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Optimal Cost-Analysis and Design of Circular Footings

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Abstract

The study pertains to the optimal cost-analysis and design of a circular footing subjected to generalized loadings using sequential unconstrained minimization technique (SUMT) in conjunction with Powell's conjugate direction method for multidimensional search and quadratic interpolation method for one dimensional minimization. The cost of the footing is minimized satisfying all the structural and geotechnical engineering design considerations. As extended penalty function method has been used to convert the constrained problem into an unconstrained one, the developed technique is capable of handling both feasible and infeasible initial design vector. The net saving in cost starting from the best possible manual design ranges from 10 to 20 %. For all practical purposes, the optimum cost is independent of the initial design point. It was observed that for better convergence, the transition parameter δ should be chosen at least 100 times the initial penalty parameter r_k .

Keywords: optimal cost analysis, sequential unconstrained optimization technique (SUMT), powell's conjugate direction method, penalty function method, circular footing

1. Introduction

Design in a sense means the allocation of the sizes of different components of an engineering system. In an endeavor to design an engineering system, the safety and economy should be of prime considerations. To arrive at the optimum cost design, a large number of alternative designs are made and the one that requires minimum cost for its implementation, and fulfills the design requirements, is selected. Generation of large number of alternative designs involve tedious repetitive computations. As such, an efficient optimization algorithm is needed to be adopted. Design of shallow foundation consists of two interrelated steps: (a) Selection of shape, size and depth of foundation and (b) Detailed analysis and structural design for the selected geometry of the foundation. Structural design of the foundation has drew adequate attention from the engineers but the proportioning of the foundation has not received the same. Common practice of the design of footings is to initially estimate the size of the footing from geotechnical engineering point of view, and subsequently carrying out the structural design. Tilt of the foundation is normally restricted by limiting the total settlement; seldom detailed analysis in this respect is performed. An integrated analysis of the footing both from geotechnical and structural engineering aspects are essential to arrive at a proper design. Thus, a general procedure is developed for the optimum cost design of shallow circular foundations taking the above mentioned aspects into consideration.

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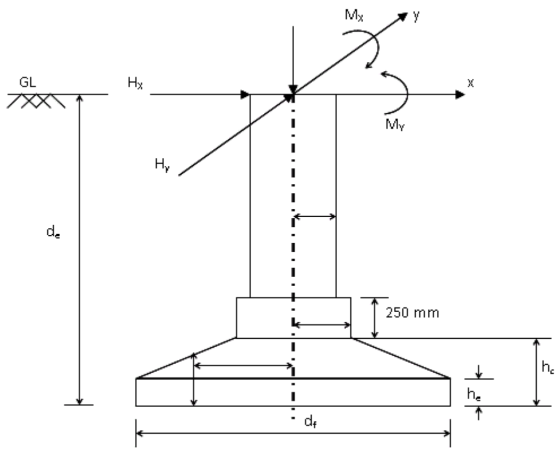
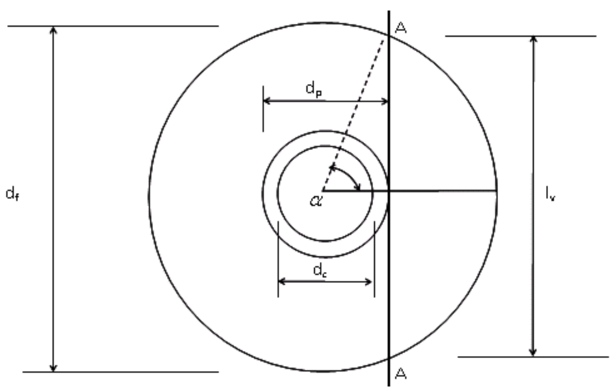
Optimization techniques have been successfully used in various structural and geotechnical problems. A brief literature review is presented on this subject. Subbarao et al. [1] proposed a method for estimating the size of the footing subjected to a uniaxial moment and an axial load. However, this study did not include the tilt of the foundation and the effect of the depth of the foundation. Bavikatti et al. [2] carried out the optimum design of isolated column footing using linear programming technique. The authors made several parametric studies and reported a net saving of 8 – 10 % in cost. However, this study did not consider the settlement aspect in the design. MadanMohan [3] has studied the optimum design of shallow footings of rectangular shape subjected to generalized loadings taking into consideration of all the structural and geotechnical engineering aspects. He also reported about the optimum plan dimensioning of a group of footings. Sequential unconstrained minimization technique was used in the analysis. Desai et al. [4] reported a cost optimum design of isolated footing and different parametric studies. However, the study considered only the axial loading on the footing.

2. Statement of the Problem

Fig. 1 shows the plan and elevation of an isolated circular column footing. Given the loads and moments on the column and data regarding the soil profile and its geotechnical properties, the problem is to determine the dimensions of the column, footing and depth of embedment of footing in such a way that the total cost of the isolated column footing is the minimum. Analysis is carried out when the soil test results are available from common type of field tests such as Standard Penetration Test (SPT), Cone Penetration Test (CPT) and standard laboratory tests conducted on undisturbed samples of soil from various depths.



(a) Elevation



(b) Plan

Fig. 1 Plan and elevation of footing with imposed loads

3. Analysis

3.1. Design Variables

The following are the nine design variables which control the cost of the isolated circular column footing:

(a) Diameter of the footing (d_f)

(b) Depth of the embedment of the footing (d_e)

(c) Percentage of steel on column (ps_1)

(d) Percentage of steel used in the form of square grid in footing (ps_2)

(e) Percentage of steel used in the form of rings at the edge of footing (ps_3)

(f) Diameter of the column (d_c)

(g) Central thickness of footing slab (h_c)

(h) Edge thickness of footing slab (h_e), and

(i) Diameter of the pedestal (d_p)

3.2. Objective Function

The total cost of the single isolated circular column footing which includes the cost of concrete, cost of steel, cost of excavation and cost of backfilling is taken as the objective function F and is given by

$$F = (cs_1 + cs_2 + cs_3 + cs_4) \quad \text{(Rs)} \tag{1}$$

where cs_1, cs_2, cs_3 *and* cs_4 are the costs of concrete, steel, excavation and backfilling respectively; The components of the costs can be stated as follows:

$$\left.\begin{aligned} cs_1 &= R_C \times V_{CONC} \\ cs_2 &= R_S \times W_{STEEL} \\ cs_3 &= R_E \times V_{EXC} \\ cs_4 &= R_B \times V_{FILL} \end{aligned}\right\} \quad \text{(Rs)} \tag{2}$$

where R_C, R_S, R_E *and* R_B are the rate of concrete in Rs/m^3, rate of steel in Rs/kg, rate of excavation in Rs/m^3 and rate of backfilling of soil in Rs/m^3, and $V_{CONC}, V_{EXC}, V_{FILL}$ *and* W_{STEEL} are the volumes of concrete in m^3, excavation in m^3 and backfilling in m^3 and weight of steel in kg used respectively. The total volume of concrete is given by

$$V_{CONC} = \left(V_1 + V_2 + V_3\right) \quad \text{(m}^3\text{)} \tag{3}$$

where V_1 = Volume of concrete in the column, V_2 = Volume of concrete in pedestal and V_3 = Volume of concrete in the footing slab, and

$$V_1 = \pi B^2 (d_f - h_c - h_p) \quad \text{(m}^3\text{)} \tag{4}$$

$$V_2 = \pi B_1^2 h_p \tag{5}$$

$$V_3 = \pi B_1^2 h_c + \int_{r=B_1}^{d_f/2} 2\pi r \left[h_c - \left(\frac{h_c - h_e}{d_f/2 - B_1} \right)(r - B_1) \right] dr \tag{6}$$

where B and B_1 are the radius of the column and the pedestal in m respectively, h_p is the thickness of the pedestal, and r is any arbitrary distance from centre of footing.

