



## Measurement and analysis of fatigue crack deformation at the micro-scale

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**ABSTRACT.** This paper introduces the use of digital image correlation for the measurement of surface displacements in the neighbourhood of a crack tip, both at the macro- and micro- scale. Various methods of interpreting the measured data and producing a crack driving force are then discussed, including the use of the full CJP model. A reduced set of parameters are then proposed, corresponding to the three principal interaction forces between the plastic enclave and the surrounding elastic material. Our own results, and those of Vasco Olmo, previously reported in the literature are then re-analysed using this new framework, and excellent agreement between two independent experiments is obtained. Implications for the analysis of further data sets are then discussed.

**KEYWORDS.** Fatigue; Crack Driving Force; Digital Image Correlation; CJP model.



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### INTRODUCTION

Understanding the mechanics of fatigue crack propagation is important if we are to safely operate a wide range of engineering systems. Damage tolerant life prediction methods are increasingly popular and give a number of advantages over safe life approaches. Nevertheless, most of the methods are based on the Paris law relation between  $da/dN$  and  $\Delta K$ . Although this 'law' is justifiably popular, it is in essence a simple empirical curve fit to laboratory data obtained at constant remote load amplitude. Load histories in real structures can differ considerably, and may include variable amplitude cycles, hence it is important to understand how crack tips behave in these circumstances. There is an increasing realisation that simple elastic crack models may have limited applicability, particularly when there is significant plastic deformation ahead of the crack and a corresponding plastic wake.

Our developing work at Oxford in the area described above has been presented previously at the Forni di Sopra [1], Malaga [2], and Urbino [3] IJ Fatigue/FFEMS workshops. It has employed digital image correlation on the macro and micro scale to measure and analyse near tip displacements fields. In particular, at Urbino [3], we reported measurements taken during in-situ loading of a fatigue crack in a scanning electron microscope (Fig.1).

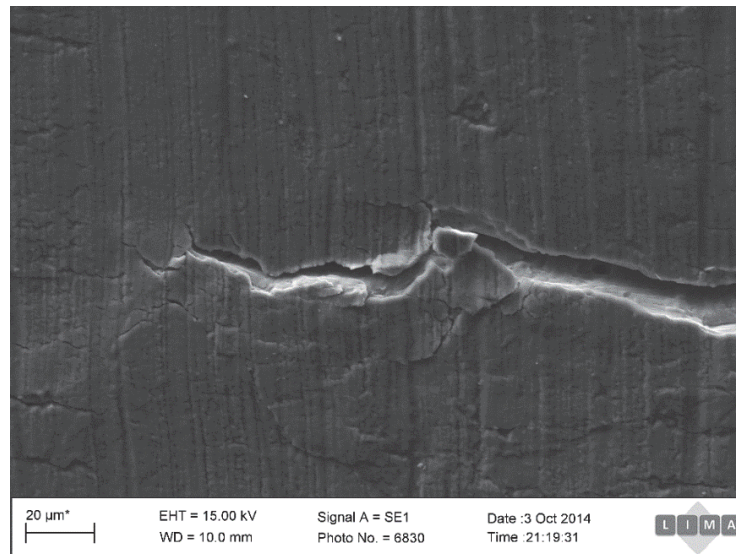


Figure 1: Typical image of a fatigue crack, during in-situ loading in the SEM

Our work focuses on measuring the variation of crack opening with distance along the line of the crack, since this quantity is straightforward to measure. Indeed, if one employs a direct measure of displacement, such as digital image correlation (DIC), then the crack flanks are the largest displacements available, once rigid body motions have been excluded. In analyzing our work, we started off by using simple elastic models [1], and showing that the technique can be used to calculate stress intensity factors by (for example), plotting the variation of crack opening against  $\sqrt{r}$ , where  $r$  is the distance from the crack tip. If the displacements vary as  $\sqrt{r}$ , then it is clear that the strains (and therefore stresses) vary as  $1/\sqrt{r}$ , and hence the  $K$ -value may readily be extracted.

Later, [2] our work extended to include a simple elasto-plastic model due to Pommier and Hammam [4]. This was shown to give a slightly better fit to the experimental results, but the absolute value of the plasticity parameter  $\rho$  (effectively the crack tip opening displacement) was found to be sensitive to the choice of crack tip position. However, it is clear from both approaches that there is a considerable amount of crack closure present, and this may be considered as a residual (negative) stress intensity factor, which may be summed with the applied  $K$  to give the actual stress intensity experienced by the crack. An example of this is shown in Fig. 2, which is data obtained for a fatigue crack growing under cyclic loading and observed in a scanning electron microscope.

