A probabilistic interpretation of the Miner number for fatigue life prediction

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ABSTRACT. The Miner number M, used as a tool for lifetime prediction of mechanical and structural components in most of the standards related to fatigue design, is generally accepted as representing a damage stage resulting from a linear progression of damage accumulation. Nonetheless, the fatigue and damage approach proposed by Castillo and Fernández-Canteli, permits us to reject this conventional cliché by relating M to the normalized variable V, which represents percentile curves in the S-N field unequivocally associated to probability of failure. This approach, allowing a probabilistic interpretation of the Miner rule, can be applied to fatigue design of mechanical and structural components subjected to variable amplitude loading. The results of an extensive test program on concrete specimens under compressive constant and load spectra, carried out elsewhere, are used. A parallel calculation of the normalized variable V and the Miner number M is performed throughout the damage progression due to loading allowing probabilities of failure to be assigned to any value of the current Miner number. It is found that significant probabilities of failure, say P=0.05, are attained for even low values of M, thus evidencing the necessity of a new definition of the safety coefficient of structural or machine components when the Miner rule is considered. The experimental and analytical probability distributions of the resulting Miner numbers are compared and discussed, the latter still providing a non-conservative prediction in spite of the enhancement. A possible correction is analyzed.

KEYWORDS. Cumulative damage; Miner number; Statistical interpretation.

INTRODUCTION

The cumulative concept proposed by Palmgren and Miner maintains that the damage level can be expressed in terms of the number of cycles applied at a given stress range divided by the number of cycles needed to produce failure for the same stress range. Failure occurs when the summation of these damage increments at several stress ranges becomes unity. After this formulation, this rule is repeatedly tested for different materials under multi-step and variable amplitude loading programs. Though its applicability has been often questioned, it has been practically adopted by all standards related to structural and mechanical fatigue design.

While Birnbaum and Saunders [1] tried to find a relation of the probabilistic distribution of the Miner number to the crack growth, Van Leeuwen and Siemes [2, 3] conducted series of tests on plain concrete and interpreted directly the scatter of the Miner number M by obtaining theoretical expressions for the mean and standard deviation values of M from the Wöhler curve. These formulae, initially derived for the simple case of constant amplitude cycling were then extended to the case of general loading. They showed that the Miner number M at failure is a stochastic variable with an approximate log-normal distribution and emphasized the importance of the study of the scatter of the Wöhler curve for constant amplitude cycling. Based on Holmen's investigation on concrete [4], Fernández-Canteli [5] justified a generalization of the Van Leeuwen and Siemes work by considering a probabilistic S-N field providing a statistical distribution of the Miner number although based on a log-normal distribution. Some theoretical advances were performed in [6] and [7].

From this, it follows that the Miner number can be used to ascertain the probability of failure, as a more suitable design criterion, rather than as a measure of a problematic and abstract "degree of damage". It can then be taken as a basis for a consistent life prediction in fatigue design, in accordance with the consideration of fatigue failure as limit state.

RESULTS FROM HOLMEN

In this Section, the fatigue results for concrete specimens under compression provided in the study of Holmen [4] are introduced and eventually adapted in order to proceed to the probabilistic interpretation of the Miner Number.

