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Application of Numerical Modelling in Underground Coal Mines — An Overview

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देश में किये जा रहे आर्थिक सुधारों को देखते हुए, भारत के भूमिगत कोयला खनन उद्योग द्वारा सुरक्षा, संपोषण एवं पारिस्थितिकी-मित्रता के साथ उत्पादन एवं उत्पादकता में सुधार आवश्यक है। इस कारण से यथानुसार अभिकल्पन का पुराना सिद्धांत खान अभिकल्पन के क्षेत्र में वैज्ञानिक एवं सुसगत शैल यांत्रिक विधियों के पक्ष में अपनी विश्वसनीयता तेजी से खो रहा है। कोयला खान अभिकल्पना में समस्याएँ अपवाद नहीं है।

सार्वभौमिक रूप में यह माना जा चुका है कि सौल्यिकी अनुरुपण शॉल-पुँज के व्यवहार एवं शैल-उत्खनन के अभिकल्पन के अनुमान का बहुमुखी साधन है। अन्य शैल-यांत्रिकी विधियों मुख्यतः विश्लेषणात्मक, आनुभविक एवं प्रेक्षणयुत विधियों के साथ यह विधि खान आयोजकों एवं उत्पादन अभियंताओं को खनन विधि, खान-तीथियों के आकार एवं दिग् विन्यास, स्तंभों की माप एवं सपोर्ट अद्रभकल्पन हेतु उपयोगी पथ-प्रदर्शन में सक्षम है।

वर्तमान आलेख के उद्देश्य साख्यिकी अनुरुपण के उस अभियांत्रिक उपयोग के बारे में बताना है जिसे कोयलाधारी सस्तर का मॉडलिंग की विशिष्ट आवश्यकताओ को ध्यान में रख कर वर्षों में विकसित किया गया है।

An analysis of stress distribution around an opening is essential to the design evaluation. If stress in rock mass exceeds its strength at a point, failure occurs. Extension of failure zone leads to roof falls, pillar spalling or floor heaving. If the failure zone is extensive, then the stability of opening is definitely endangered. Evaluation and prediction of stability are essential prior to planning and design of underground excavations. Mathematical complexities that reflect physical realities of underground mine openings necessitate a numerical approach to the analysis.

Modelling of excavation problems in rock medium requires careful considerations because of complex ground conditions. Stability of underground opening is governed by structural features, size of the opening , rock elasticity, strength properties and in-situ virgin stresses. Since the solution to each rock mechanics problem is specific to the circumstances, recognizing the rock mechanics principles and understanding of the theoretical background of numerical methods will help in selecting the right technique. Numerical modelling methods allow very quick efficient studies for various parameters alteration, so that a number of operationally feasible mining options can be evaluated, which can be used for exploring appropriate mining strategies, layouts and detailed production schemes.

NUMERICAL METHODS

Design, construction and performance evaluation of underground openings and their support require the knowledge of displacement and stress in the surrounding rock due to excavation. For the analysis of underground openings, selection of a particular numerical method is very important. Here a brief description of different numerical methods are presented.

FINITE ELEMENT METHOD (FEM)

FEM has been used for rock mechanics modelling for a number of years and has proved to be a technique which can be successfully used for modelling a wide range of geological and geotechnical environment.

In this method, the whole continuum is discretized in finite small elements (Fig.1). The elements are connected at nodal points. Locally based functions are assumed for the variation of the displacement within the element. The parameters of these functions are the nodal displacement values and they are chosen in such a way that compatibility is ensured between connecting elements.Using the virtual work principles a stiffness matrix is obtained for each element. The assembly of all elements leads to a system of equations:

$$[K]\{U\} = \{F\}$$

which is solved for the displacements at the nodes {U}. [K] is the stiffness matrix of the continuum and {F} is a vector containing the forces at the nodes. The primary results of a finite element analysis will be displacements at all nodes of the continuum. Using the displacement function the stresses inside each element can be obtained (Zinkiewicz, 1979).

