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On the use of the Theory of Critical Distances to estimate the dynamic strength of notched 6063-T5 aluminium alloy

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ABSTRACT. In this paper the so-called Theory of Critical Distances is reformulated to make it suitable for estimating the strength of notched metals subjected to dynamic loading. The TCD takes as its starting point the assumption that engineering materials' strength can accurately be predicted by directly post-processing the entire linear-elastic stress field acting on the material in the vicinity of the stress concentrator being assessed. In order to extend the used of the TCD to situations involving dynamic loading, the hypothesis is formed that the required critical distance (which is treated as a material property) varies as the loading rate increases. The accuracy and reliability of this novel reformulation of the TCD was checked against a number of experimental results generated by testing notched cylindrical bars of Al6063-T5. This validation exercise allowed us to prove that the TCD (applied in the form of the Point, Line, and Area Method) is capable of estimates falling within an error interval of ±20%. This result is very promising especially in light of the fact that such a design method can be used in situations of practical interest without the need for explicitly modelling the non-linear stress vs. strain dynamic behaviour of metals.

KEYWORDS. Theory of Critical Distances; Notches; Dynamic fracture; Al6063-T5.

INTRODUCTION

It is well-known that, at room temperature, the mechanical behaviour of engineering materials under quasi-static loading is different from their behaviour under dynamic loading [1]. Focussing attention on the material strength, it is seen from the experiments (see, for instance, Refs [2-8] and references reported therein) that the failure stress, σ_f, tends to increase as the loading/strain rate increases, this holding true both for steels and aluminium alloys. On the contrary, as far as the resistance to the propagation of cracks is concerned, examination of the state of the art [2, 8] suggests that the fracture toughness can either decrease, increase, or remain constant as the Stress Intensity Factor (SIF) rate increases. Tuning to finite radius stress concentrators, the few isolated investigations (see, for instance, Ref. [9, 10]) which have been published in the technical literature so far make it evident that the problem of designing notched metals against dynamic loading has never been studied systematically. Accordingly, there exists no universally accepted method which can be used in situations of practical interest to efficiently assess notched metallic components subjected to in-service dynamic loading.

In this complex scenario, this paper reports on a attempt of reformulating the so-called Theory of Critical Distances (TCD) [11] to make it suitable for estimating the dynamic strength of metals weakened by finite radius notches, the stress analysis being performed by accommodating the material non-linearities into a linear-elastic constitutive law.

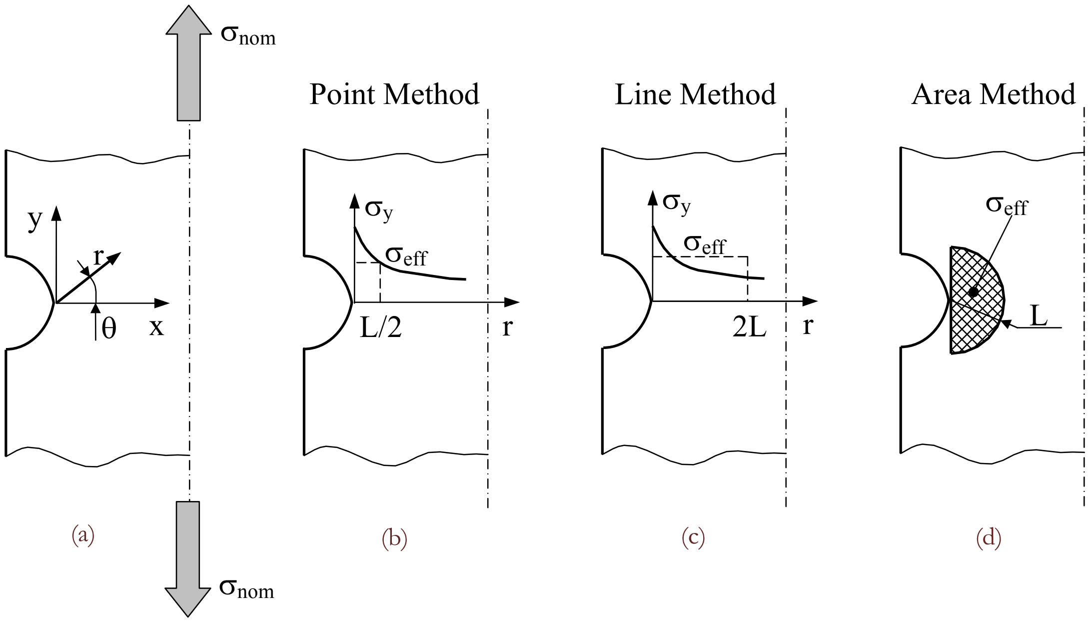

Figure 1: Definition of the local systems of coordinates (a) and effective stress, σ_{eff}, calculated according to the Point Method (b), Line Method (c), and Area Method (d).

