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# The fatigue behavior of V-notches in presence of residual stresses: recent developments and future outcomes

P. Ferro
*Department of Engineering and Management, University of Padova, Stradella S. Nicola 3, 36100, Vicenza (Italy)*
*ferro@gest.unipd.it*

M. Peron, S.M.J. Razavi, F. Berto, J. Torgersen
*Department of Mechanical and Industrial Engineering, Norwegian University of Science and Technology (NTNU), Richard Birkelands vei 2b, 7491, Trondheim, Norway.*
*mirco.peron@ntnu.no, javad.razavi@ntnu.no, filippo.berto@ntnu.no, jan.torgersen@ntnu.no*

**ABSTRACT.** Residual stresses, arising from welding processes or non-homogeneous plastic deformations, broadly influence the high cycle fatigue behavior of mechanical components. The presence of V-notches leads to singular residual stresses ahead of the notch tip and the asymptotic stress field can be described by the notch stress intensity factor (NSIF). However, plastic effects induce redistribution of residual stresses during cyclic loading and this variation is accounted in several numerical models developed for the calculation of the residual NSIFs. Due to the development of these models, the fascinating issue of predicting the fatigue strength of pre-stressed notched components has gained widely attention by the researchers and new approaches were recently developed and some of them are here reviewed.

**KEYWORDS.** High cycle fatigue; Residual stress; V-notch; Fatigue strength.

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## INTRODUCTION

It is widely reported that static and fatigue strength is affected by geometric variations, especially by sharp V-notches [1-5]. Starting from Williams' research [6], several works were written about the effect of local stress field rising from singularities, especially on fatigue strength of welded joints, that can be quantified by means of local Strain Energy Density criterion (SED) averaged over a control volume of a critical radius $R_C$ [7-11] or Notch Stress Intensity Factor criterion (NSIFs) [12-16]. Intensity and distribution of residual stresses are difficult to be quantified near the weld toe, and, moreover, residual stresses are related to welding parameters, geometry, clamping conditions, applied stress and number of cycles. As a consequence of these drawbacks, works reported just above include residual stress effects in reference curves built up from several experimental data. Despite of these complications, several numerical models were developed in order to describe the residual stress field near the weld toe, but Ferro et al. [17] were the firsts to reveal the asymptotic nature of residual stresses. They described how stationary and transient thermal loads affected thermal and residual stress fields, reporting that both of them are singular near a V-notch tip and the singularity degree matches the elastic [6] or the elastic-plastic solution [18,19], depending on the magnitude of the thermal loads and clamping conditions. Ferro [20] and Ferro and Petrone [21] investigated deeply the influence of clamping conditions and phase transformation effects (specific volume change, transformation plasticity [22] on residual stress field and they reported that phase modifications affect the sign of residual asymptotic stress distribution, leading to an enhancement or to a decrease of the fatigue strength by means of stress-relief heat treatment, depending on the material to be welded.

During low-cycle regime loading, i.e. where plastic effects are not negligible as they are instead in high-cycle regime (small scale yielding hypothesis) [23], a redistribution/relaxation occurs in the first cycle and it remains stable going ahead with fatigue cycling [23]. Reliable numerical models can provide the calculation of the residual and stationary NSIF and the residual asymptotic stress field can be seen as a 'mean stress' field as reported by Ferro [24]. The aim of this work is to review some recent improvements.

## ASYMPTOTIC RESIDUAL STRESS FIELD

Before any model is developed which quantify the influence of residual stresses on fatigue strength of pre-stressed notched components, the distribution of residual stress near a 'geometric singularity' has to be first studied. Consider the problem of the elastic equilibrium in the presence of a V-shaped notch with an opening angle 2*β* (Fig. 1).

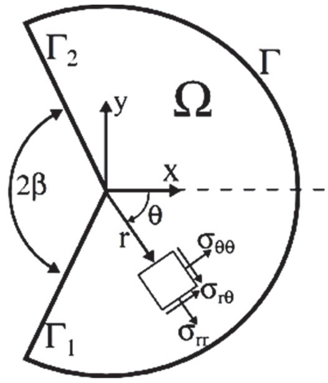


Figure 1: Domain Ω for the sharp V-notch problem.

