

# A PROBABILISTIC APPROACH FOR MODELLING OF FRACTURE IN MAGNESIUM DIE-CASTINGS

C. Dørum, D. Dispinar, O.S. Hopperstad, T. Berstad

*Quasi-static tensile tests with specimens cut from a generic cast component are performed to characterise the mechanical behaviour of the high-pressure die cast magnesium alloy AM60. The experimental data is used to establish a probabilistic methodology for finite element modelling of thin-walled die castings subjected to quasi-static loading. The cast magnesium alloy AM60 is described using an elastic-plastic constitutive model consisting of a high-exponent, isotropic yield criterion, the associated flow law and an isotropic hardening rule. A novel probabilistic approach for modelling of fracture in thin-walled magnesium die-castings using finite element analysis is developed. The Cockcroft-Latham criterion for ductile fracture is adopted with the fracture parameter assumed to follow a modified weakest link Weibull distribution. Comparison between the experimental and predicted behaviour of the cast magnesium tensile specimens gives very promising results.*

**KEYWORDS:** magnesium, die-castings, ductile fracture, weibull distribution, finite element analysis

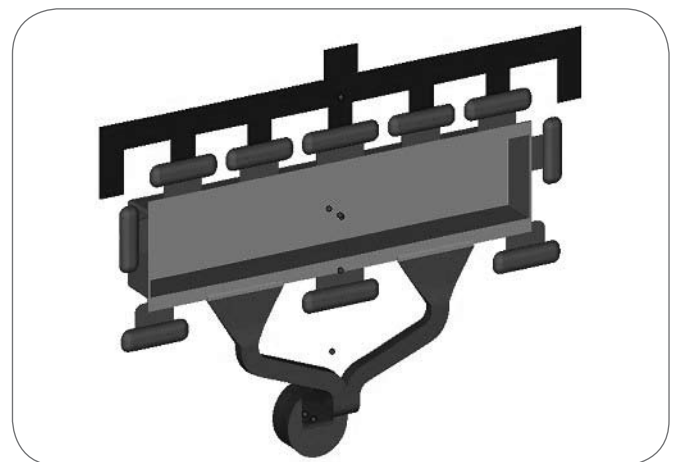
## INTRODUCTION

With increased focus on environmental issues, structural designers in the transport industry are forced to search for light-weight solutions. New materials are considered for vehicle design if they provide benefits at an affordable cost. The cold-chamber high pressure die casting method is an important production method for aluminium and magnesium castings; particular suitable for fully automatic, high productivity, high volume production of complex near net shape parts. A major challenge with this production method is to optimise the process parameters with respect to the part design and the solidification characteristics of the alloy in order to obtain a sound casting without casting defects. Unbalanced filling and lack of thermal control can cause bifilms, porosity and surface defects due to turbulence and solidification shrinkage. Consequently, the fracture behaviour of cast components can be of stochastic character.

To be efficient in the development of new products it is necessary to use finite element (FE) analysis to ensure a structural design that exploits the material. In order to be able to obtain a reli-

able prediction of the structural behaviour using such analyses, an accurate description of the material behaviour is essential. Hence, a reliable failure criterion is also required, that enables the designer to exploit the potential of the cast material. This work presents a new probabilistic approach for finite-element modelling of the structural behaviour of thin-walled cast magnesium components.

Fig. 1 shows the geometry of the generic AM60 component in-



▲  
Fig. 1

**Illustration of generic cast component: Length = 400 mm, thickness = 2.5 mm, width = 80 mm, height = 40 mm.**

*Illustrazione di un generico componente pressocolato: Lunghezza = 400 mm, spessore = 2.5 mm, larghezza = 80 mm, altezza = 40 mm.*

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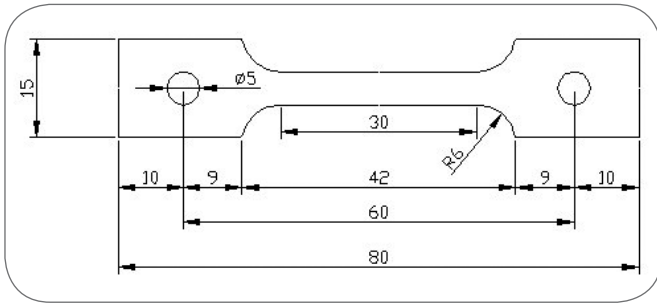
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▲  
Fig. 2

