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Research Note

Incommensurability Between Western Geometrical and Chinese Numerico-algebraic Astronomy

Takebe Katahiro's Interpretation of Planetary Limit Degrees*

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Unsolved Problem of *xiandu* 限 度 in the Planetary Theory of the *Shoushi li* 授 時 曆

During the 1960s, Yabuuti Kiyosi 薮 内 清 (1906-2000) and I worked together on the astronomical system of 1280 (*Shoushi li* 授 時 曆). We intended to publish a Japanese translation together with annotations and some research results. Yabuuti and I were concerned because we could not find the physical meaning of the concept "Limit Degrees" (*xiandu* 限 度), which appeared in a set of tables in the section on planetary motions.

Some people suggested that I publish the translation even though we did not fully understand the meaning of this term. But I could not do that, since the problem seemed to me to have some important implications in Chinese traditional astronomy. Finally, in the early part of the twenty-first century, I solved the prob-

* For revising and the English editing of this article I would like to express my gratitude to Nathan Sivin.

lem to my satisfaction, and published the Japanese translation in 2006.[1] What I was satisfied with, however, was not a true understanding of what Limit Degrees originally meant. The following conclusion reflects my best understanding so far.

Why is Limit Degrees Important?

There are no Limit Degrees in the sections on solar and lunar motions.[2] The major aim of traditional calendrical astronomy was to make a luni-solar calendar and check its validity by predicting solar and lunar eclipses. Planetary motions were of subsidiary importance, and hence seldom encouraged originality in computational astronomers.

Now, what does not exist in the movement of the sun and moon, but only in planetary motions? The answer is retrograde motion, which does not occur in solar or lunar astronomy. In order to explain planetary retrograde motion, we need some sort of model, algebraic in early Chinese astronomy, geometric in the Greek tradition. We hypothesized that Limit Degrees was a new concept added to explain planetary retrograde motion. This made its meaning important for comparison with the history of Western science. That is why I originally did not intend to publish the translation by Yabuuti and myself until I could understand this concept.

Interpretation of the Planetary Tables

Let me explain how Limit Degrees appeared in the text of the *Shoushi li* in the planetary tables. The table for Jupiter, as given in Nathan Sivin's English translation, furnishes an example.[3]

A. Grade Days (*duanri* 段 日)
B. Mean Degrees (*pingdu* 平 度)
C. Limit Degrees (*xiandu* 限 度)
D. Initial Motion Rate (*chuxinglü* 初 行 率)

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[1] Yabuuti Kiyosi 藪 内 清, Nakayama Shigeru 中 山 茂, *Jujireki, yakuchū to kenkyū* 授 時 暦 — 訳 注 と 研 究 (*Shoushi li*: Translation, Commentary and Research, 2006).

[2] There is a *ding xiandu* 定 限 度 (Corrected limit degrees) in the solar theory, and a *zhengjiao xiandu* 正 交 限 度 (Standard crossing limit degrees) in the eclipse theory, but these are not related to the planetary *xiandu.* See Sivin, *Granting the Seasons* (Secaucus, NJ: Springer, 2008), pp. 474, 507.

[3] *Yuanshi* 元 史 (History of Yuan Dynasty; Zhonghua (ed.)), 56: 1244-1245; translation in Sivin, pp. 520-521.

Grade	A, days	B, *tu*	C, *tu*	D, *tu*
Conjunction Invisibility	16.86	3.86	2.93	0.23
Dawn Hastening, Beginning	28	6.11	4.64	0.22
Dawn Hastening, End	28	5.51	4.19	0.21
Dawn Slackening, Beginning	28	4.31	3.28	0.18
Dawn Slackening, End	28	1.91	1.45	0.12
Dawn Station	24	-	-	-
Dawn Retrogradation	46.58	4.88125	0.32875	-
Evening Retrogradation	46.58	4.88125	0.32875	0.16
Evening Station	24	-	-	-
Evening Slackening, Beginning	28	1.91	1.45	-
Evening Slackening, End	28	4.31	3.28	0.12
Evening Hastening, Beginning	28	5.51	4.19	0.18
Evening Hastening, End	28	6.11	4.64	0.21
Evening Invisibility	16.86	3.86	2.93	0.22