The number of main reinforcement bar in column is decided depending on the percentage of steel in the column section. If percentage of steel is less than 1%, then number of steel bars is taken as 8. If percentage of steel is in between 1% and 4%, the number of steel bars is taken as 10. In case the percentage is more than 4%, then the number of steel bars is taken as 12. The minimum diameter of a steel bar is taken as 12 mm.

The weight of the main reinforcement in the column is given as

$$W_1 = n_{rb}\ (100 \times d_e + 12) \frac{\pi}{4} \left(\frac{d_{rb}}{10} \right)^2 \frac{7.85}{1000} \quad \text{(kg)} \tag{7}$$

where n_{rb} is the number of reinforcement bars; The diameter of the steel bar is calculated as

$$d_{rb} = \frac{4}{\pi} \left\{ \left[\frac{ps_1}{100} \pi (100B)^2 / n_{rb} \right] \right\}^{1/2} \times 10 \quad \text{(mm)} \tag{8}$$

The weight of the reinforcement in the helical tie bars in the column is calculated as follows (The details are represented in Fig. 2): The core diameter of the column is given as

$$d_{co} = (200\, B - 2\ cv_1) \quad \text{(cm)} \tag{9}$$

where $cv_1 =$ Reinforcement cover in column = 5 (cm); The pitch of the helical reinforcement is decided as follows: If $d_{co}/6$ is less than 75 mm. Otherwise, minimum pitch is taken as 75 mm [5]

$$Pitch = 10 \times \left(200\, B - 2\, cv_1 \right) / 6 \quad \text{(mm)} \tag{10}$$

The number of helical ties is calculated as

$$n_t = \left[\left(d_f - h_c - h_p \right) \times 1000 / Pitch + 1 \right] \text{ (rounded to the nearest integer)} \tag{11}$$

The length of each tie bar is given as

$$l_t = \left[\left\{ \pi\, d_{co} \right\}^2 + \left(Pitch/10 \right)^2 \right]^{1/2} \quad \text{(cm)} \tag{12}$$

The weight of steel in helical ties is

$$W_2 = n_t \times l_t\ \frac{\pi}{4} \left(\frac{8}{10} \right)^2 \frac{7.85}{1000} \quad \text{(kg)} \tag{13}$$

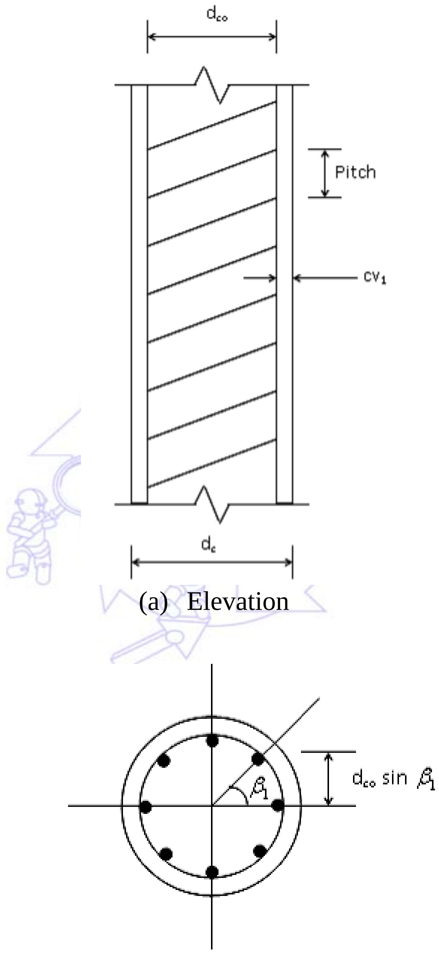
The amount of steel is estimated as follows: The footing slab will be reinforced with steel (ps_2 %) in the form of square grid. The reinforcement will not be in a position to resist circumferential tension at the edge of the slab. So, 250 mm of the outer edge of the slab will be reinforced with steel in the form of circular rings [14]. The area of reinforcement in x- direction is given as

$$Ast_x = \frac{1}{2}\left(\frac{ps_2}{100}\right)(d_f \times 100)\left(\frac{h_c + h_e}{2} - \frac{cv_2}{100}\right) \times 100 \quad (\text{cm}^2) \tag{14}$$

where cv_2 = Cover for reinforcement in slab in cm

The diameter of the reinforcement is decided as follows: If ps_2 <1%, the diameter is taken as 12 mm. If the percentage of steel is between 1 and 4%, then the diameter is taken to be 16 mm. For percentage of steel > 4%, the diameter is taken as 20mm [5]. The number of reinforcement bars in one direction is

$$n_x = \left[Ast_x \, \frac{4}{\pi}\left(\frac{10}{d_{rb}}\right)^2 + 1 \right] \quad \text{(rounded to the nearest integer)} \tag{15}$$



(a) Elevation

(b) Plan

Fig. 2 Cross-sectional plan and elevation of a column

The total length of the steel bars in one direction is

$$l_x = \left\{ \begin{array}{l} \left(100\, d_f - 2\, cv_2 + 2\, a_k\right) \\ +2 \sum\limits_{I=1}^{(n_x-1)/2} \left\{ 2 \times 100 \left[\left(d_f / 2\right)^2 - \left(I \times sp\right)^2 \right] + 2\left(a_k - cv_2\right) \right\} \end{array} \right\} \tag{16}$$

where cv_2 = Cover for reinforcement in footing = 7 cm, a_k = Length of hook for reinforcement = 50 mm for MS steel bar and 0 for deformed steel bar, and sp = Spacing of the reinforcement bars.

$$sp = \frac{d_f - 2\,cv_2\,/100}{n_x - 1} \quad \text{(m)} \tag{17}$$

The weight of reinforcement in footing is estimated as

$$W_3 = 2\,l_x\,\frac{\pi}{4}\left(\frac{d_{rb}}{10}\right)^2 \frac{7.85}{1000} \quad \text{(kg)} \tag{18}$$

The number of reinforcement bar (n_r) in the form of circular rings is taken as 4 at a spacing (sp) of 50 mm. Thus the cross-sectional area of steel is given by

$$Ast_r = ps_3\left(100h_e - cv_2\right)\times\frac{25}{100} \quad (\text{cm}^2) \tag{19}$$

The diameter of these steel bars is calculated as

$$d_{rb} = \frac{4}{\pi}\left(\frac{Ast_r}{n_r}\right)^{1/2} \times 10 \quad \text{(mm)} \tag{20}$$

However, the minimum diameter of the bar is taken to be 8 mm; The total length of these steel reinforcement is calculated as

$$l_t = \left\{\begin{matrix}\left[2\pi\left(100d_f/2 - cv_2\right) + 10\right] \\ +\sum_{I=1}^{3}\left[2\pi\left(d_f/2 - 0.05I\right)\times 100 + 10\right]\end{matrix}\right\} \quad \text{(cm)} \tag{21}$$

Thus, the weight of circular rings is calculated as

$$W_4 = n_r \times l_t\,\frac{\pi}{4}\left(\frac{d_{rb}}{10}\right)^2 \frac{7.85}{1000} \quad \text{(kg)} \tag{22}$$

Thus, the total weight of steel is

$$W_{STEEL} = \left(W_1 + W_2 + W_3 + W_4\right) \quad \text{(kg)} \tag{23}$$

The volume of the soil excavated is

$$V_{EXC} = \pi\left(d_f/2 + 0.15\right)^2 d_e \quad (\text{m}^3) \tag{24}$$

where 0.15 m is the clearance for excavation; The volume of soil for backfilling is calculated as

$$V_{FILL} = \left(V_{EXC} - V_{CONC}\right) \quad (\text{m}^3) \tag{25}$$

With the help of the above equations and the current rates for each components of cost, the total cost of the foundation system can be determined completely.

