



Combined simulation of fatigue crack nucleation and propagation based on a damage indicator

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ABSTRACT. Fatigue considerations often distinguish between fatigue crack nucleation and fatigue crack propagation. The current work presents a modeling approach utilizing one Fatigue Damage Indicator to treat both in a unified way. The approach is implemented within the framework of the Finite Element Method. Multiaxial critical plane models with an extended damage accumulation are employed as Fatigue Indicators. Locations of fatigue crack emergence are predicted by these indicators and material degradation is utilized to model local material failure. The cyclic loading is continued on the now degraded structure and the next location prone to material failure is identified and degradation modeled. This way, fatigue crack propagation is represented by an evolving spatial zone of material failure. This propagating damage zone leads to a changing structural response of the pristine structure. By recourse to the Fatigue Damage Indicator a correlation between the number of applied load cycles and the changing structural behavior is established. Finally, the proposed approach is exemplified by cyclic bending experiments in the Low Cycle Fatigue regime.

KEYWORDS. Fatigue Crack Nucleation; Fatigue Crack Propagation; Damage Indicator; Finite Element Method.



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INTRODUCTION

Fatigue failure and the mechanism leading to fatigue crack nucleation and propagation are of great concern in today's design of engineering components. There are two groups of modeling concepts towards fatigue failure in materials and structures. First the classical fatigue laws of Basquin [1] and Coffin-Manson [2,3] to predict crack nucleation, i.e. the number of cycles till some detectable crack emerges. To account for multiaxial loading conditions more complex models like critical plane approaches are utilized, e.g. Fatigue Indicator Parameters (FIP) [4]. Second, methods treating the spatial advance of a failed region like fracture mechanics concepts as the Paris law [5,6] for crack propagation or Continuum Damage Mechanics based models for material degradation under cyclic loading.

Various approaches have been proposed to merge these two considerations in one unified or generalized concept. One way to do so is the generalization of crack propagation laws. Different fatigue laws [7-12] have been proposed to find a relation between the Paris law and the fatigue laws of Basquin and Coffin-Manson.

Another approach is the combination of fatigue considerations with continuum damage mechanics [13,14] or crack propagation modeling [15]. Such attempts are often suited in the field of numerical simulations especially in the framework of the Finite Element Method (FEM). Fatigue Indicators predict crack nucleation followed by the modeling of crack propagation realized, by crack modeling or material degradation. This way, a continuous simulation of the entire fatigue failure process is obtained.

The present paper studies the estimation of the structural response due to the degradation of ductile materials. The focus is set on reversed cyclic plasticity, which is characterized by steady plastic strains from cycle to cycle with no net accumulation of directional plastic strains. This is the typical case for the Low Cycle Fatigue (LCF) regime.

A two level approach is implemented within the framework of the Finite Element Method. Cyclic loading on the structural level induces time varying multiaxial stress and strain states at the material level which, of course, are location dependent. FEM simulations are utilized to compute the local constitutive response of the material points. A critical plane method is employed at all considered material points to identify the location most prone for material failure. There, crack emergence in the structure is modeled by material degradation in the region of the critical material points. Crack propagation is obtained by repetitive application of the approach which results in an evolving spatial zone of material failure. Consequently, a changing structural behavior is modeled by this propagating damage zone.

METHODOLOGY

A computational methodology is set up in which a Fatigue Indicator Parameter (FIP) is utilized to predict crack nucleation. The same indicator is also employed to model "crack propagation" in the sense of a spatial evolving region of material failure. Depending on the fatigue damage behavior of the considered material different indicators can be appropriate [17, 18].

