Evaluation and visualization of multiaxial fatigue behavior under random non-proportional loading condition

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ABSTRACT. In cyclic multiaxial stress/strain condition under non-proportional loading in which principal direction of stress/strain are changed in a cycle, it becomes difficult to analyze stress/strain ranges because of complexity of multiaxial stress/strain states depending on time in cycles. In order to evaluate stress/strain simply and suitably under non-proportional loading, Itoh and Sakane have proposed a method called as IS-method and a strain parameter for life evaluation under non-proportional loading $\Delta\varepsilon_{NP}$. In the method, 6-components of stress/strain are converted to an equivalent stress/strain indicating the amplitude and the direction of principal stress/strain as a function of time as well as an intensity of loading non-proportionality f_{NP}. Based on IS-method, the authors also have developed a tool which enables to analyze multiaxial stress/strain condition with the non-proportionality of loading history and evaluate failure life under non-proportional multiaxial loading. The tool indicates the analyzed results on monitor and users can understand visually not only variation of the stress/strain conditions but also non-proportionality during the cycle, which helps the design of material strength.

KEYWORDS. Multiaxial fatigue; Life evaluation; Non-proportional loading; Analysis tool; Cycle counting; Additional hardening.

INTRODUCTION

Structural components and materials such as aircrafts and nuclear equipment undergo complex non-proportional multiaxial loadings where directions of stress and strain are changed into various directions under multiaxial stress and strain states. It is difficult for engineers to analyze histories of stress and strain. Therefore, development of suitable models for the design of actual components where variation of principal directions of stress and strain *vs.* time is considered 3-dimensionally is required [1-8].

To evaluate fatigue lives under multiaxial loading condition, multiaxial fatigue models which relate fatigue lives to uniaxial fatigue properties have been established. Equivalent strains and stresses based on von Mises and Tresca are considered as the most common theory. However, they lead to significant overestimation of fatigue lives under non-proportional loadings, thus a few other classical models such as critical plane approaches that correlate stress or strain with multiaxial fatigue lives have been proposed in recent years. However, some disadvantages on these approaches have been reported. Therefore, in order to avoid these disadvantages, Itoh et al. proposed a strain parameter related to material property and loading path, which are available in both of short-life and long-life fatigue regimes. The parameter takes response to material deformation and is expressed by simple calculation based on strain model [9-15].

Based on the IS-method proposed by Itoh et al., analyses for pipes subjected to cyclic multiaxial/non-proportional loading are performed. Under non-proportional loading, it is expected that the reduction of material fatigue life occurs. There are a lot of structural components that are subjected to non-proportional multiaxial loading, but the verification of shortened fatigue life has not been carried out mostly. Therefore, in order to have a guarantee for safety and soundness of structural components, an appropriate understanding and damage evaluation under non-proportional multiaxial loading are strongly urged [4-8].

In this study, on the basis of the method of Itoh-Sakane criterion (IS-method) which evaluates the state of stress and strain in a non-proportional multiaxial cyclic loading, the program is developed for the visualization and analysis of stress/strain state and the evaluation of failure life under non-proportional loading with constant or random amplitude was performed.

MULTIAXIAL FATIGUE LIFE EVALUATION MODELS

A number of multiaxial fatigue models have been proposed based on stress and strain models. Stress-based models are more widely used and are suitable for the large class of components that are operated near or below the fatigue threshold. Many of the stress-based models can be used successfully in the finite life regime if the plastic strains are small. In a word, stress based multiaxial damage criteria are suitable for infinite or high-cycle finite life evaluation.

