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Angular Velocity Analysis of an Epicyclic Gear Train Using Fundamental Circuits and a Block Diagram

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Abstract

The prediction of angular velocity is a prerequisite to investigate the power flow and mechanical efficiency of an epicyclic gear train. This paper presents a useful tool, which combines the fundamental circuit and a block diagram, to analyze the angular velocity of an epicyclic gear train. It is a visualized and straightforward method without manipulating tedious kinematic equations, which makes the angular velocity analysis of epicyclic gear trains more simplified. A compound epicyclic gear train used in a synchronous differential device is taken as an example to demonstrate the analysis process of the proposed approach. The velocity ratio of the epicyclic gear train is also derived.

Keywords: angular velocity, epicyclic gear train, fundamental circuit, block diagram

1. Introduction

An epicyclic gear train is a compact gear mechanism with the characteristics of a wide speed-ratio range, high efficiency, and light weight, which is commonly used in automotive differentials, automatic transmissions, and in-hub bicycle multi-speed transmissions [1]. For an ordinary gear train, it is relatively easy to observe the direction of motion of each gear element and the power flow through the gear train. It is more difficult, however, to visualize the kinematic behavior of an epicyclic gear train directly by observation. There are several approaches used to deal with the angular velocity analysis of epicyclic gear trains, such as: the vector loop method [2], the relative velocity method [3], the tabular method [4], the algebraic method [5], and the fundamental circuit method [6, 7], which have been proposed over the last several decades. For these methods, the fundamental circuit method seems to be a straightforward one. A fundamental circuit is made up of two meshing gears associated with a gear pair and a carrier to maintain a constant centre distance between these two gears. For a planetary gear mechanism with k gear pairs, k fundamental circuit equations can be listed. However, the velocity ratio of a planetary gear mechanism is obtained by tediously manipulating these fundamental circuit equations, which frequently causes human error. To overcome this drawback, the fundamental circuit method associated with a block diagram, which is based on the control community, is used to analyze the angular velocity of a compound epicyclic gear train and calculate the velocity ratio of this gear mechanism.

2. A Compound Epicyclic Gear Train

Fig. 1 shows an epicyclic gear train coupled with an ordinary gear train used in a synchronous differential device. The epicyclic gear train consists of two sun gears (links 1 and 2), a compound planet gear (link 4-4*), and a carrier (link 3). A reverted

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ordinary gear train, which consists of two external gearsets (links 5 and 6, links 7 and 8), is further applied to constrain the motion of the two sun gears (link 1 and 2) of the epicyclic gear train. Links 1 and 8 form an external compound gear, and links 2 and 5 also form a compound gear. Therefore, the whole compound epicyclic gear train is a 1-DOF (degree of freedom) gear mechanism, i.e., a single-input and single-output mechanism. The input link of this gear mechanism is the carrier (link 3), and the output link could be link 2 or link 1.

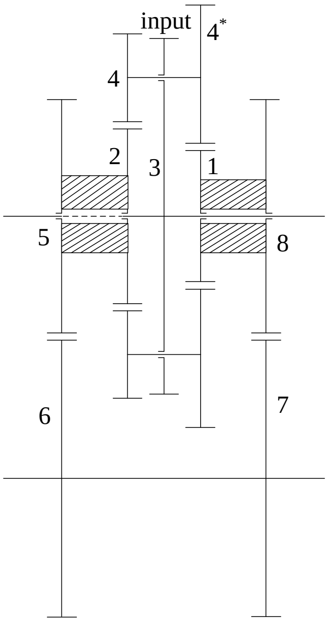

Fig. 1 An epicyclic gear train coupled with an ordinary gear train

3. Angular Velocity Analysis

3.1. Fundamental circuits

For the epicyclic gear train depicted in Fig. 1, there are two fundamental circuits. These can be identified as fundamental circuit I: (1, 4*)3 and fundamental circuit II: (2, 4)3, respectively. The related fundamental circuit equations can be listed as follows:

$$\omega_1 - \gamma_{4*1}\omega_4 + (\gamma_{4*1} - 1)\omega_3 = 0 \tag{1}$$

$$\omega_2 - \gamma_{42}\omega_4 + (\gamma_{42} - 1)\omega_3 = 0 \tag{2}$$

where ω_i is the angular velocity of link *i*, gear ratios are $\gamma_{4*1} = -Z_{4*} / Z_1$ and $\gamma_{42} = -Z_4 / Z_2$, and Z_j is the number of teeth on gear *j*. The negative sign represents the opposite rotational direction of an external gearset. Due to the geometric constraint of the reverted ordinary gear train, the relationship between angular velocities ω_1 and ω_2 for links 1 and 2 is:

$$\omega_1 = k\omega_2 \tag{3}$$

where $k = (-Z_7/Z_8)*(-Z_5/Z_6)$.