FATIGUE CRACK NUCLEATION

The Fatemi-Socie [19] Fatigue Indicator is employed in the present work which is typically used for ductile materials. This shear strain based critical plane method predicts fatigue crack nucleation and is given by,

$$\dot{\rho}_{FS} = \frac{\Delta\gamma}{2} \left(1 + k \frac{\sigma_{n,\max}}{\sigma_y} \right) \quad (1)$$

The parameters governing crack nucleation are the shear strain amplitude, $\Delta\gamma/2$, occurring on a particular plane and the maximum normal stress in a cycle, $\sigma_{n,\max}$, acting on that plane. σ_y is the yield stress and k is a factor defining the influence of $\sigma_{n,\max}$. The critical plane is identified as the plane experiencing the maximum value of the Fatigue Indicator. In case of non-proportional loading the determination of the shear strain amplitude usually requires numerical search



algorithm, which makes the identification of the FIP computational expensive. For lifetime estimations in the LCF regime the maximum FIP can be expressed as,

$$\dot{p}_{FS,max} = \gamma_f (2N_f)^c \tag{2}$$

with the number of cycles to crack emergence, N_f , and the material parameters γ_f and c . The parameters in Eqs 1 and 2 are taken for a stabilized cycle. The latter is identified by assessing the relative change of the dissipated strain energy density,

$$\delta w_{i,i-1} = \frac{\Delta w_i - \Delta w_{i-1}}{\Delta w_{i-1}} \tag{3}$$

between two consecutive cycles ($i-1$) and (i). Δw_i represents the dissipated strain energy in cycle (i) and Δw_{i-1} for cycle ($i-1$), respectively. A stabilized cycle is accepted when a defined tolerance value in a considered region is satisfied.

FATIGUE CRACK EMERGENCE

From Eq. 2 the number of cycles to crack emergence, N_f , is estimated. At the location with the lowest N_f material degradation is introduced in a small, flat region aligned with the critical plane. In this region the elastic stiffness is immediately decreased by several orders of magnitude. This way, material failure is modeled, representing an initial “crack” inside the structure.

FATIGUE CRACK PROPAGATION

The cyclic loading is continued on the now degraded structure. This leads to a change of the multiaxial stress and strain states in undamaged material points. Consequently, the FIPs are changing, too, and their new contribution to crack nucleation has to be evaluated in the next stabilized cycle. Therefore, an accumulation of the FIP for all previously identified stabilized cycles has to be done, of course for all material points, at each plane. The Palmgren-Miner [20, 21] linear damage accumulation rule,

$$\sum_i \frac{n_i}{N_i} \leq 1 \tag{4}$$

is employed, where n_i , is the number of cycles contributing to crack nucleation. N_i is the number of cycles to crack emergence, summed over i corresponding FIP magnitudes. Material points with the lowest remaining number of cycles to material degradation, n_i , are identified and selected to fail in the same way as crack emergence is modeled.

The changing stress and strain states due to the changing structural behavior lead to increasing fatigue indicators in the vicinity of failed material and, consequently, to a higher contribution to the crack nucleation process. Hence, further material is predicted to fail ahead the damaged area and “fatigue crack growth” is represented by an evolving spatial zone of material failure. Additionally, the number of applied load cycles is correlated to the changing structural behavior caused by the evolving damage zone.

IMPLEMENTATION

The described procedure is implemented within the framework of the Finite Element Method (FEM) to simulate the nonlinear structural fatigue behavior. Python scripts have been developed in [16] and are utilized for the FIP computations. The repetitive nature of the approach leads to an alternating utilization of FEM simulations and

FIP computations. Cyclic loading is applied in the FEM model and the simulation is halted after a stabilized cycle is found.

Regions for material degradation modeling are assumed to have a finite size of a disc like shape. This discs are aligned with the critical plane of the critical material point determined by the FIP. Elements intercepting this plane around the critical material point by a diameter of $2\Delta a$ are selected to fail. Due to this element based selection method averaged element values of the stress and strain states are utilized for the FIP computation.

The cyclic loading is continued and the next stabilized cycle is searched. This alternating computations are continued until structural degradation has advanced to a specified stopping criterion.

