Focussed on Multiaxial Fatigue and Fracture

Systematic validation of experimental data usable for verifying the multiaxial fatigue prediction methods

J. Papuga
Ú12105 FME CTU in Prague, Technická 4, 166 07 Prague 6, Czech Republic
papuga@pragtic.com, http://orcid.org/0000-0001-9569-4997
Evektor, spol. s r.o., Letecká 1008, 686 04 Kunovice, Czech Republic
jpapuga@evektor.cz

S. Parma
Evektor, spol. s r.o., Letecká 1008, 686 04 Kunovice, Czech Republic
origon@seznam.cz

M. Růžička
Ú12105 FME CTU in Prague, Technická 4, 166 07 Prague 6, Czech Republic
milan.ruzicka@fs.cvut.cz

ABSTRACT. The paper discusses some of the issues, the researchers interested in verifying various multiaxial fatigue limit estimation solutions are facing to. Even recently, newly proposed criteria have been or are tested on dozens of experimental inputs. Papuga in [1] pointed out, that applicability of the most often used test batch is limited and only half of these data items is worth using for such purposes. This paper extends that analysis by describing the weak points of various data sets used in this domain for validating new proposals on multiaxial fatigue limit estimates. The conclusion from the extensive analysis is that the researchers should adopt other test sets only if they very well know their background.

KEYWORDS. Multiaxial fatigue; multiaxial fatigue limit; fatigue prediction; fatigue prediction validation; multiaxial fatigue limit experiment.

Citation: Papuga, J. Parma, S., Růžička, M., Systematic validation of experimental data usable for verifying the multiaxial fatigue prediction methods, Frattura ed Integrità Strutturale, 38 (2016) 106-113.

Received: 27.04.2016
Accepted: 20.06.2016
Published: 01.10.2016

INTRODUCTION

When Papuga wrote the paper [1] on various multiaxial fatigue limit estimation methods and their validation, he referred to a set of 407 experimental items on various materials and obtained from different papers. At that moment, he cared little about the quality of the data obtained in that way. The reason for including a data item into the validation set was a simple stating of the experimenter or of any other researcher, that the data items are usable for such a validation.

Already during preparation of that data set, some bad practice or misleading data interpretation was uncovered and discussed in [1]. Anyhow, only a proper analysis [4] of the most often cited and reused data set defined by Papadopoulos et al. [2] and of other often used data items from the paper by Nishihara and Kawamoto [3] made a start point for an extensive research on evaluating the data quality.

METHOD OF EVALUATION

This paper further extends the analyses provided in [4] by updating them and enlarging the scope of the data check. Due to the multitude of papers available for validation, it can be in no way complete, and some other such papers can be found. On the other hand, it is covering now all data sources found, which were used by the cited authors in their effort to validate new or older multiaxial fatigue limit estimation methods, and which were used for such a purpose by at least one researcher not related to the team of experimenters. Some newer sets from fresh experiment campaigns were reported meantime, but the scope of a conference paper cannot give a thorough overview.

Data presented here are quickly touched and only their weak points are commented. The experiments are very valuable, their realization took a lot of time, a lot of work and a lot of funds. Anyhow, only short evaluation can get here. Thus the sets are criticized when necessary, so that the other researchers were aware of potential problems if accepting them, while the detailed description or appraisal cannot be provided here.

The measure of acceptability of experimental results for validation purposes has been already discussed in [4]. ASTM [5] e.g. recommends between 6 and 12 specimens to be used for an S-N curve applicable for an exploratory research. Papuga in [4] proposed a system of weights, where 8 and more finished experiments per the S-N curve result in a full weight (1.0) of the item, while the weight decreases linearly to 0.2 for the case of 4 specimens per the S-N curve, and experiments with fewer test points are simply discarded. In this paper, we do not evaluate the appropriateness to the validation in another way than by the check if the S-N curve is described with minimum 4 specimens. The question of the data dispersion, applicability of experiments tested on one load level only, of the importance of the basic material curves in fully reversed push-pull and fully reversed torsion, the comparison between the statistical representativeness of the fatigue limit derived from an S-N curve and from the staircase analyses are only a few of the questions that should be further raised, but cannot be discussed in such a short paper.