Advantages of FEM

- Different openings, shapes and sequence of excavation may be conveniently adopted.
- FEM may be used incorporating practically any type of rock media and rock behaviour (e.g. linearly elastic, elastoplastic or plastic).
- Progressive failure of mine pillars and excavation can also be simulated.

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Disadvantages of FEM

- If preprocessors are not available, then preparation of input data is time consuming and laborious.
- If the problem is large and geometrically complex, computation time and manpower become costlier.

BOUNDARY ELEMENT METHOD (BEM)

BEM is a powerful numerical technique in static and dynamic problems. The methods basically consist of dividing the boundary of the continuum into boundary element (Fig.2). BEM uses the known fundamental solution of stresses and displacements due to point source acting on an infinite body. To make the method work, it is necessary to have a close form simultaneous equations. The equations are solved for unknown displacements and/or stresses at the boundary. Results at interior points (field points) can be obtained by super position using the fundamental solution.

There are mainly two different types of BEM, viz.,

i) Indirect BEM

a) Fictitious stress method,

b) Displacement discontinuity method.

ii) Direct BEM or boundary integral equation method.

In fictitious stress method the stress conditions on the boundary are found first, displacement and stresses are found from linear combination of the boundary stresses (Crouch & Starfield, 1983). The displacement discontinuity method is based on analytical solution to the problem of a constant discontinuity in displacement over a finite line segment of an infinite elastic solid. For the direct BEM, Betty's reciprocal theorem can be used to obtain integral functions and numerical integral equations on the surface. Using the element shape functions and numerical integration, the integral equations can be written as a system of linear equations:

$$[A]\{a\} = [B]\{b\}$$

where, {a} contains displacements at all boundary element nodes and {b} contains the traction values at these nodes. [A] and [B] are fully populated nonsymmetric matrices but have dominant diagonal terms. Equations can now be solved for either known tractions or known displacements (Banerjee & Butterfield, 1981).

Advantages of BEM

- BEM may be used for any shape of rock opening and 3D problems in case of elastic medium.
- It is very easy and generally gives quite accurate results.
- Input data preparation is easy.

Disadvantages of BEM

- It is very difficult to incorporate discontinuities and complexity or rock behaviour.
- It is not suitable for nonelastic or anisotropic material.

FINITE DIFFERENCE METHOD (FDM)

FDM is an important technique for static elasticity of rock mass. In this method, the whole continuum is discretised into small elements, called Zones (Fig.3). Over each region the differential equation of equilibrium is approximated. This results into a system of simultaneous equation which is solved by interation methods.

Advantages of FDM

- The technique is simple to program and use.
- Progressive failure can be simulated.

Disadvantages of FDM

- The formulation results into a conditionally stable problem. Convergence depends on the equation of solution method.

Distinct Element Method

DEM is a recent numerical technique developed for discontinuous analysis of jointed rock mass. In this method whole domain is discretised in blocks or elements by the presence of natural joints (Fig. 4). The real mechanical behaviour of joints like rotation and sliding one block relative to other blocks can be simulated. The DEM is based on a time domain algorithm which solves equations of motion of the block system by an explicit finite difference method (Candall, 1987).

Advantages of DEM

- DEM is best suited of those problems where the mechanical behaviour of discontinuities play a major role on the response to the medium (Hart, 1991).

Disadvantages of DEM

- The DEM is ideally suited for micro computers because of explicit time stepping algorithm which does not require matrix solution.
- This method is inefficient if the joint structure has too many joint sets and the spacing is small.

HYBRID METHODS

In complex geomining conditions two of the above mentionod methods can be combined to get the realistic solutions.

