



The effect of residual stress on the nonsingular T-stresses

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ABSTRACT. The effect of residual stresses on surface fatigue crack propagation and fracture mechanics parameters of intersecting orthogonal cracks is discussed. Mutual influence of intersecting cracks and biaxiality affects the nonsingular parameters T_{xx} and T_{zz} . At the same time, the effect on the stress intensity factor is negligible. Two-parameter fracture mechanics is employed for an analysis of fatigue crack propagation taking into account residual stresses. The final configuration of the crack front is close to the semi-elliptical configuration.

KEYWORDS. Residual Stress; Fatigue Surface Crack; T-stresses; Intersecting Cracks; Finite Element Simulation.



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INTRODUCTION

An analysis of propagation of surface cracks is considered in a number of publications [1-5], where the results of the experimental and numerical analysis of a flat semi-elliptical crack growth under cyclic loading are presented. The Paris formula or some subsequent modifications is used for calculations of fatigue crack increment. For assessment of the crack front configuration, the principle of self-similarity is used in most investigations.

In this paper, the numerical method and the appropriate program realization are presented for the analysis of the surface crack growth in weld joint of a pipeline, including the calculation of changes in the crack front configuration due to action of the active cyclic loading, influence of non uniformly distributed residual stresses and the T-stress.

It should be noted that the results of the mutual influence of intersecting perpendicular cracks on the parameters of fracture mechanics are not enough presented in the literature [6]. It is clear that the effect of the T-stress can be significant and the two-parameter fracture mechanics should be employed.

The analysis of mutual influence intersecting cracks on the values and distribution of singular (the stress intensity factor) and nonsingular (the T-stress) components of the stress field along the crack fronts of intersecting cracks are also presented.



FATIGUE CRACK PROPAGATION UNDER CYCLIC LOADING AND RESIDUAL STRESS

Crack propagation law

The method is based on the modified Forman expression which includes the nonsingular T-stress

$$\frac{dl}{dN} = \frac{C \cdot q_T}{\frac{K_c}{K_{\max}} - 1} \cdot (\Delta K)^m \quad (1)$$

where $\Delta K = K_{\max} - K_{\min}$ is the stress intensity factor (SIF) range and $R = K_{\min}/K_{\max}$ is the SIF ratio. The correction function q_T can be represented taking into account the T-stress as follows [3]

$$q_T = e^{-C_T \cdot \left(1 + \frac{T}{\sigma_Y}\right)} \quad (2)$$

The constants C , m , C_T , as well as the critical stress intensity factor K_c and yield strength σ_Y are the properties of the material. It should be noted that in this case, the SIF ratio varies along the crack front (here, it is not a characteristic of external loads) due to the effect of the residual stress (RS). Since, the fatigue crack increment occurs when load is increased to maximum values at the cycle, the T-stress in Eq. (2) should be determined as T_{\max} . Therefore, Eq. (1) becomes

$$\frac{dl}{dN} = \tilde{C}(K_{\max}, T_{\max}) \cdot (\Delta K)^m \quad (3)$$

Numerical simulation of three-dimensional planar cracks can be constructed as a gradual process of crack increments (after finite number of the cycles ΔN) in a limited set of the crack front points, the position of which will be characterized by a dimensionless local coordinate s that runs along the crack front from one external points to other. An infinitely small value dN can be replaced by a finite cycle increment ΔN and dl replaced by Δl in Eq. (1). So, the crack increment at the current crack front configuration (after N cycles) can be calculated according to the follow equation

$$\Delta l(N, s) = \tilde{C}(K_{\max}(N, s), T_{\max}(N, s)) \cdot (\Delta K(N, s))^m \cdot \Delta N \quad (4)$$

For the successful solution of the crack propagation problem under the action of residual stresses and taking into account the constraint along the crack front by means implementation of Eq. (4), the principle of remeshing procedure of finite elements in the vicinity of the crack front is proposed.

