



## Scaling of compression strength in disordered solids: metallic foams

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**ABSTRACT.** The scaling of compression strength with porosity for aluminium foams was investigated. The Al 99.96, AlMg1Si0.6 and AlSi11Mg0.6 foams of various porosity, sample size with and without surface skin were tested in compression. It was observed that the compression strength of aluminium foams scales near the percolation threshold with  $T_f \approx 1.9 - 2.0$  almost independently on the matrix alloy, sample size and presence of surface skin. The difference of the obtained values of  $T_f$  to the theoretical estimate of  $T_f = 2.64 \pm 0.3$  by Arbabi and Sahimi and to Ashby estimate of 1.5 was explained using an analogy with the Daoud and Coniglio approach to the scaling of the free energy of sol-gel transition. It leads to the finding that, there are two different universality classes for the critical exponent  $T_f$ : when the stretching forces dominate  $T_f = f = 2.1$ , respectively when bending forces prevail  $T_f = v.d = 2.64$  seems to be valid. Another possibility is the validity of relation  $T_f \leq f$  which varies only according to the universality class of modulus of elasticity in foam.

**KEYWORDS.** Metallic foams; Aluminium foams; Compression; Compression strength; Percolation.

### INTRODUCTION

The compression strength is a key property, which determines the industrial applications of the disordered solids, such as natural rocks, porous ceramics or metallic foams [1-8]. The traditional approaches to the failure phenomena of solid materials are mostly valid for the solids that are macroscopically homogeneous. Moreover, they deal with the problem without considering the effect of the microscopic disorder. In disordered materials, the presence of pores of various sizes, shapes and orientations makes the problem more complex. The initiation and growth



of the cracks, which leads to the macroscopic failure in the disordered system is in general a non-equilibrium and non-linear phenomenon.

The scaling model [9] considers that the compression strength of the disordered material is a function of the compression strength of solid material and the degree of disorder (porosity or volume fraction of solid material). Benguigui, Ron and Bergman [10] in 2D and Sieradzki and Li [11] in 3D found experimentally that the compression strength  $\sigma_{CS}$  of disordered solids scales according to the power law:

$$\sigma_{CS} \propto (p - p_c)^{T_f} \tag{1}$$

where  $p$  is the degree of disorder,  $p_c$  is a percolation threshold, i.e., the value of disorder below which the compression strength vanishes, and  $T_f$  is a critical exponent for the compression strength. In 3D Sieradzki and Li [11] obtained  $T_f = 1.7 \pm 0.1$  for the system composed of a 2 mm thick aluminium plate with holes punched at positions corresponding to the triangular networks. On the other hand, Bergman [12] proposed the theoretical bounds for  $T_f$ , which in 3D give  $2.58 < T_f$  ( $d=3$ )  $< 2.76$ . Using computer simulation, Sahimi and Arbabi [13] found  $T_f \sim 2.64 \pm 0.3$  in 3D, which agrees with the bounds proposed by Bergman. They assumed that the lower value of  $T_f = 1.7 \pm 0.1$  by Sieradzki and Li [11] was due to the fact that the measurements were done far from  $p_c$  and also due to the significant size effect. Recently, author [14] used the scaling model for the study of the tensile strength of copper and nickel foams. The obtained results for  $T_f$  agree reasonably with the Sahimi and Arbabi estimated value  $T_f \sim 2.64$ . Unfortunately, the data are far from the percolation threshold and the obtained values can be over or under estimated.

The scope of this work is to study the scaling of the compression strength of the aluminium foams prepared by powder metallurgical route [15]. The metallic foams are the disordered solids consisting of a metal matrix filled with gas pores. The metallic foams around 0.1 and less volume fraction of metal can be prepared thus enabling to study the scaling of the compression strength near the percolation threshold.

## EXPERIMENTAL INVESTIGATIONS

To investigate the scaling of the compression strength, the metallic foams of various densities were prepared by powder metallurgical route [15]. The samples were made from aluminium alloy powders Al 99.96, AlMg1Si0.6 and AlSi11Mg0.6, which were mixed together with a foaming agent (0.4 wt.% of titanium hydride), CIP-ed and then continuously hot extruded at 450 °C into a foamable precursor. The precursor was expanded into the porous cellular solid by hydrogen, which is released from the foaming agent, during the heating of the precursor above the melting temperature of the metal matrix. Then the rapid cooling process takes place to freeze the obtained cellular structure.

