is the way in which these differentiate between two very different conceptions of time, one continuous, the other discrete. Thus Martzloff challenges the suggestion that the Chinese concept of time was one comprised of discrete cycles.

Earlier, Martzloff was involved with the Grand dictionnaire Ricci de la langue Chinoise (2001), and the philological experience that project required with respect to mathematics, astronomy, and the calendar is reflected on virtually every page of this book. Moreover, with the CNRS group studying Chinese civilization, as noted above, Martzloff has also worked on aspects of divination and the description, classification and dating of the fifty or so related manuscripts from Dunhuang, preserved in the Bibliothèque nationale and the British Library, and this experience also proves crucial to the success of this book. In fact, one of the most important discoveries related to the history of Chinese mathematics concerns what Martzloff has discovered about the origins of a written form of zero. Before the Song dynasty (960-1279), the Chinese had a nonwritten representation of zero that was purely operational-whenever numbers were put down on a surface or "counting table" with rods representing numbers according to their decimal place values, an empty space indicated any position that was zero in value.

A written form of zero in China is another matter. Above all, it is necessary to appreciate the fact that for a Chinese mathematician, numbers had a physical reality – the counting rods. In the case of written numbers there is no material existence, and if the result of a calculation does not contain tens or hundreds, for example, in writing 1007, a Chinese mathematician would simply record this as "one-thousand and seven." This would make it clear that there were no tens or hundreds. The written

language can express any number without ambiguity, with no need for a zero symbol. What is interesting is that the earliest occurrence of a Chinese symbol for zero is to be found in the official astronomical canons.

Prior to 1247, when zero first appeared in a printed work as a circle in the Shushu jiuzhang 數書九章 (Book of Calculations in Nine Chapters), the Chinese not only knew of the zero as an empty space, essential for the practical operations of arithmetic and independent of the writing of numbers, but they also knew of two written forms of zero. At the beginning of the Tang dynasty (618-907), a single text survives indicating the Chinese knew of a written zero, in the form of a point, but this occurs in an astrological text and may well be the result of Indian influences. Here Martzloff makes the very interesting remark that Chinese mathematicians regarded the Indian method of writing down numbers and doing their mathematics in writing as extremely fallible, one author declaring that if they reached the correct result, it was purely by chance. The Chinese preferred the accurate algorithmic character of their moveable counting rods that were placed on a flat surface in place-valued positions. The rods were easily moved to perform the basic arithmetic operations mechanically, quickly and accurately, and hence a reason for disregarding the written form of zero as a point.

The other very wide-spread use of a written zero was due to the demands of recording astronomical data. For this a form of zero was introduced that has escaped notice in general histories of Chinese mathematics, and Martzloff provides detailed evidence for its widespread use. This written zero, the character *kong* 空, has the general meaning of "void," and appears in astronomical tables of the *Dayan li* 大 衍 曆 (729-761). It is well-attested in the official astronomical canons of the Song (960-1279) and Yuan (1279-1368) dynasties, and almost always in astronomical tables whether lunar, solar, or planetary. There is good reason why this use of *kong* for zero appears in astronomical texts. In fact, where there is no context to make explicit the decimal value of a string of numbers, the need in tables to represent data concisely, but without any ambiguity, explains why the use of a single character for zero was not only useful but a necessity.

Apart from such details about the mathematics Chinese astronomers used, the major aim of Martzloff's book on Chinese calendars is to use the mathematics of calendar construction to reveal the principal underlying structures of the computational techniques required to establish the official Chinese historical calendar. The period Martzloff addresses begins in 104 BC and concludes in 1644. These dates are dictated in part by the state of manuscript and printed sources. Prior to 104 BC, no treatises survive, not even fragmentary accounts explaining how to calculate the calendar, although a number exist after 104 BC. 1644, on the other hand, marks a dramatic break with the preceding traditional period of Chinese astronomy, with the successful introduction of a totally new calendar constructed by astronomers of the Jesuit mission in China. Whereas the Chinese calendars relied on sophisticated mathematical algorithms, the computational methods for the calendars inspired by the Jesuits depended upon geometric techniques going back directly to the Greek Ptolemaic tradition. Given that Martzloff is interested in the classic Chinese methods of calculation, it is entirely fitting that he brings his study of Chinese computational astronomy to an end with the first adoption of a Jesuit calendar in 1644.