The basic reference for the next evaluation is Tab. 1, where the individual papers describing validation of some multiaxial fatigue limit estimation method(s) are referred to, and the data sets used for this purpose are marked by “x“ symbol. The papers often just reuse data from a second-hand source, so the fact that some paper uses Nishihara‘s and Kawamoto’s data [3] need not mean that there are not error in interpreting it or in the transcription. The table also does not describe, if the authors referring to a particular data set used in a complete form or if they selected only its parts.

The individual test sets described in Tab. 1 are evaluated in the next section including also the test set marks extensively used in FatLim and Finliv databases [6-7]. Due to the limited space available for Tab. 1, these marks could not be preserved there as well, but they can be deduced from the context.

DATA SETS

Nishihara and Kawamoto (NKc, NKd, NKh, NKm - [3])

The fact that the data items of this data set on fatigue limits obtained for hard steel, mild steel, cast iron and duralumin are the 1st, 5th, 8th and 9th as regards the frequency of their use shows its importance in validating of the multiaxial fatigue limit estimation methods. The careful analysis of the set in [4] anyhow uncovers that only two S-N curves obtained for the hard steel material can be seen as applicable, if accepting the low coefficient of determination $R^2 < 0.75$ obtained during the regression analysis of data by the Basquin formula. For the other materials, either the material curves (mild steel, cast iron) or the multiaxial experiments (duralumin) dispose of too few specimens, in some cases limited even to only one finished and one unfinished experiment. Though the set is so broadly used for validation practice, the analysis of the original paper shows that applicability of so poorly documented S-N curves is inadequate.

Paper by		McDiarmid	Papadopoulos	Stefanov	Papadopoulos et al.	Carpinteri, Spagnoli	Mamiya, Araújo	Banvillet et al.	Cruz, Zouain	Goncalves et al.	Goncalves et al.	Liu, Mahadevan	Ninic	Ninic, Stark	Braccesi etal	Papuga, Růžička	Shariyat	Li, Reis, Freitas	Vu et al.	Zhang, Yao	Carpinteri et al.	Papuga	Kenmeugne et al.	Augustins	Carpinteri et al.	Matsubara, Nishio	Araújo et al.	Golos, Debski et al.	Bruun, Härkegård	Number of occurrences
Year		1987	1994	1995	1997	2001	2002	2003	2003	2004	2005	2005	2006	2007	2008	2009	2009	2009	2010	2010	2011	2011	2012	2013	2013	2013	2014	2014	2015	
Material	Ref	[9]	[2]	[10]	[2]	[40]	[41]	[42]	[43]	[44]	[45]	[46]	[47]	[48]	[49]	[50]	[51]	[52]	[20]	[53]	[54]	[1]	[55]	[56]	[31]	[57]	[21]	[58]	[26]	
cast iron	[3]	x				x							x	x							x	x			x				x	8
duralumin	[3]	x												x							x	x			x					5
hard steel	[3]	x	x		x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x			x	x	x	x	x	24
mild steel	[3]	x				x			x				x	x		x			x	x	x	x			x			x	x	13
GGG-40	[8]	x																												1
GTS-45	[8]	x		x																										2
30NCD16	[11,12]		x		x		x	x		x	x	x		x	x	x	x	x	x	x	x	x	x		x	x		x	x	21
34Cr4	[14]		x		x		x		x	x	x	x			x	x	x	x	x	x		x	x			x	x		x	18
42CrMo4V	[15]				x		x		x		x	x			x	x	x		x	x		x				x		x	x	14
25CrMo4	[17]						x										x	x	x			x	x	x		x	x		x	10
FGS700/2	[22]							x											x			x							x	4
30NCD16	[11,12]							x								x			x		x	x	x		x			x	x	9
XC18	[23]							x											x			x							x	4
6082-T6	[38]												x	x																2
76S-T6	[25]													x							x	x			x				x	5
steels and cast irons	[27,28]													x								x							x	3
0.35%C steel	[30]													x							x				x					3
S65A	[29]															x			x			x							x	4
42CrMo4V	[32]																x										x			2
St35	[32]																x										x			2
St35	[33]																x					x		x		x			x	5
25CrMo4	[19]																		x								x			2
34CrMo4	[34]																					x							x	2
Ck 35	[34]																					x							x	2
St60	[35]																					x							x	2
34Cr4	[14]																					x		x					x	3
34Cr4	[14]																					x							x	2
XC48	[39]																					x	x	x						3
25CrMo4	[18]																					x					x		x	3
cast iron	[36]																							x						1
En24T	[37]																										x			1
GG30	[34]																												x	1