3.3. Structural Design Constraints

The structural design of the footing conforms to the specifications of the code of practice [5]. The stress conditions in the column must satisfy the following specifications:

$$\frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,cal}}{\sigma_{cbc}} \le 1.0 \tag{26}$$

$$\sigma_{cbt,cal} \leq 0.25(\sigma_{cc,cal} + \sigma_{cbc,cal}) \tag{27}$$

$$\sigma_{cbt,cal} \leq 0.75\tau_{rup} \tag{28}$$

where $\sigma_{cc,cal}$, $\sigma_{cbc,cal}$ and $\sigma_{cbt,cal}$ are calculated direct, bending compressive and tensile stresses in concrete respectively; σ_{cc} and σ_{cbc} are the permissible direct and bending stresses in concrete respectively; and τ_{rup} is the 7-days rupture strength of concrete. $\sigma_{cc,cal}$, $\sigma_{cbc,cal}$ and $\sigma_{cbt,cal}$ are calculated as follows:

Area of cross section of one bar

$$As_1 = \frac{\pi}{4}\left(\frac{d_{rb}}{10}\right)^2 \quad (\text{cm}^2) \tag{29}$$

Area of cross section of column in terms of concrete is given by

$$a_t = \pi(100B)^4 + (1.5m_c - 1)\left(\frac{ps_1}{100}\right)\times\pi(100B)^2 \quad (\text{cm}^2) \tag{30}$$

Moment of inertia of column section in equivalent of concrete is given by

$$i_t = \frac{\pi}{4}(100B)^4 + \sum_{I=1}^{(n_r-2)/2}\left\{2\times(1.5m_c - 1)\times As_1\times\left[(100B - cv_2)\sin(I\times\beta_1)\right]^2\right\} \quad (\text{cm}^4) \tag{31}$$

where m_c= Modular ratio of concrete (=E_s/E_c), β_1 = Angular spacing of steel bars= $(360° / n_\gamma)$

$$\therefore \sigma_{cc,cal} = \frac{P_v \times 10^3}{1.05a_t \times 100} \quad (\text{N/mm}^2) \tag{32}$$

where P_v = Axial load on column in kN

The factor 1.05 in the denominator of the above expression is to allow for 5% increase in the permissible load on column reinforced with helical tie bars.

$$\sigma_{cbc,cal} = \frac{m_m \times 10^6}{i_t \times 10^4}\times(100B) \quad (\text{N/mm}^2) \tag{33}$$

where m_m = Resultant maximum moment on the column in kNm and is calculated as

$$m_m = \left[\left\{m_x + h_y(d_e - h_c)\right\}^2 + \left\{m_y + h_x(d_e - h_c)\right\}^2\right]^{1/2} \quad (\text{kN-m}) \tag{34}$$

Where $m_x m_y$ are applied moments in kNm about X and Y axes respectively, h_x, h_y are applied horizontal loads in kN along X and Y axes respectively; Tensile stress due to bending is calculated as

$$\sigma_{cbt,cal} = (\sigma_{cbc,cal} - \sigma_{cc,cal}) \quad (\text{N/mm}^2) \tag{35}$$

Bearing stress in the pedestal should satisfy the following requirement

$$\sigma_{cbr,cal} \leq \sigma_{cbr,per} \tag{36}$$

where $\sigma_{cbr,cal}, \sigma_{cbr,per}$ are respectively the average calculated and permissible bearing stresses in N/mm^2 in pedestal, and are estimated as follows [5]

$$\sigma_{cbr,cal} = \frac{P_v \times 10^3}{\pi \times (1000B_1)^2} \quad \text{(N/mm}^2\text{)} \tag{37}$$

$$\sigma_{cbr,per} = 0.25\, f_{ck} \sqrt{\frac{B_1}{B}} \quad \text{(N/mm}^2\text{)} \tag{38}$$

where f_{ck} = Characteristic strength of concrete in N/mm^2; and the maximum value of the ratio (B_1/B) is 2. The stress conditions in concrete and steel due to maximum radial moment must satisfy the following requirements

$$\sigma^r_{cbc,cal} \le \sigma_{cbc,per} \tag{39}$$

$$\sigma^r_{st,cal} \le \sigma_{st,per} \tag{40}$$

where $\sigma_{cbc,per}, \sigma_{st,per}$ are permissible stresses in concrete and steel respectively, depending on the type of concrete and steel. $\sigma^r_{cbc,cal}, \sigma^r_{st,cal}$ are the calculated stresses in concrete and steel respectively as follows:

$$\sigma^r_{cbc,cal} = \frac{m_{rm} \times 10^6}{i_{t1} \times 10^4} k_c \left(100h_c - cv_2\right) \times 10 \quad \text{(N/mm}^2\text{)} \tag{41}$$

$$\sigma^r_{st,cal} = \sigma^r_{cbc,cal} \frac{m_c (1-k_c)}{k_c} \quad \text{(N/mm}^2\text{)} \tag{42}$$

where m_{rm} = maximum radial moment in kNm in slab, k_c = Neutral axis depth factor, and i_{t1} = Moment of inertia of the section about the neutral axis.

$$i_{t1} = \left\{ \frac{100\left[k_c\left(100h_c - cv_2\right)\right]^3}{3} + m_c \frac{Ast_x}{d_f}(1-k_c)^2 \times \left(100h_c - cv_2\right)^2 \right\} \text{(cm}^4\text{)} \tag{43}$$

$$k_c = -\frac{m_c \times ps_2}{200} + \left[\left(\frac{m_c \times ps_2}{200}\right)^2 + 2\left(\frac{m_c \times ps_2}{200}\right)\right]^{1/2} \tag{44}$$

Stress conditions in concrete and steel due to maximum circumferential moment must satisfy the following requirements

$$\sigma^\theta_{cbc,cal} \le \sigma_{cbc,per} \tag{45}$$

$$\sigma^\theta_{st,cal} \le \sigma_{st,per} \tag{46}$$

where $\sigma^\theta_{cbc,cal}, \sigma^\theta_{st,cal}$ are the calculated stresses in concrete and steel respectively due to maximum circumferential moment.

$$\sigma^\theta_{cbc,cal} = \frac{m_{tm} \times 25/100 \times 10^6}{i_t \times 10^4} k_c\, (100h_e - cv_2 - 1.0) \times 10 \quad \text{(N/mm}^2\text{)} \tag{47}$$

where m_{tm} = The maximum circumferential moment in kNm

$$\sigma^\theta_{st,cal} = \sigma^\theta_{cbc,cal} \frac{m_c (1-k_c)}{k_c} \quad \text{(N/mm}^2\text{)} \tag{48}$$

where i_{t1} = Moment of inertia of the section about the neutral axis

$$i_{t1} = \left\{ \frac{25\left[k_c(100h_e - cv_2 - 1\right]^3}{3} + m_c \ Ast_r(1-k_c)^2 \left(100h_e - cv_2 - 1\right)^2 \right\} \quad (49)$$

$$k_c = -\frac{m_c \times ps_3}{100} + \left[\left(\frac{m_c \times ps_3}{100}\right)^2 + 2\left(\frac{m_c \times ps_3}{100}\right)\right]^{1/2} \quad (50)$$