Remarkably, from what might seem "a sort of impenetrable and disorganized chaos" of the many different calendars the Chinese produced over the centuries, it turns out that there are only a limited number of different techniques that the Chinese astronomers actually used to calculate their official calendars. But knowing how the calendar was calculated, it is possible to reveal its underlying structure. With this knowledge in hand, Martzloff shows how the theoretical calculations actually make it possible to reconstruct calendars that correlate well with known official calendars. Unfortunately, only a very small number of authentic official calendars have survived to the present, but using those that are available, Martzloff can evaluate the veracity of his analysis. However, it is always possible that the calculations will not necessarily mirror an actual past calendar in any given year because the Chinese calendar depended not only on calculations, but also on other arbitrary considerations, usually of a divinatory or political nature.

The basic solar year was determined as the whole number of days between two consecutive winter solstices, usually either 365 or 366 days. Thus the Chinese solar calendar begins on approximately December 22 of the Gregorian calendar. Apart from the year, the most fundamental unit of time for the Chinese calendar, the day, was further subdivided to provide more exact times for sunrise and sunset, the duration of night, full and new moons, the solstices and equinoxes, among the celestial phenomena. However, this subdivision of the day could take many forms. A good example of how complicated the calculations can become is the *Dayan li* (729-761), which divided the day into 3040 parts, meaning that the solar year was determined to be 365 + 743/3040 or 1,110,343/3040 days in all.

To determine the solar terms, essential markers on the ecliptic that determined the equinoxes, among other fundamental phenomena, it was necessary to divide the year into 24 parts, and in the *Dayan li* the solar term is given as 15 yu 餘 664 miao 秒 7 (15 + 664/3040 + 7/(3040×24))

days, where 1 day=3040 yu and 1 yu = 24 miao [Martzloff 2009: 115]. Here the *yu* and the *miao* are analogous to minutes and seconds, but with what appear to be rather peculiar equivalences between them. At first this seems quite odd, but Martzloff accounts for the rationale based upon the numerological significance of 3040. As the Dayan li itself explains, the calculations that lead to this number are the following: 1200/4=300, and 300×10 = 3000, 5×8 = 40, and 3000+40 = 3040. This is based on numerological aspects of the Book of Changes, the Yijing. These numbers are further explained in the canon as follows: the 1200 comes from the "numbers of heaven and earth" by doubling the numbers from 1 to 10 divided into two groups of 5 elements, added and then multiplied together yielding (1+2+3+4+5)(6+7+8+9+10) = 600, and this when doubled gives 1200. This is divided by 4 because the divinatory technique mentioned in the same work divides the set of divinatory rods into 4 equal groups. Thus the solar year is calculated to be 365 + 743/3040 or what amounts to 1,110,343/3040 days long.

In another calendar, the *Jingchu li* 景 初 曆 (237-451), the lunar month is equal to 134,630/4559 days, determined according to the original text by dividing the day into 4559 parts. The reason for this "bizarre" divisor is not known, but as in the case of the *Dayan li*, Martzloff suggests it is without doubt the result of arithmetic manipulations of a numerological order. Remarkably, the value is correct for the length of the lunar month. This may seem surprising, but in reality, what may appear as a "confusion of genres" is explained by the fact that given any numerator or denominator, there is always a corresponding fraction that can give a good approximation to the value of the lunar month.

The great variability of these numbers from one canon and its calendar to another and the different units Chinese astronomers employed to express subdivisions of the day may suggest that they are not governed by any regular principle. However, this is not the case, and Marzloff shows that analysis of the arithmetic structure of the numerical expressions reveals that the numerical decompositions can be obtained through a quasimechanical series of predetermined operations. In the example of the Dayan li, the results arise from division where the divisors are successively equal to 3040, 24, 72, and finally, 120. What Martzloff stresses is that these should not be considered as metrical units but more like operational units, i.e. where the value of the *yu* or *miao* is determined by arithmetic operations. The terminology in fact bears this out. The fact that the first order unit, yu, means "remainder," shows that the remainder here is not a unit of measure but the result of an arithmetic operation. And the second unit, the miao, is also the result of an arithmetic operation. It varies constantly depending upon the arithmetic

operations carried out to obtain the decompositions of the solar year, for example, into a given number of parts, resulting from the structure of the calendar.

Sexagesimal divisions appear for the first time in the *Jiuzhi li* 九 執  $\mathbb{B}$ , a predictive astronomical treatise adapted from Indian sources at the beginning of the eighth century AD. They also occur in the *Huihui li* 回  $\mathbb{B}$ , in astronomical tables that the Chinese imported from the Islamic world at the beginning of the Ming dynasty (1368-1644), and then often in the seventeenth and eighteenth centuries when the Jesuits introduced mathematics and astronomy that ubiquitously used base 60.