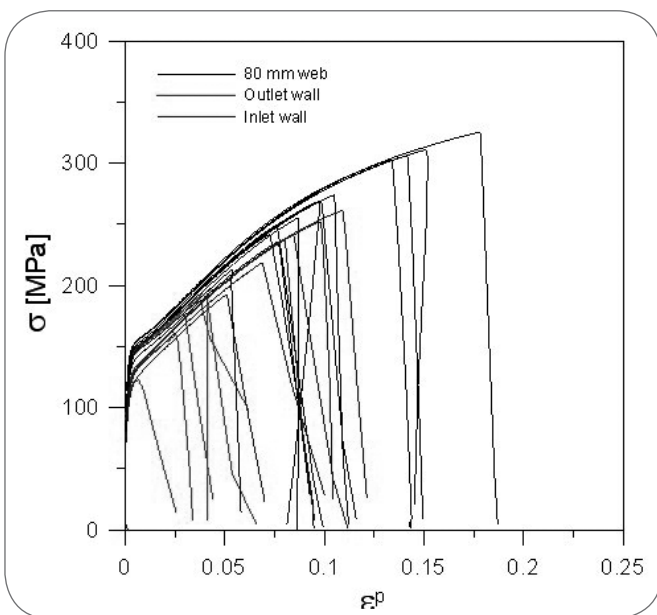
### Geometry of tensile test specimen.

Geometria del provino per la prova di trazione.

investigated in this study, together with the corresponding gating system. The length of component is 400 mm and the wall thickness is approximately 2.5 mm. In previous studies [1, 2], the AM60 material was characterized using uniaxial tensile tests, uniaxial compression tests, and plate bending tests. The results from the uniaxial tensile tests showed that the scatter in elongation at fracture is quite large. The poorest area is the outlet side, where values of effective plastic strain at fracture as low as 2-3% were measured. The best areas were found to be the 80 mm flange in front of the gates, where values of effective plastic strain at fracture as high as 22% were measured [2].

### TENSILE TESTS

The tensile tests were carried out in a hydraulic testing machine under displacement control. Force and displacement/strain were continuously measured. The displacement rate was adjusted to obtain a strain rate approximately equal to  $2 \times 10^{-3} \text{ s}^{-1}$ . All tests were carried out at ambient temperature.



▲  
Fig. 3

### Cauchy stress vs. logarithmic plastic strain curves for AM60.

Tensione Cauchy vs. curve logaritmiche di deformazione plastica per l'AM60.

Uniaxial tension specimens were cut from the inlet wall and the outlet wall in the longitudinal direction, and from the 80 mm web of the casting in both the longitudinal and transverse direction. The geometry of the tensile specimens is shown in Fig. 2. The strain in the length direction was measured by an extensometer with 25 mm gauge length. Cauchy stress versus logarithmic plastic strain curves are provided in Fig. 3. for different parts of the component.

The figure shows the strong variation of the strain to fracture with position in the casting and between duplicate tests. It is further observed that there is also variation in the flow stress level between the various tests, but this variation is less significant.

### METALLURGICAL CONSIDERATIONS

The mechanical properties of an alloy depend on the defects that may be present in the matrix. These defects could be point, line, surface or volume defects. Among these defects, volume defects (porosity, secondary phases or inclusions) are known to be the most significant ones and may affect the mechanical properties dramatically.

Inclusions, basically oxides, play an important role in casting operations. Particularly in high pressure die casting operations, with casting speeds of minimum 15 m/s up to 40 m/s, the liquid metal advances into the mould in jets that introduces the surface oxide to become incorporated into the melt. However, the oxide inclusions can not exist in melts as a single, because the only way they can become incorporated into the liquid is by entrainment action [3]. During such a simple folding action, the two non-wetted oxide surfaces come in contact to form a bifilm that acts as a crack in the casting. Therefore, in high pressure die castings, the casting will have a spatial distribution of casting defects. The size and population of these defects are critical since they act as the initiation points for porosity nucleation and also as stress risers. As seen from Fig. , pictures using scanning electron microscope (SEM) show that the fracture surface has a high density of crack-like pores. By closer examination, it can be confirmed that the crack-like pores are all oxides.

