Equivalent configurations for notch and fretting fatigue

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ABSTRACT. Under the typical partial slip conditions under which fretting fatigue takes place, the amount of superficial damage is small. Therefore, the substantial reduction in fatigue life caused by fretting, when compared to plain fatigue, may well be more associated with the stress concentration and the stress gradient phenomena generated by the contact problem than to the superficial loss of material. In this setting, notch stress-based methodologies could, in principle, be applied to fretting in the medium/high cycle fatigue regime. The aim of this work was to investigate whether it is possible to design fretting and notch fatigue configurations, which are nominally identical in terms of damage measured by a multiaxial fatigue model. The methodology adopted to carry out this search considered a cylindrical on flat contact and a V-notch. Load and geometry dimensions of both configurations were adjusted in order to try to obtain the "same" decay of the Multiaxial Fatigue Index from the hot spot up to a critical distance. Positive results of such simulations can lead us to design an experimental program that can bring more firm conclusions on the use of pure stress-based approaches, which do not include the wear damage, in the modeling of fretting fatigue.

KEYWORDS. Fretting fatigue; Notch fatigue; Stress gradient; Multiaxial fatigue.

INTRODUCTION

The term fretting denotes a small oscillatory movement between two solid surfaces in contact. In many industrial designs, one or both components of the assembly may also be subjected to a bulk fatigue load, the phenomenon is then called fretting fatigue. Fretting failure of components such as splines, the dovetail fixing between blade and disc in fans of aeroengines [1] and riveted skins of the aircraft fuselage [2] have become a major design concern. Indeed, experimental evidence has shown that the conjoint action of fretting and fatigue may produce strength reduction factors varying from 2 up to 10 compared to plain fatigue [3]. The most severe reduction of life is observed in partial slip conditions [4]. Indeed gross sliding can fastly remove small nucleated cracks before they can further propagate.

Under fretting conditions, the loads involved generate a time varying non-proportional multiaxial stress field under the contact [5]. There is usually a high stress concentration on the contact interface but it is limited to a small region and hence decays fastly as one moves from the surface to the interior of the component [6]. In this setting, the use of non-local approaches, initially developed to estimate the fatigue endurance of notched specimens, have been extended to the fretting fatigue problem [7]–[9]. Araújo et al. [8] used the Theory of Critical Distances (TCD) in conjunction with the Modified Whöler Curve Method (MWCM) and showed that it was capable of estimating the results of fretting fatigue experiments showing a contact size effect with a good degree of accuracy (±20% error band).

It is worthy noticing that notched components and mechanical assemblies under time varying loads present quite similar characteristics. In both configurations there are usually severe stress gradients and the state of stress becomes complex (multiaxial) as the analysis moves inside the material. The general aim of this work is to investigate whether it is possible to design fretting and notch fatigue configurations, which are nominally identical in terms of damage measured by a multiaxial fatigue index. Further, these designs must be such that the specimens can be machined within a good level of accuracy and with tight tolerances in both configurations (notch and fretting), turning the future experimental campaign reproducible and reliable. The advantage of proving that such experimental campaign can be generated is that it consists in a way to somehow isolate the role of the surface fretting wear in diminishing the material fatigue resistance of mechanical assemblies under partial slip. As both fatigue problems are equivalent in terms of fatigue loading over a material process zone, their resistance should be essentially similar, unless the influence of the small surface damage caused by the fretting wear, and which does not exist in the notch configuration, is greater than the authors expect it will have. Another quite important aspect that such experimental campaign could clear out is the fact that one can use the same fatigue modeling approach to design either industrial components containing geometrical discontinuities or mechanical couplings. This would avoid the need for lengthy and costly experimental programs considering complex geometries and specific test rigs to calibrate the fatigue material constants.

MATERIAL

The material considered in this study was a 7050-T7451 aluminium alloy. Material properties were taken from Araújo et al. [10] and are presented in Tab. 1. Fatigue strength for fully reversed (stress ratio, R=-1) and repeated loading (R=0) are quoted for 10^7 cycles.

Young's Modulus	Poisson's ratio	Yield strength	σ_{-1}	σ_0	$\Delta K_{th,\, R=-1}$
73.4 GPa	0.33	454 MPa	161 MPa	120 MPa	4.5 MPa√m

Table 1: Material properties of 7050-T7451 Al alloy.