REFORMULATING THE TCD TO DESIGN NOTCHED METALS AGAINST DYNAMIC LOADING

As far as notched materials subjected to quasi-static loading are concerned, the TCD postulates that static strength can accurately be estimated by directly post-processing the entire stress field acting on the material in the vicinity of the stress raiser being assessed [11]. The most important peculiarity of such a design method is that the required stress analysis can be performed by adopting a simple linear-elastic constitutive law, this holding true independently not only from the level of ductility characterising the mechanical behaviour of the material under investigation, but also from the degree of multiaxiality of the applied system of forces/moments [12-15].

According to the different formalisations of the TCD, the effective stress, σ_{eff}, to be used to perform the static assessment can be calculated in terms of either the Point, the Line, or the Area Method as follows [11]:

$$\sigma_{eff} = \sigma_y\left(\theta = 0, r = \frac{L}{2}\right) \qquad \text{Point Method (PM)} \qquad (1)$$

$$\sigma_{eff} = \frac{1}{2L}\int_0^{2L} \sigma_y(\theta = 0, r)dr \qquad \text{Line Method (LM)} \qquad (2)$$

$$\sigma_{eff} = \frac{4}{\pi L^2}\int_0^{\pi/2}\int_0^{L} \sigma_y(\theta, r)drd\theta \qquad \text{Area Method (AM)} \qquad (3)$$

The meaning of the adopted symbols as well as of the effective stress determined according to these three different strategies is explained in Fig. 1. In the definitions reported above, the critical distance, L, is a material property whose value can directly be estimated by combining the plane strain fracture toughness, K_{Ic}, with the so-called inherent material strength, σ_0, as follows [11-15]:

$$L = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_0}\right)^2 \tag{4}$$

Therefore, according to the TCD's *modus operandi*, a notched engineering material subjected to in-service quasi-static loading does not fails as long as the effective stress, σ_{eff}, is lower than the material inherent strength, σ_0 [11], i.e.:

$$\sigma_{eff} \le \sigma_0 \tag{5}$$

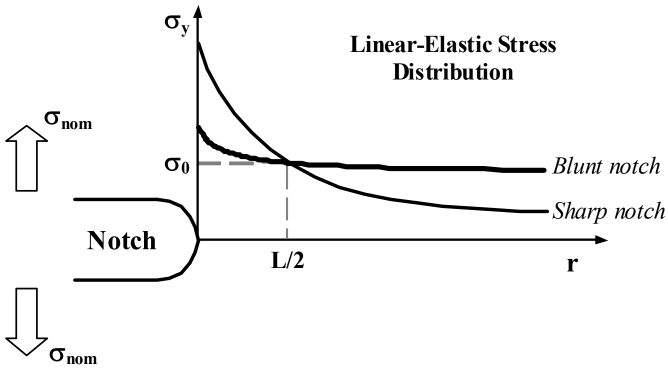

Figure 2: Determination of length scale parameter L and inherent strength σ_0 through experimental results generated by testing notches of different sharpness.

Eq. (4) and (5) make it evident that inherent material strength σ_0 plays a role of primary importance when the TCD is used to design notched components against static loading. As far as brittle materials are concerned, the inherent material strength is seen to be very close to the material ultimate tensile strength, σ_{UTS} [16]. On the contrary, when the fracture process zone experiences large scale plastic deformations, in general, σ_0 reaches a value which is somewhat larger than the ultimate tensile strength [11]. The latter consideration applies also to metallic materials, although, for certain metals, σ_0 is seen to be so close to σ_{UTS} [12] that the static assessment can accurately be performed by simply taking $\sigma_0=\sigma_{UTS}$. The considerations reported above clearly suggest that the only way to determine σ_0 is by testing notched specimens containing stress risers whose presence results in different stress distributions in the vicinity of the tested geometrical features [12-15]. Such a procedure is summarised in Fig. 2. In particular, according to the PM, the point at which the two linear-elastic stress-distance curves, plotted in the incipient failure condition, intersect each other allows both L and σ_0 to be estimated directly.