Table 1: List of papers referring to individual test sets.

Neugebauer (NbA, NbB - [8])

The research report of Neugebauer on GGG-40 and GTS-45 was extracted by McDiarmid [9] and later by Stefanov [10] to support their findings. The data are reported in graphs of the S-N curves only (mostly two load levels up to 6-13 data points in total) and have to be derived by an image analysis. The weak point of the test set is hidden in the text [8], where Neugebauer notes that all pure torsion experiments were run with deformation control. The set thus mixes outputs of these deformation-controlled experiments with the common load-controlled experiments for all other load variants.

Froustey and Lasserre (FLA, FLB – [11], [12])

The test sets have been already discussed in [4], and only the major conclusions are provided here. Froustey and Lasserre ran the tests on three different lots of 30NCD16, while only one of these lots (used for FLB test set here) is complete as regards the uniaxial experiments, while the FLA test set lacks such material data. This is probably the reason, why Dubar [13] later attempted to prepare a unified test set on 30NCD16 by weighting the results of individual batches in an undefined way, while thus confusing other researchers by seeming wholly new set.

Heidenreich, Zenner et al. (HeG, Hei, HRZ, HZ – [14], [15])

The authors repeatedly published results of their ongoing experimental campaign mainly on three different lots of 34Cr4 steel. They also reported tests on GGG-60 cast iron (HeG), but these are not anyhow present in Tab. 1. The test set has been already commented in [4] as being acceptable for validation purposes, and only some individual curves should be sorted out (the authors used lower numbers of specimens for the staircase method). The only unacceptable exception is the HZ test set, where no reversed torsion experiments were run, and the test set is not thus applicable to testing for most of the fatigue limit estimation criteria.

Lempp (Lem – [16])

Lempp's tests on 42CrMo4V steel are commonly reused for validation purposes thanks to Papadopoulos' test set [2], and thus they have been already evaluated in [4]. The originally reported fatigue limit estimates are mostly based on extrapolations to cycle counts far exceeding the last finished experiment. Even if the number of cycles is decreased to define another section point for fatigue strength determination, other issues become obvious:

- Lempp describes pronounced anisotropy of the material (83% of fatigue strength of specimens oriented in the transverse direction compared to longitudinal ones).
- The alternate torsion fatigue curve is defined by 7 data points, but the scatter of data is so high, that it leads to the coefficient of determination $R^2 = 0.624$.
- The test case of repeated axial loading is not covered at all. The prediction methods including also the fatigue limit in repeated axial loading cannot be validated by this test set.

Mielke (Mie – [17]), Troost, Akin and Klubberg (TAK – [18]), Kaniut (Kan – [19])

Mielke's set of fatigue limits on hardened 25CrMo4 steel tested by the staircase method is also often reproduced in various validation campaigns. Its weak point can be found in the intrinsic unequal size effect involved in various load combinations. The push-pull tests were realized on tiny bars of 1.9 mm diameter, while the torsion test sets on hollow specimens with outer diameter 20 mm and 2 mm wall thickness and the multiaxial tests on hollow specimens with 34 mm outer diameter and 1 mm wall thickness.