A final step in the rationalization of traditional calendrical computations was made with the *Shoushi li* 授 時 曆 (1281-1367) and its successor, the *Datong li* 大 統 曆 (1368-1644), the last two official astronomical canons before the Jesuits became seriously involved in determining the Chinese calendars. For both the *Shoushi li* and the *Datong li* calendars, the times and angular distances are more or less regular in adopting a centesimal system, whereby 1 day = 100 ke 刻; 1 ke = 100 fen 分, 1 fen = 100 miao 秒. The lunar month, expressed in Chinese as 29 days, 53 ke, 5 fen, and 93 miao, is therefore easily converted into a decimal fraction: 29.530593.

In addition to the lunar month, there were also the "solar terms" mentioned previously that were determined by dividing the ecliptic into 24 parts, called *jieqi* 節 氣, sometimes translated as "solar nodes" ("souffles solaires" in French; this reflects the role of qi as "air" or "breath," a fundamental principle in Chinese cosmology and medicine thought to animate everything in terms of fluctuations of the primordial opposites of *yin* and *yang*). Martzloff denotes the 24 solar terms  $q_1, q_2, ..., q_{24}$ , which follow one another in intervals of 15 or 16 days. Together, the dates of the two solstices, two equinoxes, and the beginnings of the four seasons determine the bajie 八 節, or "eight nodes," each determined by a particular solar term. In addition to the four seasons of the astronomical calendar determined by the solstices and equinoxes, there are the four seasons of the civil calendar determined by the "four debuts," which start one-andone-half months after the corresponding solstice or equinox, i.e. the fall of the civil calendar begins not with the autumnal equinox, but six weeks later. Other solar terms are used to mark the beginning of the hot and cold periods, and of various atmospheric types of precipitation or humidity. The full list of 24 solar terms first appeared in the OAC of the Han shu (History of the Han Dynasty). At various times there were slight variations in their order or the names applied to the solar terms, although they always reflected particular meteorological or agricultural situations, and bore such evocative names as "height of summer," "beginning of au-

#### 106

tumn," "clear light," "insects awakening," "full grain," and "bearded grain," among others.

Another division of the solar year was a further refinement of the 24 qi into 72 *hou*  $\not\in$ , or phases, additional seasonal indicators separated usually by 5 but sometimes 6 days. These are attested in Chinese antiquity, in the chapter on rituals in the *Liji* (Record of Rites). Like the 24 solar terms, each of the 2 *hou* has a particular name usually related to natural phenomena associated with changes in the seasons "where nature is the theatre of the course of the year." These refer, for example, to the melting of ice, rumbling of thunder, budding of flowers, blossoming of peaches, arrival of swallows. Other names with more original references include the metamorphosis of the eagle into a turtle dove, the mole into a quail, the sparrow into a scallop, or a bird into an oyster (presumably when they dove into the sea). Spontaneous generation is also included, as is a name reflecting the creation of fire-flies or glow-worms from decaying plants.

Calculation of any particular OAC was founded on all sorts of numerical constants. Martzloff explains that these are like the pieces of a puzzle that need to be assembled before it is possible to discern an intelligible structure. The durations of different sorts of months and years needed for the calculation of the calendar are almost always presented as separate pieces, the results of mechanical computations at various stages in the process of determining the full calendar for any given year. Numbers seem to appear out of nowhere, a given numerator or a particular constant may be introduced at a certain point, but with no indication of the connection they bear to the numbers from which they are derived.

To make matters worse, the astronomical nomenclature associated with the Chinese numerical toolkit is prolix and varies from one OAC to another. The same term may or may not refer to constants. In order to know the value of a particular constant, it is necessary to reconstruct the various steps computationally that were followed to produce the many elements reported separately, as a numerator or denominator, or a whole number of days cited alone, knowing that sometimes the same concept may be explained with the help of many different units, and may appear many times in the same list but under different forms.

To establish the calendar for a given lunar year with the help of an official astronomical canon, the calculations begin by determining the mean values of its fundamental elements, lunar and solar. The true values of the winter and summer solstices – defined by the solar terms  $q_1$  and  $q_{13}$  – are always equal to their mean values. Beginning with the Tang (618-907), the phases of the moon begin to be calculated in true values and the mean values are subject to a double correction – solar and lunar – making the calculations more complex.