Development of strain-based models started in 1970s, which are more useful for low-cycle fatigue analysis. They may be written in strain alone or some product of stress and strain. When principal directions of strain correspond to axes of coordinates employed, the most common equivalent strain models are [3-8]:

Maximum normal strain theory $\Delta\varepsilon_1$:

$$\Delta\varepsilon_1 = \text{Max}\left[\varepsilon_1^A - \varepsilon_1^B, \ \varepsilon_2^A - \varepsilon_2^B, \ \varepsilon_3^A - \varepsilon_3^B\right] \tag{1}$$

Maximum shear strain theory $\Delta\gamma_1$:

$$\frac{\Delta\gamma_1}{2} = \text{Max}\left[\frac{\gamma_1^A - \gamma_1^B}{2}, \ \frac{\gamma_2^A - \gamma_2^B}{2}, \ \frac{\gamma_3^A - \gamma_3^B}{2}\right] \tag{2}$$

Where ε_1^i, ε_2^i, ε_3^i (i=A, B) and γ_1^i, γ_2^i, γ_3^i are principal strains and principal shear strains at arbitrary times A and B in a cycle and 'Max' takes maximum value in brackets.

Mises' equivalent strain theory $\Delta\varepsilon_{eq}$:

$$\Delta\varepsilon_{eq} = \sqrt{(\Delta\varepsilon)^2 + \frac{1}{3}(\Delta\gamma)^2} = \frac{\sqrt{2}}{3}\sqrt{(\Delta\varepsilon_1 - \Delta\varepsilon_2)^2 + (\Delta\varepsilon_2 - \Delta\varepsilon_3)^2 + (\Delta\varepsilon_3 - \Delta\varepsilon_1)^2} \tag{3}$$

These models correlate multiaxial data for certain materials under some loading conditions, but they cannot be considered generally applicable to life evaluation under non-proportional loading.

Energy models based on the plastic work per cycle are proposed as a parameter for life to crack nucleation. Energy is a scalar quantity and does not address the result of the observation where cracks nucleate and propagate along specific planes.

Critical plane models attempt to correlate fatigue damage with physical observations of the nucleation and growth of cracks. The concept of critical planes was first proposed by Brown and Miller. The approach defined a failure plane as a critical plane in terms of stage I (shear-type) or stage II (tensile-type) cracks. Firstly, the history of strain on the critical plane is analyzed, and then strain parameters are used to quantify the damage parameters on the critical plane. Various terms have been proposed for different materials and experimental results. The Brown and Miller critical plane approach is expressed as $\gamma_{max} + S\,\varepsilon_n = C$, where γ_{max} is the maximum shear strain amplitude, ε_n is the amplitude of the normal strain acting on the γ_{max} plane, and S is a constant. It is reasoned that γ_{max} governs crack growth and its direction, and that ε_n further assists crack propagation.

Socie considered that the fatigue life criterion should be based on a physical mechanism. For a material with tensile-type failures, Socie modified the Smith–Watson–Topper (SWT) parameter by considering that crack growth was perpendicular to the maximum tensile stress. The parameters which control damage were the maximum principal strain amplitude $\Delta\varepsilon_1^{max}/2$ and the maximum principal stress σ_1^{max} on the maximum principal strain plane [16-21],

$$\sigma_1^{max}\frac{\Delta\varepsilon_1^{max}}{2} = \varepsilon_f'\sigma_f'\left(2N_f\right)^{b+c} + \frac{\sigma_f'^2}{E}\left(2N_f\right)^{2b} \tag{4}$$

where ε_f' is the fatigue ductility coefficient and σ_f' is the fatigue strength coefficient, while b is the fatigue strength exponent and c is the fatigue ductility exponent. However, some disadvantages on these approaches can be listed as: i) Stress based critical plane approach advocated by Findley or McDiarmid is not available for short fatigue lives regime. ii) Strain based critical plane approach advocated by Brown-Miller has no reflection of material behavior, such as non-proportional cyclic hardening. iii) Strain-stress based critical plane approach advocated by Fetemi-Socie or Smith-Watson-Topper is complex on calculating the stresses from the multiaxial strain. Therefore, in order to avoid these disadvantages, Itoh et al. proposed one strain parameter related to material property and loading path, which are available in both of short-life and long-life fatigue regimes, takes response to material deformation and is expressed by simple calculation based on strain model.