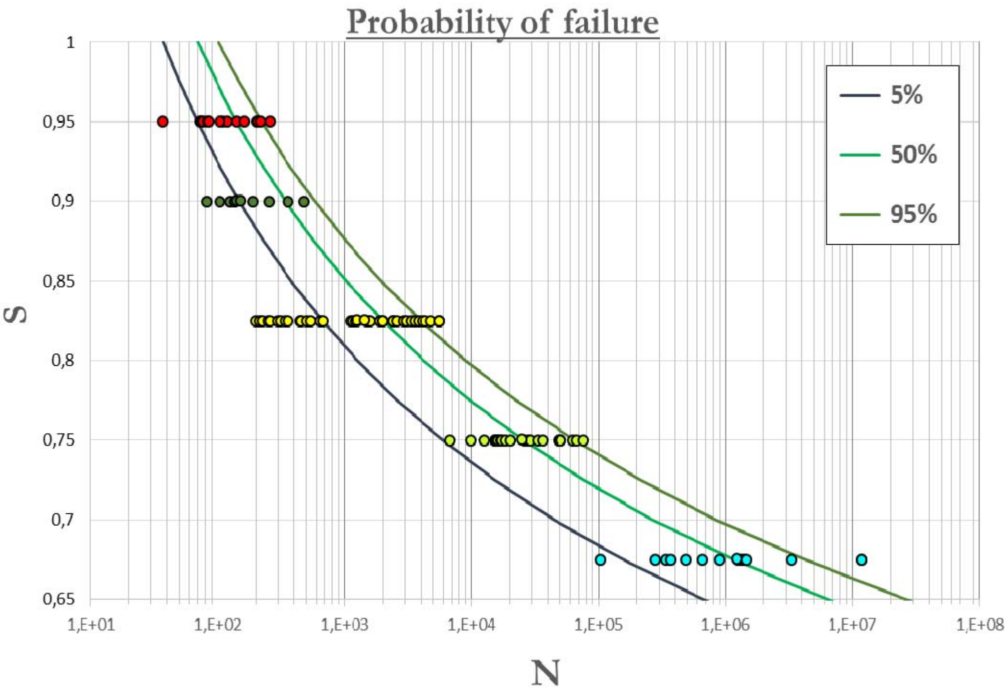


Figure 1: S-N field fitted with the Mc Call model [7] for the normalized fatigue results for concrete under compression from Holmen [4].

S-N fatigue results for constant stress level

Fig. 1, shows the S-N field resulting from the fatigue data obtained in Holmen's fatigue experimental program, according to Tab. 1, using the procedure proposed by Mc Call [8]. The tests were carried out for constant stress range $S=S_{max}-S_{min}=(\sigma_{max}-\sigma_{min})/\sigma_R$ and constant minimum stress level ($S_{min}=\sigma_{min}/\sigma_R=0.05$), where σ_{max} and σ_{min} are, respectively, the maximum and minimum stress applied during the test and σ_R is the fracture stress of the concrete This model, based on fitting a regression hyperbola linking the respective percentiles values resulting from the cumulative distribution functions (cdf) at any of the intervening stress ranges seems to be applicable only when a large number of results are at disposal, as this is the case, with an extensive amount of data. Nevertheless, it does not fulfil "a priori" the requested physical and statistical requirements for a model to be valid, in particular, the compatibility between the cumulative distribution functions of the number of cycles given stress range $F[N;\Delta\sigma]$ and of the stress range given number of cycles $E[\Delta\sigma;N]$. As an alternative, the S-N resulting under consideration of the Weibull (or Gumbel) probabilistic model proposed by Castillo and Fernández-Canteli is shown in Fig. 2. The parameters estimated using the latter model allows us to define the normalized variable $V=(\log N\text{-}B)(\log \Delta\sigma\text{-}C)$ as a Weibull (or Gumbel) distribution for minima, thus facilitating the relation of the V values to the probability of failure.