Calculation of the solar and lunar correctives calls into play not only the synodic month but the mean anomalistic lunar month. Martzloff explains that Chinese texts never define this, but it can be identified from its mean numerical value, 27.5546 days, and by its role in the calculations. Its value is equal to the mean time separating two successive passages of the moon from its perigee (or apogee), i.e. its point closest to (or furthest from) the earth, and it takes into account two new temporal parameters, the *ruqi*  $\lambda \neq$  and the *ruli*  $\lambda \not\equiv$ , all of which Martzloff explains in considerable detail.

Thanks to his meticulous mathematical analysis of the Chinese calendar, Martzloff succeeds in revealing the structural coherence of the approach the Chinese took to the construction of their calendars. He is then able to apply this technical apparatus to recreating the Chinese calendar for any given year. In doing so, he chooses a representative sampling of calendars that hold special interest for various reasons, and Part III of his book is devoted to their detailed discussion, including the *Sifen li* (25-220), the *Jingchu li* (237-451), the *Xuanming li* (822-892), the *Shoushi li* (1281-1367), and the *Datong li* (1368-1644).

The *Sifen li* 四 分 曆 (25-220) was adopted in 85 AD by the Late Han (25-220) and is notable for its longevity, 179 years, and its ability to survive changes in dynasty. After the division of China into three kingdoms, the Wei (220-265) and Shu (221-263) continued to use the *Sifen li*. Martz-loff applies his mathematical analysis of calendar computation to determine the calendar for the year 119. The *Sifen li* is a particular case of the Chinese metonic OAC. Chinese astronomers knew that 19 solar years are very nearly equivalent to 235 lunar months, or 12 ordinary years of 12 months with seven years containing an intercalary thirteenth month. Denoting this metonic cycle as 19/7, Martzloff points out other comparable cycles used by various calendars, including one of 391 years with 144 intercalary months (391/144); other combinations lead to such non-classic metonic cycles as 410/151, 429/158, 448/165, 505/186, 562/207, 600/221, 619/228, 657/242, and 676/249.

The Jingchu li 景 初 曆 (237-451) is an OAC comparable in importance to the Sifen li. It was also adopted by many dynasties in the course of more than two centuries. Officially promulgated for the first time in the Wei (220-265), which replaced the Sifen li with the Jingchu li in 237, it was also the calendar adopted by the Northern Wei (398-451). Since there are actual calendars that survive for the two years 450 and 451 from the Northern Wei, Martzloff works out the calendars for both of these years and offers a partial translation of the manuscript for 450, which contains

#### 108

all of the fundamental lunisolar components. The text, however, is not always legible, and errors in the text are clearly the result of faulty transcription from the original.

The *Xuanming li* 宣 明 曆 (822-892) is the astronomical canon adopted in China during 71 years of the Tang dynasty, and was also used in Korea in the course of the ninth and tenth centuries, and in Japan for 823 years, from 862 until 1684. Relative to other official astronomical canons, the *Xuanming li* is important because it is the only one for which there exists an almost complete official calendar, or more precisely, an almanac. Thus the dates from the almanac can be directly compared with those that calculations using the *Xuanming li* produce for the year 877. The numerical constants of the *Xuanming li* are similar to those of more ancient official astronomical canons, like the *Sifen li* or the *Jingchu li*, but the *Xuanming li* is not metonic. Its only innovation is the appearance of a new type of lunar month, the anomalistic month, which had appeared earlier in another calendar, the *Linde li* 麟 德 曆 (665-728). Martzloff calculates all of this data for the *Xuanming li* for 877, including its new moons, the 24 solar terms  $q_i$  and 72 seasonal indicators *hou*, among others.

As mentioned earlier, the *Shoushi li* (1281-1367) and the *Datong li* (1368-1644) are the last two traditional Chinese OAC before the astronomical reforms introduced at the beginning of the Qing Dynasty (1644-1911) with the help of the Jesuits. Often considered as the summit of the traditional OACs before the arrival of the Europeans in China—especially in the influential opinion of Ruan Yuan 阮 元 (1764-1849) in his *Chouren zhuan* 疇 人 傳 (Biographies of Astronomers and Mathematicians)—the *Shoushi li* lasted nearly a century, from 1281-1367. Its successor, the *Datong li*, remained in service much longer, through almost three centuries for the totality of the Ming (1368-1644). Beyond China, it served Korean and Japanese astronomers in the course of the following centuries, but it was in Japan where it was studied most actively from the second half of the seventeenth century.