The samples of various geometry were prepared: cylinders with the diameter of 20 mm and the length of 10, 20, 25, 30 and 40 mm without surface skin, with the diameter of 28 mm and the length of 32 mm with surface skin and finally with the diameter of 40 mm and the length of 51 mm with surface skin. The prepared foam samples were usually from the range of 0.07 - 0.45 volume fraction of metal (see Tab. 1). The metallic surface skin always covers the foam prepared by the powder metallurgical route. The surface skin was removed via the electric discharge machining thus excluding the possible effect of the skin on the compression strength scaling. It enabled also to evaluate the effect of the surface skin using a couple of samples with the skin. The compression test was carried on INSTRON testing machine with constant ram speed of 10 mm/min. From the load-deflection curve, the stress-strain curve was obtained and the compression strength of the foam sample was determined (see Fig.1).

## RESULTS AND DISCUSSIONS

To model the compression strength of aluminium foams, the following assumption was made: because the foams with 0.1 and less volume fraction of metal can be successfully prepared, the percolation threshold was set to zero and Eq. (1) was rewritten in the following way:

$$\sigma_{CS} = \sigma_{CS0} \left( \frac{\rho}{\rho_0} \right)^{T_f} \tag{2}$$

where

$\sigma_{CS0}$  is the compression strength of the solid material without porosity;

$\rho$  is the density of foam;

$\rho_0$  is the density of solid material;

$\rho/\rho_0$  is the volume fraction of metal.

The fitting of the experimental results to Eq. (2) showed that the scaling exponent  $T_f$  is in the range of 1.89 – 2 depending on alloy composition, sample size and surface skin presence (see Tab. 1 and Figs. 2 and 3). The results indicate that it is a universal critical exponent: it is independent on the compression mechanisms of the cell-walls and is almost the same for both brittle compression of casting aluminium alloy AlSi11Mg0.6 (see Fig. 1.a) and plastic deformation of wrought aluminium alloys Al 99.96 and AlMg1Si0.6 (see Fig. 1.b). The lowest values of  $T_f$  were obtained for the samples with surface skin, because the skin oriented parallel to the applied load increases the sample strength at constant volume fraction of metal. It must be further noted, that the data for the compression strength of metallic foams showed statistical fluctuations due to various pore sizes, shapes and orientations at constant porosity (see Fig. 4).

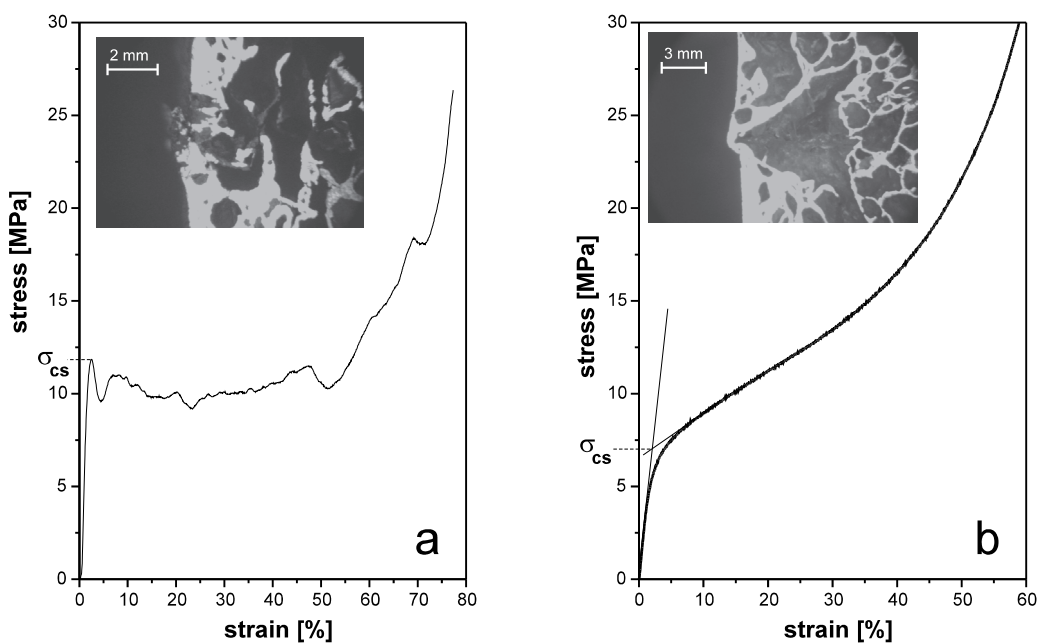


Figure 1: Typical stress-strain curves for, the definition of the compression strength of and the macroscopic structural changes at compression strength for: (a) brittle AlSi11Mg0.6 foam, 0.18 volume fraction of metal, (b) ductile Al 99.96 foam, 0.31 volume fraction of metal.