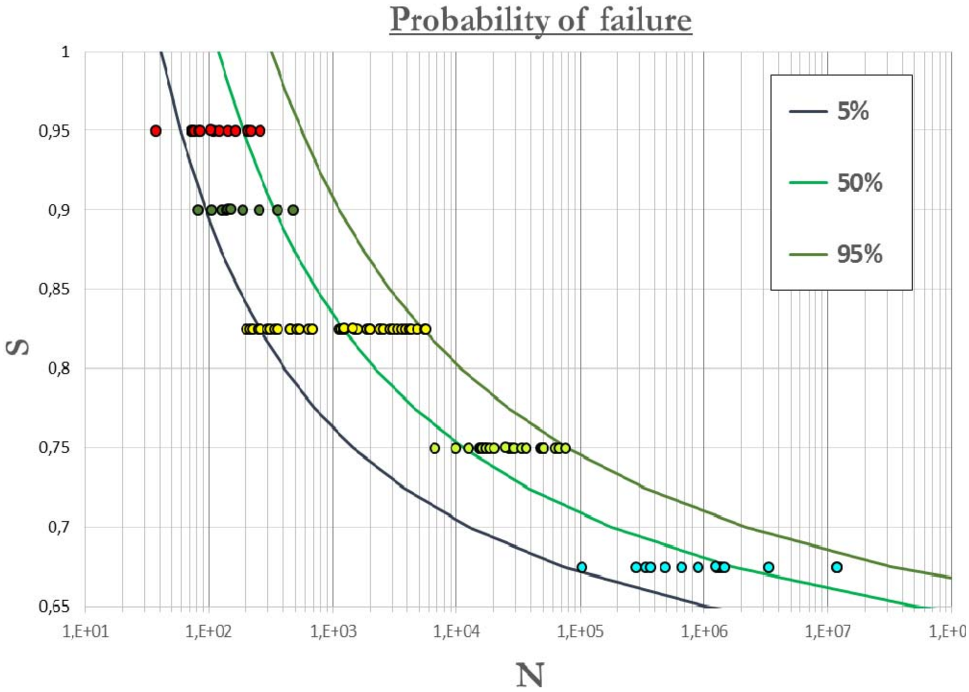


Figure 2: S-N field fitted with the Weibull model of Castillo-Canteli [6] for the normalized fatigue results for concrete under compression from Holmen [4].

S	Number of cycles to failure
0.95	257; 74; 105; 120; 206; 83; 123; 109; 37; 76; 143; 85; 203; 72; 217
0.90	356; 201; 295; 252; 680; 509; 540; 311; 257; 457; 216; 226; 451; 1129; 342
0.825	1246; 2590; 5560; 4820; 2410; 2400; 4110; 3590; 3330; 1460; 1258; 5598; 3847; 1492; 2903
0.75	16190; 27940; 67340; 1860; 12600; 6710; 26260; 50090; 15570; 9930; 20300; 48420; 24900; 36350; 17280
0.675	3294820; 1459140; 1329780; 1241760; 339830; 896330; 280320; 102950; 658960; 1399830; Run-Out; 485620; 366900; 1250200; 11784100 (Run-Out); Run-Out

Table 1: Results of the fatigue tests under constant stress range loading for S_{min}=0.05. From Holmen [4].

Load collective and basic loading block used

When a continuous load collective is used for fatigue design or fatigue testing as a practical representation of the real random or pseudo-random load history, it may be discretized as a histogram and handled as a multi-step load sequence. In the variable amplitude tests of Holmen's investigation [4], the histogram called loading model 3, represented in Fig. 3a, was derived from a loading collective, consisting in 30 steps of the maximum normalized stress $S_{max}= \sigma_{max}/ \sigma_R$. After truncation at level 18 and omission of stress ranges below level 9, which implies to discard the lower stress levels, presumably without causing damage, the definitive stress collective in Fig. 3b, considered as representative of the effective stress being supported by the concrete in the real off-shore structure, was applied to the specimens in a pseudo-random sequence.