Adaptive parametric finite elements model with varying crack configuration

The parametric finite element model (FE-model) of crack area in the form of a prismatic region is developed in ANSYS software. The model includes the regions with different structures of the element mesh and its sizes. Embedding the crack volume into the FE-model is carried out by means of the macro "Crack" which contains the following features [7]:

- configuration of the crack front as a line can be arbitrary, but must have smoothness;
- an mapped mesh of the singular elements is created along the crack front.
- crack front may pass through several geometric volumes. It is especially convenient for modeling of cracks intersection.

Numerical procedure of the SIF and T-stress determination

The elastic crack-tip displacement and stress fields of mode I crack can be represented as follows [8]



$$\begin{aligned}
 u &= \sum_{n=1}^{\infty} \frac{r^{\frac{n}{2}}}{2\mu} a_n \left[\left(\kappa + \frac{n}{2} + (-1)^n \right) \cos \frac{n}{2} \vartheta - \frac{n}{2} \cos \left(\frac{n}{2} - 2 \right) \vartheta \right] \\
 v &= \sum_{n=1}^{\infty} \frac{r^{\frac{n}{2}}}{2\mu} a_n \left[\left(\kappa - \frac{n}{2} - (-1)^n \right) \sin \frac{n}{2} \vartheta + \frac{n}{2} \sin \left(\frac{n}{2} - 2 \right) \vartheta \right] \\
 \sigma_x &= \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n \left[\left(2 + \frac{n}{2} + (-1)^n \right) \cos \left(\frac{n}{2} - 1 \right) \vartheta - \left(\frac{n}{2} - 1 \right) \cos \left(\frac{n}{2} - 3 \right) \vartheta \right] \\
 \sigma_y &= \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n \left[\left(2 - \frac{n}{2} - (-1)^n \right) \cos \left(\frac{n}{2} - 1 \right) \vartheta + \left(\frac{n}{2} - 1 \right) \cos \left(\frac{n}{2} - 3 \right) \vartheta \right] \\
 \tau_{xy} &= \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} a_n \left[\left(\frac{n}{2} - 1 \right) \sin \left(\frac{n}{2} - 3 \right) \vartheta - \left(\frac{n}{2} + (-1)^n \right) \sin \left(\frac{n}{2} - 1 \right) \vartheta \right]
 \end{aligned}
 \tag{5}$$

where r , θ and x, y are local polar and Cartesian coordinates related to the crack tip, the plane xOy is perpendicular to the crack front line; κ is the parameter of the stress state. It should be noted that $K_I = a_1 \sqrt{2\pi}$, $T = 4a_2$.

The program in the MATLAB environment, so called "Williams", in conjunction with macro "Crack" provides the calculation of the coefficients of stress field expansion terms (Eq. (5) in the vicinity of the crack tip.

The "Williams" interacts with the macro in ANSYS software, which enables collection of the displacements and stresses at measuring points surrounding the crack tip in plane xOy . The determination of the coefficients of expansion (5) is carried out by means of least squares method that provides comparison of these data and data calculated via expression (5). These calculations are performed in many planes which are perpendicular to the crack front for estimation of the SIF and T-stress distribution along front.

Algorithm of numerical crack propagation simulation

Control program (with GUI) that provides collaboration between program elements of ANSYS and MATLAB software is developed in MATLAB. A schematic diagram of its implementation is given in Fig. 1. The data files with special structures are formed at the beginning for initialization of different data, namely, sizes and geometry of the model, the crack, elements, material properties, options of loading (minimum and maximum value of cyclic loading and residual stresses distribution along thickness) and other data.

The solution begins with the start of the control program and "Williams". Next step is the launching of ANSYS in the background mode for creation of the FE-model and solution of minimum and maximum values for the loading cycle. The collected data in the analyzed points are transiting to "Williams".