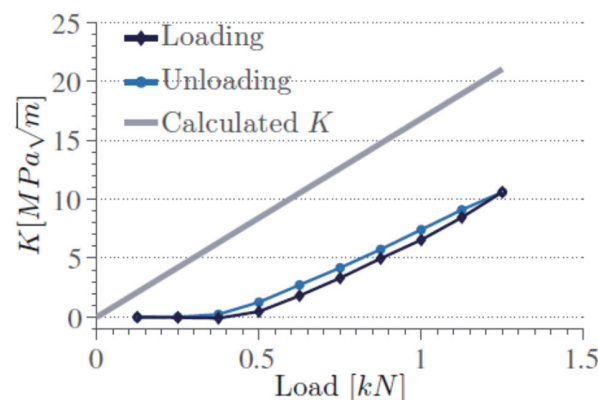


Figure 2: Variation of measured and calculated stress intensity factor with load, for a typical DIC experiment [5]

In Fig.2., the residual  $K$  is apparent as an (almost constant) offset between the theoretical and experimental lines, once the crack is open.



Since the effect of the wake is clearly important in the characterization of crack tip conditions for a fatigue crack, we decided to investigate a third possible analysis method, using a model with a wake representation built in. The Christopher/James/Patterson (CJP) model [6],[7] was selected because of its relatively simple formulation.

### DISCUSSION OF THE CJP MODEL

The CJP model attempts to capture some of the additional phenomena generated by a fatigue crack with a plastic wake. This approach leads to a crack description with four parameters as follows:

- $K_F$  The ‘Forward Stress Intensity Factor’, which is essentially similar to the applied  $K_I$  in a conventional analysis.
- $K_R$  A ‘Retardation Stress Intensity Factor’, which arises as a result of the residual stress field set up by the wake, and which might be thought similar to the offset shown in Figure 2.
- $K_S$  A ‘Shear Stress Intensity describing’ the shear present between the plastic wake and the surrounding elastic material.
- $T$  The conventional T-stress or bounded term in the Williams expansion.

It is easy to misunderstand some of these terms, so a brief further explanation will be given here with the aid of a diagram (Fig. 3) modified from that presented by James et al. in [7]. First, it is essential to understand that the plastic zone at the tip of the crack creates a wake along the crack faces, as the crack propagates. Hence, there is a plastic enclave of the approximate shape shown in grey in Fig. 3. If one then considers the boundary between this enclave and the surrounding elastic material there may be x, y and shear forces transmitted across this interface and in each case, the force *on* the enclave by the elastic hinterland will be equal and opposite to that exerted *by* the enclave on the elastic material. Figure 3 shows the forces exerted *on* the plastic enclave.

#### Forces on plastic field

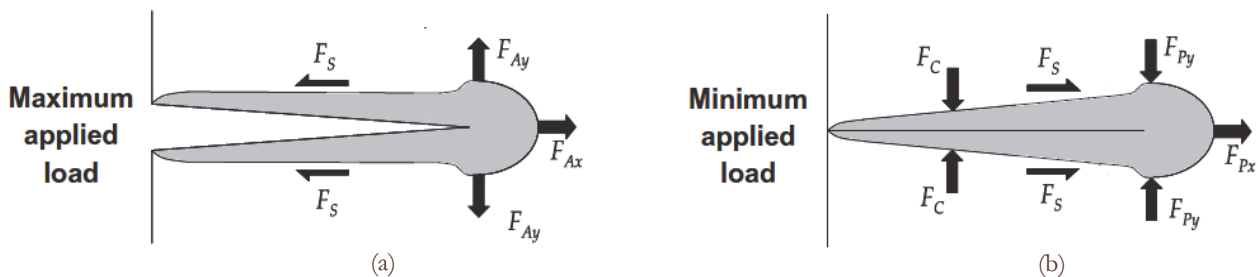


Figure 3: Forces between the plastic enclave (grey), and the surrounding elastic material (after [7]).