The shear stresses in the footing slab must satisfy the following requirements

$$\tau_{V1,cal} \le \tau_{V1,per} \quad (51)$$

$$\tau_{V2,cal} \le \tau_{V2,per} \quad (52)$$

where $\tau_{V1,per}$ is the permissible one way shear stress depending on the percentage of steel in footing slab, $\tau_{V2,per}$ is the permissible punching shear stress, and $\tau_{V1,cal}$, $\tau_{V2,cal}$ are calculated average one-way and two-way shear stresses respectively estimated as follows:

$$\tau_{V1,cal} = \frac{v_v}{1000h_c} \quad (\text{N/mm}^2) \quad (53)$$

where v_v = Maximum one way shear force per unit length and $\tau_{V1,per}$ = Permissible one way shear stress depending on the percentage of steel in footing slab and is estimated as

$$\begin{bmatrix} 0.0 < ps_2 \le 1.0\% \Rightarrow \tau_{V1,per} = 0.25 \ \left(N/mm^2\right) \\ 1.0 < ps_2 \le 2.0\% \Rightarrow \tau_{V1,per} = 0.45 \ \left(N/mm^2\right) \\ ps_2 > 2.0\% \Rightarrow \tau_{V1,per} = 0.51 \ \left(N/mm^2\right) \end{bmatrix} \quad (54)$$

The depth of the slab at a distance $h_c/2$ from the face of column which is critical section for a two-way (punching) shear is

$$h_r = h_c - \left(\frac{h_c - h_e}{d_f/2 - B_1}\right)\left(h_c/2 + B - B_1\right) \quad (\text{m}) \quad (55)$$

$$\tau_{V2,cal} = \frac{P_v - p \times \pi\left(B + h_c/2\right)^2}{2\pi(B + h_c/2)h_r} \times 10^{-3} \quad (\text{N/mm}^2) \quad (56)$$

where P = Uniform pressure produced by axial load P_v; The permissible punching shear stress is estimated as

$$\tau_{V2.per} = 0.16\sqrt{f_{ck}} \quad (\text{N/mm}^2) \quad (57)$$

The percentages of steel in column, at the central portion of the slab and at the edge of the slab must be within the permissible limits. Hence, the following constraints have to be satisfied:

$$ps_{\min 1} \le ps_1 \le ps_{\max 1} \quad (58)$$

$$ps_{\min 2} \le ps_2 \le ps_{\max 2} \quad (59)$$

$$ps_{\min 3} \le ps_3 \le ps_{\max 3} \quad (60)$$

where $ps_{\min}$ and $ps_{\max}$ are the minimum and maximum permissible steel reinforcement respectively; For the feasibility of geometry of foundation system, the following restrictions need to be imposed

$$h_{em} \le h_e \le h_c \tag{61}$$

$$d_c \le d_p \tag{62}$$

$$d_{fm} \le d_f \tag{63}$$

$$d_{cm} \le d_c \tag{64}$$

where h_{em} = Minimum specified edge thickness in m, d_c is the diameter of the column and d_{fm}, d_{cm} = Specified minimum diameter of footing and column respectively in m.

3.4. Geotechnical Design Constraints

A meaningful design of foundation must satisfy the following constraints from the consideration of geotechnical engineering aspects:

$$p_{\max} \le Q_{safe} \tag{65}$$

where P_{max} = Maximum pressure on the soil and Q_{safe} = Safe bearing capacity of foundation

$$p_{\max} = \left[\frac{1.1P_v}{\pi\left(d_f/2\right)^2} + \frac{4m_m}{\pi\left(d_f/2\right)^3} \right] \quad (\text{kN/m}^2) \tag{66}$$

Where the factor 1.1 with P_v is due to the assumption that the dead weight of the footing is 10% of the column load that is carried by the column itself. For the purpose of estimating the bearing capacity and settlement, only those soil layers are considered effective which are included within a depth equal to that of the twice the diameter of the footing measured from bottom of the footing.

Case (i): When c' and ϕ' values of soils are supplied from standard laboratory tests

For the calculation of bearing capacity weighted average values of c' and ϕ' (effective cohesion and angle of internal friction values for a soil) are estimated over the effective soil layers as

$$\bar{c} = \frac{\sum C_i' H_i}{\sum H_i} \quad \text{and} \quad \overline{\Phi} = \frac{\sum \Phi_i' H_i}{\sum H_i} \tag{67}$$

where H_i are height of layers, such that $(\Sigma H_i - d_e) \ge 2d_f$; The bearing capacity factors, shape factors and the inclination factors are calculated as per Winterkorn and Fang [6]

$$\left.\begin{aligned} N_q &= e^{\pi tan\overline{\Phi}}.\,tan^2\left(\frac{\pi}{4}+\frac{\overline{\Phi}}{2}\right) \\ N_c &= \left(N_q - 1\right)cot\overline{\Phi} \\ N_r &= 2\left(N_q + 1\right)tan\overline{\Phi} \end{aligned}\right] \tag{68}$$

$$\left.\begin{aligned} \xi_c &= 1 + N_q/N_c \\ \xi_q &= 1 + \tan\overline{\Phi} \\ \xi_\gamma &= 0.6 \end{aligned}\right] \tag{69}$$

$$\begin{bmatrix} \xi_{qi} = \left(1 - \dfrac{P_h}{P_v + B^{'} \times L^{'} \times \bar{c}\cot\overline{\Phi}}\right)^2 \\ \xi_{ci} = \xi_{qi} - \dfrac{1-\xi_{qi}}{N_c \tan\overline{\Phi}} \\ \xi_{\gamma i} = \left(1 - \dfrac{P_h}{P_v + B^{'} \times L^{'} \times \bar{c}\cot\overline{\Phi}}\right)^3 \end{bmatrix} \tag{70}$$

where $L^{'}$, $B^{'}$ are the effective length of P_h is the resultant horizontal force on the column transferred to the footing

$$P_h = \sqrt{\left(H_x^2 + H_y^2\right)} \tag{71}$$

where H_x, H_y are the horizontal forces on the column x- and y- directions; Based on the general bearing capacity [15], the ultimate bearing capacity of the foundation is calculated as

$$q_{ult} = \left[\bar{c}N_c\xi_c\xi_{ci} + \bar{q}N_q\xi_q\xi_{qi} + 0.5\gamma BN_\gamma\xi_\gamma\xi_{\gamma i}\right] \tag{72}$$

where $\bar{q}$ is the effective overburden pressure at foundation level; The safe bearing capacity is given by

$$Q_{safe} = q_{ult} / FS \tag{73}$$

where FS is the factor of safety for bearing capacity

Case (ii): When soil data are given as CPT values, a weighted average cone resistance value is calculated for effective zone of soil layers as follows:

$$\overline{q_c} = \frac{\sum q_{ci}H_i}{\sum H_i} \tag{74}$$

where q_{ci} is the cone resistance value at each soil layer; Allowable bearing capacity is calculated from $\overline{q_c}$ as per Bowles [7]

$$\left.\begin{array}{ll} q_a = \dfrac{\bar{q}_c}{30}\dfrac{\rho_{per}}{25} & B \le 1.2m \\ q_a = \dfrac{\bar{q}_c}{30}\left(\dfrac{B_{\min}+0.3}{B_{\min}}\right)^2\dfrac{\rho_{per}}{25} & B > 1.2m \end{array}\right] \tag{75}$$

where B_{min} is the least dimension of the footing, and ρ_{per} is the maximum permissible settlement.