STRESS TENSOR FOR ELASTIC CONTACT OF CYLINDERS UNDER PARTIAL SLIP AND IN THE PRESENCE OF A BULK FATIGUE LOAD

The first step towards a solution for the subsurface stress field is to solve the contact problem itself, i.e., to find the magnitude and distribution of the surface tractions. In the present problem, the pad radius, R, the normal load per unit length, P, and the specimen thickness were defined so that each solid could be considered as an elastic half-space and the solution for the pressure distribution was Hertzian [11]. For pure fretting, the time varying tangential force $Q(t)$ generates a shear traction described firstly by Cattaneo [12] and independently by Mindlin [13]. In partial slip condition, the contact is characterized by a central stick zone bordered by two sliding zones. In fretting fatigue, the effect of the bulk tension is to offset this stick zone. By superposing the normal and tangential contributions, it is possible to directly evaluate the resulting stress tensor using Muskhelishvili's potential theory [14].

For synchronous and in phase load combinations, at the instants of maximum and minimum bulk/shear loads, the stress tensor turns out to be:

$$\frac{\boldsymbol{\sigma}\left(\frac{x}{a},\frac{y}{a},t\right)}{p_0}=\left(\frac{\boldsymbol{\sigma}^n\left(\frac{x}{a},\frac{y}{a}\right)}{p_0}\right)\pm f\left(\frac{\boldsymbol{\sigma}^t\left(\frac{x}{a},\frac{y}{a}\right)}{fp_0}\right)\mp f\frac{c}{a}\left(\frac{\boldsymbol{\sigma}^t\left(\frac{x-e}{c},\frac{y}{c}\right)}{fp_0}\right)+\frac{\boldsymbol{\sigma}_B(t)}{p_0} \tag{1}$$

For any other time instant *t*, during loading or unloading, the appropriate analytical solution is stated as:

$$\frac{\boldsymbol{\sigma}\left(\frac{x}{a},\frac{y}{a},t\right)}{p_0}=\left(\frac{\boldsymbol{\sigma}^n\left(\frac{x}{a},\frac{y}{a}\right)}{p_0}\right)\pm f\left(\frac{\boldsymbol{\sigma}^t\left(\frac{x}{a},\frac{y}{a}\right)}{fp_0}\right)\mp 2f\frac{c'(t)}{a}\left(\frac{\boldsymbol{\sigma}^t\left(\frac{x-e'(t)}{c'(t)},\frac{y}{c'(t)}\right)}{fp_0}\right)\pm f\frac{c}{a}\left(\frac{\boldsymbol{\sigma}^t\left(\frac{x-e}{c},\frac{y}{c}\right)}{fp_0}\right)+\frac{\boldsymbol{\sigma}_B(t)}{p_0} \tag{2}$$

In the above equations, p_0 is the peak pressure, c and e are the stick zone half width and its offset from the center of the contact at the instant of maximum or minimum shear load. At any other time instant, c' and e' correspond to the stick zone half width and its offset from the center of the contact. The superscripts n and t stand for the stress components due to the normal and the tangential load, respectively. Finally, $\boldsymbol{\sigma}_B$ is the stress tensor associated with the bulk fatigue load. Plane strain conditions are assumed. Explicit expressions to compute c, e, c', e', $\boldsymbol{\sigma}^n$ and $\boldsymbol{\sigma}^t$ are given in a convenient form by Hills and Nowell [15].

STRESS TENSOR AROUND A V-NOTCH

The elastic solution for the stress field of plane V-notched specimens can be derived using complex potential [14] or bi-harmonic potential functions [16]. Recently, the Williams' crack solution [17] was re-addressed by Lazzarin and Tovo [18] and Filippi et al. [19]. Such approach was used in this work to evaluate the stresses along the bisector of a notch loaded in mode I.