MULTIAXIAL FATIGUE LIFE EVALUATION MODEL: IS-METHOD

Definition of stress and strain

I n non-proportional multiaxial fatigue, principal directions of stresses and strains vary during a cycle. In such a case, strain and stress ranges and mean strain and stress cannot be easily determined. It is necessary to represent the principal stress/strain and the direction of principal axis as a function of the time. The principal stress/strain vectors at time t are set as $\boldsymbol{S}_I(t)$, the index I equals to 1, 2 and 3, that is, maximum, median and minimum principal stress or strain vectors, respectively. In addition, $\boldsymbol{S}$ is the symbol denoting either stress (σ) or strain (ε). Fig. 1 illustrates three principal values, $\boldsymbol{S}_I(t)$, applied to a cube at time t together with xyz-coordinates (spatial coordinates). $S_I(t)$ is the maximum amplitude of the principal stress or strain at the time t in the IS-method [2, 4], which is defined by the following equation.

$$S_I\left(t\right) = \left|\boldsymbol{S}_I\left(t\right)\right| = \text{Max}\left[\left|\boldsymbol{S}_1\left(t\right)\right|, \left|\boldsymbol{S}_2\left(t\right)\right|, \left|\boldsymbol{S}_3\left(t\right)\right|\right] \tag{5}$$

The maximum value of $S_I(t)$ during a cycle is taken as the maximum principal value S_{Imax}, which is defined as,

$$S_{Imax} = \left|\boldsymbol{S}_I\left(t_0\right)\right| = \text{Max}\left[S_I\left(t\right)\right] \tag{6}$$

That is, t_0 is the time when $|\boldsymbol{S}_1(t)|$ or $|\boldsymbol{S}_3(t)|$ takes the maximum value in one cycle.

Fig. 2 illustrates two rotation angles, $\xi(t)/2$ and $\zeta(t)$, to express the direction change of principal value in XYZ-coordinates, where XYZ-coordinates are the material coordinates taking X-axis in the direction of SI_{max} and the other two

axes in arbitrary directions. The rotation angle of ξ(t)/2 is the angle between the SI$_{max}$ and SI(t) directions and the rotation angle of ζ(t) is the angle of SI(t) from the Y-axis in X-plane. The two angles of ξ(t)/2 and ζ(t) are written as,

$$\frac{\xi(t)}{2} = \cos^{-1}\left(\frac{S_I(t_0) \cdot S_I(t)}{|S_I(t_0)||S_I(t)|} \right) \qquad \left(0 \le \frac{\xi(t)}{2} \le \frac{\pi}{2} \right) \tag{7}$$

$$\zeta(t) = \tan^{-1}\left(\frac{S_I(t) \cdot e_Z}{S_I(t) \cdot e_Y} \right) \qquad \left(0 \le \zeta(t) \le 2\pi \right) \tag{8}$$

where dots in Eqs 7 and 8 denote the inner product, t_0 is the time to take SI$_{max}$ and e_Y and e_Z are the unit vectors in Y and Z directions, respectively. $S_I(t)$ is a vector of principal value and the subscript i takes 1 or 3, e.g., i takes 3 when $S_{Imax} = |S_3(t_0)|$.
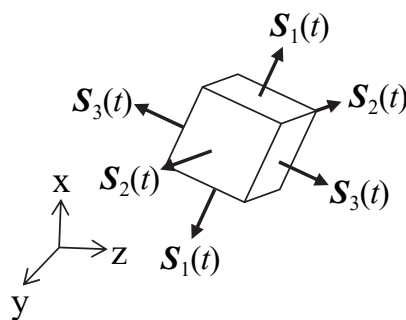

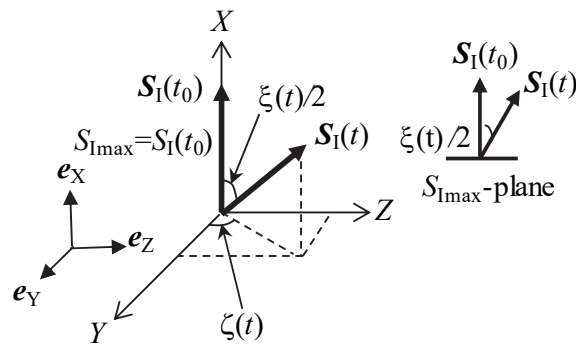Figure 1: Principal stress and strain in xyz coordinates.

