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Consensus via Adaptive Gain Controllers Considering Relative Distances for Multi-Agent Systems

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Abstract

In this paper, for multi-agent systems (MASs) with leader-follower structures, we present a linear matrix inequality (LMI)-based design method of an adaptive gain controller considering relative distances between agents. The proposed adaptive gain controller consists of fixed gains and variable ones tuned by time-varying adjustable parameters. The objective of this paper is to derive enough conditions for the existence of the proposed adaptive gain controller which achieves consensus for each agent. The advantages of the proposed adaptive gain controller are as follows; The proposed controller can be obtained by solving LMI, and the proposed control system can achieve consensus and formation control, even if uncertainties are included in the information for relative distances. In this paper, we show that the design problem of the proposed adaptive gain controller can be reduced to the solvability of LMI. Finally, simple numerical examples are included to illustrate the effectiveness of the proposed adaptive gain controller for MASs.

Keywords: multi-agent systems (MASs), consensus, relative distance, adaptive gain controller, linear matrix inequality (LMI)

1. Introduction

When we consider designing control systems for dynamical systems, it is necessary to derive a mathematical model for the controlled system, and one can see that optimal control is well known to be a powerful strategy in modern control theory. LQ regulator for linear systems is a typical controller, and it ensures asymptotical stability for closed-loop systems with good robustness provided that a mathematical model for a control system describes precisely [1,2]. However, there always exist some gaps between the mathematical model and the controlled system, and the gaps are referred to as “uncertainty”. Therefore, controller design methods dealing with uncertainties explicitly have been required, and robust control for uncertain dynamical systems has been extensively studied. One can see that robust control can be classified into “robust stability analysis” and “robust stabilization”, and lots of existing results for robust control strategies have been presented [3-6] and quadratic stabilizing controllers and control are well known robust control strategies[7,8]. Note that the conventional robust controller consists of fixed gains which are derived by considering the worst-case variations for uncertainties. In contrast with the conventional robust control with fixed gains, some researchers have presented variable gain robust controllers for uncertain systems [9-11]. Such variable gain robust control strategies are more flexible and adaptive comparing with the conventional robust control with fixed gains.

On the other hand, the practical systems in modern society have become large-scale and complex due to rapid development of technologies, and such systems are referred to as “large-scale interconnected systems”. Since it is difficult to apply centralized control strategies to such large-scale interconnected systems, design problems of decentralized control for

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large-scale interconnected systems have been well studied (see [12] and references therein). For instance, one can see that large interconnected power distribution systems which have strong interactions, transportation and traffic systems with lots of external signals, water systems which are widely distributed in the environment, energy systems, communication systems and so on are large-scale systems. Moreover, formation control has recently attracted much attention, and a multi-agent system, in general, can be described as a network of a few of coupled dynamic units that are called agents. The design problems of formation control for MASs are considered as one of the decentralized control problems and it is well-known that MASs can achieve various task efficiently. For design problems of formation control for MASs, the consensus problem has received a lot of attention, because this problem has drawn substantial attention from various fields such as vehicle formations, unmanned aerial vehicles, mobile robots, sensor networks, and so on. Moreover, a consensus which means the states of all agents are driven to a common state by implementing distributed protocols is well accepted as one of the most important and fundamental problem in formation control. Thus, a large number of existing results for consensus problem have been presented (e.g. [13-16]). In the work of Olfati-Saber et al. [13], the consensus problem for a network of first-order integrators with directed information flow and fixed/switching topology has been studied, and convergence analysis of a consensus protocol for a class of networks of dynamic agents with fixed topology have been shown [14]. Zhang and Tian have studied the mean-square consensus for MASs composed of second-order integrators [15], and the matrix inequality-based stabilization condition and consensus algorithm for MASs have been presented [16]. Also, a number of the existing results for the leader-follower consensus for MASs has been presented([17-19]). "Leader-follower" refers to defining a leader (whether real or virtual) and controlling another agent (follower) to follow the leader. In these results for consensus problem for MASs, controllers have fixed gain parameters only, and relative distances between agents cannot be considered explicitly. There are few results of the consensus problem via adaptive gain-based controller considering relative distances between agents for MASs.

From the above, this paper deals with a consensus problem for MASs with leader-follower structures. In this paper, we present a design method of an adaptive gain controller giving considering relative distances between agents. The adaptive gain controller consists of fixed gains and variable ones tuned by time-varying adjustable parameters. In this paper. we show that enough conditions for the existence of the proposed adaptive gain controller can be reduced to linear matrix inequality (LMI). The proposed adaptive gain control strategy has advantages as follows; The proposed controller design approach can handle relative distances between agents explicitly. Furthermore, even if the information for relative distances between the other agents and the leader is unknown, but their upper bounds are known, the controller can achieve consensus. Furthermore, the proposed consensus control system can be designed by solving LMI. Finally, simple numerical examples are included to illustrate the effectiveness of the proposed formation control systems.