From further analysis of reports, it can be found that Troost, Akin and Klubberg [18] continued on tests on the same batch of material, thus forming the TAK set. They tested both specimen size configurations and nicely confirmed the size effect. Thanks to that, they results can be combined with Mielke's multiaxial data, so that Mielke's set could be used for validation purposes.

Kaniut studied at the same university, and he reported experiments realized on the same material with identical material parameters. It can be deduced so that he also worked on the same lot of 25CrMo4. His main focus was nevertheless set to determining the fatigue limit values for two-channel loading with unequal frequencies applied in each load channel. Because he used the staircase method for obtaining the fatigue limits, there are no material curves available for fatigue damage accumulation, while the unequal frequencies on individual load channels are likely to form more than one load cycle per a complete period of the total load cycle. Papuga highlighted this point in [1], and decided not to include similar load conditions into the validation test set, but other researchers [20-21] do not find such reasoning important and work with this test set as well.

Bennebach (Ben – [22]), Palin-Luc (PaL – [23])

As usually in the case of tests realized at ENSAM in Bordeaux, staircase philosophy was adopted for fatigue limit determination for these both cases. Because the first international publication of the test set was found in the paper by Morel and Palin-Luc [24], test sets were originally marked MPC and MPB in [1].

Bennebach's PhD thesis [22] allows the reader to define just two fatigue limits. Such a low number differs from the MPC set, where four items are available. One more fatigue limit can be traced back to Palin-Luc [23], where unfortunately no information is present, whether the same lot of material was used. The origin of the last item in MPC test set has not been found yet.

Findley (Fin – [25])

Papuga reported in [1] this test set three times, because he defined three different S-N curve section cuts at 10^6, 10^7 and 10^8 cycles in order to increase the number of experiments realized on aluminum alloys. Though the same strategy was accepted e.g. in [26], this simple multiplication of the same data set is not consequent, because it was not adopted while processing the other data sets.

Gough and Pollard (GPx – [27-28]), Gough (Ggh – [29])

Gough and Pollard published in the 30th of the 20th century several papers (see e.g. [27-28]) describing results of their multiaxial tests on bar specimens manufactured from various steels and cast irons. The combination of plane bending and torsion without any phase shift and without any mean stress can be found in all data sets. Apparently, the focus of their effort was mostly devoted to covering of various materials extensively, than to determining the fatigue limits precisely. Therefore, many of these experiments do not conform to the lowest standards defined here before, and big scatter of data in some curves can be found in many cases. As a result of such practice, one half of the defined multiaxial fatigue limit items could be included into the validation data set at maximum.

After the WWII, Gough published further papers (e.g. [29]), mostly devoted to more complex testing on S65A steel. Nevertheless, no information on the way the reported fatigue limits were set can be found in any of his papers from that period, and only his personal classification describing the reliability of the outputs is available.

Rotvel (Rot – [30])

Rotvel's data were used for a validation study even recently [31], though Papuga wrote in [1], that the experiments miss the case of pure torsion loading. This value has to be estimated, if the test set should be admitted for validation purposes, and this is apparently the method of determination used in [31].

Bhongbhibhat (BhO, BhA – [32])

These tests realized on specimens on large hollow specimens manufactured from 42CrMo4 (BhO) and St35 (BhA) are affected by substantially different cross-sections of specimens used for uniaxial and multiaxial fatigue tests. Additionally, the lifetime regions of individual curves in these data groups differ so substantially, that only small intersections can be found.