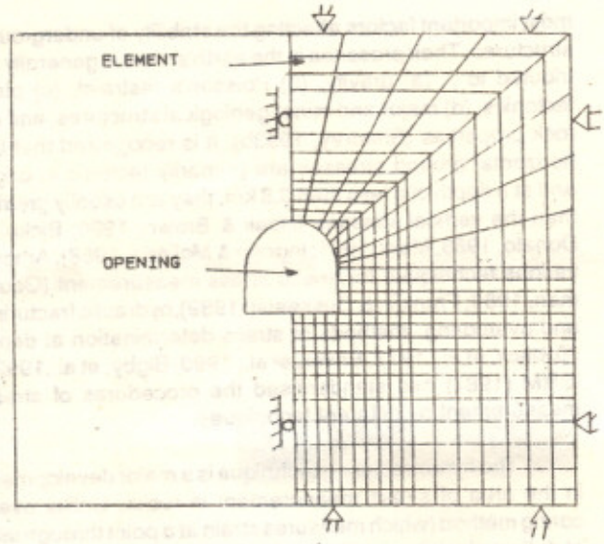

FIG.1 - FE DISCRETISATION

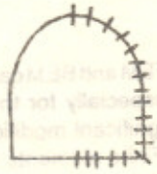

FIG.2 - BE DISCRETISATION

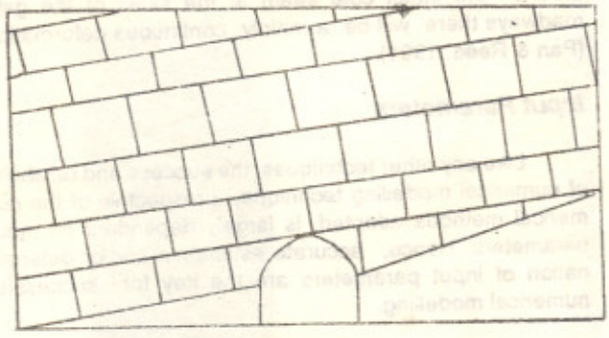

FIG.4 - BLOCK MODEL

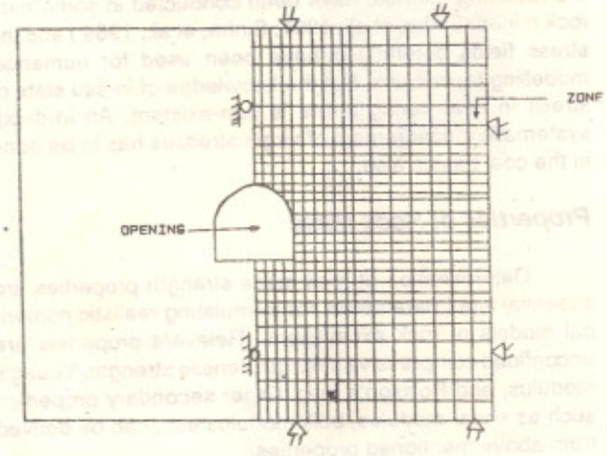

FIG.3 FD DISCRETISATION

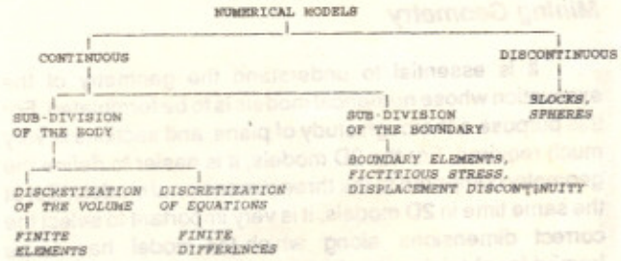

FIG.5: GEOMECHANICAL CLASSIFICATION OF NUMERICAL MODELS

Coupled FEM and BEM

The advantages of both FEM and BEM can be achieved by coupling them (FEBEM) especially for the analysis of underground openings with significant modification of rock properties near the opening. Finite elements may be used near excavation surface and boundary elements may be used away from it. FEBEM gives more accurate results than FEM (Varadarajan, et al., 1985).

Coupled DEM and FEM

In this hybrid numerical method DEM and FEM are coupled, so that distinct elements are used in part of mesh, while the remainder part consists of finite elements. For example, in modelling the rock mass deformation around an advancing longwall coal face, the rock in the roof of the region behind the face will fail in a blocky manner, while around the intact coal seam at the sides of the gate roadways there will be a mainly continuous deformation (Pan & Reed, 1991).

Input Parameters

Like any other techniques, the success and reliability of numerical modelling technique, irrespective of the numerical methods adopted, is largely dependent on input parameters. Hence, accurate estimation and/or determination of input parameters are the key for successful numerical modelling.