MULTIAXIAL CRITERION

Susmel and Lazzarin [20] observed that the multiaxial High-Cycle Fatigue behaviour of metallic materials could successfully be estimated by using a simple τ_a vs $\sigma_{n,max}/\tau_a$ relationship. The model has been described by Susmel and co-workers in a series of articles [20-22]. Briefly, the so-called Modified Wöhler Curve Method (MWCM) can be formalized as follows:

$$\tau_a(\phi^c,\theta^c)+\kappa\frac{\sigma_{n,max}}{\tau_a}(\phi^c,\theta^c)\leq\lambda \tag{3}$$

where τ_a is the equivalent shear stress amplitude in the critical plane (ϕ^c,θ^c), $\sigma_{n,max}$ is the maximum stress perpendicular to this plane, and the parameters κ and λ are material constants that can be obtained from two fatigue strengths generated under different loading conditions. For instance, these constants can be evaluted using the fatigue limits σ_{-1} and σ_0, generated respectively under fully-reversed (R =-1) and under repeated (R = 0) uniaxial loading, as follows [21]:

$$\kappa=\frac{\sigma_{-1}-\sigma_0}{2}\qquad\lambda=\sigma_{-1}-\frac{\sigma_0}{2} \tag{4}$$

which gives $\kappa=20.8MPa$ and $\lambda=101.5MPa$ for the 7050-T7451 Al alloy. A key aspect in using such model for non-proportional loadings, as it is the case for fretting fatigue, is the computation of τ_a relative to a material plane. We propose here to use the maximum rectangular hull concept developed by Araújo et al. [23-25]. It consists of rotating rectangular hulls engulfing the shear stress vector path in a material plane, and computing the square root of the sum of the squares of each rectangle' half-sides. The greatest of these values is then defined as the shear stress amplitude, τ_a.

IDENTIFICATION OF THE CRITICAL DISTANCE

The use of the MWCM in conjunction with the Theory of Critical Distances (TCD) is based on the assumption that all the physical processes leading to crack initiation are confined within the so-called structural volume. The size of this volume is assumed not to be dependent on either the stress concentration feature weakening the component or the complexity of the stress field damaging the fatigue process zone [26]. In the TCD fatigue endurance is assumed to occur when:

$$\frac{1}{V}\int_V F(\boldsymbol{\sigma})dV \leq c \tag{5}$$

where $\boldsymbol{\sigma}$ is stress tensor within a volume of material V around the stress raiser, c is a material parameter associated with its fatigue resistance, and $F(.)$ denotes an effective stress that appropriately characterizes the fatigue loading. Eq. (5) can be simplified by substituting the material volume by a line (LM) or by a single point (PM) at a certain distance from the hot spot. To obtain this length parameter, the critical distance, a third material parameter is needed, the threshold stress intensity factor range under R=-1, ΔK_{th}. Considering a material point at a distance l from the crack tip and on the bisector line of a sharp crack it is possible to compute the stress tensor at this position at any time instant of the loading history. If now one computes the τ_a and $\sigma_{n,max}$ variables of the multiaxial model from this state of stress, it is possible to find equations relating l, ΔK_{th} and $\Delta\sigma_{-1}$ for the point and the line methods:

$$l_{PM} = \frac{1}{2\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_{-1}}\right)^2 \tag{6}$$

$$l_{LM} = \frac{2}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_{-1}}\right)^2 \tag{7}$$

Using material properties reported in Tab. 1, one can use Eq. (6) to calculate $l_{PM} = 0.031$ mm. An alternative manner to interpret the critical distance is to associate it with the length of non-propagating cracks in crack like notched specimens under fatigue limit conditions. Therefore, from a phenomenological point of view, some authors such as Susmel [27] and Taylor [28] assume the critical distance as a material property. However, it is clear that it depends on the effective stress used to model the multiaxial problem.

METHODOLOGY

The first step to try to establish equivalent notch/fretting fatigue experiments is to define a reference configuration. In this case, a fretting fatigue data tested in the apparatus of the University of Brasília was considered as the starting point. This test was carried out using two cylindrical fretting pads, which were loaded against a flat dogbone specimen. Tab. 2 lists the loads, pad radius, the coefficient of friction, f, and the life for complete fracture of the especimen for this test. The cross section of the fretting specimen is schematically shown in Fig. 1 (13 x 13 mm). The characteristics of the fretting apparatus are well detailed by Martins et al. [29], hence it will not be described here. Now one can plot the multiaxial fatigue index S_{eq}, defined in Eq. (8), from the trailing edge of the contact (hot spot for the fatigue index) along the distance y in the interior of the fretting specimen. The computation is carried up to a distance equivalent to the characteristic length provided by the line method, l_{LM}.