Issler (Iss – [33])

Issler's data set suffer from the doubtful way the fatigue limits were determined. No clue is provided by the author, but based on an analysis of his results, it seems that he either used the section cut at 2 000 000 cycles or the stress level of the last finished experiment with the longest lifetime. Section cuts at lower number of cycles are recommended to avoid potential extrapolations.

The S-N curves for multiaxial load combinations are described by graphs with data points only. Unfortunately, the uniaxial fatigue curves are not present at all, and the fatigue limits derived by the author are provided solely. Because of the way the fatigue limits were determined in other cases, it cannot be anticipated how precise these values are.

Baier (BaB, Bai, BaG – [34])

The data on both steels (BaB – Ck35, Bai – 34CrMo4) are very interesting for validation purposes. The tests were realized as S-N curve tests on sufficient numbers of hollow specimens in most cases. Anyhow, the tests on GG30 cast iron (BaG test set), for which any fully reversed torsion tests are missing, should not be used.

El-Magd and Mielke (EMM – [35])

The authors did not run any reversed torsion fatigue tests, and the later transcription of this set seems to be accompanied by some kind of an estimate of this missing value. Any more information about this test set, or other papers referring to these experiments would be welcomed.

Roš and Eichinger (RE – [36])

Though the data set of these two authors is very huge, and relates to many various materials, there is no single note on the way the fatigue limits were derived. The number of specimens used to determine them can only be guessed. No fully reversed torsion sets are reported.

McDiarmid (McD – [37])

The data do not refer to any fatigue experiments realized in pure torsion mode.

CONCLUSION

If comparing the content of Tab. 1 with the previous subsections, it can be concluded, that only a few from the commonly used data sets can be used for validation purposes without any hesitation. Mostly, two major reasons can be found for rejecting a particular test set from the validation:

1. There are not enough experiments in order to adequately define the fatigue limits, or the statistical parameters of the regression outputs are not representative enough.
2. The torsion load case is not covered at all. The authors of the present paper understand such deficiency as too substantial, because they define the fatigue limit in fully reversed torsion as a standard input parameter. Another potential mode of validation could be to define the weight parameters of individual stress parameters from the least square error analysis realized on the complete test set. According to the authors, such solution is not adequate, because it uses the weight coefficients derived from the best fit analysis. Such strategy can be rarely used in engineering practice, because only minimum information is often available in these cases. It is thus reasonable to persuade the engineering audience about validity of tested prediction methods, while using standardized inputs of material parameters, and to convince it to ensure availability of the fully reversed fatigue limits in axial and torsion loadings at least.

The authors of new criteria should be aware of various problematic issues of the described test sets. They should carefully analyze the reasons to include the particular data sets into their validation sets.

ACKNOWLEDGEMENTS

The authors thank to the Czech Science Foundation and to the Technology Agency of the Czech Republic for the support of their work in preparing this paper by the grant projects No. GA15-18274S and TA01011274 respectively.

REFERENCES

[1] Papuga, J., A survey on evaluating the fatigue limit under multiaxial loading, Int. Jnl. of Fatigue, 33 (2011) 153-165.
[2] Papadopoulos, I.V., Davoli, P., Gorla, C., Filippini, M. Bernasconi, A., A comparative study of multiaxial high-cycle fatigue criteria for metals, Int. Jnl. of Fatigue, 19 (1997) 219-235.
[3] Nishihara, T., Kawamoto, M., The strength of metals under combined alternating bending and torsion with phase difference, Mem. College Engng., Kyoto Imperial University, 11 (1945) 85-112.
[4] Papuga, J., Quest for Fatigue Limit Prediction Under Multiaxial Loading, Procedia Engineering, 66 (2013) 587-597.
[5] Anon.. Standard Practice for Statistical Analysis of Linear or Linearized Stress-Life (S-N) and Strain-Life (e-N) Fatigue Data, [E 739 – 91, Reapproved 2004], ASTM International, West Conshohocken, (2004).
[6] Papuga, J., New technologies for sharing research data and costs, Procedia Engineering, 2 (2010) 855-864.