Once an opening is created in a rock mass, stress redistribution and concentration occurs around the opening; these are known as induced stresses. The magnitudes and directions of induced stresses largely depend on the shape and size of the opening, pre-exacavation stress environment (in-situ and virgin stress), and characteristics of the rock mass. Therefore, these input parameters have to be estimated or determined very accurately for formulation of realistic numerical models.

Mining Geometry

It is essential to understand the geometry of the excavation whose numerical models is to be formulated. For this purpose a thorough study of plans and sections is very much required. For the 2D models, it is easier to define the geometry compared to the three dimensional models. But at the same time in 2D models, it is very important to select the correct dimensions along which the model has to be framed for obtaining the desired output. This would largely depend on type of the problems to be solved. To simulate a complicated geometry it is quite a common practice to idealize the geometry. Small details which are expected to have insignificant bearing on the analysis are ignored. This makes the model simpler and saves the computation time. Once the geometry is decided, it is defined by the coordinates in the model.

In-situ Virgin Stresses

The magnitude and directions of virgin stress are the most important factors affecting the stability of underground structures. Their presence in the earth's crust is generally attributed to : (a) gravity, (b) Poisson's restraint, (c) plate tectonics, (d) major and minor geological structures, and (e) rock properties (Sheorey, 1993b). It is recognized that the horizontal ground stresses are primarily tectonic in origin and at a depth of a less than 0.8 km, they are usually greater than the vertical stresses (Hoek & Brown, 1980; Bickel & Donato, 1988, Mark, 1991: Ingram & Molinda, 1988). Among various techniques for in-situ stress measurement (Goodman, 1980; Franklin & Dusseault, 1989), hydraulic fracturing and overcoring methods of stress determination at depth (Zoback, et al., 1977, Enver, et al., 1990; Bigby, et al., 1992). ISRM (1987) has standardised the procedures of stress measurement by different techniques.

The hydrofracturing technique is a major development in the area of stress measurement in rocks. Unlike overcoring method (which measures strain at a point through use of delicate instrumentation in the test hole), it directly determines average stresses over large areas by recording two hydraulic pressures, one necessary to open a segment of crack in the test-hole and the other required to keep the fracture open. To do so, it uses simple down-hole mechanical tools so that the method can be employed at any depth from the surface. Elementary elastic relationship exists between recorded pressure and in-situ stresses, and fracture direction and stress orientation (Haimson, 1978; Enver, et al., 1992). Numerous numerical models have been developed to simulate this process (Boone, et al., 1991, Curren, et al., 1985; Boone, et al., 1986; Cleary & Wang, 1985, 1985; Boone & Detournay, 1990; Detournay & Cheng, 1988).

In India, in-situ stress measurements, mainly by hydrofracturing method, have been conducted in some hard rock mines (Sinha, et al., 1991; Sinha, et al., 1989) and the stress fields determined have been used for numerical modelling techniques. But the knowledge of in-situ state of stress in coal baring strata is non-existent. An in-depth systematic measurement of virgin stresses has to be done in the coal basins also.

Properties of Rock Mass

Determination of rock mass strength properties are essential input parameters for formulating realistic numerical models of rock excavations. Relevant properties are unconfined compressive strength, tensile strength, Young's modulus, and Poisson's ratio. Other secondary properties such as shear modulus, bulk modulus etc., can be derived from above mentioned properties.

A considerable number of methods has been developed over the years for the purpose of determining the mechnical properties of rock mass. By and large, four groups can be distinguished as follows :

- Mathematical model
- Classification
- Large scale testing, and

- Back calculation.

Rock mass classification systems are mainly used to evaluate the rock mass strength (Krauland, et al., 1989). Rock mass characterization is a vast subject area concerned with comprehensive description of rock masses. The characterization of rock mass has been tackled by many researchers though the use of classification system(Beiniawski, 1984; Beiniawski, 1973, Barton, et aal., 1974, Franklin, et al., 1988, Rocha, 1981; Nguyen & Ashoworth, 1985; Bello 1988; Udd & Wang, 1985, Turk & Dearman, 1985, Venkateswarlu, et al., 1989, Sheorey, 1991, Sen, 1990, Sen 1990b, Laubscher, 1990). Each of the various systems attempts to classify a rock mass either qualitatively or quantitatively in groups.