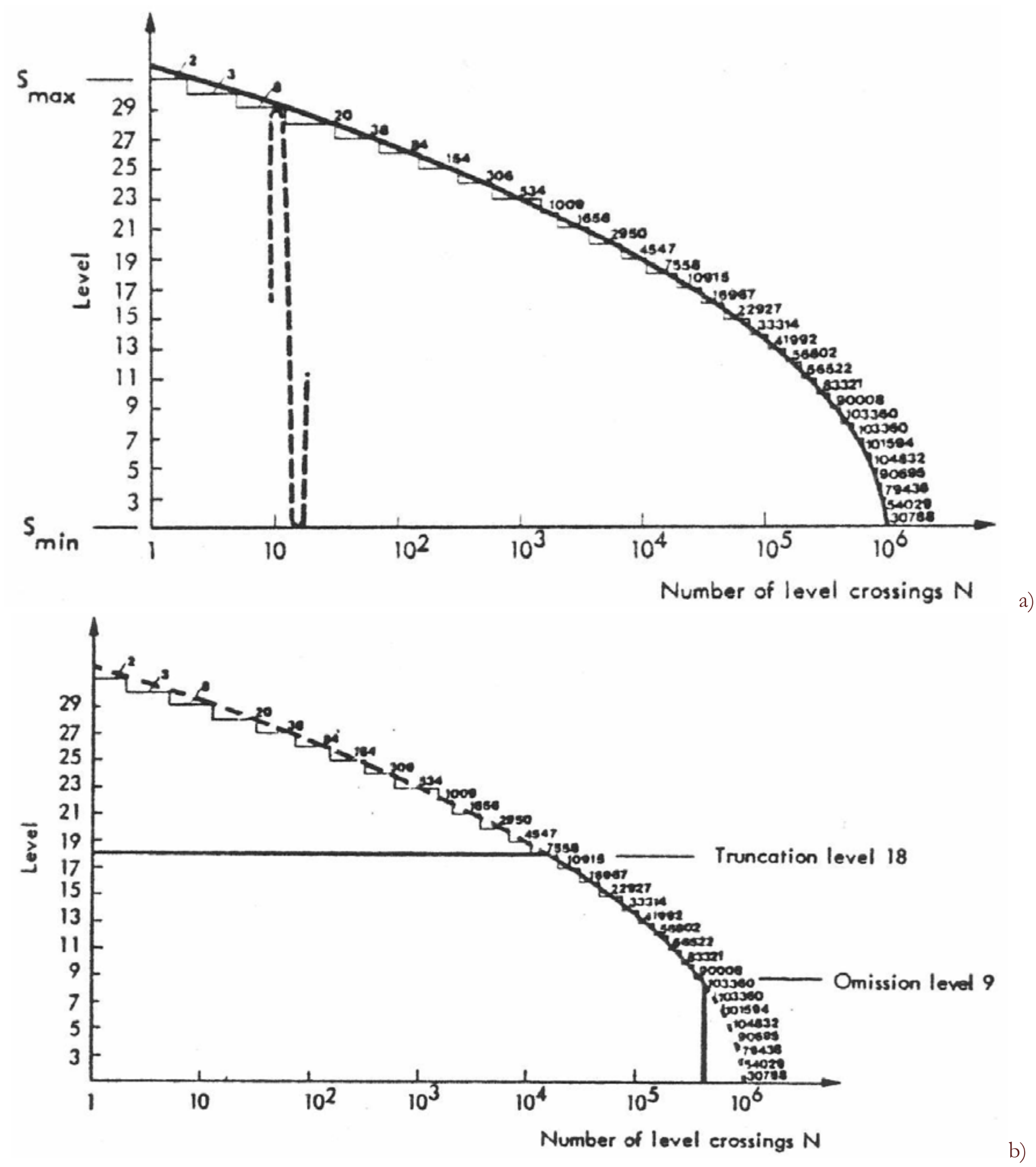


Figure 3: Holmen's load collectives from [4]: a) Original load histogram consisting in 30 stress steps and b) Loading model 3 with truncation of higher stress amplitudes and omission of lower stress amplitudes applied in the fatigue variable testing.