$$S_{eq} = \tau_a + \kappa\frac{\sigma_{n,max}}{\tau_a} \tag{8}$$

Fig. 2 (a) shows the gradient of S_{eq} with respect to y for this test and the limit for crack initiation defined by the threshold parameter $\lambda = 101.5MPa$ for this alloy. Notice that, considering the point method as the critical distance, the index is higher than λ (at the distance l_{PM}), and is then expected the test would break before 10^7 cycles. Indeed the test broke around 1.2 x 10^6 cycles.

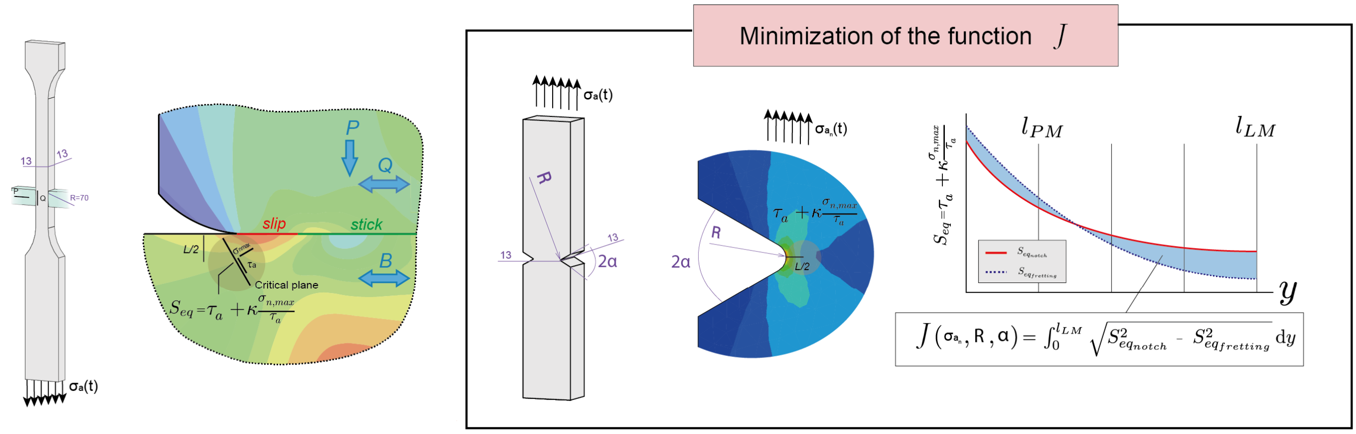

Figure 1: Scheme of the methodology proposed to find equivalent notch/fretting fatigue configurations.

To carry on with the analysis the following step is to search for the notch configuration that will provide an equivalent decay for the multiaxial fatigue index over l_{LM}. To do so, a minimization technique based on the difference between the indexes $S_{eq,notch}$ and $S_{eq,fretting}$ along the distance l_{LM} can be applied (function J in Fig. 1). Notch radius, notch opening angle and the remote fatigue load are the variables of the minimization. The range of variation of such variables must be such that one can guarantee that the notch specimens can be machined and mainly its notch radius lies within a tight tolerance. In this setting, notch radius could vary between 0.1 and 1 mm (in steps of 0.05 mm), its opening angle between 30° and 90° (15° increments) and the notch depth was fixed in 5 mm. The limitation for the choice of remote bulk load range was defined by the operational capability of the servohydraulic machine. It can accurately apply and control load amplitudes varying from 1 to 100 kN (a 1kN increment was defined).

Pad radius	Peak pressure	σ_a $(R=-1)$	Q/fP	f	Frequency	Life (cycles)
70mm	300 MPa	55 MPa	0.33	0.54	5Hz	1119774

Table 2: Parameters and observed life of the fretting fatigue test used to design an equivalent notch fatigue test.