Matrix alloy	Geometry [mm]	Range of volum fraction	$\sigma_{CS0}$ [MPa]	$T_f$ [-]	$\chi^2$ [-]	N [-]
	$\phi$ 20 x 25	0.09 - 0.44	$69.8 \pm 2.6$	$1.96 \pm 0.11$	6.67	16
Al 99.96	$\phi$ 28 x 32 (skin)	0.18 - 0.41	$69.9 \pm 0.8$	$1.94 \pm 0.04$	0.65	8
	$\phi$ 40 x 51 (skin)	0.17 - 0.32	$70.0 \pm 0.8$	$1.89 \pm 0.04$	0.68	12
AlMg1Si0.6	$\phi$ 20 x (10-40)	0.07 - 0.40	$114.9 \pm 1.5$	$1.97 \pm 0.03$	2.20	65
AlSi11Mg0.6	$\phi$ 20 x (10-40)	0.10 - 0.89	$305.0 \pm 4.1$	$2.00 \pm 0.04$	40.0	77

Table 1: Composition, sample geometry and observed scaling exponents for investigated metal foams determined via Eq. (2).  $\chi^2$  is the error of the fitting. N is the number of measured foam samples used for fitting.

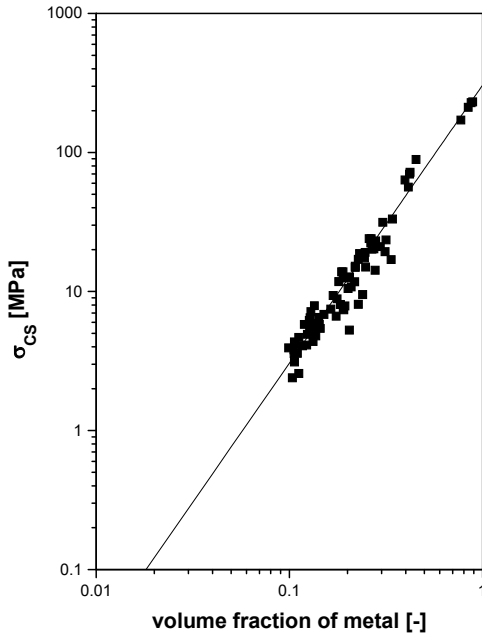


Figure 2: Scaling of the compression strength for AlSi11Mg0.6 foam without surface skin  $\phi$  20 x {10, 20, 30, 40} mm<sup>2</sup>.

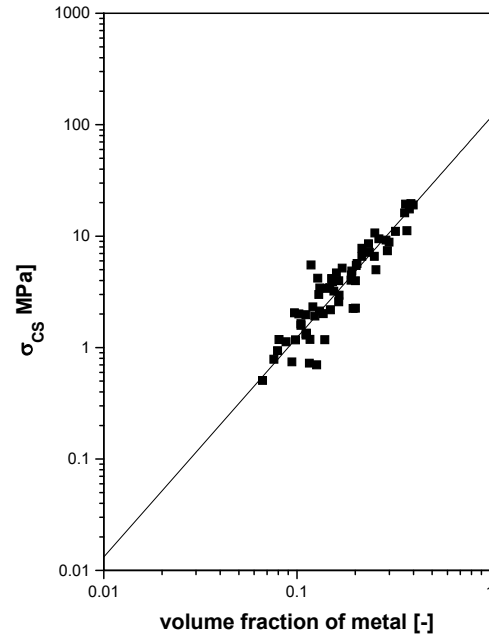


Figure 3: Scaling of the compression strength for AlMg1Si0.6 foam without surface skin  $\phi$  20 x {10, 20, 30, 40} mm<sup>2</sup>.