If the material is homogeneous and isotropic, under the hypothesis of linear, thermo-elastic theory and plane-strain conditions, the equations representing the stress field near the V-notch, are independent of the thermal terms and match the solution obtained by Williams in [6] ([17]). Whatever the load applied (thermal or mechanical), under linear-elastic hypothesis and plane-stress or plane-strain conditions, the induced stress field near the notch tip (by relating only to the first term of the Williams' solution and mode I of V-notch opening), is described by the following asymptotic equation:

$$\sigma_{ij}\left(\theta\right)=\frac{K_I^{th,m}}{r^{1-\lambda_I}}\,g_{ij}\left(\theta\right)\quad\left(i,j=r,\theta\right)\tag{1}$$

where $g_{ij}(\theta)$ are the angular functions, $\lambda_I$ is the first eigenvalue obtained from Eq. (2),

$$\lambda\sin\left(2\beta\right)+\sin\left(2\beta\lambda\right)=0\tag{2}$$

and $K_I^{th,m}$ is the NSIF due to a thermal (*th*) or mechanical (*m*) symmetrical load (opening mode I). According to Gross and Mendelson's definition [25]:

$$K_I^{th,m}=\sqrt{2\pi}\lim_{n\to 0}r^{1-\lambda_I}\sigma_{\theta\theta}\qquad\left(r,\theta=0\right)\tag{3}$$

The first eigenvalue depends only by the V-notch angle ($2\beta$) and varies in the range between 0.5 and 1. The eigenvalue is 0.5 in the crack case ($2\beta = 0$), and increases to 0.674 and 0.757 when the notch opening angles are equal to 135 and 150 degrees, respectively. By simulating the solidification of a fusion zone (FZ) near the tips of a double V-notched plate ($2\beta = 135°$), the asymptotic nature of residual stresses is revealed (Fig. 2).