Starting with Fig. 3a, at maximum there is clearly an applied force opening the crack and causing tensile yield. This is labelled  $F_{Ay}$  in the figure. Similarly, there may be shear at both top and bottom of the enclave, and these forces (which for mode I must be in the same direction) are labelled  $F_S$  in Fig.3. Since the enclave must be in equilibrium, these two shear components must be reacted by a horizontal force, termed  $F_{Ax}$ . The authors include a separate term for the T-stress, but this is clearly not a net force across the interface. Hence, it is perhaps more helpful to think of  $T$  as being a bounded response to  $F_{Ax}$ , which clearly does not cause any stress intensity at the crack tip. At the minimum applied load, James et al consider the crack at least partially closed (Fig. 3b). They then show  $F_S$  reversed in sign, which seems plausible, since the crack has reached this state by unloading from the maximum load. The  $F_x$  force however, is not reversed in sign, so that if Fig. 3b is considered as a free body diagram, then the plastic enclave would not be in equilibrium. What James et al do, however, change is the notation for the x-direction force, calling it now  $F_{Px}$  presumably to show that the plastic zone is somehow the entity responsible for the force, rather than the applied load. Of course, in the general case, both applied load, and plastic zone resistance contribute to the force across the interface, and it is not possible (or perhaps helpful) to separate them. In the vertical direction, the sign of the main vertical force *is* reversed, *and* the notation is changed. Further, an additional force  $F_C$  is introduced to represent the contribution to vertical force caused by crack closure. James et al. [6], [7], then go on to collapse the plastic enclave onto the line of the crack, and develop a Muskhelishvili stress function for the surrounding elastic material. This eventually results in the four terms defined above.

Our own work has so far focused on simpler one or two parameter models, and our view is that the introduction of different forces with different notations and signs in Fig. 3., whilst intended to link the model to physical phenomena, is not particularly helpful. What is helpful, however, is the concept of a plastic wake and its effect on the surrounding material. Hence, there is no need to introduce different y-direction forces at minimum load, but one can simply stick with the system of forces shown in Fig. 3a and drop the double subscripts. So,  $F_y$  causes crack opening and a stress intensity,  $F_s$ , exerts shear on the areas above and below the crack, and this is reacted by  $F_x$ , which may be thought responsible for the bounded stress component in the x-direction (T-stress). These three forces neatly map onto the four terms tabulated on the previous page:

- $F_s$  Is responsible for  $K_S$ . Physically, its link to crack propagation rates is difficult to see, so that one might consider it, at most a secondary effect.
- $F_x$  Is responsible for the T-stress. Again, a secondary effect, though perhaps more important than  $K_S$ .
- $F_y$  Must clearly be responsible for  $K_F$  and  $K_R$ , and which may be separated using the CJP approach, or if preferred may remain as a single  $K_I$  term.

## EXPERIMENTAL RESULTS AND DISCUSSION

We have essentially already extracted the dominant  $K_I$  term in the above simplified version of the CJP model, and an example result has already been presented in Fig. 2. To compare this with the results of the full model, we will use some results produced by Vasco Olmo [8]. He used a full-field DIC technique to extract  $K_F$  and  $K_R$  for the CJP model, using essentially the same material (Al4%Cu) and specimen geometry (Compact Tension) as in our own work. Figure 4 gives his results for a test conducted a load ratio,  $R = 0$ . It can be seen that the measured  $K_F$  value is very close to the nominal elastic  $K$ , calculated using the usual standard solution. The  $K_R$  value starts close to zero, but then becomes negative, and increases in magnitude until the peak load, decreasing again during unloading.

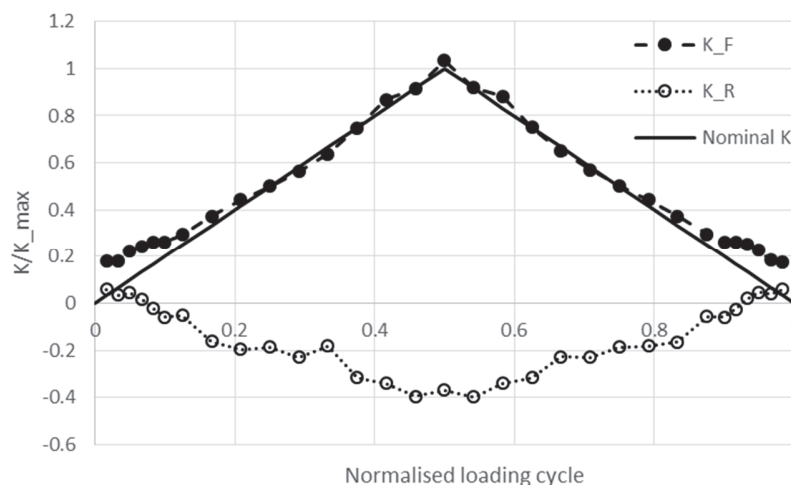


Figure 4: Results obtained by Vasco Olmo [8], showing the variation of  $K_F$  and  $K_R$  through a load/unload cycle for an Al4%Cu CT specimen. Results are normalized with respect to the nominal elastic  $K$ .