Grade Days (*duanri*, column A) is the number of days in each grade of planetary motion (*duanmu*) within the synodic period, increasing up to retrogradation and then decreasing symmetrically until it returns to its initial value. Mean Degrees (*pingdu*, column B) is the mean motion of the planet per day in Chinese degrees (*du* 度) during the grade, whether direct or retrograde (*tui* 退). Whether the values for Mean Degrees come from observation or computation is not relevant to the problem.

In column C, Limit Degrees (*xiandu*) rises and falls in parallel with Mean Degrees, but is smaller. Mean Degrees is proportional to Limit Degrees, but only within the grades of forward motion. From the appearance of this parallel arrangement, Yabuuti and I thought the two quantities belonged to the same category. We hastily conjectured that their direct functional relationship yielded true degrees (*dingdu* 定 度, literally "corrected degrees"). This was the major source of our misunderstanding. Except during retrograde motion, the ratio between the Mean Degree and Limit Degrees is always the same (1.32:1); in retrograde motion, Mean Degrees is negative and Limit Degrees is small but positive. Yabuuti also noted that, like Mean Degrees, Limit Degrees in retrograde motion is numerically arranged to make its total equal one synodic period.

The source of our misunderstanding was that Mean Degrees and Limit Degrees appear side by side in the table. If Limit Degrees were the equation of motion, added to Mean Degrees to yield True Degrees, it would be easy to use, but in that case the planet could never retrograde. Hence, such a simple interpretation is obviously wrong.

Takebe Katahiro's Commentary on Limit Degrees

There were only two Japanese scholars in the Edo period who paid special attention to the Limit Degrees concept, the great shogunal astronomer Takebe Katahiro 建部賢弘 (1664-1739) and Nakanishi Takafusa 中西敬房 (fl. late eighteenth century).

Takebe, who wrote the best annotated version of the *Shoushi li*, especially warned readers that 'Limit Degrees is the degrees a planet traversed from the beginning of the each phase, not real degrees of planetary position. Limit Degrees is the degree of each phase to be calculated and determined from the position of the mean sun.'[4] As Sivin, who was familiar with this commentary, put it more concisely, "Limit Degrees is the mean increase in the planet's distance from the sun over the period of the grade, in *tu*," i.e., *du*.[5] Takebe did not give an example of calculation using Limit Degrees. Since his book was voluminous and never published, remaining in manuscript to this day, his interpretation went largely unnoticed.

Nakanishi's Discussion

Independent of Takebe's commentary on the *Shoushi li*, Nakanishi wrote *Rekigaku Hōsūgen* (Mathematical Foundations of Calendrical Science 暦学法数原, 1787). To the end of this treatise he appended "Ron Gosei Kakudan Gendo" 論五星各段限度 (On the planetary Limit Degrees for each grade). He confessed that he could not understand the mathematical basis of Limit Degrees. Mean Degrees can be obtained by actual observation. When the planet is in retrograde motion and Mean Degrees is negative, Limit Degrees is always positive. This is beyond comprehension, he confessed. Finally he commented that "the theory of Limit Degrees may be erroneous."