Figure 2: Definition of principal stress and strain directions in XYZ coordinates.

Definitions of principal stress and strain in polar Coordinates
Fig. 3 shows the trajectory of $S_I(t)$ in 3D polar coordinates for a cycle where the radius is taken as the magnitude of $S_I(t)$, and the angles of $\xi(t)$ and $\zeta(t)$ are the angles shown in Fig. 2. A new coordinate system is used in Fig. 3 with the three axes of S$_I$^{1}, S$_I$^{2} and S$_I$^{3}, where S$_I$^{1}-axis directs to the direction of $S_I(t_0)$. The rotation angle of $\xi(t)$ has double magnitude compared with that in the specimen shown in Fig. 2 considering the consistency of the angle between the polar coordinates and the physical plane. The figure on the right side in Fig. 3 shows the waveform of loading which is the projection value to path length of $S_I(t)$ projected to S$_I$^{1}-axis. The loading waveform represents the magnitude of the normal strain or stress acting on the plane (S_{Imax}-plane) perpendicular to the direction of $S_I(t_0)$. The maximum range (ΔS_I) and mean value (S_{Imean}) can be obtained from Fig. 3. ΔS_I and S_{mean} are calculated as follows,

$$\Delta S_I = \text{Max}[S_{Imax} - S_I(t)\cos\xi(t)] = S_{Imax} - S_{Imin} \tag{9}$$

$$S_{Imean} = 0.5 \left(S_{Imax} + S_{Imin}\right) \tag{10}$$

As mentioned above, the reference axis ($S_I{}^1$) is the direction of maximum principal stress or strain. In the model, it is also possible to use Tresca and von Mises stress or strain as *S*. By utilizing the IS-method, even complex non-proportional multiaxial cyclic loading can be replaced by simple waveform, similar to uniaxial loading case.
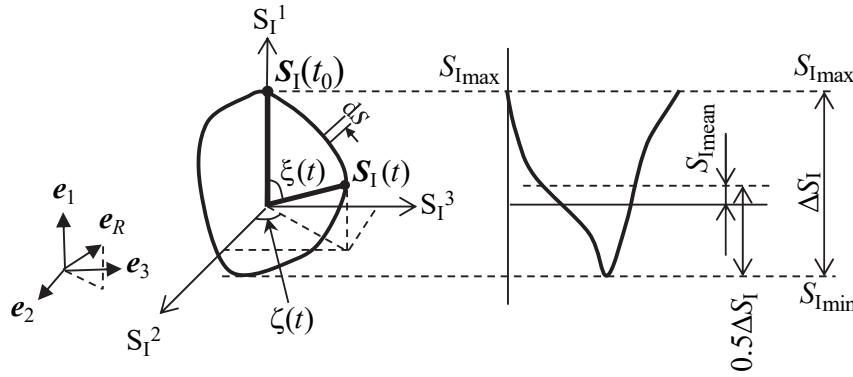

Figure 3: Definition of principal range and mean principal values.

Strain Parameter for Life Evaluation

In order to evaluate the fatigue life of non-proportional multiaxial loading, Itoh et al. have proposed life evaluation equation, $\Delta\varepsilon_{NP}$, taking into account material dependence and strain paths [1-5].

$$\Delta\varepsilon_{NP} = \left(1 + \alpha \, f_{NP}\right)\Delta\varepsilon_I \tag{11}$$

In the equation, $\Delta\varepsilon_I$ is the principal strain range stated previously as ΔS_I and α is a material parameter expressing the amount of additional hardening by non-proportional loading, which is defined as the ratio of stress amplitudes between circular loading and push-pull loading. f_{NP} is the non-proportional factor which expresses the severity of non-proportional loading and is defined as,

$$f_{NP} = \frac{\pi}{2\varepsilon_{Imax} \cdot L_{path}} \int_C \varepsilon_I(t) \left| e_1 \times e_R \right| ds \tag{12}$$

where, e_1 and e_R are the unit vectors of $S_I(t_0)$ ($\varepsilon_I(t_0)$) and $S_I(t)$ ($\varepsilon_I(t)$), respectively as shown in Fig. 3. L_{path} is the full length of loading path, C is the integration of strain paths, d*s* is the increase in the strain path, '×' shows the cross product. f_{NP} totally evaluates the severity of non-proportional loading in a cycle. Therefore, the maximum non-proportionality ($f_{NP}=1$) occurred in circular straining and the minimum non-proportionality ($f_{NP}=0$) in proportional loading.