Case (iii): When soil data are given as SPT values, a weighted average N-values calculated for effective zone of soil layers as follows:

$$\bar{N} = \frac{\sum N_i H_i}{\sum H_i} \tag{76}$$

From $\bar{N}$ value, allowable bearing capacity is calculated as per Bowles [7]

$$\left.\begin{aligned} q_a &= \frac{\bar{N}}{0.05} K_d \frac{\rho_{per}}{25} \ kN/m^2 \qquad B \le 1.2m \\ q_a &= \frac{\bar{N}}{0.08} \left(\frac{B+0.3}{B}\right)^2 \frac{\rho_{per}}{25} \ kN/m^2 \qquad B > 1.2m \end{aligned}\right] \tag{77}$$

where

$$K_d = 0 + 0.33 \frac{d_e}{B} \le 1.33 \tag{78}$$

To safely carry the axial load (P_v), the requirement to be satisfied is

$$P_v \le P_{ALL} \tag{79}$$

where P_{ALL} = Allowable load on footing

$$P_{All} = (Q_{safe} \times A_{ef}) \ \text{(kN)} \tag{80}$$

where A_{ef} = Effective area of the foundation

$$A_{ef} = L' \times B' \ (\text{m}^2) \tag{81}$$

where L', B' = Effective dimensions of footing calculated as

$$\left.\begin{aligned} \frac{B'}{B} &= 0.86 - 1.9\frac{e}{B} \qquad 0 \le \frac{e}{B} \le 0.2 \\ \frac{B'}{B} &= 0.80 - 1.6\frac{e}{B} \qquad 0.2 \le \frac{e}{B} < 0.5 \end{aligned}\right\} \tag{82}$$

$$\left.\begin{aligned} \frac{L'}{B} &= 0.86 \qquad 0 \le \frac{e}{B} < 0.15 \\ \frac{L'}{B} &= 0.806 + 1.244\frac{e}{B} - 0.569\left(\frac{e}{B}\right)^2 \qquad 0.15 \le \frac{e}{B} \le 0.5 \end{aligned}\right\} \tag{83}$$

where e = Eccentricity of loading. To prevent the overturning of footing, the requirement to be satisfied is

$$q \le p \tag{84}$$

where q and p = Maximum soil pressures induced due to resultant moment and axial forces respectively. The settlement of footing should be limited as follows:

$$\rho_{c,cal} \le \rho_{per} \tag{85}$$

where ρ_{per} = Maximum permissible settlement in mm [8], $\rho_{c,cal}$ = Calculated settlement at the centre of footing in mm. The total settlement of the footing is calculated as the sum of the compression in the individual layers. For calculation of compression in each layer, Steinbrenner's [9] approximate method is used:

$$\rho_i = \rho_i^{(A)} - \rho_i^{(B)} \tag{86}$$

where P_i = Settlement of layer i, and $P_i^{(A)}$, $P_i^{(B)}$ are the displacements at the levels of top and bottom of the layer i, considering a hypothetical soil layer of semi-infinite extent with the same elastic properties as the layer i, which are calculated using Mindlin's equation [10] for displacement under a concentrated load

$$\rho_z = \frac{(1+\mu_s)P}{8\pi(1-\mu_s)E_s}\left[\begin{array}{l}\dfrac{3-4\mu_s}{R_1}+\dfrac{5-12\mu_s+8\mu_s^2}{R_2}+\dfrac{(z-c)^2}{R_1^3}\\ +\dfrac{(3-4\mu_s)(z+c)^2-2cz}{R_2^3}+\dfrac{6cz(z+c)^2}{R_2^5}\end{array}\right] \tag{87}$$

where notations are shown in Fig. 2, ρ_z = Vertical displacement at depth z, E_s= Young's modulus for soil, and μ_s= Poisson's ration for soil.

The force on the soil is not concentrated but distributed over the contact area of the footing. The contact area is divided into a large number of sufficiently small segments as shown in the Fig. 3. In the figure, a typical segment with dimensions Δr and $\Delta\theta$ with centre-coordinate (r, θ) is highlighted. Distributed load in this segment is replaced by an equivalent concentrated load at the centre of the segment. Thus, the total contribution towards the settlement of all such segment can be calculated as

$$\rho_{c,cal} = \sum_{i=1}^{N_\theta}\sum_{j=1}^{N_r}\left(\rho_{Zij}^{(A)} - \rho_{Zij}^{(B)}\right) \tag{88}$$

where N_θ, N_r = Number of divisions in the θ, r directions; Equivalent concentrated load for the segment with central coordinate (r, θ) is calculated as

$$P = (r\,\Delta r\,\Delta\theta)\left(p + \frac{2qr}{d_f}\cos\theta\right) \tag{89}$$

Increase of stress on a soil element is calculated as

$$\Delta p = \sum_{i=1}^{N_\theta}\sum_{j=1}^{N_r}\sigma_{Zij} \tag{90}$$

where σ_{zij} is the increase in stress on the soil element due to the load on the segment $i-j$ using Mindlin's equation as follows

$$\sigma_z = \frac{P}{8\pi(1-\mu_s)}\left[\begin{array}{l}\dfrac{(1-\mu_s)(z-c)}{R_1^3}-\dfrac{(1-2\mu_s)(z-c)}{R_2^3}+\dfrac{3(z-c)^3}{R_1^5}\\ +\dfrac{3(3-4\mu_s)z(z+c)^2-3c(z+c)(5z-c)}{R_2^5}+\dfrac{30cz(z+c)^3}{R_2^7}\end{array}\right] \tag{91}$$

where c is the depth at which the concentrated load is acting; Distortion settlement of foundation must satisfy the following requirement

$$\rho_{diff,cal} \le \theta_{per} \times d_f / 2 \times 1000 \quad \text{(mm)} \tag{92}$$

where θ_{per} = Specified permissible rotation of foundation [8], and $\rho_{diff,cal}$ = Calculated differential settlement of foundation estimated as

$$\rho_{diff,cal} = \left(\rho_{c,cal} - \rho_{e,cal}\right) \quad \text{(mm)} \tag{93}$$

where $\rho_{c,cal}$ = Calculated settlements at the centre and the edge of the footing respectively; For the safety of the horizontal force, the following condition should be satisfied

$$\tau_{fr,cal} \le \tau_{fr,per} \quad \text{(mm)} \tag{94}$$

where $\tau_{fr,per}$ = Specified permissible contact shear stress in kN/m^2, and $\tau_{fr,cal}$ = Calculated contact shear stress in kN/m^2

$$\tau_{fr,cal} = \frac{4P_h}{\pi\left(d_f\right)^2} \quad (\text{kN/m}^2) \tag{95}$$

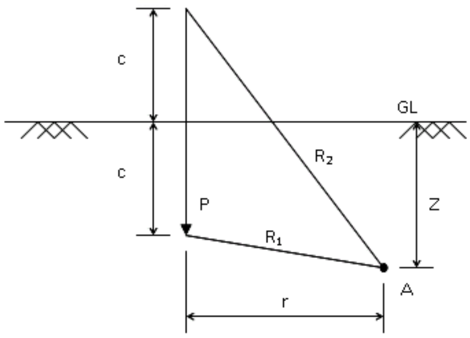
$$\tau_{fr,per} = 0.25 p_S \tag{96}$$

where P_s = Average pressure intensity in kN/m^2; The depth of the foundation should satisfy the following requirements

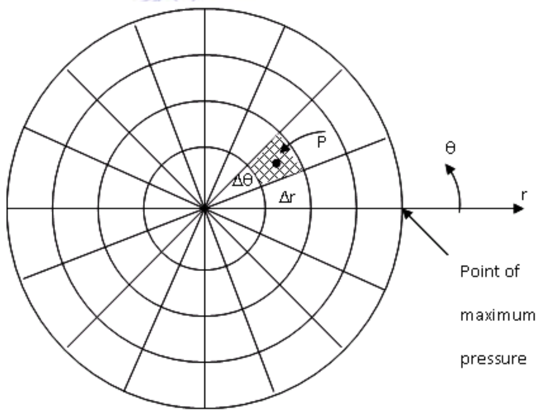
$$d_e \le 2d_f \tag{97}$$

$$d_{em} \le H_{col} \tag{98}$$

where d_{em}= Specified minimum depth of foundation [8], and H_{col} = Height of the column.