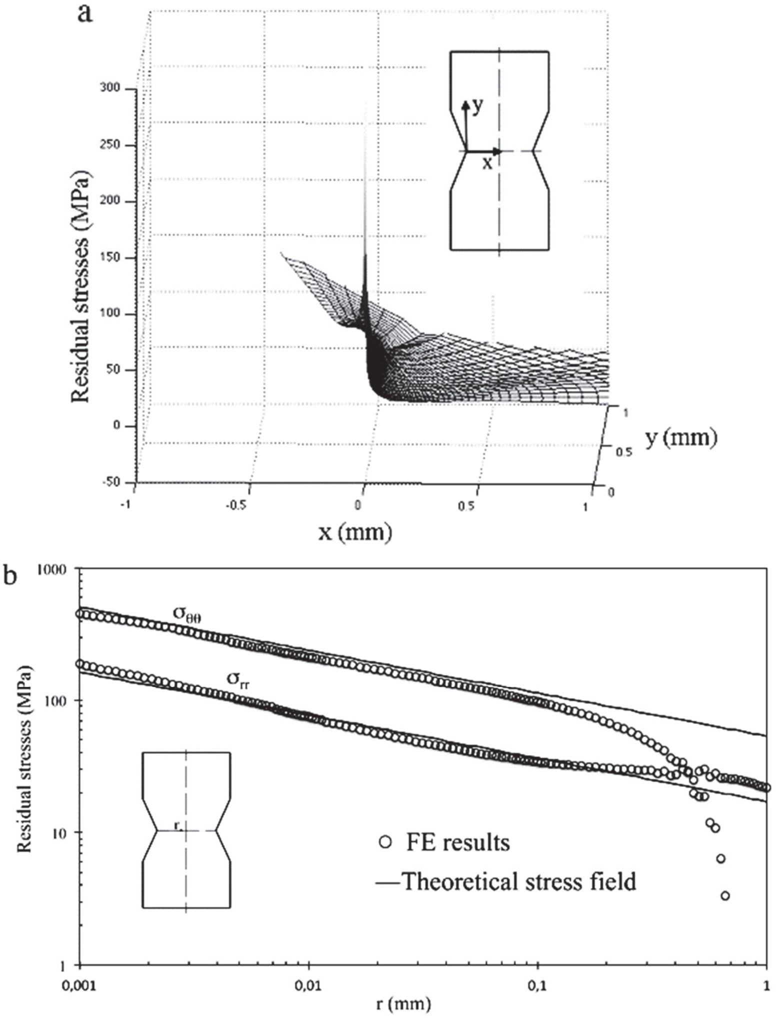


Figure 2: (a) in-plane distribution of residual stresses (radial component, $\sigma_r$) near the notch tip; (b) Tensile residual stresses along the bisector of the V-notch ($2\beta$=135°), ($K_I^{res}$ =135 MPa.mm$^{0.3264}$) (material: ASTM 11 SA 516, free edges) [24].

### *Influence of phase transformation on residual stress field*

More in depth investigations were carried out in order to evaluate the influence of phase transformations on residual stresses. During solidification and cooling of a multi-phase material, the variation of the specific volume and the 'transformation plasticity' [22] associated to phase transformation, influence the thermal and residual stresses induced by thermal loads. It was shown that such effects are so high that any numerical model of welding process that doesn't take into account phase transformation effects fails in calculating the thermal and residual stress filed [26]. When the scale of observation is focused on about one tenth of the notch depth, it was observed that phase transformation changes the sign of residual stresses if compared to the results obtained in a simplified mono-phase material [20]. This means that stress relief heat treatments may decrease the fatigue strength at high-cycle regime when the sign of the asymptotic residual stress is negative. Fig. 3 shows the asymptotic residual stress fields (θ component) along the bisector of the V-shaped weld toe calculated by taking into account phase transformation effects, volume change only and no phase transformations. As a general rule it was found that as-welded mono-phase materials, such as austenitic or ferritic stainless steels, are characterized by a compressive singular residual stress field, while multi-phase material such as carbon steels shows a tensile asymptotic residual stress field (under free-edge clamping condition).