If we now choose to plot a single crack driving force,  $K_F + K_R$ , the results are transformed to those shown in Fig. 5. Finally, we note that in our own work, the datum for displacements was the unloaded specimen *with the crack present*, so that we are unable to detect any pre-existing residual  $K$ . Hence the appropriate parameter to plot is  $\Delta(K_F + K_R)$ , and this is given in Fig. 6, along with our own experimental data (Fig. 2), re-plotted on the same axes. The comparison between the two sets of data, obtained independently on two different specimens in two different laboratories is striking, particularly when one considers that different DIC algorithms are used, and different post-processing routes were adopted.

Comparison of these two sets of data, suggest that the four parameters of the mode I CJP model may usefully be reduced to three if one combines the  $K_F$  and  $K_R$  parameters, and that the three remaining terms each map clearly onto the effects of a force transmitted across the interface between the plastic zone and resulting wake, and the surrounding elastic hinterland. When viewed from this perspective, the combined  $K_F + K_R$  parameter appears to be very similar to the measured delta  $K$  in our own experiments carried out under similar conditions. Both our own laboratory and that of Vasco Olmo and Diaz Garrido in Jaén have a wealth of similar data, carried out for different loading conditions and specimen geometries, and more time is needed to make further comparisons of a similar nature. However, before we do so, it would be useful if the community could take a view on whether the full complexity of the



CJP model is helpful (i.e. is the splitting of delta K into separate  $K_F$  and  $K_R$  terms justified from the point of view of predicting crack behavior).

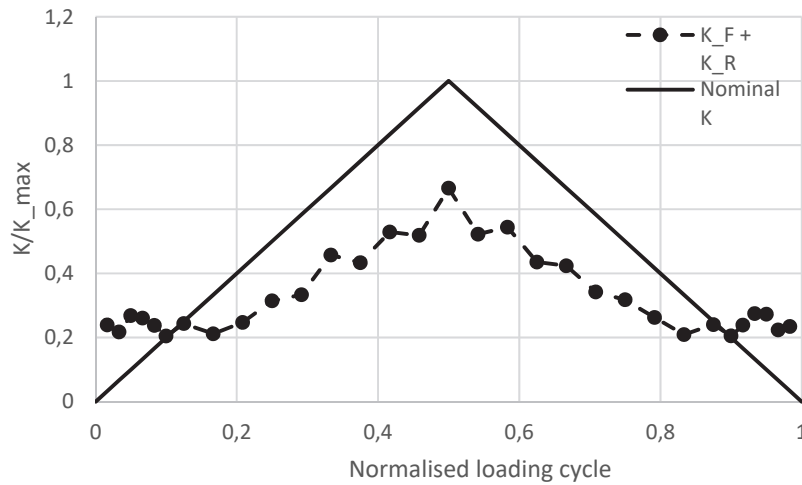


Figure 5: Results from Fig. 4, obtained by Vasco Olmo [8], re-plotted to show the variation of  $(K_F + K_R)$  with loading cycle

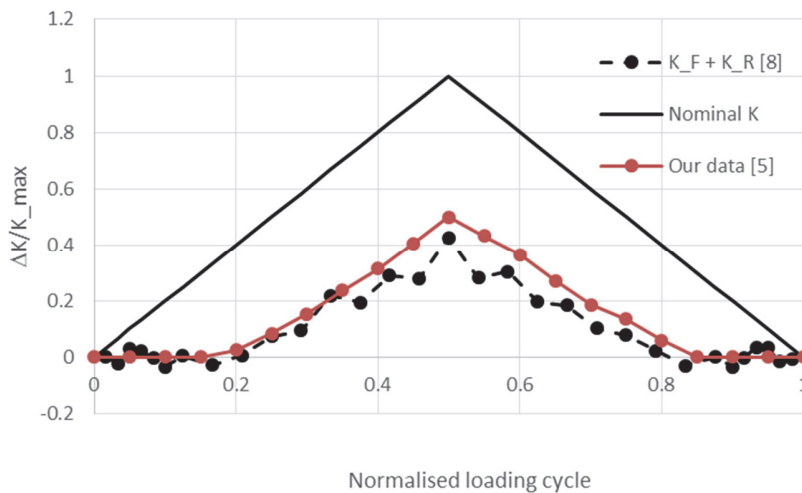


Figure 6: Vasco Olmo's data [8] plotted as delta K against loading cycle and compared against our own [5] (also shown in Fig.2.)

## CONCLUSIONS

This paper has introduced and discussed the CJP model for fatigue crack displacement, strain, and stress fields, and proposed a simplification, which seems to fit more readily with the forces acting between the plastic enclave and the surrounding elastic material. A comparison has been made between one of our own experiments, reported previously [5], and one from the group at Jaén [8]. Excellent agreement was found between the two sets of data and further collaboration to examine the full database of experimental results in a single agreed framework is proposed.

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