The observed values are also in contradiction with the prediction of the compression yield stress dependence on volume fraction of metal for closed cell foams according to Ashby et al. [2]:

$$\frac{\sigma'_{CS}}{\sigma_{ys}} = C_0 \left[ C_I \left( \frac{\rho}{\rho_0} \right)^{3/2} + C'_I \left( \frac{\rho}{\rho_0} \right) \right]. \quad (3)$$

Basically this equation is in general a combination of cell face stretching and cell edge bending. It is evident that the obtained dependence is almost the straight line in log-log space. For that reason, let us assume that the cell edge bending part can be neglected. Therefore the cell face stretching will prevail and the results ought to scale with the exponent of 3/2, but it is significantly lower value as was experimentally observed.

Why obtained values of  $T_f$  are low in comparison with the theoretical value  $2.64 \pm 0.3$  and higher than 1.5 derived by Ashby et al. [2]? One significant solution can be the fact, that with the increasing porosity, the **pore size** in metallic foams becomes comparable with sample geometry and the size effect takes place. Nevertheless, after enlarging the volume of samples by 30% **no increase of  $T_f$**  was observed (see Al 99.96 foams in Tab. 1 and Fig. 5).

It is evident, that the question why the compression strength of metallic foams scales near the percolation threshold with  $T_f$  of 1.89 – 2 cannot be simply explained by the sample size and the distance from the percolation threshold. It is necessary to look for another explanation. A possible solution can be found in the experimental work of Daoud and Coniglio [17]. They proposed that the free energy  $F$  of sol-gel transition scales as:

$$F \propto A \cdot \varepsilon^{\nu d} + B \cdot \varepsilon^f, \quad \varepsilon = \left| \frac{p - p_c}{p_c} \right| \quad (4)$$

where  $A$ ,  $B$  are numerical constants,  $p$  is the volume fraction of sol,  $p_c$  is the percolation threshold,  $\nu$  is the critical exponent for the correlation length,  $d$  is dimension of the problem and  $f$  is the critical exponent for the modulus of elasticity. The critical exponent  $f$  possesses two different universality classes in 3D [16]:  $f = 2.1$  for central-force model when stretching forces dominate, and  $f = 3.76$  for bond-bending model when bending forces dominate. The first term in Eq. (4) represents the contribution of the finite gel clusters below and above the percolation threshold. The second term to the free energy is the contribution of an infinite gel cluster. In 3D  $\nu = 0.88$  [9] thus giving  $\nu \cdot d = 2.64$ , which coincide with the theoretical prediction for  $T_f = 2.64 \pm 0.3$  by Sahimi and Arbabi and also with Bergman's bounds on  $T_f$ .