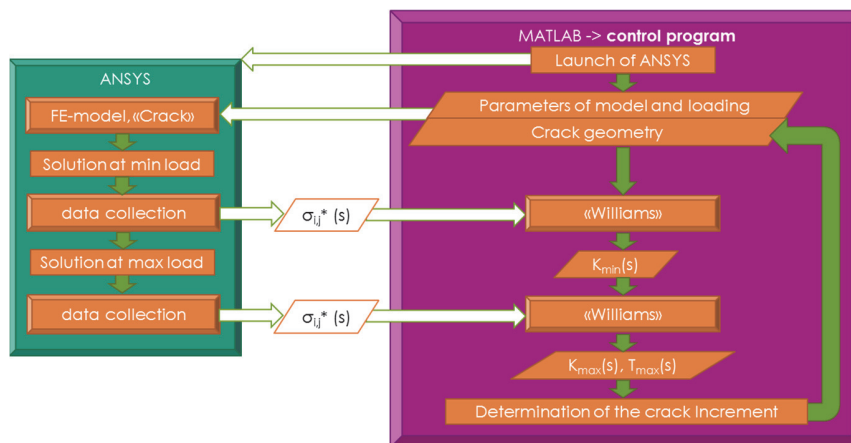


Figure 1: The algorithm of numerical solution for the crack propagation problem.

After computing the coefficients of William's expansion for the minimum and maximum loads per cycle (as well as K_{min} , T_{max} , K_{max}), the control program calculates values of the fatigue crack increment at each points of the crack front in accordance with Eq. (4). The new configuration of the crack front is formed and saved in the data file for the "Crack" macro. This specified sequence of procedures in ANSYS and MATLAB are executing repeatedly.

Internal surface crack propagation in weld joint of a pipeline

Let consider a welded zone of the pipe with outer diameter 1420 mm and wall thickness 18.7 mm. The pipeline internal pressure is 10.6 MPa and it leads to the active axial and circumference stress 200 MPa and 400 MPa, respectively. Note that axial direction is perpendicular to the plane of the weld joint. In operation, there are additional axial stresses that occur because of bending and thermal loading. The axial stresses in conjunction with the pressure pulsation lead to cyclic loading. The following parameters of the loading cycle are accepted in the work: average stress is 200 MPa, the minimum and maximum values are 133 MPa and 267 MPa, respectively.

In addition, there are residual stresses in the zone of welding. Distribution of axial residual stresses (RS) along the thickness was accepted in accordance with the results of well-known experimental study of a full-scale pipeline (Fig. 2).

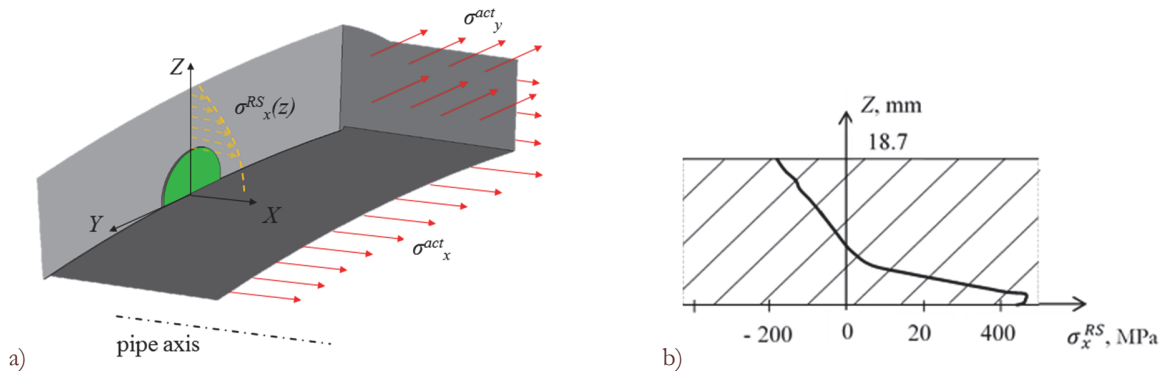


Figure 2: Loading condition of the pipe (a) and the axial residual stress distribution along pipe thickness (b)

Taking into account the small curvature of the pipe, the region with the crack is considered as thick plate. The initial front geometry of the crack located in the weld joint plane on the inner pipe surface is assumed in form of a semicircle with radius of 4 mm. Ten terms in the Williams expansion are kept to estimate the SIF and the T-stress at points of the fatigue crack front after discrete cycles $\Delta N = 10^5$. The total number of cycles, at which a calculation stop was occurred, (if previously SIF did not reach the critical value) is $N=10^7$.

The results of calculations in the form of finite configurations of the crack front at various conditions are shown in Fig. 3.