[7] Papuga, J., Parma, S., Lutovinov, M., Růžička, M., Building a Foundation for Benchmarks for Fatigue Prediction Methods – The FinLiv Database. Procedia Engineering, 101 (2015) 372-379.

[8] Neugebauer, J., Zum Schwingfestigkeitsverhalten von Gusswerkstoffen unter mehrachsiger, frequenzverschiedener Beanspruchung, [Technical report Report FB-175], LBF, Darmstadt, (1986).

[9] McDiarmid, D.L., Fatigue under out-of-phase bending and torsion, Fatigue Fract. Engng. Mater. Struct., 9 (1987) 457-475.

[10] Stefanov, S. H., The curvilinear integral method: computer realization and testing 1 (under non-proportional reversed axial force and torque), Int. Jnl. of Fatigue, 17 (1995) 567-575.

[11] Froustey, C., Fatigue multiaxiale en endurance de l'acier 30NCD16, [PhD thesis], E.N.S.A.M. CER de Bordeaux, Bordeaux, (1987).

[12] Froustey, C., Lasserre, S., Fatigue des aciers sous sollicitations combinees. Application a l'acier 30NCD16. [Technical report Rapport DRET-LAMEF-ENSAM. contrat 87/115], ENSAM, Talence, (1988).

[13] Dubar, L. Fatigue multiaxiale des aciers. Passage de l'endurance a l'endurance limite. Prise en compte des accidents geometriques, [PhD thesis], E.N.S.A.M. CER de Bordeaux, Bordeaux, (1992).

[14] Heidenreich, R., Schubspannungsintensitätshypothese - Dauerschwingfestigkeit bei mehrachsiger Beanspruchung [Forschungshefte FKM, Heft 105]. FKM, Frankfurt am Main – Niederrad, (1983).

[15] Heidenreich, R., Richter, I., Zenner, H., Schubspannungsintensitätshypothese - weitere experimentelle und theoretische Untersuchungen Konstruktion, 36 (1984) 99-104.

[16] Lempp, W., Festigkeitsverhalten von Stählen bei mehrachsiger Dauerschwingbeanspruchung durch Normalspannungen mit überlagerten phasengleichen und phasenverschobenen Schubspannungen, [PhD thesis], Technische Universität Stuttgart, Stuttgart, (1977).

[17] Mielke, S., Festigkeitsverhalten metallischer Werkstoffe unter zweiachsiger schwingender Beanspruchung mit verschiedenen Spannungszeitverläufen, [PhD thesis], RWTH Aachen, Aachen, (1980).

[18] Troost, A., Akin, O., Klubberg, F., Versuchs- und Rechendaten zur Dauerschwingfestigkeit von metallischen Werkstoffen unter mehrachsiger Beanspruchung, Materialwissenschaft und Werkstofftechnik 23 (1992) 1-12.

[19] Kaniut, C., Zur Betriebsfestigkeit metallischer Werkstoffe bei mehrachsiger Beanspruchung, [PhD thesis], RWTH Aachen, Aachen, (1983).

[20] Vu, Q., Halm, D., Nadot, Y., Multiaxial fatigue criterion for complex loading based on stress invariants, Int. Jnl. of Fatigue, 32 (2010) 1004-1014.

[21] Araújo, J.A., Carpinteri, A., Ronchei, C., Spagnoli, A., Vantadori, S., An alternative definition of the shear stress amplitude based on the Maximum Rectangular Hull method and application to the C-S (Carpinteri-Spagnoli) criterion, Fatigue Fract. Engng. Mater. Struct., 36 (2014) 764-771.

[22] Bennebach, M., Fatigue d'une fonte GS. Influence de l'entaille et d'un traitement de surface, [PhD thesis], ENSAM Cer de Bordeaux, Bordeaux, (1993).