3.2. Block diagram

The kinematic characteristics of an epicyclic gear train can be carried out by using a block diagram [8]. Eqs (1) and (2) are further expressed as:

$$\omega_3 = \omega_1 - \gamma_{4*1}(\omega_4 - \omega_3) \tag{4}$$

$$\omega_2 = \omega_3 + \gamma_{42}(\omega_4 - \omega_3) \tag{5}$$

Based on the linear relationships shown in Equations (4) and (5), the related block diagram of the gear mechanism shown in Fig. 1 is plotted as shown in Fig. 2.

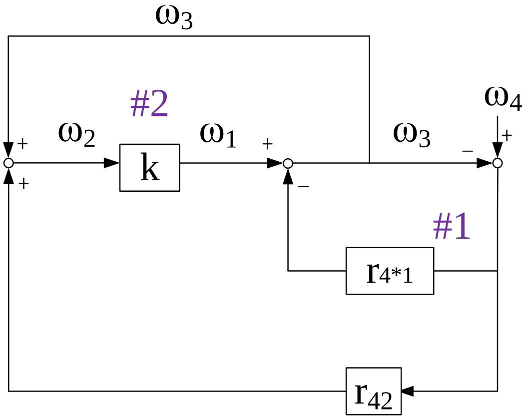

Fig. 2 Block diagram of the gear mechanism shown in Fig. 1

3.3. Velocity ratio

The velocity ratio of a gear mechanism is defined as the ratio of the output shaft speed to the input shaft speed, i.e., the transfer function. By investigating the block diagram shown in Fig. 2, the angular velocity ratios between ω_2, ω_3 and ω_4 can be easily obtained as:

$$\frac{\omega_2}{\omega_4} = \frac{\gamma_{42} - \gamma_{4*1}}{1 - \gamma_{4*1} + k\gamma_{42} - k} \tag{6}$$

$$\frac{\omega_3}{\omega_4} = \frac{k\gamma_{42} - \gamma_{4*1}}{1 - \gamma_{4*1} + k\gamma_{42} - k} \tag{7}$$

When link 2 is the output link and link 3 is the input link, the velocity ratio of this compound epicyclic gear train is

$$\frac{\omega_2}{\omega_3} = \frac{\gamma_{42} - \gamma_{4*1}}{k\gamma_{42} - \gamma_{4*1}} \tag{8}$$

Here, a set of gear-teeth, as listed in Table 1, of the presented compound epicyclic gear train shown in Fig. 1 is taken as an example to determine the velocity ratio. The related gear ratios are $\gamma_{4*1} = -16/16 = -1$ and $\gamma_{42} = -12/20 = -0.6$. In addition, the parameter $k = (-28/26)*(-24/30) = -0.86$. By applying Equation (8), the velocity ratio of the gear mechanism is $\omega_2/\omega_3 = [-0.6 - (-1)]/[(-0.86)(-0.6) - (-1)] = 0.26$. Since the value of the velocity ratio is less than 1, this gear mechanism can be used as a gear reducer when link 2 is the output link and link 3 is the input link. Due to the enginering reality, only the coaxial lnks rotate about the stationary shaft a-a, i.e., links 1, 2 and 3, can be assigned as the input or output links. There are a total of $P_2^3 = 6$ different arrangements shown in Table 2. The value of the velocity ratio for each case can be determined based on the block diagram shown in Fig. 2. As depicted in Table 2, the rotations of the output and input links for cases I, II, III and V are in opposite directions. Case IV can be used as a gear multiplier, while case VI is a gear reducer.

Table 1 The number of gear-teeth for each gear element of the compound epicyclic gear train

Z_1	Z_2	Z_4	Z_{4*}	Z_5	Z_6	Z_7	Z_8
16	20	12	16	24	30	28	26

Table 2 Six different arrangements of the input and output links for the compound epicyclic gear train

Case	I	II	III	IV	V	VI
Input link	1	1	2	2	3	3
Output link	2	3	1	3	1	2
Velocity ratio	-1.16	-4.47	-0.86	3.85	-0.22	0.26

4. Conclusion

In this paper, the fundamental circuit method associated with a block diagram is applied to analyze the velocity ratio of a compound epicyclic gear train. The kinematic equations of an epicyclic gear train can be quickly listed by using fundamental circuits. A block diagram is used to represent the linear relationship of the angular velocity of each link. The velocity ratio can be easily obtained by investigating the block diagram without tediously manipulating these kinematic equations. This provides a novel and useful tool to guide mechanical engineers in predicting the speed ratio of an epicyclic gear train.

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