2. Preliminaries

This chapter shows the mathematical notation used in this paper. $\mathbb{R}^{m\times n}$ represents an *m*-by-*n* real matrix, and I_n represents an *n*-dimensional identity matrix. For matrix A, A^T and A^{-1} represent transpose and inverse. For a square matrix A, $A>0(A\geq 0)$ indicates that A is positive definite (positive semidefinite) and $A<0(A\leq 0)$ is negative definite (negative semidefinite). $\|C\|$ is the norm of any matrix C. $diag(A_1, A_2, ..., A_n)$ gives a diagonal block matrix with matrices A_i (i = 1, 2, ..., n) on the diagonal. Element * in the matrix represents a symmetric element. If A is a $m\times n$ matrix and B is a $p\times q$ matrix, the Kronecker product $A\otimes B$ is defined as follows;

$$A\otimes B \triangleq \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix} \in \mathbb{R}^{mp\times nq} \tag{1}$$

Moreover, in this paper, we express the information path between agents based on graph theory [20]. The graph is collection of vertices and edges, and the notation of a graph is $\mathcal{G}=(\mathcal{V},\varepsilon)$, where $\mathcal{V}=\{1,2,\ldots,N\}$ is the set of N vertices in the graph, and ε is the set of edges connecting the vertices. Note that there are two types of graph, i.e. undirected graph and directed one. In this paper, we consider the directed graph. Furthermore, we introduce an adjacency matrix $\mathcal{A}$, a degree matrix $\mathcal{D}$ and a graph Laplacian $\mathcal{L}$ in order to express the graph algebraically. The adjacency matrix $\mathcal{A}$ represents the adjacency relation of each vertex of the graph. In the graph $\mathcal{G}=(\mathcal{V},\varepsilon)$ for a pair $(i,j)\in\varepsilon$ that is, there is an edge from $j\in\mathcal{V}$ to $i\in\mathcal{V}$ the vertex i is said to be adjacent to j. In this case, the adjacent set of vertices i is $\mathcal{N}_i \triangleq \{j\in\mathcal{V} \mid (i,j)\in\varepsilon\}$, and the elements a_{ij} of $\mathcal{A}=[a_{ij}]$ is defined as

$$a_{ij}=\begin{cases}1 & \text{if}(i,j)\in\varepsilon \text{ and } i\neq j\\ 0 & \text{otherwise}\end{cases} \tag{2}$$

Additionally, in the directed graph $\mathcal{G}=(\mathcal{V},\varepsilon)$, the in-degree of a vertex represents the number of edges incoming to the vertex and it is denoted as d_i^{in}. Conversely, out-degree means the number of edges outgoing from a vertex. Then for the graph $\mathcal{G}=(\mathcal{V},\varepsilon)$, the degree matrix $\mathcal{D}\in\mathbb{R}^{N\times N}$ and the graph Laplacian $\mathcal{L}$ are defined as

$$diag(A_1,A_2,\ldots,A_n) \tag{3}$$

$$\mathcal{L}\triangleq\mathcal{D}-\mathcal{A} \tag{4}$$

Furthermore, the following useful lemma is used in this paper:

Lemma 1 [21]: For arbitrary vectors α and β, matrices G and H with appropriate dimensions, the following inequality holds:

$$2\alpha^T GH\beta \le 2\|G^T\alpha\|\|H\beta\| \tag{5}$$

3. Problem Formulation

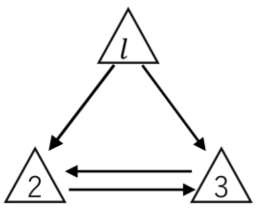

Fig. 1 multi-agent system in this paper

In Fig.1, the triangle "i" represents i-th agent(i=1,2,3). Moreover "l" means the leader and the others are followers, and arrows indicate communication paths. Then, the adjacency matrix $\mathcal{A}$, the degree matrix $\mathcal{D}$, and the graph Laplacian $\mathcal{L}$ in Fig.1 can be obtained as

$$\mathcal{A}=\begin{pmatrix}0&0&0\\1&0&1\\1&1&0\end{pmatrix},\mathcal{D}=\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix},\mathcal{L}=\begin{pmatrix}2&0&0\\-1&2&-1\\-1&-1&2\end{pmatrix} \tag{6}$$

Now we assume that i-th agent ($i=l,2,3$) can be described as the following state equation:

$$\frac{d}{dt}x_i(t) = Ax_i(t) + Bu_i(t) \quad (i = 1, 2, 3) \tag{7}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ are the vectors of the state and the control input, and the state $x_i(t) \in \mathbb{R}^n$ is given by

$$x_i(t) = \begin{pmatrix} x_{xi}(t) & v_{xi}(t) & x_{yi}(t) & v_{yi}(t) \end{pmatrix}^T \tag{8}$$

i.e. the state $x_{xi}(t)$ (resp. $x_{yi}(t)$) is the position in *x*-axis (resp. *y*-axis), and $v_{xi}(t)$ (resp. $v_{yi}(t)$) is velocity in *x*-axis (resp. *y*-axis) for the i -th agent. In (7), $A \in \mathbb{R}^{l \times n}$ and $B \in \mathbb{R}^{l \times m}$ are the system parameters which are defined as

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{9}$$

Here, in order to consider the relative positions between agents, we introduce the following vectors:

$$d_i = \begin{pmatrix} d_{xi} & \hat{v}_{xi} & d_{yi} & \hat{v}_{yi} \end{pmatrix}^T \tag{10}$$

where d_{xi} (resp. d_{yi}) is the desired relative position in *x*-axis (resp. *y*-axis) between the i -th agent and the leader agent. Similarly, $\hat{v}_{xi}$ and $\hat{v}_{yi}$ are the target velocity. Note that one can see that $d_l \triangleq 0$. Here, we consider the difference between the actual position of the agent $(x_i(t))$ and the desired relative position between the leader and the follower d_i as a new state of the system. From (7), the state equation of each follower considering the relative positions from leader to follower is expressed as

$$\frac{d}{dt}\left(x_i(t) - d_i\right) = A\left(x_i(t) - d_i\right) + Bu_i(t) \quad (i = 1, 2.3) \tag{11}$$