[23] Palin-Luc, T., Fatigue multiaxiale d’une fonte GS sous sollicitations combinées d’amplitude variable, [PhD thesis], ENSAM Cer de Bordeaux, Bordeaux, (1996).

[24] Morel, F., Palin-Luc, T., A non-local theory applied to high cycle multiaxial fatigue, Fatigue Fract. Engng. Mater. Struct., 25 (2002) 649-665.

[25] Findley, W.N., Combined-stress fatigue strength of 76S-T61 aluminum alloy with superimposed mean stresses and corrections for yielding, [Technical report NACA TN-2924], NACA, Washington, (1953).

[26] Bruun, Ø.A., Härkegård, G., A comparative study of design code criteria for prediction of the fatigue limit under in-phase and out-of-phase tension–torsion cycles, Int. Jnl. of Fatigue, 73 (2015) 1-16.

[27] Gough, H.J., Pollard, H.V., The strength of metals under combined alternating stresses, in: Proc. of Institute of Mechanical Engineering, The Institution of Automobile Engineers, London, 131 (1935) 1-103.

[28] Gough, H.J., Pollard, H.V., Properties of some materials for cast crankshafts, with special reference to combined stresses, in: Proc. of the Institution of Automobile Engineers, The Institution of Automobile Engineers, London, (1937) 821-893.

[29] Gough, H.J., Engineering steels under combined cyclic and static stresses, Journal of Applied Mechanics, (1950) 113-125.

[30] Rotvel, F., Biaxial fatigue tests with zero mean stresses using tubular specimens, Int Jnl. of Mech. Sci., 12 (1970) 597-613.

[31] Carpinteri, A., Spagnoli, A., Vantadori, S., Bagni, C., Structural integrity assessment of metallic components under multiaxial fatigue: the C–S criterion and its evolution, Fatigue Fract. Engng. Mater. Struct., 36 (2013) 870-883.

[32] Bhongbhibhat, T., Festigkeitsverhalten von Stählen unter mehrachsiger phasenverschobener Schwingbeanspruchung mit unterschiedlichen Schwingungsformen und Frequenzen, [PhD thesis], Universität Stuttgart, Stuttgart, (1986).

[33] Issler, L., Festigkeitsverhalten metallischer Werkstoffe bei mehrachsiger phasenverschobener Schwingbeanspruchung, [PhD thesis], Universität Stuttgart, Stuttgart, (1973).

[34] Baier, F., Zeit- und Dauerfestigkeit bei überlagerter statischer und schwingender Zug-Druck- und Torsionbeanspruchung, PhD thesis, Universität Stuttgart, Stuttgart, (1970).

[35] El Magd, E., Mielke, S., Dauerfestigkeit bei überlagerter zweiachsiger statischer Beanspruchung, Konstruktion, 29 (1977) 253-257.

[36] Roš, M., Eichinger, M., Die Bruchgefahr fester Körper bei wiederholter Beanspruchung – Ermüdung, [Technical report Bericht Nr. 173]. Eidgenössische Materialprüfungs- und Versuchsanstalt für Industrie, Bauwesen und Gewerbe, Zürich, (1950).

[37] McDiarmid, D.L., Mean Stress Effects in Biaxial Fatigue where the Stresses are Out-of-Phase and at Different Frequencies, in: K.F. Kussmaul, D.L. McDiarmid, D.F. Socie (Eds.), Fatigue Under Biaxial and Multiaxial Loading, Mechanical Engineering Publication, London, (1991) 321-335.

[38] Susmel, L. Petrone, N., in: A. Carpinteri, M. De Freitas, A. Spagnoli (Eds.), Proc. of 6th International Conference on Biaxial/Multiaxial Fatigue and Fracture, ESIS Publication, London, (2001) 83-104.

[39] Simbürger, A., Festigkeitsverhalten zäher Werkstoffe bei einer mehrachsigen phaseverschobenen Schwingbeanspruchung mit körperfesten und veränderlichen Hauptspannungsrichtungen, [PhD thesis], TH Darmstadt, Darmstadt, (1975).