APPLICATION

The approach is exemplified by simulation of micrometer sized cyclic bending experiments in the LCF regime, see [22]. A 3D-FEM model of the Copper cantilever beam is shown in Fig. 1. The bending sample has a cross-section of $23 \times 23 \mu\text{m}^2$ and a length of $260 \mu\text{m}$. Linear interpolated, fully integrated, eight noded, continuum elements are used for discretization. Displacement boundary conditions are used for fixation and cyclic loading. Harmonic loading with an amplitude of $19.5 \mu\text{m}$ is enforced at the neutral axis of the beam. An elastic-plastic J2 constitutive model with kinematic hardening is used to model the material behavior of Copper. The elastic and plastic material data is summarized in Tab. 1.

The fatigue model is calibrated by recourse to the experimental data from [22], to identify material parameters as needed in Eq. 2 for lifetime estimation. According to this data and neglecting the cyclic hardening behavior, a first significant change in the structural response is detectable at $N_f = 1000$ load cycles. The unified approach is calibrated in such a way that crack emergence will occur at this number of load cycles.

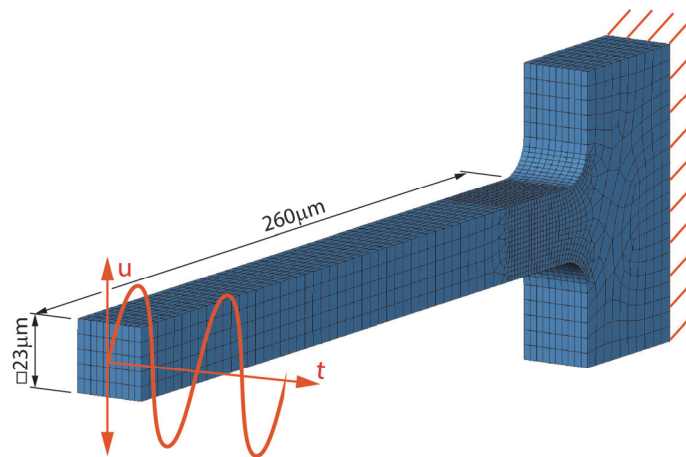


Figure 1: 3D FEM-Model of cyclic bending experiment of micrometer sized copper cantilever beam

Material	Young's modulus (GPa)	Poisson's ratio	Yield stress (MPa)	Tangent modulus (MPa)
Cu	102	0.35	104.4	3900

Table 1: Elastic and plastic properties of Cu [16]

Therefore, the first stabilized cycle is identified by the FEM simulation and a maximum FIP, $p_{FS,max} = 0.025573$, computed. Due to the few existing experimental data only this single data point is available for the lifetime estimation. Therefore, the material parameter $c = -0.7$ is assumed and finally $\gamma_f = 5.2300118$ is obtained from Eq. 2.

The Fatigue Indicator Parameter (FIP) as discussed above are computed for a stabilized cycle. The latter is accepted when the relative change of the dissipated strain energy density, Eq. 3, satisfies the criterion: $\delta w_{i,i-1} \leq 0.0001$. The dissipated



strain energy density in a cycle is computed for the region of plastic deformation in the cantilever beam. Therefore, individual material points may not completely reach the defined criterion. This is accepted to reduce the number of load cycles in the FEM simulation. The disc shape regions for material degradation modeling are defined to have a diameter, $2\Delta a$, of four times the average element length in the refined region.