Evaluation for Random Loading

When loading for life evaluation is random loading, appropriate counting method is required. Some counting methods are already proposed, e.g. peak counting, the level crossing counting, the range-pair counting and the rain-flow counting. However, the method for adapting cycle counting method to multiaxial random loading is not established. Since multiaxial random loading can be converted to equivalent uniaxial loading by using IS-method, the counting methods for uniaxial random loading can be used as they are. Fig. 4 shows loading waveform and non-proportionality before and after using cycle counting method. In the figure, the rain-flow method is used and the equivalent strain ($\varepsilon_I(t)\cos\xi(t)$) is divided into 2 cycles: O-A-BC'-D-E and B-C-C'. Accordingly, non-proportionality ($\varepsilon_I(t)\,|\,e_1\times e_R|$) is split at B and C'. Using strain range and non-proportional factor in each cycle, the strain range for evaluation of failure life under non-proportional loading is calculated by Eq (11). Applicability of this method for life evaluation under non-proportional loading will be discussed further in the future based on the test results.
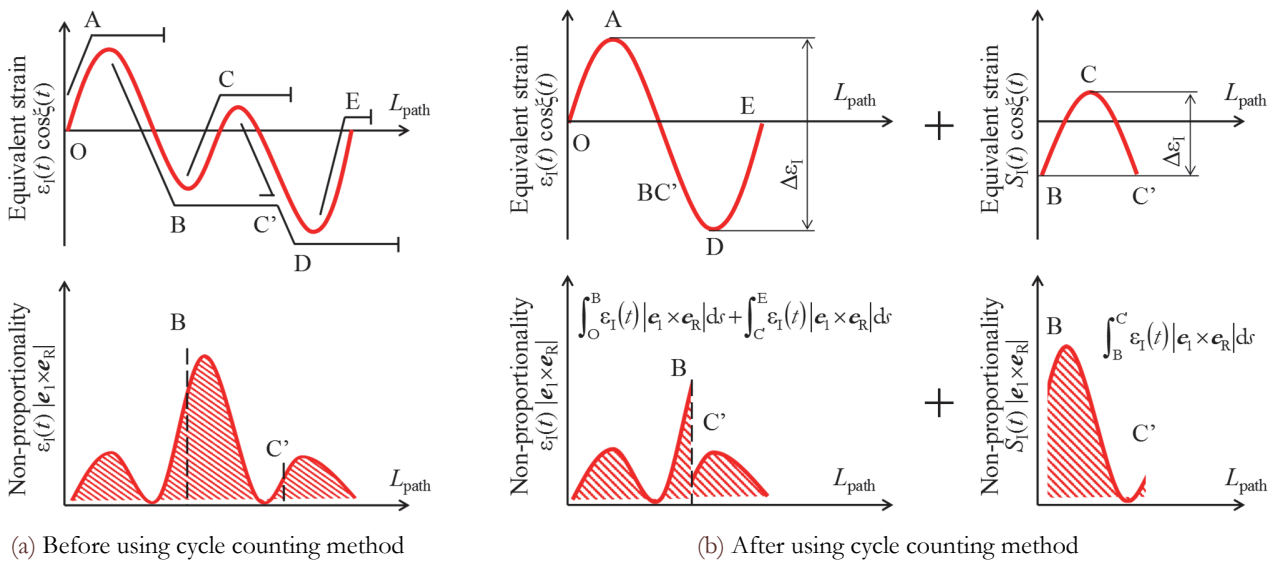
(a) Before using cycle counting method   (b) After using cycle counting method

Figure 4: Random loading before and after using counting method.