According to Nakanishi, there are three kinds of perturbation from the mean motions of the planets, that due to planetary motion, that due to solar motion, and that due to the 24 equal seasonal divisions of the tropical year *ki* 氣. The notes that accompany the *Shoushi li* specified that in case of the inner planets, Venus and Mercury, the value of the solar equation of center be doubled or tripled. Nakanishi himself calculated the positions of Mercury and Venus according to the *Shoushi li* method and found that the results did not tally with historical re-

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[4] "Juji Rekikyo" 授時暦経 (Essentials of the *Shoushi li* Compilation) *ge* 下 (vol. 2), hogosei 歩五星 (Calculation of Planetary Movements), Mokusei 木星 (Jupiter), *gendo* 限度 *(xiandu).*

[5] Sivin, p. 521.

cords of observation. This was his major reason for concluding that Limit Degrees is erroneous.[6]

Yabuuti's Distress

Yabuuti read Takebe's comment, but he did not study it in detail. Instead, he tried to explain Limit Degrees in terms of the Greek epicyclic system, but his explication was not entirely successful. He tried to manipulate various parameters and numerical values to find a satisfactory answer, but finally gave up, saying that "it is unintelligible from modern viewpoint."[7] Since the synodic period of planetary motion has nothing to do with the solar equation of center, he asked, why should the *Shoushi li* treat both Mean Degrees of planetary position and Limit Degrees as functions of the synodic period? Yabuuti's and my understanding of the *Shoushi li*'s planetary theory remained frustrated at the same level as that of Nakanishi.

Moreover, Yabuuti, like Nakanishi, could not comprehend the doubling or tripling of the equation of center for the inner planets. This is incomprehensible as long as one has a homocentric universe in mind. Yabuuti and I could only conjecture that the authors of the Yuan system were trying to force the planetary theory to correspond to observation by applying twice or three times the regular value. There are other examples of this sort of adjustment that seem equally inelegant to modern astronomers.

Qu Anjing's Work

Recently, I asked a Chinese historian of mathematics, Qu Anjing 曲安京, to work on a Yabuuti-style history of Chinese mathematical astronomy, and especially to investigate the hitherto unsolved problem of Limit Degrees. He worked out a solution in the style of a mathematician, using a purely geometrical model of Western homocentric cosmology, without investigating the historical circumstances of the *Shoushi li*.[8] Qu Anjing went beyond the narrow problem of Limit Degrees for the moment, and tried to reconstruct the Chinese planetary theory from the geometrical models of Western astronomy.

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[6] As Sivin points out, the *Shoushi li*'s flawed planetary techniques were copied with minor modifications from a predecessor, and are incapable of yielding consistently accurate results; ibid., pp.549-550.

[7] Yabuuti, ibid.

[8] Qu Anjing, "Zhongguo gudai de xingxing yundong lilun" 中国古代的行星運動理論 (Planetary Theory in Ancient China) *Ziran kexueshi yanjiu* 自然科学史研究 (Studies in the History of Natural Science) 25.1 (2006): 1–17.

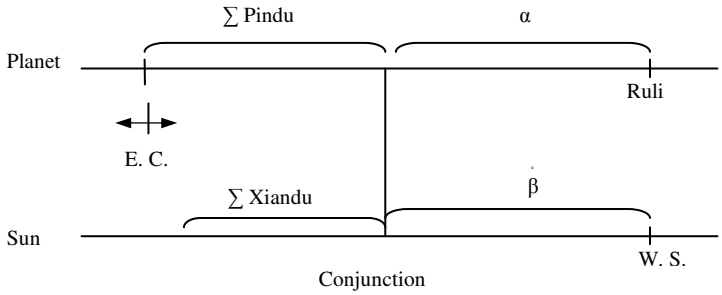
Qu finally arrived at a formula to compute the difference between mean and true conjunction. This was formally satisfactory, because it expressed the planetary position in purely synodic terms. But he was unable to validate the numerical values of Limit Degrees as listed in *Shoushi li*. On this basis, we still do not understand the meaning, use and definition of Limit Degrees, which cannot directly explain planetary retrograde motion.

My Interpretation of Takebe's Commentary

By carefully reexamining Takebe's commentary, I found that the Yuan method uses a very simple numerico-algebraic scheme, combining the solar and planetary equations of center, nothing else. As far as I know, Takebe is the only person who noted this crucial point explicitly.

I have reconstructed Takebe's commentary. It is simple when expressed in a traditional linear numerical calculation as shown below, rather than using the Western homocentric model Qu employed.