As briefly mentioned above, the state of the art [1-7] shows that, in general, the mechanical response, the mechanical properties, and the cracking behaviour of metallic materials subjected to dynamic loading are different from the ones observed under quasi-static loading. Having recalled this important aspect, the hypothesis can be formed that, similar to the dynamic failure stress, σ_f, and the dynamic fracture toughness, K_{Id}, both the inherent material strength, σ_0, and the critical distance, L, may vary as the applied loading rate increases. Therefore, according to definition (4), the critical distance under dynamic loading can be rewritten as follows:

$$L(\dot{F}) = \frac{1}{\pi}\left[\frac{K_{Id}(\dot{F})}{\sigma_0(\dot{F})}\right]^2 \tag{6}$$

where $\dot{F}$ is used to denote the loading rate. This new definition for critical distance L allows the effective stress to be determined under dynamic loading via the following definitions which are directly derived from Eqs (1) to (3):

$$\sigma_{eff}(\dot{F}) = \sigma_y\left(\theta = 0, r = \frac{L(\dot{F})}{2}\right) \qquad \text{(PM)} \tag{7}$$

$$\sigma_{eff}(\dot{F}) = \frac{1}{2L(\dot{F})} \int_0^{2L(\dot{F})} \sigma_y(\theta = 0, r)\ dr \qquad \text{(LM)} \tag{8}$$

$$\sigma_{eff}(\dot{F}) = \frac{2}{\pi L(\dot{F})^2} \int_{-\pi/2}^{\pi/2} \int_0^{L(\dot{F})} \sigma_1(\theta, r)\ r\ dr d\theta \qquad \text{(AM)} \tag{9}$$

Therefore, a notched component is supposed to be able to withstand the applied dynamic loading as long as the following condition is assured:

$$\sigma_{eff}(\dot{F}) \leq \sigma_0(\dot{F}) \tag{10}$$

Owing to the complexity of the reasoning summarised in the present section, it is evident that a set of appropriate experimental results is required to check the validity of the formed hypotheses. This will be done in the next section.

VALIDATION BY EXPERIMENTAL DATA

In order to check the accuracy of the proposed reformulation of the TCD in predicting the strength of notched metals subjected to dynamic loading, an *ad hoc* experimental investigation was run in the testing laboratory of the Sheffield University at Harpur Hill, Buxton, UK. In more detail, twenty-six cylindrical samples of Al6063-T5 were tested under both quasi-static and dynamic axial tensile loading. The loading was applied to the proximal end of the test sample, through a cross-head beam driven by pneumatic pressure. The distal end of the specimen was connected to a dynamic load cell which itself was connected to a stiff reaction frame. Quasi-static loading was generated by slowly increasing the pneumatic driving load on the cross head, whilst dynamic loading was produced by storing pressurised air in a reservoir and releasing this suddenly to the cross-head by bursting a retaining diaphragm at the outlet of the reservoir. The un-notched specimens had net diameter, d_n, equal to 5 mm and gross diameter, d_g, to 10 mm. The bluntly notched specimens had d_n=5 mm, d_g=10 mm, and notch root radius r_n equal to 4 mm, these resulting in a net stress concentration factor, K_t, of 1.25. The samples containing both the intermediate and the sharp stress concentrators had d_n=5.2 mm and d_g=10 mm, the notch root radius being equal to 1.38 mm (K_t=1.69) and to 0.38 (K_t=2.93), respectively. The generated results are summarised in Tab. 1 in terms of failure force, F_f, time to failure, T_f, and loading rate, $\dot{F}$. In particular, the failure force, F_f, was taken equal to the maximum force recorded during each test, the corresponding instant being used to define T_f. Accordingly, the loading rate, $\dot{F}$, was directly estimated via the following trivial relationship:

$$\dot{F} = \frac{F_f}{T_f} \tag{11}$$

The linear-elastic stress fields required to calculate the effective stress, σ_{eff}, according to definitions (7) to (9) were determined by solving Finite-Element (FE) models done using commercial software ANSYS®. In these linear-elastic FE models the mesh in the vicinity of the notch apices was refined until convergence occurred.

By assuming that $\sigma_0(\dot{F})$ could be taken equal to $\sigma_f(\dot{F})$ [12], the results generated by testing both the plain and the sharply notched specimens were used, via a conventional best-fit procedure, to determine both material inherent strength $\sigma_0(\dot{F})$ and critical distance $L(\dot{F})$, by obtaining:

$$\sigma_0(\dot{F}) = \sigma_f(\dot{F}) = 209.9 \cdot \dot{F}^{0.0118}\ \text{[MPa]} \tag{12}$$

$$L(\dot{F}) = 1.541 \cdot \dot{F}^{0.0368}\ \text{[mm]} \tag{13}$$

Tab. 1 summarises the accuracy of the TCD (applied in the form of the PM, LM, and AM) in estimating the strength of the notched specimens we tested under both quasi-static and dynamic axial loading, the error being calculated as follows:

$$\text{Error} = \frac{\sigma_0(\dot{F}) - \sigma_{eff}(\dot{F})}{\sigma_{eff}(\dot{F})}\ [\%] \tag{14}$$