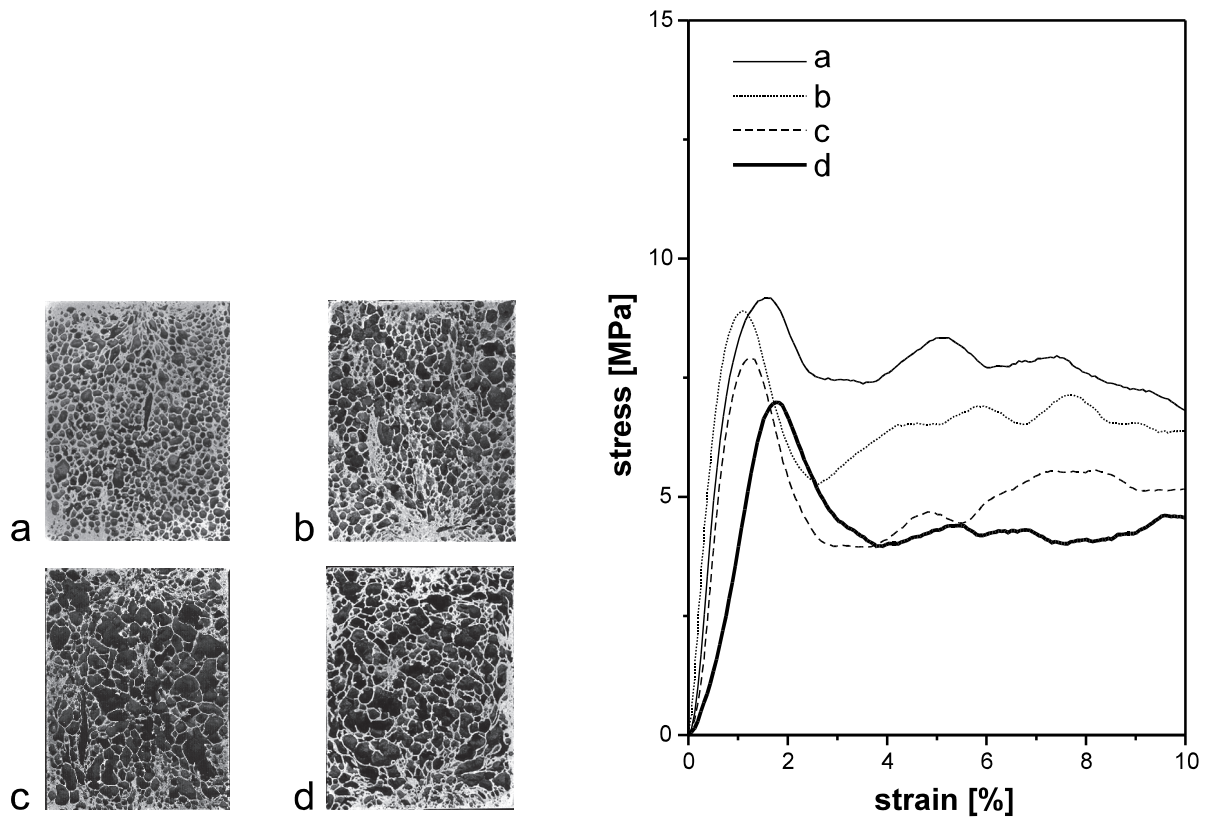


Figure 4: Effect of the pore size and orientation on the compression strength of AlSi11Mg0.6 foam with surface skin  $\phi$  40 x 51 mm<sup>2</sup> at 0.19 volume fraction of metal: vertically oriented pores with apparent diameter of (a) 2.25 mm, (b) 3.03 mm, (c) 5.94 mm and (d) randomly oriented pores with apparent diameter of 4.52 mm.

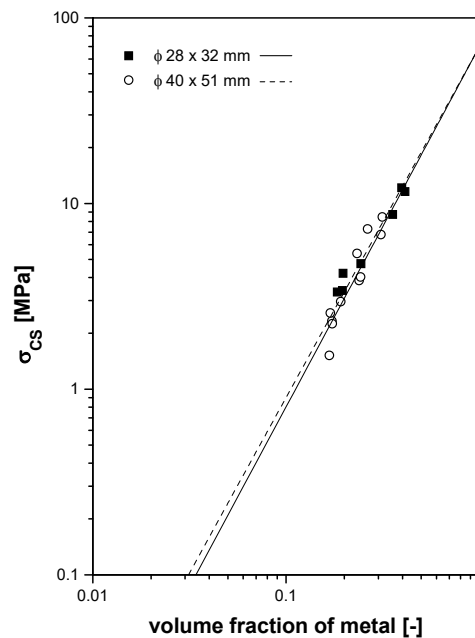


Figure 5: Effect of the sample size on the scaling of compression strength for Al 99.96 foam with surface skin.

This fact can support the following analogy: the energy necessary for the compression failure of the metallic foam is an area under the stress-strain curve and its value depends on the compression strength and foam structure. Therefore, one can suppose, that Eq. (4) can be used to describe the compression energy and also to model the scaling of the

compression strength of the metallic foams. Using this analogy, the first term can be identified with the interaction of the existing macrocracks inside the cell-walls of the foam which are not involved in foam fracture. The macrocracks (see Fig. 6) are created during the cooling process due to the hydrogen pressure within the pores and due to subsequent non-uniform cell-wall contraction during cooling. The second term can be connected with the interaction of some amount of the existing macrocracks and also microcracks initiation and growth thus leading to the breakage/plastic failure of foam sample weakest region and therefore to macroscopic failure of the sample (see Fig. 1 – start of the densification of weakest region of foam by crack propagation/plastic bending depending on alloy composition [18]).

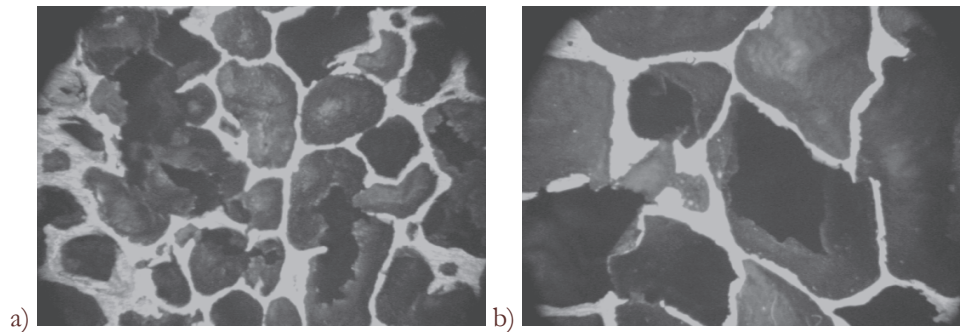


Figure 6: Macro cracks inside the cell walls in (a) AlSi11Mg0.6 foam and (b) AlMg1Si0.6 foam as a result of foaming process (0.2 volume fraction of metal).