By introducing the additional state vector $\bar{x}_i(t)$ described as

$$\bar{x}_i(t) = \left(x_i(t) - d_i\right) = \begin{pmatrix} x_{xi}(t) - d_{xi} \\ v_{xi}(t) - \hat{v}_{xi} \\ x_{yi}(t) - d_{yi} \\ v_{yi}(t) - \hat{v}_{yi} \end{pmatrix} = \begin{pmatrix} \bar{x}_{xi}(t) \\ \bar{v}_{xi}(t) \\ \bar{x}_{yi}(t) \\ \bar{v}_{yi}(t) \end{pmatrix} \tag{12}$$

Then one can see from (11) and (12) that the following state equation can be obtained:

$$\frac{d}{dt}\bar{x}_i(t) = A\bar{x}_i(t) + Bu_i(t) \tag{13}$$

Summarizing the state equations of all agents, we get the following total system:

$$\frac{d}{dt}\bar{x}(t) = A_t\bar{x}(t) + B_t u(t) \tag{14}$$

where A_t, B_t, $\bar{x}(t)$ and $u(t)$ are matrices and vectors given by

$$A_t = I_3 \otimes A = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{pmatrix}, \quad \mathrm{B}_t = I_3 \otimes B = \begin{pmatrix} B & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & B \end{pmatrix}$$
$$\bar{x}(t) = \begin{pmatrix} \bar{x}_l^T(t) & \bar{x}_2^T(t) & \bar{x}_3^T(t) \end{pmatrix}^T, \quad u(t) = \begin{pmatrix} u_l^T(t) & u_2^T(t) & u_3^T(t) \end{pmatrix}^T \tag{15}$$

Next, we consider the control input $u(t)$. Note that consensus problem, “consensus” for agents means that the following relation for $\forall_i \in \mathcal{V}$ and $\forall_j \in \mathcal{V}$ holds:

$$\lim_{t\to\infty}\left(x_i(t)-x_j(t)\right)=0 \tag{16}$$

If $F\in\mathbb{R}^{2\times 4}$ is the consensus gain, it is known that the consensus input for $x_i(t)$, $u_{Fi}(t)$ is given by [22].

$$u_{Fi}(t)=F\sum_{j\in\mathcal{N}_i}\left(x_i(t)-x_j(t)\right) \tag{17}$$

In the case of this paper, $u_{Fi}(t)$ is calculated as follows:

$$\begin{aligned} &u_{Fl}(t)=0\\ &u_{F2}(t)=F(-x_l(t)+2x_2(t)-x_3(t))\\ &u_{F3}(t)=F(-x_l(t)-x_2(t)+2x_3(t)) \end{aligned} \tag{18}$$

Namely, $u_F(t)=\left(u_{Fl}^T(t)\quad u_{F2}^T(t)\quad u_{F3}^T(t)\right)^T$ can be represented by the following matrix-vector form:

$$u_F(t)=\begin{pmatrix}0&0&0\\-F&2F&-F\\-F&-F&2F\end{pmatrix}\begin{pmatrix}x_l(t)\\x_2(t)\\x_3(t)\end{pmatrix}=(\mathcal{L}\otimes F)x(t) \tag{19}$$

Additionally, let $u_K(t)$ be the state feedback input for stabilization of the system. By using the feedback gain matrix $u_K(t)\in\mathbb{R}^{2\times 4}$, the state feedback input $u_K(t)$ can be written as

$$u_K(t)\triangleq(I\otimes K)\bar{x}(t) \tag{20}$$

Finally, we introduce a compensation input *v(t)* and consider the following control input:

$$\begin{aligned} u(t)&\triangleq u_K(t)+u_F(t)+v(t)\\ &=(I\otimes K)\bar{x}(t)+(\mathcal{L}\otimes F)x(t)+v(t)\\ &=\begin{pmatrix}K&0&0\\-F&K+2F&-F\\-F&-F&K+2F\end{pmatrix}\bar{x}(t)+\begin{pmatrix}0&0&0\\-F&2F&-F\\-F&-F&2F\end{pmatrix}d_t+v(t) \end{aligned} \tag{21}$$

where $v(t)=\left(v_l^T(t)\quad v_2^T(t)\quad v_3^T(t)\right)^T$. Note that the design method for the compensation input *v*(*t*) and the gain matrices $K\in\mathbb{R}^{2\times 4}$ and $F\in\mathbb{R}^{2\times 4}$ is discussed in the next section. From (14) and (21), we have