From the official historical treatises of the Yuan and Ming that are devoted to them, and from a few Korean and Japanese sources, it might seem that these two OAC are quite different. They don't use exactly the same technical terms, they don't express numbers in at all the same way as did previous calendars (as mentioned above, they employed a centesimal numeration system), and the most recent of the two contains geometric figures and an embryonic logical justification for the formulas for calculation it employed (see the *Mingshi* 明 史 or "History of the Ming"). Apart from the fact that the *Shoushi li* considered the tropical year to be subject to secular variations that the *Datong li* rejected, the data presented in the *Shoushi li* is for Beijing (the Yuan capital) whereas the data of the *Datong li* 

is for Nanjing (the Ming capital from 1368–1420). Nevertheless, Chinese historians of the seventeenth and eighteenth centuries responsible for the compilation of the *Mingshi* agreed that except for their treatment of the secular variations in the tropical year, the two calendars were the same. However, as Martzloff points out, the identity of the two calendars is best understood operationally, because the existence of such secular variations as the length of the tropical year in the *Shoushi li*, and the absence of such variations in the *Datong li*, inevitably produced slightly different results. Thus the calendars obtained from those two OAC cannot strictly speaking be considered to be identical. Nevertheless, because the secular variations of the *Shoushi li* are basically negligible, in analyzing the mathematical formulations of the two calendars Martzloff notes that what holds for the *Shoushi li* basically applies to the *Datong li* as well, at least when calculations are limited to a small number of centuries, as was the case historically.

One of the major distinguishing features of the *Shoushi li* is that its epoch is defined somewhat differently than in the other Chinese OAC, since it doesn't coincide with the first new moon or with the first winter solstice of a Grand Origin in the distant past, but is situated in the year 1280 AD. Uncharacteristically, with its new calendar barely in place, a change was deemed necessary in one of the constants on which it depended—the *runying* 🗒 應 (a constant of lunar displacement, which was revised to a value of 20.205 days instead of 20.1850 days beginning in 1293).

Martzloff notes that in making the calculations for true new moons from the mean new moons, another corrective factor – *jiajiancha* 加 滅 差–was applied (literally, "additive or subtractive difference") that is formally very close to what Ptolemy used in his *Mathematical Syntaxis* (*The Almagest*) to calculate the times of true lunisolar conjunctions. However, Ptolemy used ecliptic longitudes whereas the Chinese formula is expressed in purely temporal terms, and there is no analogue to mean longitudes of the sun  $\lambda_{\odot}$ .

Nonetheless, computationally the *jiajiancha* corrective factor may be considered as a simplified form of the Greek formula. The origin of the Chinese formula is not known, but Martzloff says that contacts between China, India and the Christian-Nestorian and Islamic worlds cannot be ruled out in matters of astronomy beginning with the Tang dynasty (618-907). But he also notes that whereas for Ptolemy the formula has a deductive and geometric provenance, in the *Shoushi li* it appears in a purely arithmetic form and with no indication of how it might be justified. Because the computational procedures are very similar, at least operational-

ly, Martzloff holds out the possibility that the Chinese formula could be a distant adaptation of an originally Greek idea.

The *Datong li* (1368-1644) is the official astronomical canon that was put into place for the entire duration of the Ming dynasty. As already explained, it differs very little from the canon of the Yuan dynasty, the *Shoushi li*, so that its calculations can be made in the same way in both cases. Since the *Shoushi li* took as its epoch the year in which its calendar began, the epoch for the *Datong li* is 1280, and Martzloff computes all of the primary and secondary constants needed to set the calendar for the *Datong li*: the solar year was A=365.2425 days; the solar terms  $q_i$ =A/24=14.2184375 days, the seasonal markers A/72=5.0728125 days; the synodic month =29.530593 days; the anomalistic month =27.5546 days. He then calculates the calendar for the year 1417, which he shows to have been an intercalary year, which all of the tables of concordance of the Chinese calendar confirm.

As with all the calendars with true elements, calculation of the calendar for the year 1417 is made in two successive steps. First it is necessary to establish the values of the mean elements of the calendar (among them the solar terms, seasonal indicators and phases of the moon), and then, in a second step, the additive or subtractive corrections for the mean lunar phases are determined in order to calculate the true phases of the moon for the year 1417, as well as the intercalary month. Martzloff reports that the theoretical dates as calculated indeed conform to those of the printed example of the official calendar for the year 1417. Of all the official calendars of the Ming dynasty (1368-1644) which have survived to this day, the one for 1417 (Yongle 15) is the oldest, and is preserved in the Central National Library of the Republic of China (Taipei). Printed as a book of 30 pages on very thin paper, it resembles the sort of booklets commonly printed in China since the Song (960-1279).