(a) Definition figure for Mindlin's Equation



(b) Discretization of the circular foundation

Fig. 3 Notations and discretizations for calculating the settlement of footing

4. Mathematical Programming Formulation

Minimization of the objective function (Eq. 1) subjected to the structural and geotechnical constraints can be stated as a generalized optimization problem as follows:

$$\left.\begin{aligned} &\textit{Find optimal design variable vector } \left(\vec{D}_m\right), \textit{such that,} \\ &\quad F = F\left(\vec{D}_m\right) \textit{is the minimum of } F\left(\vec{D}\right) \\ &\textit{subject to} \\ &\quad g_j\left(\vec{D}_m\right) \le 0.0, \quad j = 1, 2, \ldots, N \end{aligned}\right\} \tag{99}$$

5. Minimization Procedure

In general, most of the powerful minimization algorithms are for the unconstrained minimization problem. For this reason, the problem of constrained minimization is transformed into an unconstrained minimization problem using Penalty Function Method. In this method, a composite function is constructed by blending the constraint with the objective function and minimizing the function so obtained using Powell's conjugate direction method for multidimensional search [11] and quadratic interpolation technique for one-dimensional minimization. The Extended Penalty function used is as proposed by Kavlie & Moe [12].

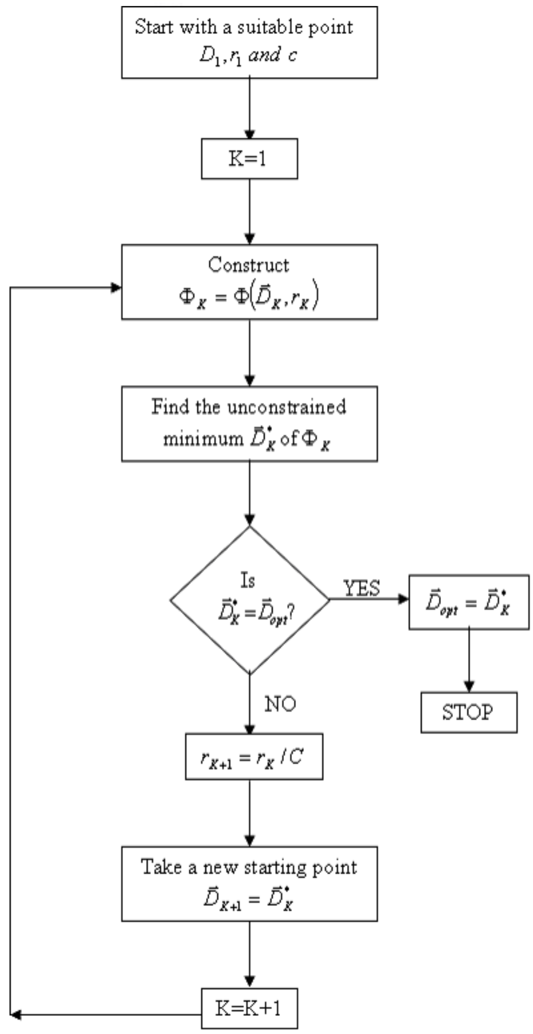


Fig. 4 Flowchart for extended Penalty function method

$$\Phi\left(\vec{D}, r_K\right) = F(\vec{D}) - r_K \times \sum_{j=1}^{m} G\left[g_j(\vec{D})\right] \tag{100}$$

where

$$G\left[g_j(\vec{D})\right] = \begin{cases} \dfrac{1}{g_j(\vec{D})} & when\ g_j(\vec{D}) \le \varepsilon \\ \dfrac{2\varepsilon - g_j(\vec{D})}{\varepsilon^2} & when\ g_j(\vec{D}) \ge \varepsilon \end{cases} \tag{101}$$

where, r_k is the initial penalty parameter, $\varepsilon = -\dfrac{r_K}{\delta}$ is a parameter that defines the transition between two types of penalty terms, and δ = a constant depending on the initial value of r_K. The flowchart of the Penalty function method has been presented in Fig. 4. The Powell's conjugate direction method is an extended version of pattern search method. Given that the function has been minimized once in each of the coordinate directions and then in the associated pattern direction, discards one of the coordinate directions in favor of the pattern direction for inclusion in the next minimization, since this is likely to be a better direction than the discarded direction.

After that, each cycle of minimizations generate a new pattern direction and again replace one of the coordinate directions. The flowchart of the method has been presented in Figure 5. The search is terminated when the relative change in the function value and the decision variables, between two consecutive cycles of minimization, is less than the desired accuracy. The quadratic interpolation technique is used to locate a minimum of a single variable function. These methods are available in standard text book on optimization [13].

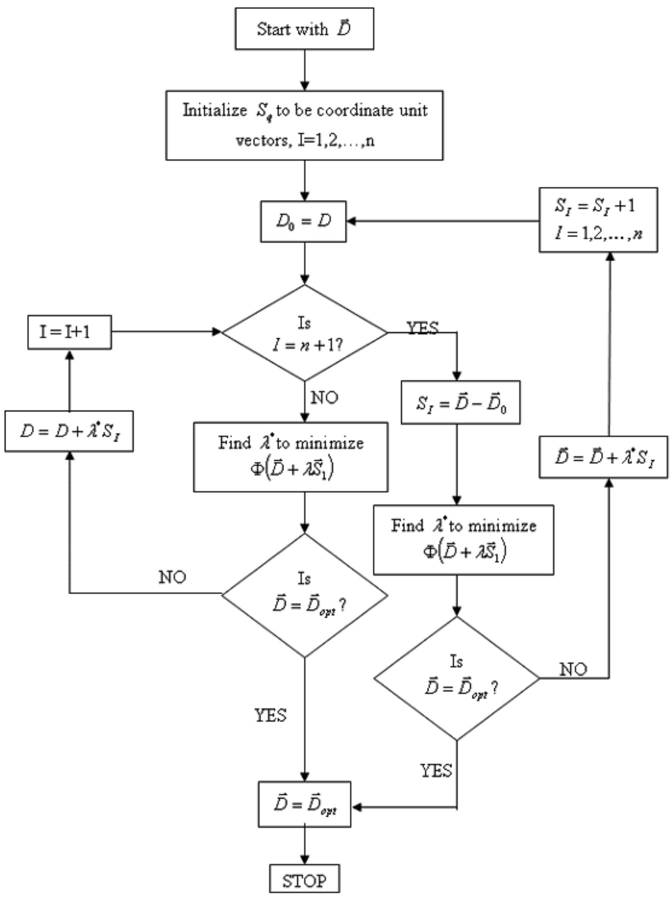


Fig. 5 Flowchart for Conjugate Direction method

6. Results and Discussions

A shallow circular footing as shown in Fig. 1 has been analyzed in the present study. Fig. 6 shows the details of the reinforcements in the column and footing needed in the analysis. A computer program has been developed for the optimum cost design of circular footings subjected to generalized loadings. To study the effectiveness of the suggested method, results on the various aspects of the present study have been obtained and presented. Table 1 provides the soil profile data obtained from the Cone Penetration Test and the Static Penetration Test.