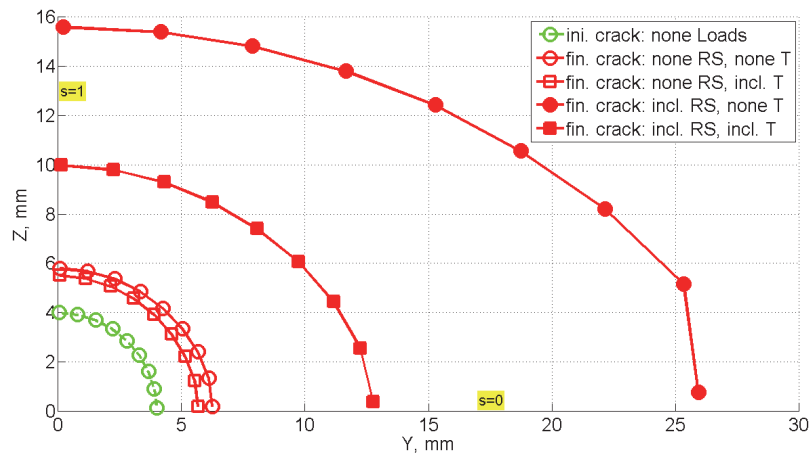


Figure 3: Final configuration of the crack front (half) under different loading conditions.

It should be noted that independently of the initial crack geometry, final configuration of the crack front is close to the semi-elliptical. As expected, the stress $T > 0$ leads to decreasing of the crack growth rate. This fact allows concluding about the conservatism of classic Paris and Forman formulas (not taking into account the effect of constraint and the residual stress). Otherwise, the residual stress (RS) affects on the rate of crack growth. Stress intensity factor distribution along the crack front strives for a constant value during crack size growing in absent of the RS. Presence of the RS leads to quite complex distribution of the SIF along the crack front. In contrast to the SIF, distribution and values of the T-stress along the crack front are weakly dependent on the RS.

INTERSECTING CRACKS IN PIPELINE WITH RESIDUAL STRESS

Fracture mechanics parameters for intersecting surface cracks in the weld joint of pipeline are analyzed below. The pipe has the following dimensions, namely, outer diameter is 325 mm; wall thickness is 16 mm. The material is corrosion-resistant austenitic steel.

The circular crack (crack A) with depth of 6 mm is assumed to be formed on the inner side of the pipe in the plane of welding zone. The semi-elliptical crack is located in the meridional section (crack B). These cracks are intersected perpendicular each other (Fig. 4). Dimensions of the elliptical crack are 8 mm (crack depth) and 12 mm (length).

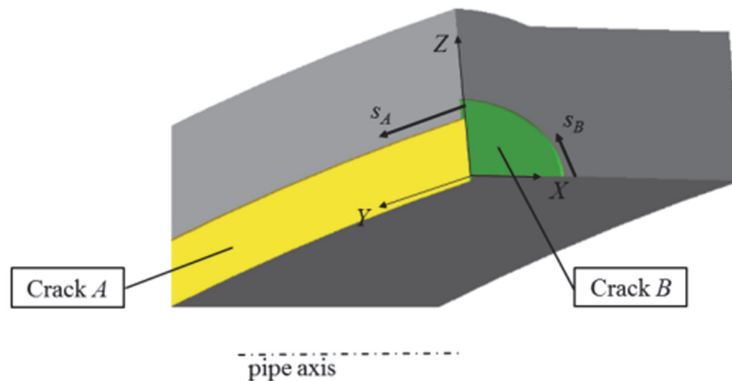


Figure 4: Intersecting cracks on inner surface of the pipe.

The active circumferential stress is 67.6 MPa and axial stress is 108.8 MPa (taking into account not only pressure, but other loading factors). In addition, there are residual stresses occurred during the welding process. It is assumed that the axial residual stress is linearly varying through the pipe thickness. The residual stress reaches the yield strength on the inside surface near the weld root and a similar value with the opposite sign on the outer surface. The circumferential residual stress is also equal to the yield strength and this stress is constant through the thickness (they are varying along the pipe).