**Figure 1: The OAC for 1417.** Reproduced with permission of the National Central Library, R.O.C. (Taiwan), from the *Datong li* of the 15th year of the Yongle reign-period (大明永樂十五年大統曆 *Daming Yongle shiwunian Datong li*).



The text of the calendar for 1417 begins with a set of preliminaries including a résumé of the calendar. Two pages of this section are devoted to a "diagram of the directions of the annual spirits" meant to reveal the activities of a normal day, good days and bad days, as well as various prohibitions correlated with the corresponding direction. The calendar as issued by the bureau of Astronomy was to be printed and diffused throughout the empire. According to the law, counterfeiters were to be beheaded, and those who denounced them, leading to their arrest, would receive a reward of 5 *liang* in silver. The following are typical entries for the calendar of 1417:

The Celestial Way is moving in the direction of the south, voyages should be undertaken towards the south and it is advisable to repair buildings located in the south [...]. This month the east wind melts the frost, the hibernating animals begin to revive, the fish come up just to the ice, the otter offers a fish in sacrifice, the wild geese arrive, the plants bourgeon. The 28, day *yimao* [#52], the sun enters [the house of Jupiter] *Juzi*.

Sign of Hunting: appropriate to make ablutions, sewing clothes is appropriate, making transactions. One should not change residences nor practice acupuncture.

# One should make sacrifices; one should not begin a trip.

In eight appendixes Martzloff provides an explanation of the sexagesimal cycle on which the naming of the years is based, the 24 solar terms, the 72 seasonal indicators or phases, a list of all the official astronomical canons with their names, as well as the official metonic canons, a list of the temporal constants for the OACs from the Grand Origin, as well as the lunar and solar constants. The last appendix is devoted to the significance of the word *li*.

Although *li* 曆 is generally associated specifically with the calendar, Martzloff argues that in reality the term corresponds to the concept of the "astronomical canon" proper. The pioneering Jesuit historian of astronomy Antoine Gaubil (1689-1759) appreciated this and did not limit *li* to the calendar but extended the term to all of astronomy. Unfortunately, in the course of time, this was forgotten. Martzloff points out that Joseph Needham in his volume on mathematics and astronomy (vol. 3, 1959) in the venerable series *Science and Civilisation in China*, dismissed the subject as follows: "The whole history of calendar-making … is that of successive attempts to reconcile the irreconcilable, and the numberless systems of intercalated months ... and the like, are thus of minor scientific interest." However, in 1960 Yabuuchi Kiyoshi and Nathan Sivin began to promote a different view, arguing that *li* was equivalent on the one hand to astronomical tables and on the other to astronomical systems. Martzloff cites in particular the recent work of the historian of astronomy Jiang Xiaoyuan, who published a study in 1991 showing in detail the diverse meanings of *li* in applications not only to the calendar but to a vast set of techniques used in astrology.

Likewise, Martzloff contends that while *li* generally may be taken to mean "calendar," i.e. "an ordered assemblage of days and months," this is only one aspect of *li*. From a strictly technical point of view, he argues that the equivalence between *li* and the calendar cannot be maintained. Consider, for example, the dynastic historical treatises thanks to which we have access to the calculations of the various Chinese calendars. Surprising though it may seem, it is clear that these treatises are not interested at all in the concrete calendar. The treatises devoted to *li* all aim to explicate the multitude of techniques for calculating not only the calendar but also creating official planetary ephimerides. The latter are not called "calendars" but "catalogues." In fact, one of them has the title: Catalogue (Martzloff's emphasis) of appearances and disappearances of the five planets for the year ten of the Jiaqing era (1531), 38th year of the sexagesimal cycle xin*mao*. The astrological uses of this type of catalogue are apparent, and it even includes tables of positions for imaginary stars (four invisible stars associated with Indian astrology and concerned primarily with nodes of the moon). The diffusion of these "catalogues," of a basically astrological character, was highly confidential, not to say clandestine, and is no doubt why they have remained unappreciated. Martzloff says that only one example, in the entire history of China until the end of the Ming, has survived to the present.