$$\begin{aligned} \frac{d}{dt}\bar{x}(t)&=A_t\bar{x}(t)+B_t\left\{\begin{pmatrix}K&0&0\\-F&K+2F&-F\\-F&-F&K+2F\end{pmatrix}\bar{x}(t)+\begin{pmatrix}0&0&0\\-F&2F&-F\\-F&-F&2F\end{pmatrix}d_t+v(t)\right\}\\ &=\left\{\begin{pmatrix}A&0&0\\ *&A&0\\ *&*&A\end{pmatrix}+\begin{pmatrix}BK&0&0\\-BF&B(K+2F)&-BF\\-BF&-BF&B(K+2F)\end{pmatrix}\right\}\begin{pmatrix}\bar{x}_l(t)\\\bar{x}_2(t)\\\bar{x}_3(t)\end{pmatrix}+\begin{pmatrix}0&0&0\\-BF&2BF&-BF\\-BF&-BF&2BF\end{pmatrix}\begin{pmatrix}d_l\\d_2\\d_3\end{pmatrix}+\begin{pmatrix}B&0&0\\ *&B&0\\ *&*&B\end{pmatrix}v(t)\\ &=\begin{pmatrix}A_K&0&0\\-BF&A_F&-BF\\-BF&-BF&A_F\end{pmatrix}\begin{pmatrix}\bar{x}_l(t)\\\bar{x}_2(t)\\\bar{x}_3(t)\end{pmatrix}+\begin{pmatrix}Bv_l(t)\\Bv_2(t)+BF(2d_2-d_3)\\Bv_3(t)+BF(-d_2+2d_3)\end{pmatrix} \end{aligned} \tag{22}$$

In (21), A_K and A_F are the matrices described as $A_K=A+BK$, $A_K=A+BK+2BF$.

From the above, the controller design objective in this study is to derive the consensus gain $F \in \mathbb{R}^{2\times 4}$, the feedback gain $K \in \mathbb{R}^{2\times 4}$ and compensation input $v(t) \in \mathbb{R}^{6}$ so that the asymptotic stability of the closed-loop system of (22) is guaranteed.

4. Main Results

In this section, the design method of the feedback gain $K \in \mathbb{R}^{2\times 4}$, the consensus gains $F \in \mathbb{R}^{2\times 4}$ and the compensation input $v_i(t) \in \mathbb{R}^2 (i = l, 2, 3)$ is shown.

We give the following theorem for determining these parameters of the overall system (22).

Theorem 1. Consider the overall system of (14) and the control input of (21). If there exist solutions $S > 0$, W_K and W_F of following LMI condition:

$$\begin{aligned}
&\begin{pmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ * & \psi_{22} & \psi_{23} \\ * & * & \psi_{33} \end{pmatrix} < 0 \\
&A_K = A + BK, A_K = A + BK + 2BF \\
&\psi_{11} = SA^T + AS + W_K^T B^T + BW_K, \quad \psi_{12} = \psi_{13} = -W_K^T B^T \\
&\psi_{22} = \psi_{33} = SA^T + AS + W_K^T B^T + BW_K + 2BW_F + 2W_F^T B^T, \quad \psi_{23} = -W_F^T B^T BW_F
\end{aligned} \tag{23}$$

then the compensation input $v(t)$ is designed as follows,

$$v(t) = \begin{pmatrix} v_l(t) \\ v_2(t) \\ v_3(t) \end{pmatrix} = \begin{pmatrix} 0 \\ -2Fd_2 - dm_3 \dfrac{\left\| F^T B^T P\overline{x}_2(t) \right\|}{\left\| B^T P\overline{x}_2(t) \right\|^2} B^T P\overline{x}_2(t) \\ -2Fd_3 - dm_2 \dfrac{\left\| F^T B^T P\overline{x}_3(t) \right\|}{\left\| B^T P\overline{x}_3(t) \right\|^2} B^T P\overline{x}_3(t) \end{pmatrix} \tag{24}$$

where the matrix P is given by $P = S^{-1}$ and the feedback gain matrix K and the consensus one F are designed as

$$K = W_K S^{-1} \tag{25}$$

$$F = W_F S^{-1} \tag{26}$$

Moreover, by applying the control input of (27) with the compensation input $v(t)$ of (24) the gain matrices K (25) and F (26) to the overall system of (14), asymptotic stability of the closed-loop system of (22) is guaranteed.

Proof: Using a positive definite symmetric matrix $P = P^T \in \mathbb{R}^{4\times 4}$, we introduce the following quadratic function as a candidate for Lyapunov function:

$$V(\overline{x}, t) = \overline{x}(t)^T (I_3 \otimes P)\overline{x}(t) \tag{27}$$

The time derivative of the quadratic function along the trajectory of the closed-loop system of (22) satisfies

$$\frac{d}{dt}V(\overline{x}, t) = \frac{d}{dt}\overline{x}(t)^T (I_3 \otimes P)\overline{x}(t) + \overline{x}(t)^T (I_3 \otimes P)\frac{d}{dt}\left(\overline{x}(t)\right) \tag{28}$$

As is well known, the stability condition for the closed-loop system is

$$\frac{d}{dt}V(\bar{x},t)<0 \tag{29}$$

and the time derivative of the quadratic function along the trajectory of the closed-loop system of (22) can be written as