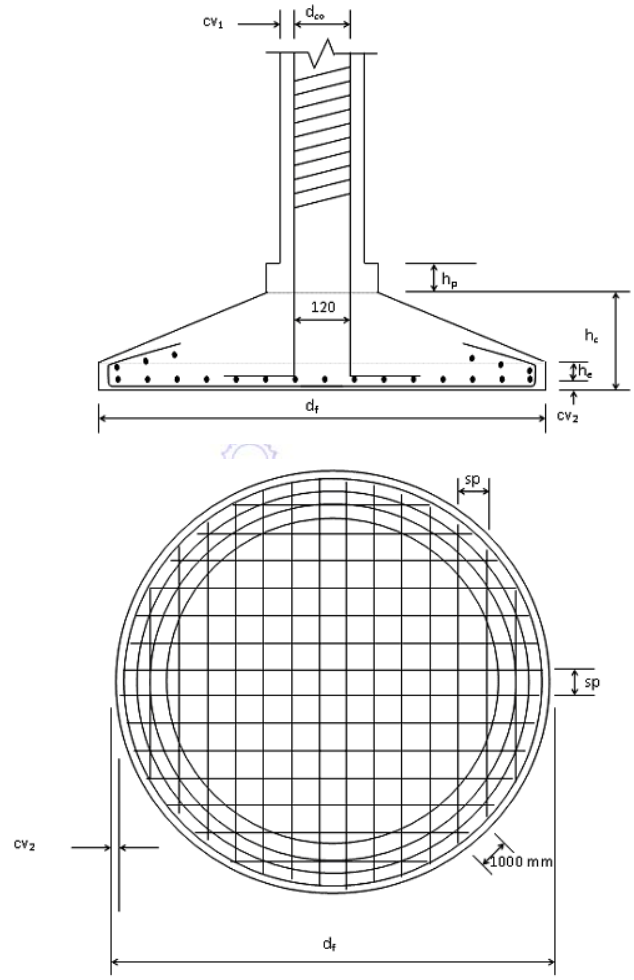


Fig. 6 Details of reinforcements in column and footing

Studies were undertaken to check the net amount of saving that can be made with the help of a computer aided optimum design. After several trials, a few design vectors that appeared to provide better results were chosen as starting points. The detailed results are presented in Table 2. It is observed that for the cases studied the percentage saving in the cost from the initial design ranges from 10 – 20 %. The lowest and highest cost for the footings have been obtained for the second and the third sets of initial design vectors respectively. The difference in the cost is only 1%. However, a comparison of the final optimum design vector for these two cases reveals that there is a significant difference in many of the corresponding design variables.

This signifies that the obtained solutions are either local minima, or the objective function which is insensitive to the changes in the design variables near the optimum point or termination of computations might have occurred on satisfying the specified convergence criterion (absolute error in each of the design variables being less than 0.0001), even before actually reaching the minimum.

To study the effect of the initial design vector on the final solution, various initial design vectors were chosen and corresponding solutions were obtained. Such studies are essential to check whether the obtained solution is a global solution or not. These aspects have been presented in Table 3 wherein the optimum values of the objective function and the penalty function starting from different design vectors have been presented. It can be concluded from the table that design vectors do not have significant influence on the optimum value of the objective function.

Table 1 Soil Profile data for CPT and SPT

Depth of layers (m)	Average cone penetration values (q_c) (kN/m^2)	Average standard penetration number (N)	Unit weight of soil (kN/m^3)	Poisson's ratio (μ_S)	
2.50	8000	27	21	0.5	0.3
2.50	8000	27	20	0.5	0.3
2.50	10000	33	21.5	0.5	0.3
2.50	10000	33	20	0.5	0.3
2.50	12000	40	22	0.5	0.3
2.5	12000	40	21.2	0.5	0.3
8.0	17000	57	21	0.5	0.3

Table 2 Optimum design variables and percentage of savings in cost for CPT data and $\mu_S = 0.5$

Test Sets		Set 1		Set 2		Set 3	
		Initial	Final	Initial	Final	Initial	Final
Design variables	d_f	2.250	2.250	2.200	2.242	3.000	2.182
	d_e	1.600	1.543	1.600	1.555	1.800	1.544
	ps_1	0.820	0.820	0.900	0.805	1.000	0.500
	ps_2	0.840	0.409	0.800	0.443	1.000	0.554
	ps	0.300	0.280	0.600	0.267	1.000	0.635
	d_c	0.790	0.769	0.801	0.778	0.900	0.786
	h_c	0.710	0.690	0.700	0.689	0.800	0.693
	h_e	0.230	0.225	0.300	0.236	0.400	0.195
	d_p	0.910	0.900	1.000	0.805	1.000	0.795
Cost Function values (Rs.)		2907	2588	3110	2570	2863	2592
Percentage of saving		12.30		20.97		10.44	
Number of function evaluations		747		906		934	

Table 3 Effect of starting point on the optimum solution for CPT data and $\mu_S = 0.5$

Test Sets		Set 1		Set 2		Set 3		Set 4	
		Initial	Final	Initial	Final	Initial	Final	Initial	Final
Design variables	d_f	2.300	2.220	2.250	2.250	2.200	2.242	3.000	2.132
	d_e	1.600	1.592	1.600	1.543	1.600	1.555	1.800	1.544
	ps_1	1.000	0.805	0.820	0.820	0.900	0.805	1.000	0.500
	ps_2	0.800	0.470	0.840	0.409	0.800	0.443	1.000	0.635
	ps	0.600	0.283	0.300	0.260	0.600	0.267	1.000	0.635
	d_c	0.800	0.790	0.790	0.769	0.801	0.778	0.900	0.786
	h_c	0.700	0.690	0.710	0.690	0.700	0.689	0.800	0.693
	h_e	0.300	0.220	0.230	0.225	0.300	0.236	0.400	0.195
	d_p	0.900	0.885	0.910	0.910	1.000	0.805	1.000	0.795
Function Evaluations	F	3295	2590	2907	2588	3110	2570	2863	2592
	Φ	3398	2591	2909	2588	3060	2571	2874	2592
No. of function evaluations		755		747		506		934	

Fig. 7 and 8 depict the variation of penalty function (Φ) and the objective function (F) with the number of function evaluations and the penalty parameter. The figures demonstrate that the minimization of the penalty successfully leads to the minimization of the objective function. Hence it can be concluded that no ill-conditioning occurs for this type of problems and the developed algorithm is efficient in locating the minimum.

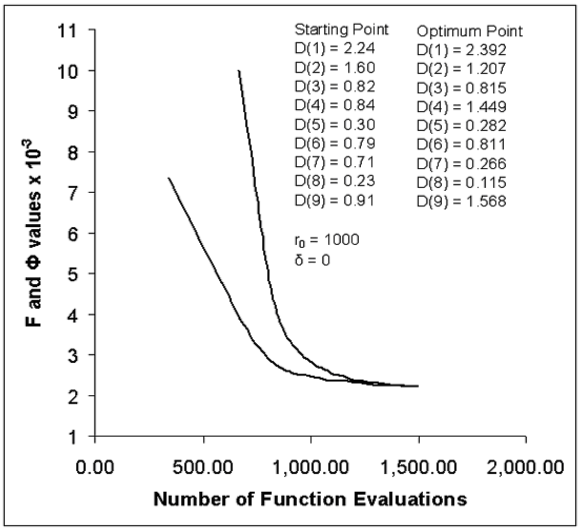


Fig. 7 Path of F and Φ functions from first iteration with the number of function evaluations

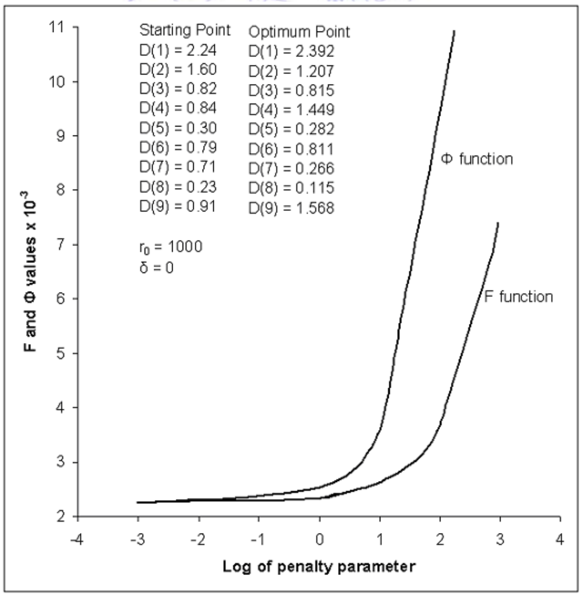


Fig. 8 Path of F and Φ functions from first iteration with change in r_k

Table 4 shows the effect of the initial value of the penalty parameter r_K and the parameter δ on the progress of the solution. It has been observed that better convergence is achieved when the parameter δ is taken at least 100 times the penalty parameter. It has also been observed that the smaller value of the initial penalty parameter requires lesser number of function evaluations for the convergence.