Figure 1



Planetary Equation of Center (EC)	=	Fp (α + Σ Mean Degrees)
Solar Equation of Center	=	Fs (β + Σ Limit Degrees), where Fp is the planetary equation of center formula, and Fs is the solar equation of center formula.

α is the arc from conjunction to the position of the planet on its orbit, which the Yuan astronomers called the "argument" *(ruli* 入曆), and

β is the arc from conjunction to the winter solstice point.

For both determinations, the computist is told to set up and solve a third-degree algebraic equation, or to consult a table.

changdu 常度	=	Σ Mean Degrees + planetary EC
dingdu 定度	=	Σ Mean Degrees + planetary EC + solar EC

Takebe was following a purely numerical approach in interpreting *Shoushi li*. He uses the language of traditional computational astronomy, such as *yingsuo* 盈縮 ("expansion and contraction" of solar motion from the mean) and *ruli* 入曆 ("the argument" of a given motion), instead of "equation of center" or "perihelion", as in Western astronomy. For the sake of easier understanding, I shall mix modern terminology in the following.

Takebe considers that there are two kinds of equation of center. Each is an adjustment to the mean synodic planetary position. One is the planetary equation of center, the other the solar equation of center. The *Shoushi li* calculates both as follows; The summation of Mean Degrees from the synodic point or conjunction *(he* 合) is added to the distance between the synodic point and the planetary perihelion. The sum of Limit Degrees from the synodic point is then added to the distance between solar perigee (actually fixed to winter solstice point in the *Shoushi li*) and the synodic point.

Mean planetary motions are the sum of Mean Degrees in each planetary grade of the synodic period. One obtains a hybrid quantity, *Changdu* 常度 ("ordinary" degrees) is obtained by adding the planetary equation of center, and True Degrees by adding the two equations of center, planetary and solar; this final result is the true planetary position.

The Spirit of Chinese Computational Astronomy: A Passion for Precision

The Chinese quest for numerical precision is exemplified in the *Shoushi li*'s determination of a basic astronomical parameter, the observation of solstitial time, which is incomparably better than pre-modern values of Greek and Islamic traditions.[9] The same spirit is responsible for the Yuan astronomers' adoption of the secular variation of tropical year length. Takebe Katahiro, computing this minute variation, reflected the same spirit.

A Chinese angle is expressed in *du*, one day's mean solar motion. It was in the beginning 365.25, gradually moving toward the Yuan value, 365.2425. Chinese astronomers were not attracted to the convenience of rounding numbers to 360 degree, as in Europe. The Japanese mathematical tradition (*Wasan* 和算), in which Takebe was a crucial figure, mainly deals with problems of geometrical

[9] Nakayama Shigeru (1963), "Accuracy of Pre-modern Determination of Tropical Year Length," *Japanese Studies in the History of Science* 2: 101–118.

figures and solves them numerically. This method actually reached a dead end in the *Shoushi li*. Because of its high accuracy for lunisolar computation, it was used officially for nearly four hundred years in China, and lasted much longer in Japan and Korea. Because its precision far surpassed practical needs, it was not replaced until the Jesuits, because they supported the Manchu invaders, introduced a geometrically-oriented methodology in the seventeenth century.

Conclusion: Incommensurability between Western Geometrical Model and Chinese Numerico-algebraic Approach

Traditional Chinese astronomy depended entirely on a numerical approach that did not depend on a geometrical scheme or model. The *Shoushi li* took this to an extreme, by abolishing the calculation of a grand conjunction as the calendrical epoch, instead placing the epoch in the recent past. It adopted the equivalent of a decimal system for all values taken from historical records and observations, as well as for parameters. The primary aim of its authors, as the Evaluation (*liyi* 曆議, included in the treatise) confirms, was to maximize accuracy by minimizing the discrepancy between observation and calculation as much as possible. That was, if not their methodology, their spirit.