It is well known that in the presence of defects or stress risers, the components may fracture at stresses far away from their nominal theoretical limits. Fig. 3 is a perfect example to such phenomena. Even within the groups (longitudinal and transverse direction, outlet and inlet side) there is a huge scatter of elongation and maximum stress values. It is important to note that even the proof stress changes considerably. Here, the oxides (or bifilms) act as a way of strengthening mechanism in the matrix which is very similar to the behaviour of metal matrix composites. It is also interesting to note that there is one sample in Fig. 3 that fractures even before reaching the proof stress. This pre-mature fracture is another example of the presence of defects (most probably bifilms) in the casting. The flow lines observed on the surface of the samples (see Fig. 5) are the proof of unstable flow of the liquid magnesium in the mould cavity. It is so interesting to observe bifilms (folded oxides) in such sizes on the outer surface of casting part.

### MATERIAL MODELLING

The cast magnesium alloy AM60 is modelled using an elasto-plastic constitutive model including a high-exponent, isotropic yield criterion, the associated flow law and isotropic hardening. Fracture is modelled by element erosion when a fracture

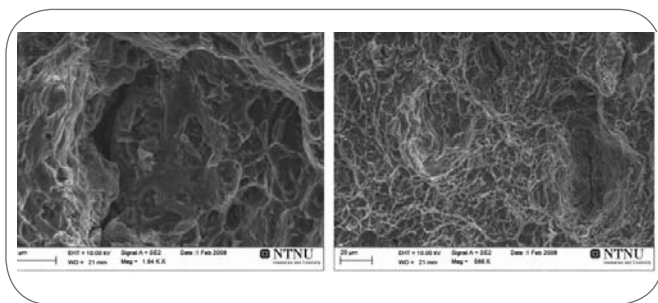


Fig. 4

**SEM images from the fracture surface of samples showing the crumpled oxides.**

*Immagini SEM della superficie di frattura dei campioni che mostrano gli ossidi.*

criterion is reached. The model has been implemented in explicit finite element code LS-DYNA [4].

The high-exponent isotropic yield criterion [5, 6] is written in the form

$$(1) \quad \phi(\sigma_1, \sigma_2) = (\sigma_1)^{2k} + (\sigma_2)^{2k} + (\sigma_1 - \sigma_2)^{2k} = 2\sigma_y^{2k}$$

where  $\sigma_1$  and  $\sigma_2$  are principal stresses in plane stress and  $k$  is a material parameter. The flow stress  $\sigma_y$  is defined by the isotropic hardening rule

$$(2) \quad \sigma_y = \sigma_0 + \sum_{i=1}^2 Q_i (1 - \exp(-C_i \varepsilon_e))$$

where  $\varepsilon_e$  is the effective plastic strain,  $\sigma_0$  is the proportionality limit, and  $Q_i$  and  $C_i$  are hardening parameters. Using a least squares method, the hardening parameters were determined from the Cauchy stress versus logarithmic plastic strain curves in Fig. . Any variation in flow stress with position in the casting was not accounted for in the FE simulations, and thus a mean hardening curve was applied.

In the present model, a criterion of ductile fracture proposed by Cockcroft and Latham [7] is added. The fracture criterion is coupled with the element-erosion algorithm available in LS-DYNA [4]. As the fracture criterion is reached in an element, this element is removed (eroded) from the finite element model. The fracture criterion can be expressed as

$$(3) \quad W = \int \max(\sigma_1, 0) d\varepsilon_e \leq W_c$$

where  $\sigma_1$  is the maximum principal stress and  $W_c$  is the critical value of the integral  $W$ . Hence, fracture occurs when  $W = W_c$ . Henceforth,  $W_c$  will be referred to as the fracture parameter, while  $W$  will be denoted the Cockcroft-Latham integral. It is seen that fracture cannot occur when the maximum principal stress is compressive and that neither stresses nor strains alone are sufficient to cause fracture. Furthermore, the fracture strain increases with decreasing stress triaxiality (in the shear tests, the stress triaxiality is significantly reduced compared to the uniaxial tension test).