Because preparation of ephemerides and official calendars presupposed all sorts of calculations, the Chinese dynastic histories devoted considerable attention to them, and so Martzloff suggests the term li should really be synonymous with *shu* (i), another word laden with meanings akin to "stratagem, astuteness, artifice, technique, process, receipt," and even "divinatory technique." In fact, in the spirit of Martzloff's work, he suggests that the two words, *shu* and *li*, are interchangeable. Nevertheless, *shu* has a wider meaning, because it can apply to medicine, arithmetic, martial arts, and divination, among others. The *li* are more akin to mathematics, comprised of aggregates of numerical constants, tables of instructions and formulaic calculations applying a wide variety of technical terms to predict or account for all sorts of astronomical, astrological, and calendrical phenomena. The results of the cal-

culations of *li* are always presented in the same way, as calendars, almanacs, or ephemerides. As Martzloff surmises:

This means that the conception of the Chinese for the calendar and astronomy between 104 BC and 1644 differs considerably from that prevalent in Europe, where the calculations needed to establish the calendar were relegated to computists, whereas those of astronomy depended on techniques and principles that were radically different, which historians of astronomy and of the calendar have never confused: the *De Revolutionibus* of Copernicus is not the treatise of Clavius relative to the reform of the Gregorian calendar. There is clearly a world of difference between the ecclesiastical moon of Clavius, having no interest but from a computational point of view, and the astronomical moon of Copernicus.

In terms of computations, one question Martzloff's book raises concerns the remarkable accuracy that Chinese astronomers achieved in their measurements of such fundamental data as the length of the month or year, and even in their calculation of *pi* to an accuracy that was not matched in the West until the end of the sixteenth century, when more accurate computations were made by the German/Dutch mathematician Ludolf van Ceulen. The accuracy of Chinese methods derives in the first instance from the nature of their empirical measurements, and from the fact that in the Chinese histories the lifa treatises always emphasize the significance of the most accurate measurements possible. For example, in considering the Shoushi li, the determination of the winter solstice was achieved by means of a gnomon to which was attached a yingfu 景符 or "shadow definer" as Needham terms it, something like a pin-hole device that permitted very accurate measurements of the shadow cast by the sun, upon which determination of the solstices depended, but as astronomers well knew, was not easy to determine precisely. Chinese astronomers early on learned that it was best to begin measurements a few days ahead of the expected solstice, and continue for a few days afterwards, and then by interpolation determine when the exact moment of the solstice must have occurred. These matters have been studied in detail by Franz and Margaret Bruin and by Chen Meidong in his detailed survey: Zhongguo kexue jishu shi, tianwenxue juan 中国科学技术史,天文学卷 (History of Chinese Science and Technology, Volume on Astronomy) (2003). In an especially impressive account of the accuracy of observations of solstices in thirteenth-century China, Raymond Mercier was even able to use the precise measurements of gnomon shadow lengths given in the Yuanshi or

"History of the Yuan" (measured down to the unit *hao*, about .03 mm) to determine the exact location of the Yuan dynasty observatory as having been in Beijing, despite assumptions previously that it was located in Yangcheng in Henan province. As a measure of precision, it is worth remembering that the mean value for the tropical year determined by Guo Shoujing 郭 守 敬 for the year 1280 is identical with that used in the Gregorian calendar (365.2425 days).

The accurate determination of the ratio of the circumference to the diameter of the circle is another matter, but one related closely to astronomy. Surprisingly, given the observational accuracy of their data for which Chinese astronomers aimed, the general acceptance of 3 as an approximate value for *pi* is at first hard to understand. Although Liu Hui in his commentary on the Nine Chapters offered a more precise set of rates for the circumference and diameter as 157:50, and Li Chunfeng 李 淳 風 in the Tang dynasty improved on that value to an even more precise rate of 22:7, it was not in a mathematical text but in an astronomical document written as part of the 隨 書 Suishu (History of the Sui) that the calculation of *pi* by Zu Chongzhi 祖 冲之 as being between 3.1415926 and 3.1415927 was noted, in fact a value of pi carried to a level of precision not surpassed in Europe until van Ceulen's computations. The general reliance upon 3 as a value for *pi* in most Chinese texts, both mathematical and astronomical, is explained by Martzloff in his study of the history of Chinese mathematics, where he points out that even the Shoushi li used a value of *pi* equal to 3, largely due to a situation that provided a compensation of sorts for another built-in set of errors, namely the use in Chinese astronomy of formulae for arcs and chords that were themselves only approximations, and under these circumstances, a more accurate value for *pi* would not have rendered correspondingly better results [Martzloff, 1997: 334-335].