In Eq.(4),  $\varepsilon$  is in the range of [0, 1], thus implying that the term with lower exponent represents the dominant part of the compression energy. Therefore, when the stretching forces dominate, the leading term is the second one with  $f = 2.1$ . In this case, the foam fails mostly via interaction and growth of some macrocracks and after sufficient concentration of microcracks is generated in foam cell walls by stretching (compression/tension). On the contrary, when microcracks are created by bending forces with  $f = 3.76$ , material fails by the growth and interaction of the existing macrocracks and the first term ought to prevail with  $v.d = 2.64$ .

The critical exponent  $f = 1.66 \pm 0.07$  has been experimentally found for the modulus of elasticity of identical aluminium foams [15]. The obtained value belongs to the universality class of the central-force model 2.1. The lower value of  $f$  was found due to the significant size effect and the anisotropy of the samples. It implies that the metallic bonds inside the cell walls of the aluminium foams fail predominantly by the stretching forces and confirms  $T_f \approx 1.9 - 2.0$ .

One can conclude that, the theory seems to be correct. However, it is necessary to check the proposed ideas. The validity can be simply proved using the metallic foams or other disordered solids that belongs to the bond-bending universality class in 3D with  $f = 3.76$ . In this case, the compression strength of such material ought to scale with  $T_f$  approximately 2.64.

Summarising, the critical exponent of the compression strength seems to possess two different universality classes in 3D:

$$T_f \leq f = 2.1 \tag{5}$$

when central-forces dominate (stretching forces). When bond-bending forces dominate, the compression strength depends on the correlation length of macrocracks, and

$$T_f = v.d = 2.64 \tag{6}$$

This scaling relation coincides with the limits proposed by Bergman and is in agreement with the theoretical prediction by Sahimi and Arbabi [13]. Latest research in the field of Ti–6Al–4V foams [19] produced by additive manufacturing gives  $f = 2.96$  and  $T_f = 2.81$  (yield point). Also in the field of replicated microcellular materials, one finds  $f = 2.6-3$ , and the value of  $f$  tends to increase as percolation threshold decreases [20, 21]. Therefore it can be expected that these materials fails in compression via bending mechanism and therefore their scaling exponents possess higher values as powder metallurgical metallic foams.

Another possibility is following: Eq. (5) is usually found experimentally in the form of:

$$T_f \leq f \tag{7}$$



where  $f$  varies according to the stretching/bending forces acting in the foam microstructure and therefore exponent for compression strength depends on the corresponding universality class for modulus of elasticity.

As a final remark it can be mentioned that the compression strength of the solid material determined using Eq. (2) (see Tab. 1) more or less coincide with tensile strength of corresponding aluminium alloys prepared by PM route.

## CONCLUSIONS

Summarising, the compression strength of aluminium foams scales near the percolation threshold with  $T_f \approx 1.9 - 2.0$  almost independently on the matrix alloy, sample size and presence of surface skin. The obtained values of  $T_f$  are the same for the brittle and ductile aluminium alloys thus proving that the scaling exponent  $T_f$  is a universal critical exponent. However,  $T_f \approx 1.9 - 2.0$  is in contradiction with the theoretical estimate of  $T_f = 2.64 \pm 0.3$  by Arbabi and Sahimi, and also significantly higher than Ashby estimate of 1.5. This problem was solved, using an analogy with the Daoud and Coniglio approach to the scaling of the free energy of sol-gel transition. It leads to the finding that, there are either two different universality classes for the critical exponent  $T_f$ : When the compression of the cell-walls is caused by stretching forces  $T_f = f = 2.1$ . When bending forces prevail, the scaling relation  $T_f = v.d = 2.64$  seems to be valid. Another possibility is the validity of relation  $T_f \leq f$  which varies only according to the universality class of modulus of elasticity in foam.

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