The uniaxial tensile test specimens failed before the point of diffuse necking for the AM60 alloy, and, accordingly, the stress and strain field are uniform up to fracture. Hence, the fracture parameter is obtained as the area under the work-hardening curve.

Zhou and Molinari [8, 9] propose a micro-cracking model for brittle materials (ceramics) considering the stochastic distribution of internal defects. The model introduces a Weibull distri-



Fig. 5

**Picture of test specimens with flow lines on the surface.**

*Immagine dei campioni di prova con linee di scorrimento sulla superficie.*

bution [10] of the local strength of cohesive elements. Thus, the probability of introducing a weak cohesive element increases with the cohesive element size. Inspired by this idea, the fracture parameter of a finite element is assumed to follow a modified weakest-link Weibull distribution in the current study.

The Weibull distribution gives the fracture probability  $P(\sigma)$  of a material volume to under effective tensile loading, i.e.

$$(4) \quad P(\sigma) = 1 - \exp\left[-\left(\frac{V}{V_0}\right)\left(\frac{\sigma}{\sigma_0}\right)^m\right]$$

where  $V$  is the volume,  $V_0$  is the scaling volume,  $\sigma_0$  is the scaling stress, and  $m$  is the Weibull modulus. Since cast magnesium is not a brittle material, the use of a critical fracture stress is not justified. Instead, the Cockcroft-Latham ductile fracture criterion is adopted, and the fracture probability of a material volume is recast as

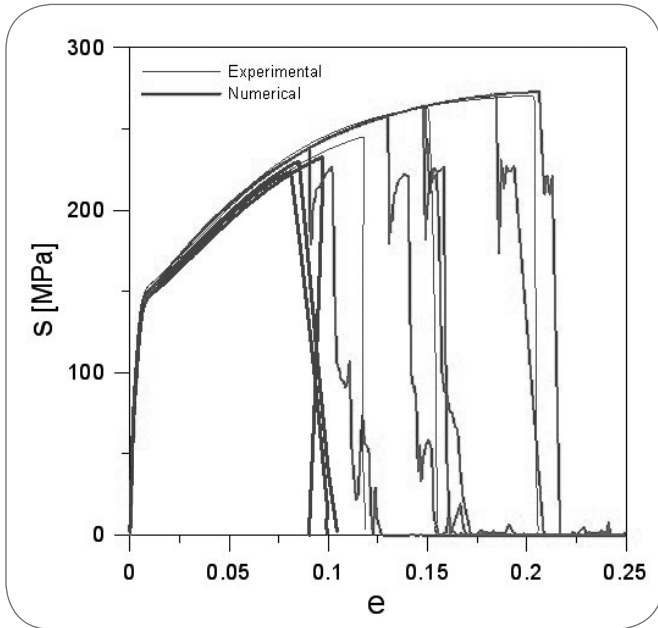
$$(5) \quad P(W_c) = 1 - \exp\left[-\left(\frac{V}{V_0}\right)\left(\frac{W_c}{W_{c0}}\right)^m\right]$$

where  $W_{c0}$  is the scaling value of the fracture parameter. By using a random number generator and inverse sampling, this Weibull distribution of fracture parameters can then be assigned to the integration points in the FE mesh. With this approach, a small element in the FE model will most probably be given more ductile material properties than a larger element.

Fig 6. compares the numerical predictions with the experimental results from the tensile tests on cast magnesium AM60. Here, the uniaxial tension test specimens were modelled by 720 shell elements (i.e., a characteristic element size equal to 1.0 mm) It is seen that the observed experimental scatter is well reproduced numerically.