Minimization of the difference between the two gradients was achieved by using a 0.5mm notch radius specimen, with a notch opening angle of 60° and a fully reverse fatigue stress amplitude of 42.5 MPa. Now Fig. 2 (a) is again invoked to depict the result of the optimized notch configuration. It can be seen that the variation of the multiaxial fatigue index against distance from the notch root matches quite well the curve obtained for the fretting fatigue configuration. Fig. 2(b) depicts the points representing the shear stress amplitude, τ_a, against the normalized maximum stress, $\sigma_{n,max}/\tau_a$, for both configurations. These points do not lay one over the other but they keep the same distance from the continuous threshold line dividing safe (under the line) and unsafe domains. In Fig. 2(c) an interesting behavior is observed. The shear stress amplitude and the maximum normal stress are traced against the distance for the notch and for the fretting configuration. While the amplitude of shear stress on the critical plane for the fretting problem is always higher than for the notch one along the distance, the opposite happens to the maximum normal stress. Therefore, although the decay of the multiaxial fatigue index is essentially the same for both configurations, the same can not be said about the individual variables that are present in the index computation.

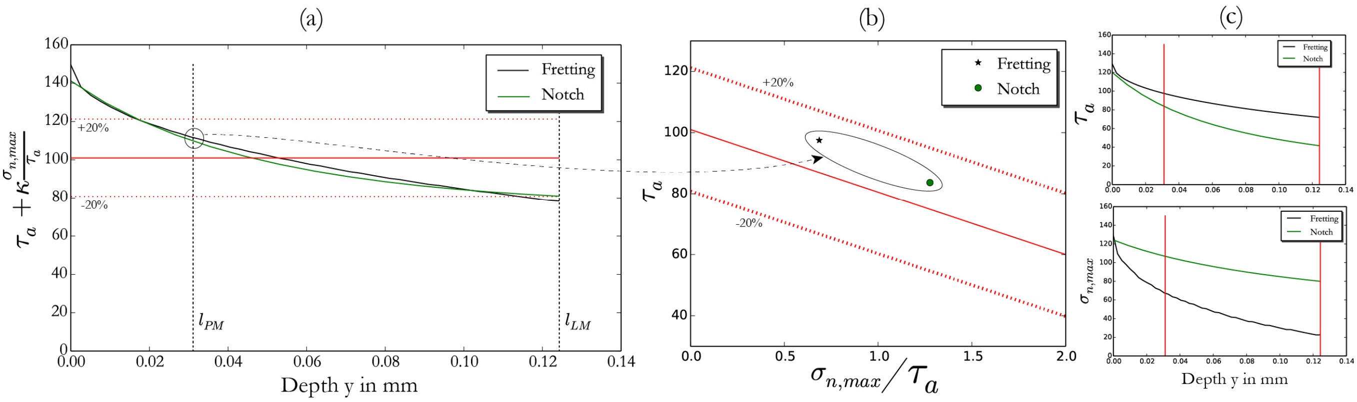

Figure 2: (a) Variation of the multiaxial fatigue index against distance in the specimen (from the hot spot) for the fretting fatigue test and for the optimized notch configuration. (b) Points representing the shear stress amplitude, τ_a , against the normalized maximum stress, $\sigma_{n,max} / \tau_a$, for both configurations. (c) Variation of τ_a and $\sigma_{n,max} / \tau_a$ against distance for fretting and notch problems.

CONCLUSION

It was shown that it is possible to design a series of fretting and notch fatigue tests for a 7050-T7451 Al alloy, which are nominally equivalent in terms of multiaxial fatigue conditions along a process zone. The notch geometry and fretting configuration found can be machined and reproduced within a given tolerance, and closed form solutions for the cyclic stress field under plane strain are available for both. It was pointed out however that equivalent fatigue index parameters does not mean equivalent shear stress amplitude and maximum normal stress. The design of this test campaign can constitute a key initiative to improve our knowledge on the influence of the surface damage on the life of components under fretting fatigue conditions. Further, should the results of these tests confirm the equivalence hypothesis between this two apparently distinct fatigue problems, a single unified stress based approach could be used to design them. Future work involves the conduction of the experimental tests.

ACKNOWLEDGMENTS

The supports provided by Safran/Snecna, by CNPq (contracts 310845/2013-0 and 309748/2013-5) and by FINATEC are gratefully acknowledged.

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