VISUALIZATION OF STRESS AND STRAIN PATH AND LIFE EVALUATION

Based on the definition of stress/strain mentioned above, a program for visualization of stress/strain path and life evaluation under non-proportional loading was developed. C# was used as the programming code. Fig. 5 shows a flowchart of the program, that was divided into 8 categories as shown below.

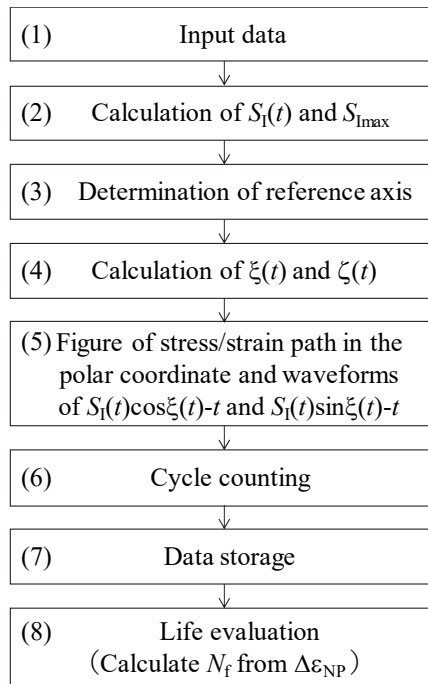| |
|---|
| (1)   Input data |
| (2)   Calculation of $S_I(t)$ and S_{Imax} |
| (3)   Determination of reference axis |
| (4)   Calculation of $\xi(t)$ and $\zeta(t)$ |
| (5) Figure of stress/strain path in the polar coordinate and waveforms of $S_I(t)\cos\xi(t)$-t and $S_I(t)\sin\xi(t)$-t |
| (6)   Cycle counting |
| (7)   Data storage |
| (8)   Life evaluation (Calculate N_f from $\Delta\varepsilon_{NP}$) |

Figure 5: Calculation flow in the program.

(1) Input data

The stress/strain data *vs.* time is put into the program. The input data consist of the 6 components of stress/strain and time at each column. Additional columns such as temperature can be added. Fig. 6 is the table of the input data of non-proportional multiaxial random loading formatted for the program.

	A	B	C	D	E	F	G	H
1	2013/8/12							
2	Ritsumeikan University							
3	Time(s)	temp(D)	Stress(GPa)					
4	0	25	9.93E-05	-2.98E-05	-2.98E-05	0.02231	0	0
5	1	25	0.000298	-8.94E-05	-8.94E-05	0.018782	0	0
6	2	25	0.000894	-0.00027	-0.00027	0.013416	0	0
7	3	25	0.000646	-0.00019	-0.00019	0.009297	0	0
8	4	25	0.000695	-0.00021	-0.00021	0.008597	0	0
9	5	25	0.001986	-0.0006	-0.0006	0.003279	0	0
10	6	25	0.000149	-4.47E-05	-4.47E-05	-0.00109	0	0
11	7	25	0.000646	-0.00019	-0.00019	-0.00452	0	0
12	8	25	0.000646	-0.00019	-0.00019	-0.00482	0	0
13	9	25	0.001044	-0.00031	-0.00031	-0.00666	0	0
14	10	25	0.003175	-0.00095	-0.00095	-0.00651	0	0
15	11	25	0.027627	-0.00829	-0.00829	-0.00448	0	0
16	12	25	0.033739	-0.01012	-0.01012	-0.00477	0	0
17	13	25	0.043925	-0.01318	-0.01318	-0.00322	0	0

Figure 6: Formatted input data for random loading.

(2) Calculation of $S_I(t)$ and S_{Imax}

$S_I(t)$ and S_{Imax} are calculated based on IS-method.

(3) Determination of reference axis

The reference axis for defining $\xi(t)$ and $\zeta(t)$ is determined. Although the determination of reference axis is only one kind in the IS-method, that can select three methods (Method 1, 2, 3) in this program for visualization. In Method 1, the reference axis corresponds to the direction of S_{Imax} as described in section of "*Definition of stress and strain*". In Method 2, the reference axis is determined based on the maximum accumulated damage. Accumulated damage K is estimated using following equation, and the reference axis is determined to be the direction where K takes its maximum value.

$$K = \int_C \left(S_I(t) \cdot \left| \sin \xi(t) \right| \cdot \left| \sin \zeta(t) \right| \right) ds \tag{13}$$

In Method 3, the reference axis can be determined as any direction by users at their request.