In order to reduce as much as possible the influence of the sequence, a proportional fraction of the number of cycles represented in the histogram, the so called "basic loading block", will repeatedly applied until failure. The smaller the length of the basic loading block, the closer the random nature of real load applied, and the better the agreement between the total number of cycles applied during the test and its correspondence to the different stress ranges intervening in the load collective for the purpose of the Miner number calculation. An improvement of the real sequence could be achieved by arranging the stress range sequence intervening in the basic stress block in a random manner. After direct conversion of the load to stress for the cylindrical specimens used in the test, the stress ranges corresponding to the 18 levels considered in the load collective, the number of cycles in the original block and those in the basic stress block are exposed in Tab. 2. The number of total cycles to failure for the tests performed under variable loading according to Holmen and the corresponding number of replications of the basic stress blocks necessary to achieve failure are displayed in Tab. 3.

Stress range level	Normalized stress range	Number of cycles corresponding to the original histogram	Number of cycles corresponding to the basic loading block	Accumulated number of cycles applied in the original histogram	Accumulated number of cycles applied in the basic loading block
18	0.7750	18869	25	18869	25
17	0.7324	10915	14	29784	39
16	0.6897	16967	22	46751	61
15	0.6471	22927	30	69678	91
14	0.6044	33314	44	102992	135
13	0.5618	41992	55	144984	190
12	0.5191	56802	75	201786	265
11	0.4765	66522	88	268308	353
10	0.4338	83321	111	351629	464
9	0.3912	90008	120	441637	584
8	0.3485	103360	137	544997	721
7	0.3059	103360	137	648357	858
6	0.2632	101594	135	749951	993
5	0.2206	104832	139	854783	1132
4	0.1779	90695	120	945478	1252
3	0.1353	79438	105	1024916	1357
2	0.0926	54029	72	1078945	1429
1	0.0500	30788	41	1109733	1470

Table 2: Stress ranges corresponding to the 18 levels considered in the load collective and number of cycles in the original block and basic stress block

Test ref.	S_{max}	N_f	Miner	Number of basic load blocks
1	0.775	294670	0.6	201
2	0.775	291911	0.58	199
3	0.775	687112	1.38	468
4	0.775	76328	0.15	52
5	0.775	143992	0.29	98
6	0.775	548075	1.12	373
7	0.775	809727	1.64	551
8	0.8	49698	0.22	34
9	0.8	64508	0.3	44
10	0.825	25537	0.25	18
11	0.825	55710	0.53	38
12	0.825	92798	0.93	64
13	0.825	102330	1.04	70
14	0.836	31890	0.5	22
15	0.836	31971	0.51	22
16	0.836	20388	0.22	14
17	0.836	83638	1.11	57
18	0.775	41038	0.18	71
19	0.775	189486	0.86	325
20	0.775	213384	0.83	366
21	0.775	161450	0.58	277
22	0.775	28714	0.13	50
23	0.775	43345	0.19	75
24	0.775	72929	0.34	125
25	0.775	32715	0.14	57
26	0.8	28717	0.3	50
27	0.8	39697	0.41	68
28	0.825	33856	0.75	58
29	0.825	9615	0.2	17

Test ref.	S_{max}	Nf	Miner	Number of basic load blocks
30	0.825	4416	0.08	8
31	0.825	30583	0.68	53
32	0.836	8724	0.25	15
33	0.836	34492	1.04	60
34	0.836	20299	0.61	35
35	0.836	47683	1.47	82
36	0.836	27093	0.78	47
37	0.836	12300	0.41	22
38	0.743	34490	0.07	60
39	0.743	178077	0.35	305
40	0.743	79856	0.16	137
41	0.743	153162	0.3	263
42	0.743	75187	0.15	129
43	0.743	85593	0.17	147
44	0.743	96517	0.19	166
45	0.743	192318	0.38	330
46	0.756	120790	0.41	207
47	0.756	22476	0.08	39
48	0.779	49445	0.4	85
49	0.779	15298	0.14	27
50	0.779	66704	0.55	115
51	0.779	38960	0.31	67
52	0.79	55944	0.64	96
53	0.79	106438	1.29	183
54	0.79	58068	0.69	100
55	0.79	40810	0.48	70
56	0.79	51615	0.6	89
57	0.79	21427	0.25	37

Table 3: Number of total cycles N_f applied until failure, experimental Miner numbers resulting from the Holmen's variable loading tests and corresponding number of replications of the basic stress blocks necessary to achieve failure.