[40] Carpinteri, A., Spagnoli, A., Multiaxial high-cycle fatigue criterion for hard metals, Int. Jnl. of Fatigue, 23 (2001) 135-145.

[41] Mamiya, E.N., Araújo, J.A., Fatigue limit under multiaxial loadings: on the definition of the equivalent shear stress, Mechanics Research Communications, 29 (2002) 141-151.

[42] Banvillet, A., Palin-Luc, T., Lasserre, S., A volumetric energy based high cycle multiaxial fatigue criterion, Int. Jnl. of Fatigue, 25 (2003) 755-769.

[43] Cruz, I., Zouain, N., A shakedown model for high-cycle fatigue, Fatigue Fract. Engng. Mater. Struct., 26 (2003) 123-135.

[44] Gonçalves, C.A., Araújo, J.A., Mamiya, E.N., A simple multiaxial fatigue criterion for metals, Comptes Rendus Mécanique, 332 (2004) 963-968.

[45] Gonçalves, C.A., Araújo, J.A., Mamiya, E.N., Multiaxial fatigue: a stress based criterion for hard metals, Int. Jnl. of Fatigue, 27(2005) 177-187.

[46] Liu, Y., Mahadevan, S., Multiaxial high-cycle fatigue criterion and life prediction for metals, Int. Jnl. of Fatigue, 27 (2005) 790-800.

[47] Ninic, D., A stress-based multiaxial high-cycle fatigue damage criterion, Int. Jnl. of Fatigue, 28 (2006) 103-113.

[48] Ninic, D., Stark, H.L., A multiaxial fatigue damage function, Int. Jnl. of Fatigue, 29 (2007) 533-548.

[49] Braccesi, C., Cianetti, F., Lori, G., Pioli, D. (2008). Int. Jnl. of Fatigue 30, 1479-1497.

[50] Papuga, J., Růžička, M., Two new multiaxial criteria for high cycle fatigue computation, Int. Jnl. of Fatigue, 30 (2008) 58-66.

[51] Shariyat, M., Three energy-based multiaxial HCF criteria for fatigue life determination in components under random non-proportional stress fields, Fatigue Fract. Engng. Mater. Struct., 32 (2009) 785-808.

[52] Li, B., Reis, L., de Freitas, M., Comparative study of multiaxial fatigue damage models for ductile structural steels and brittle materials, Int. Jnl. of Fatigue, 31 (2009) 1895-1906.

[53] Zhang, C.-C., Yao, W.-X., An improved multiaxial high-cycle fatigue criterion based on critical plane approach, Fatigue Fract. Engng. Mater. Struct., 34 (2011) 337-344.

[54] Carpinteri, A., Spagnoli, A., Vantadori, S., Multiaxial fatigue assessment using a simplified critical plane-based criterion, Int. Jnl. of Fatigue, 33 (2011) 969-976.

[55] Kenmeugne, B., Soh Fotsing, B.D., Anago, G.F., Fogue, M., Robert, J.-L., Kenne, J.-P., On the evolution and comparison of multiaxial fatigue criteria, International Journal of Engineering and Technology, 4 (2012) 37-46.

[56] Augustins, L., An empirical multiaxial high cycle fatigue criterion for automotive cylinder head design, Engineering Failure Analysis, 28 (2013) 264-274.

[57] Matsubara, G., Nishio, K., Multiaxial high-cycle fatigue criterion considering crack initiation and non-propagation, Int. Jnl. of Fatigue, 47 (2013) 222-231.

[58] Golos, K.M., Debski, D.K., Debski, M.A., A stress-based fatigue criterion to assess high-cycle fatigue under in-phase multiaxial loading conditions, Theoretical and Applied Fracture Mechanics, 73 (2014) 3-8.