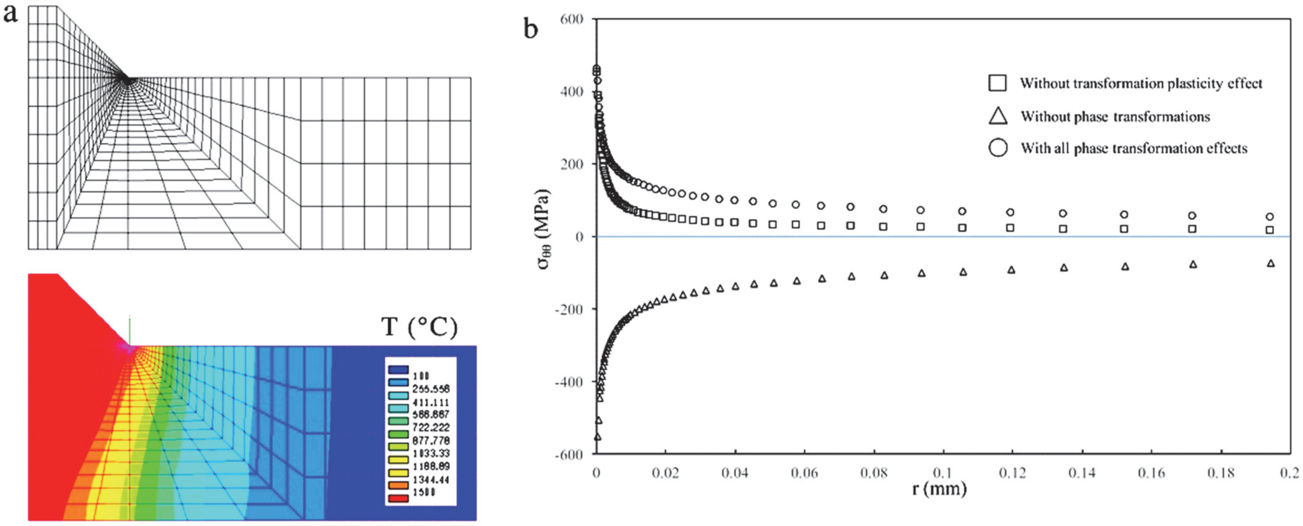
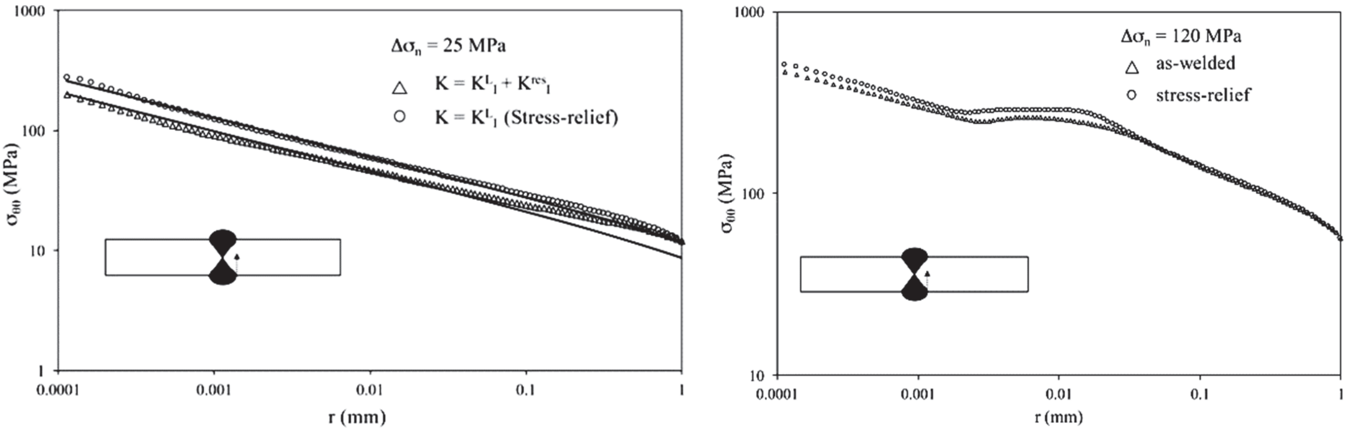


Figure 3: (a) Mesh of the numerical model and fusion zone dimension and shape (butt-welded joint); (b) Phase transformation effects on residual stresses along the bisector of the V-notch ($2\beta = 135°$) (material: ASTM 11 SA 516, free edges).

### *Residual stresses redistribution*

It is well known that residual stresses redistribute during cyclic load due to plastic effect. However, such redistribution is completed after few cycles and a residual NSIF (R-NSIF) stationary value is reached. It was found that the residual stress redistribution under fatigue loading is negligible in the high-cycle regime since the zone that experiments plastic deformation is restricted to about one tenth of the zone dominated by the elastic asymptotic residual stress distribution (small scale yielding hypothesis) [24]. On the other hand, stress redistribution increases as the remotely applied stress increases.

At high stress amplitudes, plasticity 'erases' the pre-existing residual stress field so that there is not difference between the fatigue strength of a stress-relieved joint and an as-welded joint. This means that the superposition property can be applied only in the high-cycle regime where the experimental results show that fatigue strength is sensitive to pre-existing residual stresses. In this case the R-NSIF can be summed to the stress-induced NSIF as shown in Fig. 4a. It can be noted that in that case, residual stresses are negative (single-phase material, AA 6063) so that they decrease the maximum transverse stress filed (Fig. 4a). Fig. 4b shows that at high remotely applied stress amplitude, the plastic effects make the maximum transverse stress filed almost insensitive to the pre-existing residual stress field.