(4) Calculation of $\xi(t)$ and $\zeta(t)$

$\xi(t)$ and $\zeta(t)$ are calculated in accordance with the IS-method.

(5) Figure of stress/strain path in the polar coordinate and waveforms of $S_I(t)\cos\xi(t)$-t and $S_I(t)\sin\xi(t)$-t

Based on the above-mentioned calculations and analyses, stress/strain path and waveform are calculated. User can understand magnitude and angular variation quantity of loading in the three-dimensional shape. In addition, reference axis can be changed manually in polar coordinate based on these results. Fig. 7 shows a graph of input stress as a function of time. In the graph, σ_x, σ_y, σ_z, τ_{xy} are presented. The magnitude of σ_y is equal to that of σ_z, and the magnitude of τ_{xy} is equal to that of σ_x, while the phase difference between σ_x and τ_{xy} is 180º. This waveform of input data is 1 block cycle which consists of 40cycles. Fig. 8 shows the waveforms of $S_I(t)\cos\xi(t)$-t and $S_I(t)\sin\xi(t)$-t. $S_I(t)\cos\xi(t)$-t indicates stress/strain waveform, while $S_I(t)\sin\xi(t)$-t indicates the magnitude of non-proportionality on time. It is apparent that the applied stress path has high non-proportionality.

(6) Cycle counting (In the case of random loading)

In the case of random loading, waveform of stress/strain is analyzed using the rain-flow method, a kind of cycle counting method. It should be noted that damage has to be evaluated by a cumulative damage rule, i.e. Miner's rule. Stress/strain waveform under non-proportional loading was already reduced to the simple waveform through categories (1)−(5). Therefore, the cycle counting and cumulative damage rule used in uniaxial loading can be applied to non-proportional loading.

(7) Data storage

In this step, figures and tables can be graphed and saved to memory. From the figures, users can understand stress/strain range, multiaxiality and non-proportionality of stress/strain state simultaneously.

(8) Life evaluation (Calculate N_f from $\Delta\varepsilon_{NP}$)

Based on the results from categories (1)−(7), life evaluation can be performed. In the life evaluation, Manson-Coffin equation (Eq. (14)) obtained from uniaxial proportional loading fatigue test and the IS-method are employed with the rain-flow counting method.