## CONCLUDING REMARKS

The quasi-static behaviour of high-pressure die cast magnesium alloy AM60 has been studied through tensile tests. The specimens were taken from various positions in the cast profile. The experimental data were used to develop a probabilistic method for finite element modelling of thin-walled die castings subjected to quasi-static loading. The ductility of the specimens cut from the castings depends on the position in the casting. There are also significant variations in ductility when comparing the measured characteristics of specimens cut from different castings that were cast under equal casting conditions. Thus, as a result of unstable flow of the liquid magnesium in the mould cavity, the mechanical properties of the casting are of stochastic nature. By combin-



▲  
Fig. 6

**Experimental and numerical engineering stress vs. engineering strain data.**

Valori di tensione sperimentali e calcolati numerica vs. valori di deformazione.

ing the Cockcroft-Latham fracture criterion and the Weibull statistical distribution function, the fracture parameter was defined as a stochastic Weibull distributed parameter. Repeated finite element simulations of the tensile tests were carried out, giving predictions very similar to the experimental behaviour. Accurate numerical prediction of the mechanical capacity (especially in terms of ductility) of castings requires that the inhomogeneous distribution of defects is included. A coupling of die-casting process simulations and the current approach should be investigated

to establish a deterministic-stochastic approach that can model both the variations in ductility depending on the material's position in the casting as well the stochastic aspects.

#### ACKNOWLEDGEMENTS

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#### REFERENCES

- 1] Dørum, C., Hopperstad, O.S., Lademo, O.-G., Langseth, M.. Aluminium and magnesium castings – experimental work and numerical analyses, *Int. J. Crash*. 8 (2003) 455-470.
- 2] Dørum, C., Hopperstad, O.S., Lademo, O.-G., Langseth, M. Numerical modelling of the structural behaviour of thin-walled cast magnesium components using a through-process approach. *Materials and Design* 28 (2007) 2619-2631.
- 3] Campbell, J., *Castings*, 2nd Ed., Oxford, UK, Butterworth-Heinemann, 2003.
- 4] Hallquist, J.O. LS-DYNA, v.970, Livermore Software Technology Corp., 2003.
- 5] Hersey, A.V., Dahlgren, V.A. The plasticity of an isotropic aggregate of anisotropic face-centered cubic crystals. *J. Appl. Mech. Phys. Solids* 76 (1954) 241-249.
- 6] Hosford, W.F. A generalized isotropic yield criterion. *J. Appl. Mech.* 39 (1972) 607-609.
- 7] Cockcroft, M.G., Latham, D.J. Ductility and the workability of metals. *J. Inst. Metals* 96 (1968) 33-39.
- 8] Zhou, F., Molinari, J.-F. Stochastic fracture of ceramics under dynamic tensile loading. *Int. J. Solids and Struct.* 41 (2004) 6573-6596.
- 9] Zhou, F., Molinari, J.-F. Dynamic crack propagation with cohesive elements: a methodology to address mesh dependency. *Int. J. Numer. Meth. Engng.* 59 (2004) 1-24.
- 10] Weibull, W. A statistical distribution function of wide applicability. *J. Appl. Mech.* 18 (1951) 293-297.

## ABSTRACT

### APPROCCIO PROBABILISTICO PER LA MODELLIZZAZIONE DELLA FRATTURA NEI PRODOTTI PRESSOCOLATI IN MAGNESIO

*Parole chiave:* magnesio e leghe, pressocolata, caratterizzazione, frattura

Nel presente lavoro sono state effettuate prove di trazione quasi-statiche su campioni ricavati da un componente generico pressocolato al fine di caratterizzare il comportamento meccanico della lega di magnesio AM60 pressofusa ad alta pressione. I dati sperimentali sono stati utilizzati per stabilire una metodologia probabilistica per la modellazione agli elementi finiti di

un pezzo a pareti sottili pressocolato, sottoposto a carico quasi-statico. La lega di magnesio AM60 pressocolata è stata descritta mediante un modello costitutivo elasto-plastico costituito da un criterio ad alto esponente di resa isotropa; dalla legge di flusso associata da un regime di incrudimento isotropo. Mediante l'analisi agli elementi finiti è stato sviluppato un nuovo approccio probabilistico per la modellazione della frattura di pressocolati in magnesio a parete sottile mediante l'analisi agli elementi finiti. Per la frattura duttile è adottato il criterio di Cockcroft-Latham assumendo che il parametro di frattura segua una distribuzione di Weibull modificata ad anello più debole. Per i campioni di magnesio pressocolato il confronto tra il comportamento tensile sperimentale e quello previsto per i campioni tensili di magnesio pressocolato ha fornito risultati molto promettenti.