(a) (b)

Figure 4: Maximum transverse stress field of stress-relief and as-welded joint after ten cycles at different values of the remotely applied stress amplitude ($\Delta\sigma_n$) (a) $\Delta\sigma_n$ = 25 MPa; (b) $\Delta\sigma_n$ = 120 MPa; $2\beta = 135°$, $K_I^L$ = NSIF induced by load, $K_I^{res}$ = NSIF induced by the solidification of the FZ: AA 6063, free edges [23].

## QUANTIFICATION OF THE INFLUENCE OF RESIDUAL STRESSES ON FATIGUE STRENGTH OF WELDED JOINTS

On the basis of the above mentioned developments, a model which quantifies the influence of residual stresses on fatigue strength of welded joints or pre-stressed notched components was finally developed [24], and experimentally validated [23]. The model uses the concept of strain energy density (SED) averaged over a control volume of radius $R_C$ [7,8,11], which under plane strain conditions and mode I loading takes the form:

$$\bar{W} = \frac{e_I}{E}\left[\frac{K_I^{th,m}}{R_C^{1-\lambda_I}}\right]^2 \tag{4}$$

In Eq. (4), the parameter $e_I$ depends on V-notch opening angle (2β), Poisson's ratio (ν) of the material and failure hypothesis. Under Beltrami failure hypothesis (total strain energy density) and plane strain conditions, ν = 0.34 (aluminum alloy AA 6063) and $2\beta = 135°$, $e_I$ is equal to 0.111. The control radius ($R_C$) is a material characteristic length that for Al-alloy welded joints was found to be equal to 0.12 mm [8,11]. Now, residual stresses have the effect of modifying the local load ratio (R). As a matter of fact, the following relationships hold true:

$$\left.\begin{aligned} \Delta K &= K_{I_{\max}}^{m} - K_{I_{\min}}^{m} \\ \mathrm{R} &= \frac{K_{I_{\min}}^{m} + K_I^{res}}{K_{I_{\max}}^{m} + K_I^{res}} \\ \mathrm{R} &= \frac{K_{I_{\min}}^{m}}{K_{I_{\max}}^{m}} \end{aligned}\right\} K_{I_{\min}}^{m} + K_I^{res} > 0 \tag{5}$$

$$\left.\begin{aligned} \Delta K &= K_{I_{\max}}^{m} + K_I^{res} \\ \mathrm{R} &= 0 \end{aligned}\right\} K_{I_{\min}}^{m} + K_I^{res} \le 0 \tag{6}$$

where $K_I^{res}$ is the R-NSIF which characterizes the residual stress field. $R^m$ and $R$ correspond to the local load ratio of the nominal and real cycle, respectively. Starting from Eqs (4-6), for $R = 0$, the following equation is obtained (details about the analytical frame employed are published in [24]):

$$\Delta\sigma_n = \frac{R_C^{1-\lambda_I}\left[\frac{E}{e_I}\left(\frac{C}{N}\right)^{1/z}\right]^{1/2}}{k_I t^{1-\lambda_I}} - \frac{K_I^{res}}{k_I t^{1-\lambda_I}} \tag{7}$$

where $\Delta\sigma_n$ (= $\sigma_{n,max}$) is the nominal stress amplitude. Similarly, for $R>0$ the following relationship is obtained:

$$\sigma_{\max}^{m} = \frac{\left(\left(K_I^{res}\right)^2 + \frac{\left(1+\mathrm{R}^m\right)}{\left(1-\mathrm{R}^m\right)}\frac{E}{e_I}\left(\frac{C}{N}\right)^{1/z}\right)^{1/2}}{k_I t^{1-\lambda_I}\left(1+\mathrm{R}^m\right)} - \frac{K_I^{res}}{k_I t^{1-\lambda_I}\left(1+\mathrm{R}^m\right)} \tag{8}$$

where $C$ is a constant and $z$ is the slope of the fatigue data expressed in terms of local strain energy density experimentally calculated $z = \log(N_{D_1}/N_{D_2})/Log(\Delta\bar{W}_{D_2}/\Delta\bar{W}_{D_1})$, subscripts $D_1$ and $D_2$ indicate two points of the curve $\Delta\bar{W}(N)$; $k_I$ is a non-dimensional coefficient, analogous to the shape functions of cracked components calculated by using the following equation:

$$K_I^m = k_I \sigma_n t^{1-\lambda_I} \quad (9)$$

where $\sigma_n$ is the remotely applied stress, and $t$ is a geometrical parameter of the plate, according to [16]. Eqs. (7,8) are applied in high-cycle fatigue regime where the redistribution of the pre-existing residual stresses is considered negligible [24]. By using experimental results of fatigue strength of butt-welded stress-relieved and as-welded joints in AA 6063 [27], the model was validated in[23]. Under the condition $K_{I_{min}}^m + K_I^{res} \leq 0$, Fig. 5 shows an estimation of the fatigue resistance of the stress-relieved and as-welded component predicted by means of the proposed model, Eq. (7). Due to the negative value of the R-NSIF, an improvement of fatigue strength of as-welded joints is observed experimentally and predicted by the model compared to the fatigue strength of the stress-relief specimens. It is worth mentioning that in this model the fatigue strength of as-welded joints in the low-cycle regime was set equal to that of stress-relieved specimens according to the redistribution/relaxation induced by high remotely applied stress amplitudes (Fig. 4b). Some extensions related to creep are also possible [28].

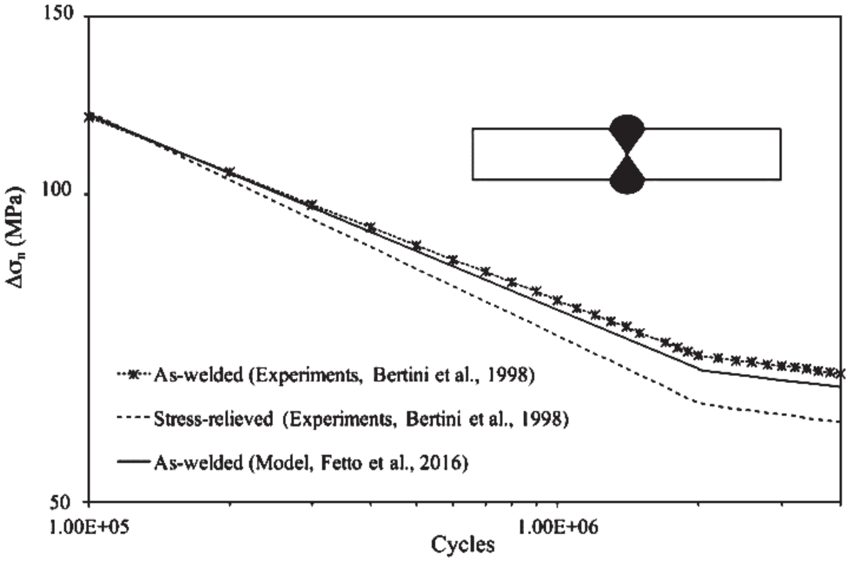


Figure 5: Residual stress influence on fatigue strength of the AA 6063 butt-welded joint predicted by Eq. (7).

## CONCLUSIONS

Starting from 2006, the asymptotic residual stress fields in notched components have been studied extensively with the aim to develop a model which quantifies the influence of residual stresses on fatigue strength of welded joints. Such asymptotic residual stress fields were found strongly influenced by mechanical constraints, geometry, process parameters and material. In particular, the sign of the residual stress field depends on phase transformations effects such as volume changes and transformation plasticity. For this reason, these effects cannot be neglected in any reliable numerical model of welding process. Furthermore, residual stresses redistribute during the fatigue load application because of the plastic deformation that occurs near the weld toe at high remotely applied stress amplitudes. On the contrary, when such remotely applied stress amplitudes are low, the stress redistribution is not expected and the superposition principle can be applied. In such conditions, the residual stress field near the notch tip works in the same way as a mean stress and its influence on fatigue strength is quantified by the proposed model.

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