The physico-mechanical properties i.e., modulus of deformation and various strengths of intact rock specimen determined in the laboratory do not represent the real properties of rock mass since the effects of weakness planes, joints, etc., are not taken into consideration (Hirt & Shakoor, 1992). Several empirical approaches have been developed to correlate the intact rock properties and geotechnical properties of rock mass for estimating various rock mass properties (Li-Zhou, 1985; Richards & Hustrulid, 1985; Cregger, et al., 1985, Glynn, 1987; Bieniawski, 1984,; Laubscher, 1990; Serafim & Pereira, 1983, Sheorey, et al., 1989).

FORMULATION OF MODEL

The modelling process begins by dividing the selected mining geometry into a number of distinct geological and geotechnical horizons, based on borehole information and geological sections that represents material of similar properties. The horizon are then represented by rows of isotropic or anisotropic elements, each of which is homogeneous and assigned suitable deformation properties. For coal measures strata each layer in the model is generally considered to contain bedding planes, to which a frictional strength is assigned. Similarly compresisve and tensile strengths of rock mass are assigned to each layer. Thus the model effectively considers three modes of failures; compressive, tensile and shear. Having obtained physical property values, consideration is then given to the mesh densities desirable in different parts of the model. In general, greater densities are required in regions of greater distortion, such as the immediate vicinity of an excavation. A value about 0.5m between nodes has been found to be satisfactory for fine mesh in most simulation (Payne & Isaac, 1985; Pande, et al., 1990.

Creation of underground opening causes a change in the state of stress, which results in fracturing of rock. The influence of joints on the distribution stresses may be incorporated by using two approaches; either changing the material properties or duplicating rock discontinuities by sets of special finite elements (Szwedzicki, 1981). Desai & Christon (1977) have also discussed the method of simulating regularly distributed planes of weakness. Goodman & Shi (1985) have described the method for simulating joint planes using joint element method.

The underground mining problem is in fact a three dimensional one. However, due to large storage area required and the cost of computing time, two dimensional representations are preferable to three dimensional models. Therefore, if the geomining condition is not complicated and the assumption of plain strain condition is quite realistic, a 2D model would be adequate. While choosing the size of model depth (above and below coal seam) and width (number of openings to be included) should be taken care of (Wang, et al., 1985).

The rock media may be either massive or jointed and may contain major shear zones, fault planes etc.. (Hoek & Brown, 1980). Further, the method and the sequence adopted during excavation may significantly modify the behaviour of the rock adjoining the opening. Numerical models most commonly used to suit different geomechanical conditions can be classified as shown in (Fig. 5). Two of these models can be combined together to calculate the stress distribution in a complicated mine structure.

APPLICATIONS OF NUMERICAL TECHNIQUES

Ravi, et al., (1992) pointed that numerical modelling in mine design and ground control should be taken as a tool, not as a solution. From the review of the available literature, it has been found that numerical simulation can be applied for the following aspects of underground coal mines:

- The finite element simulation, stress control via yielding pillars, and specialized techniques for geomechanical behavioural monitoring of underground openings can be combined to gain increased safety, decreased costs of artificial support, increased rates of recovery, and increased rates of advance (Gardner, et al., 1985); Janakiraman & Das, 1987; Dhar, 1991; Dhar & Srivatava, 1988; Rao, et al., 1978; Park & Ash, 1985; Park, 1992; Serata, et al., 1985, Pariseau, 1981; Sheorey 1993).
- Integration of finite element analysis and field instrumentation for application of stress control method in underground coal mines is very much effective (Serata, et al., 1985; Pariseau & Sorensen, 1979; Su, et al., 1969; Fama & Follington, 1992).
- The fracture propagation in rock mass including time-dependent behaviour of fracture can be analysed by using both fracture mechanics and FEM (Matsuki, 1985; Mclennan & Picardy, 1985; Haghighi, et al., 1985; Heuz & Ingaeffea, 1980). Better understanding of dynamic effects of various mining parameters on coal strata may lead to both productivity and safety improvements by appropriate design of mining operations (Prucz & Fu, 1989).
- FEM can be used for modelling the behaviour of overburden rock masses over longwall mines panels for predicting surface subsidence (Siriwardane, 1985; Summers & Jeffery, 1992; Shankar & Dhar, 1988).
- FEM can be used for analysing the effect of staggering longwall panels (Dhar, et al., 1985; Dhar & Srivastava, 1988).