$$\begin{aligned}
\frac{d}{dt}V(\bar{x},t)&=\left\{\begin{pmatrix} A_K & 0 & 0 \\ -BF & A_F & -BF \\ -BF & -BF & A_F \end{pmatrix}\bar{x}(t)+\begin{pmatrix} Bv_l(t) \\ Bv_2(t)+BF(2d_2-d_3) \\ Bv_3(t)+BF(-d_2+2d_3) \end{pmatrix}\right\}^T (I_3\otimes P)\bar{x}(t) \\
&+\bar{x}(t)^T(I_3\otimes P)\left\{\begin{pmatrix} A_K & 0 & 0 \\ -BF & A_F & -BF \\ -BF & -BF & A_F \end{pmatrix}\bar{x}(t)+\begin{pmatrix} Bv_l(t) \\ Bv_2(t)+BF(2d_2-d_3) \\ Bv_3(t)+BF(-d_2+2d_3) \end{pmatrix}\right\} \\
&=\bar{x}(t)^T\begin{pmatrix} A_K^TP+PA_K & -F^TB^TP & -F^TB^TP \\ -PBF & A_F^TP+PA_F & -F^TB^TP-PBF \\ -PBF & * & A_F^TP+PA_F \end{pmatrix}\bar{x}(t) \\
&+\bar{x}(t)^T P_tB_t\begin{pmatrix} v_l(t) \\ v_2(t)+F(2d_2-d_3) \\ v_3(t)+F(-d_2+2d_3) \end{pmatrix}+\left\{P_tB_t\begin{pmatrix} v_l(t) \\ v_2(t)+F(2d_2-d_3) \\ v_3(t)+F(-d_2+2d_3) \end{pmatrix}\right\}^T\bar{x}(t)
\end{aligned} \tag{30}$$

where $P_t\in\mathbb{R}^{12\times 12}$ is the following symmetric positive definite matrix:

$$P_t=I_3\otimes P=\begin{pmatrix} P & 0 & 0 \\ * & P & 0 \\ * & * & P \end{pmatrix} \tag{31}$$

Here, by introducing the matrix $\Phi(P,K,F)$ and the scalar function $\omega(P,F,v(t))$ which are defined as

$$\begin{aligned}
\omega(P,F,v(t))&\triangleq\bar{x}(t)^T P_tB_t\begin{pmatrix} v_l(t) \\ v_2(t)+F(2d_2-d_3) \\ v_3(t)+F(-d_2+2d_3) \end{pmatrix}+\left\{P_tB_t\begin{pmatrix} v_l(t) \\ v_2(t)+F(2d_2-d_3) \\ v_3(t)+F(-d_2+2d_3) \end{pmatrix}\right\}^T\bar{x}(t) \\
&=2PB\left(PBv_l(t)\right)^T\bar{x}_l(t)+2PB\left[PB\left\{v_2(t)+F\left(2d_2-d_3\right)\right\}\right]^T\bar{x}_2(t) \\
&+2PB\left[PB\left\{v_3(t)+F\left(-d_2+2d_3\right)\right\}\right]^T\bar{x}_3(t)
\end{aligned} \tag{32}$$

$$\Phi(P,K,F)\triangleq\begin{pmatrix} A_K^TP+PA_K & -F^TB^TP & -F^TB^TP \\ * & A_F^TP+PA_F & -F^TB^TP-PBF \\ * & * & A_F^TP+PA_F \end{pmatrix} \tag{33}$$

one can see that the stability condition for the closed-loop system of (22) is reduced to

$$\frac{d}{dt}V(\bar{x},t)=\bar{x}^T(t)\Phi(P,K,F)+\omega(P,F,v(t))<0 \tag{34}$$

Namely, if the matrix $\Phi(P,K,F)$ is negative definite and $\omega(P,F,v(t))<0$ are satisfied, then the quadratic function $V(\bar{x},t)$ becomes a Lyapunov function. For leader agent, there is no need the compensation input, i.e. $v_l(t)=0$ then we have

$$\begin{aligned}
&\omega_l(t)=PBv_l(t)=0 \\
&\omega_2(t)=\left[PB\left\{v_2(t)+F(2d_2-d_3)\right\}\right]^T\bar{x}_2(t)=v_2^T(t)B^TP\bar{x}_2(t)+2d_2F^TB^TP\bar{x}_2(t)-d_3F^TB^TP\bar{x}_2(t) \\
&\omega_3(t)=\left[PB\left\{v_3(t)+F\left(-d_2+2d_3\right)\right\}\right]^T\bar{x}_3(t)=v_3^T(t)B^TP\bar{x}_3(t)-d_2F^TB^TP\bar{x}_3(t)+2d_3F^TB^TP\bar{x}_3(t)
\end{aligned} \tag{35}$$

where $\omega_i(t)$ $(i=l,2,3)$ is the i-th term in the right-hand side of (32). Since $v_l(t)=0$, we consider the design problem of $v_2(t)$ and $v_3(t)$. Let $\xi_3(t)$ be an auxiliary input for reducing the effect of d_3. In this paper, d_3 means the relative position between the leader and the follower 3, and it is unknown to the follower 2. The follower 2 can obtain the information for the upper bound dm_3 for the relative position, i.e. dm_3 satisfies $\|d_3\| \le dm_3$. Therefore, we consider

$$v_2(t) = -2Fd_2 + \xi_3(t) \tag{36}$$

and one can see that for the third term in the right-hand side of $\omega_2(t)$ in (35) the following inequality holds:

$$\begin{aligned} -d_3F^T B^T P\overline{x}_2(t) &\le \|-d_3\| \left\|F^T B^T P\overline{x}_2(t)\right\| \\ &\le dm_3 \left\|F^T B^T P\overline{x}_2(t)\right\| \end{aligned} \tag{37}$$