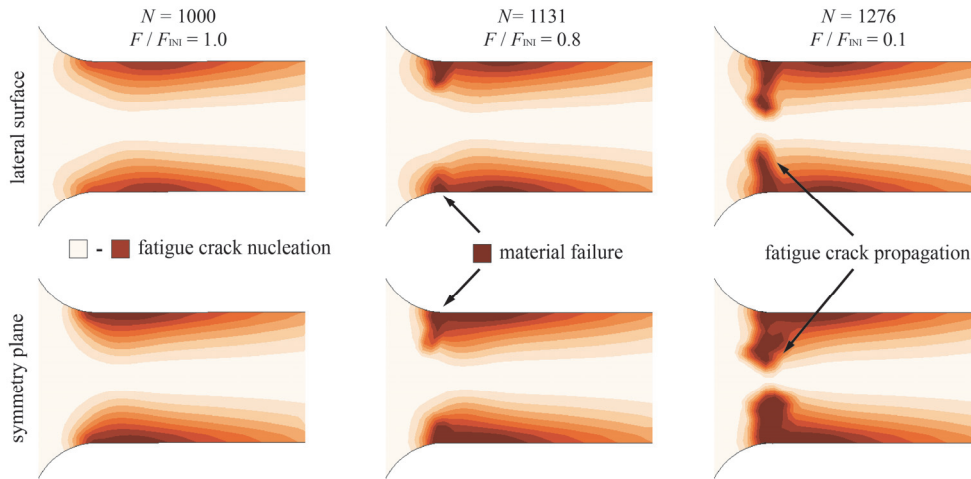


Figure 2: Fatigue damage accumulation resulting in material degradation is indicated by contour plots on the lateral surface and the symmetry plane of the beam. Left, the FIP after the first stabilized cycle is indicating crack emergence. Rightwards, the spatial evolution of material failure is visualized for an increasing number of load cycles.

RESULTS

Figure 2 presents the fatigue damage process. The Fatigue Indicator, Eq. 1, is depicted by contour plots, in the top row for the lateral surface and in the lower row for the symmetry plane of the beam. The fatigue damage accumulation is depicted after three different numbers of load cycles. Dark regions indicate an advanced damage accumulation. Left, for $N = 1000$ load cycles the FIP in the first stabilized cycle is displayed. This image represents the undamaged structure indicating locations prone for crack emergence. The elements most prone to fail are located at the symmetry plane at the end of the shoulder. Here, material failure is first introduced into the structure. The spatial evolution of this material degradation is depicted after $N = 1131$ load cycles. The structural change due to the evolving damage zone is indicated by the decreasing reaction force F in relation to the initial one F_{INI} in the first stabilized cycle. The simulation is continued until the reaction force decreased under ten percent of the initial one. This criterion is fulfilled after $N = 1276$ load cycles are reached.

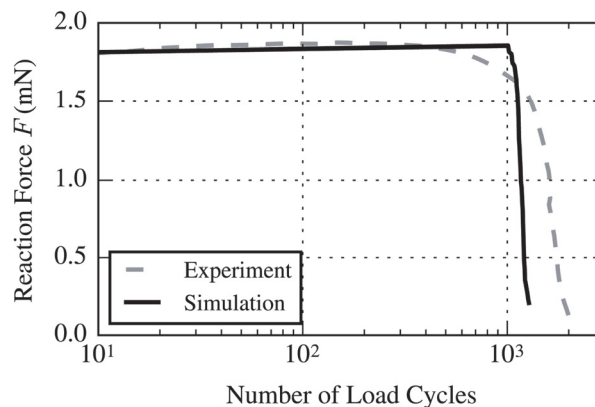


Figure 3: Structural degradation indicated by the decreasing reaction force compared to the number of applied load cycles – Simulation vs. Experiment [22].



The corresponding spatial material failure is depicted in the right images. The damage zone is evolving from the location of crack emergence towards the neutral axis and is decreasing from the midplane to the outside of the beam. The decreasing reaction force versus the number of load cycles is displayed in Fig. 3. The experimental results [22] are compared to the numerical simulation. After one thousand load cycles material failure modeling starts and a degrading structural behavior is obtained. Compared to the experimental observation the degradation in the simulation is progressing more quickly than the degradation of the test specimen.

CONCLUSION

A fatigue damage modeling approach capable of predicting fatigue crack nucleation and propagation in one combined numerical simulation has been presented. A multiaxial critical plane concept combined with damage accumulation is utilized as Fatigue Damage Indicator to identify locations of crack emergence. Local material degradation is modeled in the pristine structure and the cyclic loading is continued until the next occurrence of material failure is detected. Applying this procedure in a repetitive way, an evolving spatial zone of material failure is simulated, representing fatigue crack propagation. This way, a changing structural response of the initial perfect structure with respect to the number of applied load cycles is obtained. The combination of fatigue crack nucleation and crack propagation has been successfully exemplified in a numerical FEM analysis of a cyclic loaded cantilever beam.

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