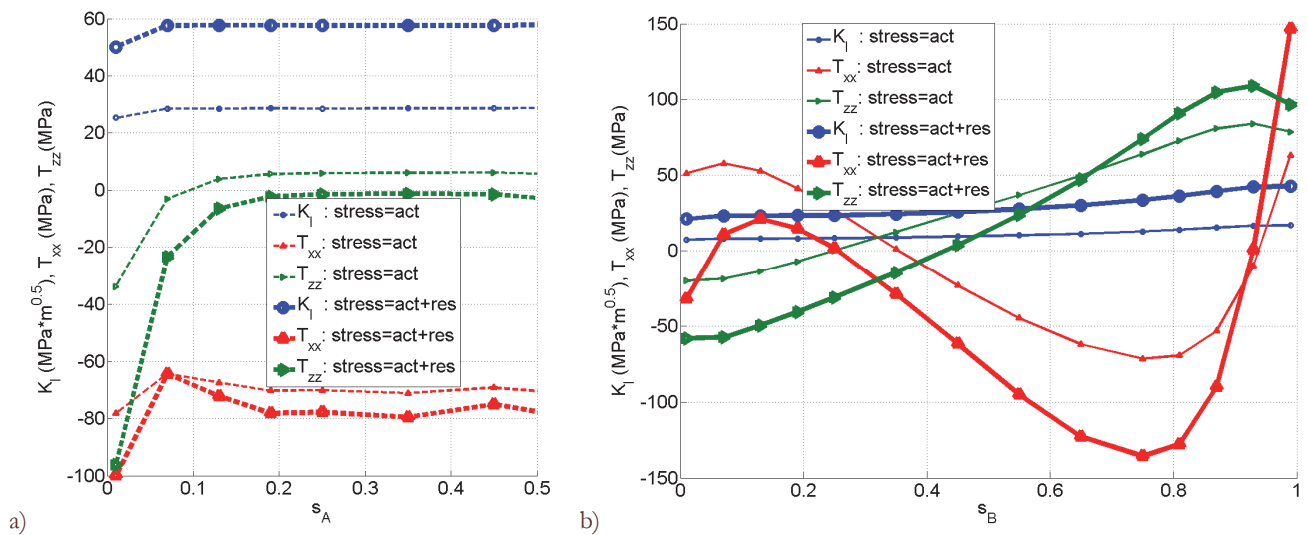


Figure 5: Distribution of K_I , T_{xx} , T_{zz} along the crack front A (a) and crack front B (b) under different loading conditions.

The computed results are summarized in Fig. 5. Note that s_A and s_B are dimensionless local coordinates (Fig. 4) transiting along a front of the crack A and B, respectively. Distributions of the T_{zz} -stress [9] as well as the SIF and the T_{xx} -stress are determined by means of the program "Williams" for only active external stress ("act") as well as in combination with the residual stress ("res"). An analysis of fracture mechanics parameters allows concluding the following. The axial crack (B) has a great influence on the T_{xx} -stress and the T_{zz} -stress. The values of T_{xx} and especially T_{zz} are decreasing (staying more



negative). Strong influence of the RS on the SIF of the crack A is not observing. However, the residual stress acts on SIF values in more manner then on T-stress values of crack A. The distribution of fracture parameters and the effect of the RS for the crack B are more complex. The SIF has an almost constant value along the crack front, whereas T-stresses have values great varied. It should be noted that there is the influence of the crack A on the state of the crack.

CONCLUSIONS

The effect of residual stresses on surface fatigue crack propagation and fracture mechanics parameters of intersecting orthogonal cracks in the pipeline is discussed. The method on the basis of modified Foreman equation and program software for realization of gradual remeshing of the finite element model during incremental crack growth is developed. It is implemented for an numerical analysis of fatigue surface crack propagation in welded area of pipeline taking into account nonlinear distributed residual stress and constraint effect in the crack tip employing two-parameter fracture mechanics. In spite of the fact that the residual stress and the nonsingular T-stress have a significant influence on the crack growth rate. Final configuration of the crack front is close to the semi-elliptical configuration. Mutual influence of intersecting surface cracks and biaxiality effects on the nonsingular parameters T_{xx} and T_{∞} is observed. At the same time, their effect on the stress intensity factor is negligible.

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