116

beginning of the Inscription, it notes that "Heaven tired of the disorder on Earth," and this is a more telling reason for discrepancies, that when the terrestrial world was in disarray, then the heavens might reflect that disorder by its own digression from the usually harmonious motions of the heavens. Towards the end of the Inscription, the usual orderly motions of the planets are ascribed to harmony in the universe: "The seven heavenly bodies follow their paths smoothly. Yin and Yang are in balance." But when the Yin and Yang were out of balance, their lack of harmony might well be reflected in erratic motions of the heavens. This is very different from the Greek notion of the heavens being strictly subject to the laws of nature, as Heraclitus says: "All events proceed with the necessity of fate. ... The sun will not outstep the measure of his path; or else the goddesses of Fate, the handmaids of Justice, will know how to find him."

On this view a mathematically accurate clockwork model of the heavens was a natural corollary, but in terms of Chinese cosmology, where in Martzloff's picturesque description quoted earlier, the sun's motion at times displayed a "liberté cinématique," it was impossible to accord this with any laws of nature. Because they did not conceive of the heavens as a grand clockwork mechanism, this precluded any possibility of Chinese astronomers even considering let alone finding some universal law (through time) that would account at every moment for the observed arrangement of the heavens. Contrary to the deeply ingrained neo-Pythagorean/Platonic view in the West that the very essence of nature was inherently mathematical, Martzloff emphasizes that from a Chinese perspective, "the grand book of nature has not been edited in the language of mathematics." Instead, for the Chinese mathematics was simply a tool, one among many that could be applied in their analysis of the heavens, but which had no special standing or metaphysical significance on its own terms. The numerological significance that was accorded to numbers, on the other hand, was quite different and did play a role in the kinds of numbers that were held to be significant in calendrical computations.

In other words, from a purely Chinese perspective, the book of nature was not written in the strict language of mathematics, and it was not assumed, as did Copernicus or Galileo, for example, that the world having been created according to mathematical laws, mathematics must then mirror and indeed represent the actual motions of the heavens. This essential difference may help to explain a large part of the Needham question, given that when compared with the West, the status of mathematics has always been very different in China, as clearly emerges from an understanding of the role it played in the construction of the Chinese calendar. As Martzloff goes on to explain, the fact that for the Chinese nature was *not* essentially mathematical went hand-in-hand with "la conviction chinoise selon laquelle les mathématiques ne constituent qu'un artifact parmi d'autres et aucunement la représentation définitive de la réalité physique." As Martzloff has noted elsewhere: "The idea of the intrinsic imitation of mathematics was repeatedly asserted in the treatises on the computus of the official histories. … Thus Chinese traditional astronomy was in some sense more comparable with actual meteorology than ancient Western astronomy whose fundamental concern was the explanation rather than the prediction of the phenomena" [Martzloff, 2000: 380].

This difference in how the Chinese conceived of the relations between knowledge and nature has been described by Francesca Bray as follows: "But the guiding program of Chinese scholars was quite different from that of Western scientists. Their ultimate aim was not the understanding of nature but the regulation of human society... History, not mathematics, was the queen of their sciences" [Bray, 1991: 208]. A possible exception to this general view, however, may be the case of 沈 括 Shen Kuo (Shen Gua), who Mark Elvin suggests believed that the cosmos was indeed regulated in a very precise way, but the degree of subtlety was such that human abilities were inadequate to understand exactly and account accurately for phenomena in the heavens [Elvin 1993–1994: 5–6]. For a discussion of the limits of empirical knowledge in Chinese science, see [Sivin 1989].

In closing, it should be noted that Le calendrier chinois ends with more than the usual bibliography, but concludes with a vade mecum of sorts. The bibliography is not just a collection of works cited, but as Martzloff says, it is designed to promote research in the history of the Chinese calendar and of astronomy generally in China. Thus he includes the most important dictionaries, encyclopedias, library catalogues and other research tools essential to serious study of these subjects. He also provides very helpful guides to chronological tables of Chinese calendars, along with primary sources used, manuscript sources, Chinese, Japanese and Korean sources, and thirty pages of secondary sources that will be of value to anyone interested in pursuing this very interesting but technically complex material. This is a demanding, rigorous book to read, but anyone who wants to achieve a full technical understanding of the computational foundations of the Chinese calendar will find this book worth the concentrated study it requires. The rewards are not only in the details, but in the general overview that Martzloff provides of the great accomplishments of traditional Chinese astronomy and the long tradition of technical calendar-making that was sustained from dynasty to dynasty.

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120