The stress and settlement calculations have been carried out by discretizing the circular footing into a large number of elements as shown as in Fig. 8, and summing up the effects of all the elements. A reasonable element size has been obtained by studying the effect of the element size on the accuracy of stress calculations as shown in Table 5. Based on the results, the element size has been decided to be $\Delta r = 0.20$ and $\Delta\theta = 22.5°$ for diameter of footing around 3 m presenting a ratio of $\Delta r/d_f = 0.067$. For settlement computations, Mindlin's equations [10] for stress and displacement with Steinbrenner's approximate [9] method have been used.

Table 4 Effect of parameters r_0 and δ on the optimum solution for CPT data and $\mu_S = 0.5$

Test Sets		Set 1		Set 2		Set 3		Set 4	
		Initial	Final	Initial	Final	Initial	Final	Initial	Final
Design variables	d_f	2.250	2.250	2.250	2.253	2.240	2.389	2.240	2.349
	d_e	1.600	1.530	1.600	1.516	1.600	1.220	1.600	1.288
	ps_1	0.820	0.820	0.820	0.809	0.820	0.823	0.820	0.819
	ps_2	0.840	0.420	0.840	0.539	0.840	1.847	0.840	0.763
	ps_3	0.300	0.269	0.300	0.298	0.300	0.324	0.300	0.423
	d_c	0.789	0.780	0.790	0.785	0.790	0.803	0.790	0.796
	h_c	0.710	0.660	0.710	0.631	0.710	0.297	0.710	0.380
	h_c	0.230	0.225	0.230	0.224	0.230	0.167	0.230	0.182
	d_p	0.910	0.890	0.910	0.838	0.910	1.354	0.910	1.060
Parameter	r_0	0.01		0.01		1000		1000	
	δ	0		12		100000		120	
Function value		3126	2546	2908	2528	3136	2340	3136	2442
No. of function evaluations		946		1280		1930		1928	

Table 5 Effect of element size on the accuracy of stress calculation (d_f = 3.0 m, d_e = 0.0 m)

Depth (m)	Stress by Bussinesq's formula (kN/m^2)	Calculated stresses (kN/m^2) and percentage error for different element sizes							
		Δr=0.2 m $\Delta\theta$=22.5º		Δr=0.3 m $\Delta\theta$=22.5º		Δr=0.15 m $\Delta\theta$=11.25º		Δr=0.2 m $\Delta\theta$=45º	
		Stress	Error	Stress	Error	Stress	Error	Stress	Error
0.5	109.5	112.4	2.63	115.9	5.82	110.9	1.22	112.4	2.63
1.0	93.8	94.6	0.79	95.3	1.58	94.2	0.38	94.6	0.79
1.5	73.1	73.5	0.5	73.8	0.99	73.3	0.24	73.5	0.5
2.0	55.5	55.4	0.37	55.6	0.73	55.3	0.18	55.4	0.37
2.5	41.8	41.9	0.28	42.0	0.56	41.8	0.14	41.9	0.28
3.0	32.1	32.1	0.22	32.3	0.44	32.2	0.11	32.2	0.22
3.5	25.2	25.3	0.17	25.3	0.35	25.3	0.08	25.3	0.98
4.0	20.2	20.3	0.14	20.3	0.28	20.2	0.07	20.3	0.14

Table 6 presents the optimum design, starting from the same initial design vector for both the CPT and SPT data. The study reveals that the optimum cost obtained from using CPT values is lower than that obtained from SPT values. The reason for the same lies probably in the difference in the approach of estimation of the elastic parameters from these tests. However, the percentage difference in the obtained minimum costs ranges from 15 to 20%.

Table 6 Effect of soil properties and soil data

Test Sets		Set 1		Set 2		Set 3		Set 4	
		Initial	Final	Initial	Final	Initial	Final	Initial	Final
Design variables	d_f	2.240	2.335	2.240	2.330	2.240	2.440	2.240	2.450
	d_e	1.600	1.123	1.600	1.125	1.600	1.260	1.600	1.260
	ps_1	0.820	0.813	0.820	0.810	0.820	0.821	0.820	0.823
	ps_2	0.840	1.246	0.840	1.258	0.840	1.729	0.840	1.738
	ps_3	0.300	0.207	0.300	0.209	0.300	0.337	0.300	0.339
	d_c	0.790	0.780	0.790	0.780	0.790	0.810	0.790	0.818
	h_c	0.710	0.260	0.710	0.256	0.710	0.304	0.710	0.306
	h_e	0.230	0.148	0.230	0.149	0.230	0.165	0.230	0.166
	d_p	0.910	1.627	0.910	1.636	0.910	1.287	0.910	1.395
Type of soil data		CPT		CPT		SPT		SPT	
Poisson's ratio, μ_s		0.5		0.3		0.5		0.3	
Function value		3136	2058	3130	2169	3123	2367	3485	2536
No. of function evaluations		1038		1250		1573		1722	

7. Conclusions

A computer program have been developed for the optimum design of shallow circular footing subjected to generalized loadings, using sequential unconstrained minimization technique in conjunction with Powell's multidimensional search and quadratic interpolation method for one dimensional minimization. The developed program has been found to be quite efficient in its functioning. The developed computer program can take care of common types of soil data namely SPT and CPT data. The savings in cost of the shallow circular footing using the developed technique has been found to be in the tune of 10 – 20 %.

The methodology adopted works on the error estimation and perturbation technique. In the first instance, a set of design variables (referred as the initial design vector) are chosen. These variables are then used to estimate the constraint criteria. Each of the constraint is either equality constraint or inequality constraint. For equality constraints, the estimated value must be equal to the constraint value, while for the inequality constraint, the estimated value should satisfy the inequality with a positive tolerance.

Once the error of the constraint estimation is determined based on the satisfaction of the constraints, the perturbation in each of the design variable is carried out (by one-directional search) sequentially to attempt to satisfy the constraint. Once all the design variables are perturbed, the global error of the system of equations is checked. If the global tolerance is satisfied, the program terminates by the estimation of the objective function with the final set of design variables being the optimum set. Hence, in this manner, the optimum set of the particular problem is determined. Since the methodology adopted is relatively free from the influence of the chosen initial design vector (as elaborated in Table 3 of the article), one does not need to be thoughtful about the parametric values of the initial vector. Any values of the initial vector will be able to provide the converging solution, but the only difference being in the number of function evaluations to reach the final solution.

When started from different initial design points, it has been observed that the variation in the final optimum cost is not very much significant even though there are some variations in the optimum values of the design variable. It has been observed that for better convergence, the δ parameter should be chosen at least 100 times the initial penalty parameter r_k. Smaller values of r_k, of the order of 1.0 or 0.1 require less number of function evaluations. But a better convergence may be achieved with higher values of r_k, of the order of 100 or 1000, at the expense of large number of function evaluations. The developed computer program has been found to be quite efficient for these types of problems and has the ability to accept either a feasible or an infeasible initial design vector for constrained minimization. For the stress and settlement computations, the size of the elements of the discretized footing is very important. It has been found that an element having $\Delta r / d_f$ ratio as 0.067 and $\Delta\theta = 22.5^\circ$ gives reasonable accurate results.

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