Tab. 1 makes it evident that the use of the proposed reformulation of the TCD resulted in estimates falling within an error interval of ±20% (that is, in an accuracy level similar to the one which is usually obtained when the TCD is used in other ambits of the structural integrity discipline [11]). Such a high level of accuracy is certainly promising, especially in light of the fact that it was reached without the need for explicitly modelling the stress vs. strain dynamic behaviour of the tested aluminium alloy. Accordingly, if the reliability and accuracy of the proposed approach were further confirmed by considering different metallic materials, this reformulation of the TCD could allow practitioners to perform the dynamic assessment of notched ductile components by remarkably reducing the time and costs of the design process.

Code	d_g [mm]	d_n [mm]	r_n [mm]	K_t	F_f [kN]	T_f [s]	$\dot{F}$ [kN/s]	**Error** [%] **PM**	**LM**	**AM**
S1 T1	10.0	5.0	Plain	1.00	3.8	4.1	0.9268			
S1 T2	10.0	5.0	Plain	1.00	4.8	0.08	60.00			
S1 T3	10.0	5.0	Plain	1.00	4.1	35	0.1171			
S1 T5	10.0	5.0	Plain	1.00	4.6	0.05	92.00			
S1 T6	10.0	5.0	Plain	1.00	4	0.02	200.0			
S1 T7	10.0	5.0	Plain	1.00	4.4	0.006	733.3			
S1 T8	10.0	5.0	Plain	1.00	4.5	0.005	900.0			
S1 T11	10.0	5.0	Plain	1.00	4.1	0.004	1600			
S1 T12	10.0	5.0	Plain	1.00	4.7	0.01	470.0			
S1 T9	10.0	5.2	0.38	2.93	5.4	22	0.2455	-0.1	-2.2	4.4
S1 T10	10.0	5.2	0.38	2.93	6.7	0.004	1675	-1.4	3.0	3.9
S2 T1	10.0	5.2	0.38	2.93	6.8	0.007	971.4	1.4	5.4	6.9
S2 T2	10.0	5.2	0.38	2.93	6.7	0.007	957.1	0.0	3.9	5.3
S1 T17	10.0	5.2	1.38	1.69	4.6	29	0.1586	-1.5	-11.3	-3.0
S1 T18	10.0	5.2	1.38	1.69	6.2	0.003	2066.7	5.2	2.5	3.4
S2 T5	10.0	5.2	1.38	1.69	5.3	21	0.2524	12.2	1.2	10.5
S2 T6	10.0	5.2	1.38	1.69	5.1	16	0.3188	7.3	-3.0	5.7
S2 T7	10.0	5.2	1.38	1.69	6.7	0.007	957.1	15.9	11.8	13.8
S2 T9	10.0	5.2	1.38	1.69	6.2	0.009	688.9	8.1	3.9	6.2
S2 T10	10.0	5.2	1.38	1.69	6.9	0.007	985.7	19.3	15.1	17.1
S2 T11	10.0	5.2	1.38	1.69	4.9	11	0.4455	2.3	-7.4	0.7
S2 T12	10.0	5.2	1.38	1.69	5.2	16	0.3250	9.4	-1.1	7.7
S2 T13	10.0	5.2	1.38	1.69	6	0.007	857.1	4.1	0.2	2.2
S2 T14	10.0	5.2	1.38	1.69	5.9	0.009	655.6	3.0	-1.1	1.2
S1 T15	10.0	5.0	4.00	1.25	3.7	30	0.1233	-9.1	-16.4	-9.7
S1 T16	10.0	5.0	4.00	1.25	3.5	23	0.1522	-14.3	-21.2	-15.0

Table 1: Summary of the generated experimental results and accuracy of the proposed re-formulation of the TCD in estimating the quasi-static/dynamic strength of the tested notched samples of Al6063-T5.

CONCLUSIONS

1) The proposed reformulation of the TCD was seen to be successful in estimating the strength of the notched specimens of Al6063-T5 we tested under both quasi-static and dynamic axial loading.
2) Thanks to its specific features, the proposed design methodology allows real notched components to be designed against dynamic loading by directly post-processing the relevant stress fields estimated via conventional linear-elastic FE models. This implies that an accurate assessment can be performed without the need for explicitly modelling the material response under dynamic loading.

3) More work needs to be done in this area to further check the accuracy and reliability of the proposed design methodology against experimental results generated by testing, under dynamic loading, notched metallic materials characterised by different levels of ductility.

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