Thus, by selecting $\xi_3(t)$ defined as

$$\xi_3(t) = -dm_3 \frac{\left\|F^T B^T P\overline{x}_2(t)\right\|}{\left\|B^T P\overline{x}_2(t)\right\|^2} B^T P\overline{x}_2(t) \tag{38}$$

we can obtain

$$\begin{aligned} \omega_2(t) &\le -2d_2F^T B^T P\overline{x}_2(t) + \left(-dm_3 \frac{\left\|F^T B^T P\overline{x}_2(t)\right\|}{\left\|B^T P\overline{x}_2(t)\right\|^2} \overline{x}_2(t) PB \right) B^T P\overline{x}_2(t) + 2d_2F^T B^T P\overline{x}_2(t) + dm_3 \left\|F^T B^T P\overline{x}_2(t)\right\| \\ &\le 0 \end{aligned} \tag{39}$$

Similarly, we consider the following compensation input for follower 3:

$$v_3(t) = -2Fd_3 + \xi_2(t) \tag{40}$$

For the third term in the right-hand side of $\omega_3(t)$ in (35), we find that the inequality

$$\begin{aligned} -d_2F^T B^T P\overline{x}_3(t) &\le \|-d_2\| \left\|F^T B^T P\overline{x}_3(t)\right\| \\ &\le dm_2 \left\|F^T B^T P\overline{x}_3(t)\right\| \end{aligned} \tag{41}$$

is satisfied, and thus $\xi_2(t)$ is designed as

$$\xi_2(t) = -dm_2 \frac{\left\|F^T B^T P\overline{x}_3(t)\right\|}{\left\|B^T P\overline{x}_3(t)\right\|^2} B^T P\overline{x}_3(t) \tag{42}$$

Then we can obtain the following inequality:

$$\omega_3(t) \le 0 \tag{43}$$

Consequently, if $v(t)$ is designed as

$$v(t) = \begin{pmatrix} v_l(t) \\ v_2(t) \\ v_3(t) \end{pmatrix} = \begin{pmatrix} 0 \\ -2Fd_2 - dm_3 \dfrac{\left\|F^T B^T P\overline{x}_2(t)\right\|}{\left\|B^T P\overline{x}_2(t)\right\|^2} B^T P\overline{x}_2(t) \\ -2Fd_3 - dm_2 \dfrac{\left\|F^T B^T P\overline{x}_3(t)\right\|}{\left\|B^T P\overline{x}_3(t)\right\|^2} B^T P\overline{x}_3(t) \end{pmatrix} \tag{44}$$

then we have

$$\omega(P,F,v(t)) = 2\left(\omega_2(t) + \omega_3(t)\right) \le 0 \tag{45}$$

Once again considering the asymptotic stability condition of (34), the quadratic form term of $\bar{x}(t)$ should satisfy

$$\bar{x}^T(t)\Phi(P,K,F)\bar{x}(t) < 0 \tag{46}$$

The inequality of (46) is equivalent to the following condition;

$$\Phi(P,K,F) < 0 \tag{47}$$

In order to design the consensus gain F , and the feedback gain K , we introduce the symmetric positive definite matrix S satisfying $S = P^{-1}$ and change of variables $W_K = KS \in \mathbb{R}^{2\times4}$ and $W_F = FS \in \mathbb{R}^{2\times4}$. Moreover, pre- and post-multiplying (47) by $(I_3 \otimes S)$, we get.

$$\begin{aligned}(I_3 \otimes S)\Phi(P,K,F)(I_3 \otimes S) &= \begin{pmatrix} S & 0 & 0 \\ * & S & 0 \\ * & * & S \end{pmatrix}\begin{pmatrix} A_K^T P + PA_K & -F^T B^T P & -F^T B^T P \\ * & A_F^T P + PA_F & -F^T B^T P - PBF \\ * & * & A_F^T P + PA_F \end{pmatrix}\begin{pmatrix} S & 0 & 0 \\ * & S & 0 \\ * & * & S \end{pmatrix} \\ &= \begin{pmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ * & \psi_{22} & \psi_{23} \\ * & * & \psi_{33} \end{pmatrix} < 0 \\ &(A_K = A + BK, A_K = A + BK + 2BF) \\ &\psi_{11} = SA^T + AS + W_K^T B^T + BW_K, \quad \psi_{12} = \psi_{13} = -W_K^T B^T \\ &\psi_{22} = \psi_{33} = SA^T + AS + W_K^T B^T + BW_K + 2BW_F + 2W_F^T B^T, \quad \psi_{23} = -W_F^T B^T BW_F \end{aligned} \tag{48}$$

This inequality of (48) is linear matrix inequality (LMI) for S , W_K and W_F . If the solution of the LMI of (48) exists, the asymptotic stability of the closed-loop system of (22) is guaranteed, and the feedback gain K and the consensus gain F can be obtained as

$$K = W_K S^{-1}, \ \ F = W_F S^{-1} \tag{49}$$

From the above discussion, the proof of Theorem1 is accomplished.

Remark 1: In this paper, we approached the case of network topology such as fig.1 as an example, but the other topological structures can be handled if similar theoretical development is applied. However, it is inevitable that LMI will increase in size and complexity by the number of agents and the topology becomes complicated.

Remark 2: When the relative position between the leader and the follower is considered explicitly, it is often uncertain or unknown about the relative position between the leader and the other followers. Thus, construction of the state equation is generally not easy. As a result, there are not much exiting results which have explicitly dealt with the relative position in the dynamics as far as we know. On the other hand, in this study, it is possible to discuss LMI-based control system design that clearly indicates relative positional relationship by adding an input using the maximum value of relative distance that is known when performing the desired formation.