- 3 D finite element modelling of longwall using progressive failure concept can be used to get realistic results (Ash & Park, 1987).

- Comprehensive integrated approach (geomechanical characterization, finite element modelling and field monitoring) can provide indication for all important geomechanical effects associated with underground mining such as entry stability, floor heave, overburden impacts and surface subsidence (Hasenfus & Su, 1992; Aggson, 1978).

- Finite element ground water flow model can be developed to study the movement of water in coal seam in which large cavities are created (Contractor & Eftekharsadeh, 1985).

- Prediction of the extent of the strata movement, their interactions with support systems, and the effects of the progressive excavation can be studied by using FEM (Pan, et al., 1989).

- BEM can be used for determining stresses and displacement around long openings (Brady & Bray, 1978).

- The finite difference numerical model has shown flexibility in modelling different geological environments. The modelling techniques provide a useful means of predicting gate-road conditions in situations where small coal pillars are used as an integral part of the support system, for modelling the interaction between the support and pillar simultaneously (Payne & Isaac, 1985; Isaac & Payne, 1985).

- Discrete fracture modelling of fractured rock masses can be used for evaluation of shaft inflow and mine drainage systems (Deshowitz & Schrauf, 1987). Time and cost saving solution can be obtained for multilined pressure shafts (Yufin, et al., 1985).

- Rock bolting has become more popular as a means of supporting underground openings. Asai, et al., (1985) proposed the Rigidbody Joint-element Method (RJM) as a general numerical method for analyzing the behaviour of discontinuous rock mass and effects of rock bolting. RJM can help us to determine effective places where rock bolts are to be installed. The spacing and length of rock bolts for underground openings in jointed rock mass can be evaluated by numerical analysis (Crawford, et al., 1985; Siddall & Gale, 1992).

- Numerical modelling can be used for assessment of stability of inter-panel pillars (chain pillars) for coal mine (Fama & Wardle, 1987).

- Numerical modelling can be applied for nonelastic and large deformation simulation of coal pillar behaviour (Vervoort, 1992; Pan & Hudson, 1991).

- Numerical modelling can be done for assessing the cause of cutter roof failure, predicting the probability of its occurrence and for selecting optimum control method (Hill, 1986; Ahola, et al., 1991; Aggson & Mouyard, 1988; Bauer, 1990).

CONCLUDING REMARKS

The application of numerical modelling techniques for stability analysis and mine design requires a thorough understanding of the main characteristics of ground media namely, in-situ virging stress state, rock mass properties and deformation response. Hydraulic fracturing is a recognised method for rapid and low cost stress measurement in underground mines (Hunger & Morgenstren, 1980; Boone, et al., 1991; Starfield & Cundall, 1988). Presently, the knowledge of in-situ state of stress is non-extistent in the coal basin of this country. Therefore, there is a strong need to measure the in-situ virgin stresses by hydraulic fracturing method. This will significantly increase the reliability of numerical modelling techniques. Various approaches for estimating rock mass properties, discussed earlier, can be applied for Indian coal measures rock with some site specific modifications as and when required.

Numerical modelling is a powerful technique for parametric studies of different geometric configurations on improving safety, production, productivity and recovery in mines. Perspectives on numerical simulation in underground coal mining have changed dramatically in many countries. It is an important tool, increasingly applied in mine planning and evolving production strategies vis-a-vis different geomining environments. In tune with the fast changing technoeconomic scenario, decision makers and planners of Indian coal mining industry should take ample initiative to promote the application of this versatile tool in an extensive and useful manner.

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