So much for the assumption, frequent in the older generation of historians of astronomy, that even though geometrical models do not appear on traditional calendrical theory, the astronomers must have used them implicitly. Our investigation suggests that such a hypothesis is unwarranted.

Qu rigorously applied a modern geometrical analysis to Limit Degrees, but could not derive a clearly expressed geometrical concept. In other words, he proved that Limit Degrees could not be translated into modern geometry. This is the real significance of his work. It showed that when we compare a modern Western geometrical approach with the Chinese numerico-algebraic one, they may be fundamentally *incommensurable*. One can see easily the incommensurability of the equation of center as expressed in a Western quasi-trigonometrical function and in a third degree algebraic formula in China.

Beyond the Conclusion: Is the Yabuuti School Whiggish?

In 1950s when 'Scientific Revolution' was still a great issue among historians of science, Joseph Needham broke the news that China was a great scientific power, and Nathan Sivin later showed that more than one scientific revolution took place there. Ever since 1957, when I noticed that Needham's *Science and Civilisation*

in China ignored calendrical astronomy,[10] I have tried to introduce this tradition to the Western world by cooperating with Yabuuti, who spent his lifetime in that quest. That is because I consider Chinese mathematical astronomy the exact science of East Asia, with its crowning achievement in the *Shoushih li*, comparable to the Keplerian-Newtonian tradition of the West. But at this time, near the end of my life, I have begun to question the Yabuuti school's methodology as Whiggish, a term used pejoratively by historians.

Yabuuti's style evolved from interpretations based on his mastery of modern astronomy. Comparing this understanding with traditional ones, we can identify the characteristic way of East Asian conceptual development, even though the original texts did not specify any underlying procedures. The approach we used for the Limit Degrees problem was typical of the "Yabuuti school". Nevertheless, Yabuuti was unable to give a theoretical rationale for the Limit Degrees concept, and concluded that it is impossible to understand in terms of modern astronomy.

Positivist historians of mathematics define their task as translating ancient mathematical astronomy into the terminology of modern science. They assume that if it is translatable, it has scientific value. Yabuuti and I took that approach, hoping to discover and explain the physical meaning of Limit Degrees. This was, in other words, a Whiggish project.

We attempted for nearly forty years to comprehend Limit Degrees while translating the *Shoushi* calendar. In trying to reduce it to a geometrical concept, we missed the point that Takebe Katahiro emphasized in his annotation of the *Shoushi li*. We assumed that the physical meaning would be a geometrical concept. When Yabuuti said that he could not understand Limit Degrees from the viewpoint of modern science, it was actually a geometrical concept that he was unable to explicate.

Up to that point we were satisfied with the Yabuuti School's ability to elucidate traditional concepts by reducing them to concepts and models that present-day readers could readily understand in modern terminology. But the case of Limit Degrees shows that is not always feasible. Our method turned out to be too Whiggish to be reliable.

Does this mean that we should discard all of the Yabuuti's achievements as Whiggish, because in one exceptionally rare case his approach did not work? To reject entirely the fruits of Whiggism would be to discard the foundations of today's history of science. It would be foolish for historians brought up and educated in the modern world to be certain that they can faithfully reproduce the

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[10] Needham, *Science and Civilisation in China,* vol. 3 (Cambridge University Press, 1959); Sivin, "Why the Scientific Revolution Did Not Take Place in China—Or Didn't It? The Edward H. Hume Lecture, Yale University," *Chinese Science* 5 (1982): 45-66; Shigeru Nakayama (2000), "Recollections of Joseph Needham with Sidelights on Yabuuti Kiyosi," a brochure published by National Science and Technology Museum Taiwan for a Symposium on the History of Science in Commemoration of the Centennial of the Birth of Joseph Needham).

thought processes of an astronomer in China seven hundred years ago. As we strive to understand those thought processes, it would be foolish to renounce the advantages of hindsight. After all, history is the conversation between past and present.