5. Numerical Simulation

Firstly, by solving LMI (48), we have symmetric positive definite matrices $S \in \mathbb{R}^{4\times4}$, $P \in \mathbb{R}^{4\times4}$ and matrices $W_K \in \mathbb{R}^{2\times4}$ and $W_F \in \mathbb{R}^{2\times4}$ which are given by

$$S = \begin{pmatrix} 1.1574 & -4.2577\times10^{-1} & 2.0326\times10^{-3} & -3.4553\times10^{-3} \\ -4.2577\times10^{-1} & 1.1574 & -3.4553\times10^{-3} & 2.0326\times10^{-3} \\ 2.0326\times10^{-3} & -3.4553\times10^{-3} & 1.1574 & -4.2577\times10^{-1} \\ -3.4553\times10^{-3} & 2.0326\times10^{-3} & -4.2577\times10^{-1} & 1.1574 \end{pmatrix} \tag{50}$$

$$P = \begin{pmatrix} 9.9925\times10^{-1} & 3.6760\times10^{-1} & 2.3596\times10^{-4} & 2.4249\times10^{-3} \\ 3.6760\times10^{-1} & 9.9925\times10^{-1} & 2.4249\times10^{-3} & 2.3462\times10^{-4} \\ 2.3596\times10^{-4} & 2.4249\times10^{-3} & 9.9925\times10^{-1} & 3.6760\times10^{-1} \\ 2.4249\times10^{-3} & 2.3462\times10^{-4} & 3.6760\times10^{-1} & 9.9925\times10^{-1} \end{pmatrix} \tag{51}$$

$$W_K = \begin{pmatrix} -1.1651 & -1.9053 & -9.7081\times10^{-3} & -1.6958\times10^{-18} \\ -9.7077\times10^{-3} & -2.1926 & -1.1651 & -1.9053 \end{pmatrix} \tag{52}$$

$$W_F = \begin{pmatrix} 6.9246\times10^{-6} & 7.9258\times10^{-1} & 6.9246\times10^{-6} & 5.6843\times10^{-1} \\ 6.9246\times10^{-6} & 5.6843\times10^{-1} & 6.9246\times10^{-6} & 7.9258\times10^{-1} \end{pmatrix} \tag{53}$$

Then the feedback gain $K \in \mathbb{R}^{2\times4}$ and the consensus gain $F \in \mathbb{R}^{2\times4}$ can be calculated as

$$K = \begin{pmatrix} -1.8646 & -2.3321 & -1.4595\times10^{-2} & -6.8409\times10^{-3} \\ -8.2057\times10^{-1} & -2.1978 & -1.8699 & -2.3327 \end{pmatrix} \tag{54}$$

$$F = \begin{pmatrix} 2.9273\times10^{-1} & 7.9213\times10^{-1} & 2.1088\times10^{-1} & 5.6819\times10^{-1} \\ 2.1088\times10^{-1} & 5.6819\times10^{-1} & 2.9274\times10^{-1} & 7.9213\times10^{-1} \end{pmatrix} \tag{55}$$

In this example, initial values for the closed-loop system of (22) are selected as follows;

$$x_l(0) = \begin{pmatrix} 5 \\ -2 \\ 3 \\ 2 \end{pmatrix}, \qquad x_2(0) = \begin{pmatrix} 6 \\ 4 \\ 0 \\ -3 \end{pmatrix}, \qquad x_3(0) = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 4 \end{pmatrix} \tag{56}$$

Furthermore, let $r(t)$ be the leader's reference input and $\overline{x}_l(t)$ be $\overline{x}_l(t) = x_l(t) - r(t)$. In this example, $r(t)$ gives the leader to go around a circle of radius 3 with 20[s]. Also, give d_2 and d_3 are so that the followers 2 and 3 leaves the leader by (2, 2) and (-2, 2).

$$r(t) = \begin{pmatrix} 3\cos(0.1\pi t) \\ -0.3\sin(0.1\pi t) \\ 3\sin(0.1\pi t) \\ 0.3\cos(0.1\pi t) \end{pmatrix}, \qquad d_2 = \begin{pmatrix} 2 \\ 0 \\ -2 \\ 0 \end{pmatrix}, \qquad d_3 = \begin{pmatrix} -2 \\ 0 \\ -2 \\ 0 \end{pmatrix} \tag{57}$$

Additionally, dm_2 and dm_3 are selected as $dm_2 = dm_3 = 6$.

The simulation result of this numerical example is shown in Figs. 2 - 7. In Figs.2 - 5, show the state trajectory of each agent and the shape of the formation every 5[s]. Figs.6 and 7 show the time histories of each agent in the *x* and *y*-axes, respectively. From Figs.2-5, we can see that the leader follows the given trajectory, and the followers 2 and 3 follow the desired relative position. Also, from fig.6 and fig.7, looking at the transition of the position coordinates of each agent, it can be seen that the follower moves away from the leader's movement locus by the desired position as time passes. Namely, it can be said that the proposed formation control system has been designed, and thus we have shown the effectiveness of the proposed formation control systems.