$$\Delta\varepsilon_{NP} = A N_f^{-0.12} + B N_f^{-0.6} \tag{14}$$

The result of life evaluation is also presented in Fig. 8. In the right side of Fig. 8, the name of material used and parameters A and B in Manson-Coffin are inputted. By comparing Eq. (11) and Eq. (14), Newton method for nonlinear equation was applied for life evaluation. Although the detail of the method is omitted here, the solution of Eq. (14) was obtained within 5 times of repeated calculations. In this study, the fatigue life obtained by above calculations was 21136 blocks. In the future study, the result of life estimation will be compared with experimental results. In the present paper, visualization and life estimation method was established based on IS-method and it was applied for the non-proportional multiaxial random fatigue loading. The method enables the calculation of stress and strain states under non-proportional multiaxial random loading and realizes life estimation under highly non-proportional random fatigue loading.
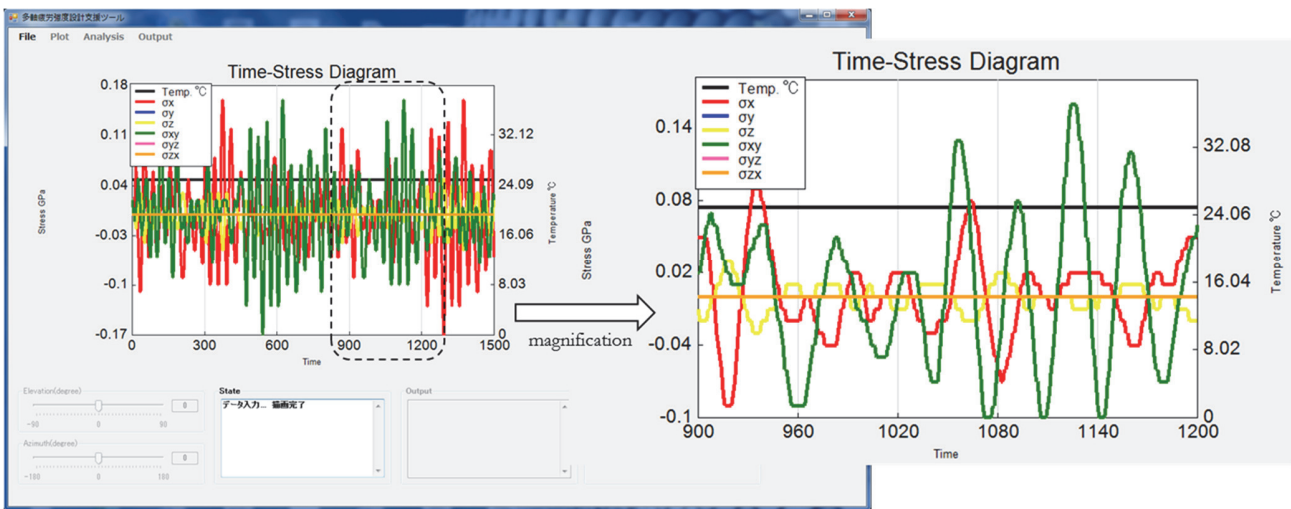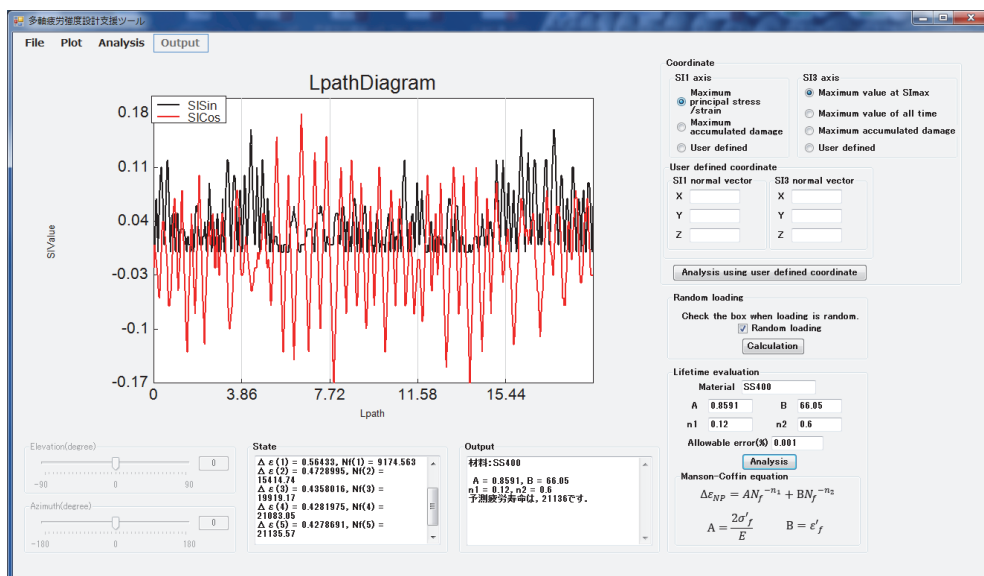

Figure 7: Stress waveforms of input data.

Figure 8: Output data and life evaluation result.

CONCLUSIONS

A software to visually indicate the stress and strain states, the non-proportionality of loading and life evaluation was developed based on the IS-method. It was applied for the life evaluation under the multiaxial random fatigue loading, and the theoretical life was estimated based on Manson-Coffin equation. The software enables researchers and engineers without specific knowledge on multiaxial fatigue loading to assess fatigue strength under non-proportional multiaxial fatigue condition.

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