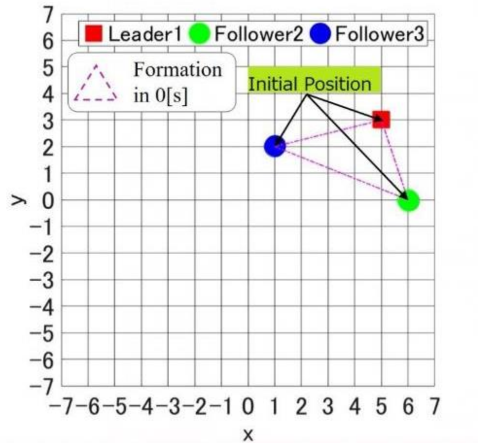

Fig. 2 Movement of each agent in 0[s]

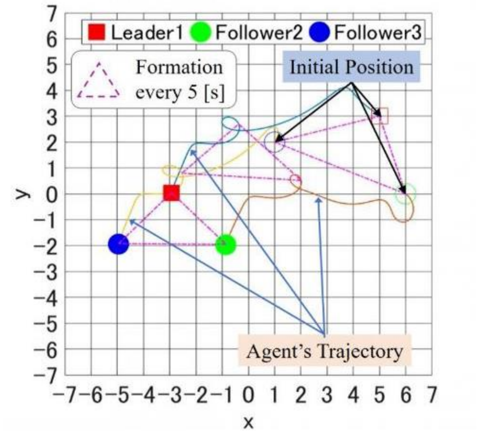

Fig. 3 Movement of each agent in 10[s]

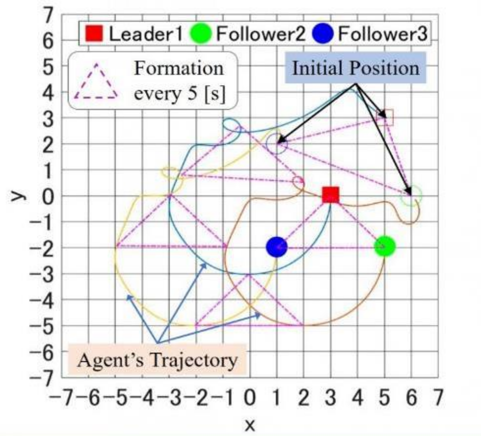

Fig. 4 Movement of each agent in 20[s]

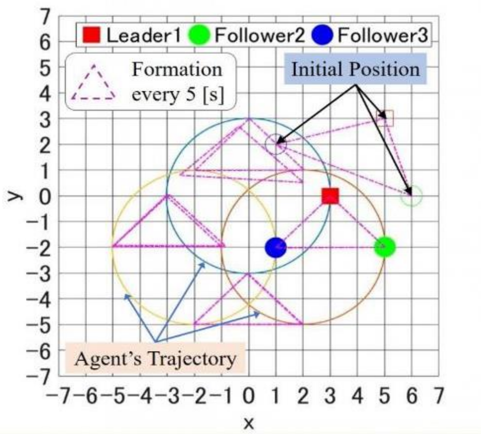

Fig. 5 Movement of each agent in 40[s]

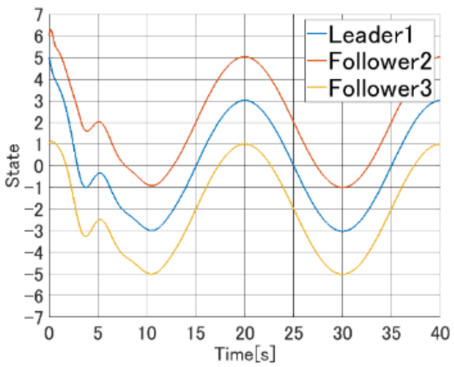

Fig.6 Time histories of $\mathbf{x}_{xi}(\mathbf{t})$

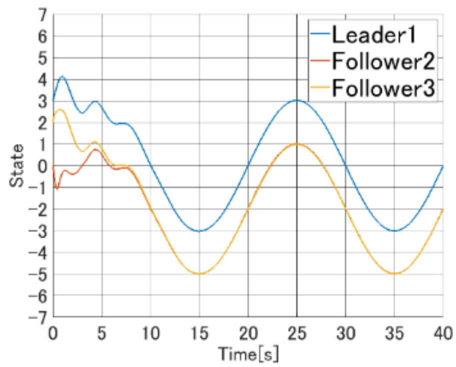

Fig.7 Time histories of $\mathbf{x}_{yi}(\mathbf{t})$

6. Conclusions

In this paper, we present a design method of an adaptive gain controller considering relative distances between agents for MASs with leader-follower structure. The proposed adaptive gain controller consists of the state feedback laws with fixed gains and compensation input with adaptive gains which are adjusted by updating rules. We have shown that the sufficient conditions for the existence of the proposed adaptive gain controller are reduced to the solvability of LMI, i.e. the proposed controller can be designed by using software such as MATLAB's LMI Control Toolbox, Scilab's LMITOOL and so on. In the proposed control strategy, there is no need the information on the target value of the other followers and the information about the upper bound on relative positions is only required. Furthermore, the effectiveness of the proposed formation control system has been shown through a simple numerical example.

The future research subjects are an extension of the proposed design to such a broad class of systems as discrete-time systems and output feedback systems. Moreover, for the proposed adaptive gain controller, improvement of transient performance and guaranteeing disturbance attenuation level are our important future research subjects. Additionally, we will study the conservativeness of the proposed controller design and extend the proposed controller synthesis to such a broad class of control systems as a formation for MASs consisting of more general agent's dynamics with uncertainties and consensus via output feedback controllers.

Conflicts of Interest

The authors declare no conflict of interest.

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