Experimental Miner number	Miner number (basic stress block based)	Experimental Miner number	Miner number (basic stress block based)	Experimental Miner number	Miner number (basic stress block based)
0.08	0.118	0.30	0.300	0.60	0.577
0.07	0.074	0.29	0.281	0.55	0.586
0.08	0.083	0.30	0.341	0.64	0.655
0.14	0.137	0.50	0.441	0.68	0.779
0.13	0.143	0.51	0.441	0.69	0.682
0.15	0.149	0.41	0.441	0.75	0.852
0.14	0.164	0.31	0.341	0.78	0.942
0.20	0.250	0.30	0.323	0.93	0.940
0.22	0.280	0.34	0.359	0.58	0.795
0.25	0.264	0.35	0.375	1.04	1.028
0.25	0.301	0.41	0.464	1.11	1.142
0.15	0.159	0.40	0.433	0.86	0.932
0.22	0.232	0.53	0.558	1.04	1.202
0.16	0.168	0.38	0.406	0.83	1.050
0.18	0.204	0.48	0.477	1.12	1.070
0.17	0.181	0.41	0.438	1.29	1.248
0.19	0.215	0.61	0.701	1.38	1.342
0.25	0.252	0.58	0.571	1.47	1.643
0.19	0.204	0.60	0.607	1.64	1.581
0.08	0.118	0.30	0.300	0.60	0.577

Table 4: Comparison between the Miner numbers estimated using the basic stress block approach proposed in this work and those directly overtaken from Holmen (shown in increasing order).

The probabilistic S-N field

For the damage assessment of the variable loading test results when the Miner approach is applied, the fatigue Weibull regression model proposed by Castillo and Fernández-Canteli the derivation of which is extensively justified in [6]. The consideration of the compatibility condition between the lifetime and stress range distributions, see Fig. 4, besides other physical and statistical considerations, leads to a functional equation, the solution of which provides the following S-N field:

$$F\left(N;\Delta\sigma\right)=1-\exp\left\{-\left[\frac{\left(\log N-B\right)\left(\log\Delta\sigma-C\right)-\lambda}{\delta}\right]^{\beta}\right\} \quad ; \quad \left(\log N-B\right)\left(\log\Delta\sigma-C\right)\geq\lambda, \tag{1}$$

where B and C are, respectively, a limit or threshold number of cycles and fatigue limit for $N\rightarrow\infty$ and β, λ and δ are, respectively, the Weibull shape, location and scale parameters. The percentile curves are hyperbolas sharing the asymptotes $\log N = B$ and $\log\Delta\sigma = C$ (see Fig. 4), with the zero percentile curve representing the minimum possible required number of cycles to achieve failure for different values of $\log\Delta\sigma$.

The model parameters can be determined with the free software program ProFatigue [9] in a two-step procedure: first B and C, then the Weibull parameters β, λ and δ using well-established methods described in the literature. As soon as the five parameters are estimated, the whole S-N field is analytically defined enabling a probabilistic prediction of the fatigue failure under constant amplitude loading to be achieved, see Fig. 5.

From Eq. (1) it is apparent that the probability of failure for an element subject to a stress range $\Delta\sigma$ during N cycles depends only on the product $V=(\log N-B)(\log\Delta\sigma-C)$. This illustrates that, as soon as B and C are known, V becomes a normalizing variable of the data results permitting the whole experimental data set to be pooled in a single Weibull distribution with the same shape parameter β as they would pertain to a single sample. This statistical normalization proves to be a suitable and powerful procedure increasing the reliability of the parameter estimation, allowing the whole *S-N* field to be described by a unique Weibull distribution function.

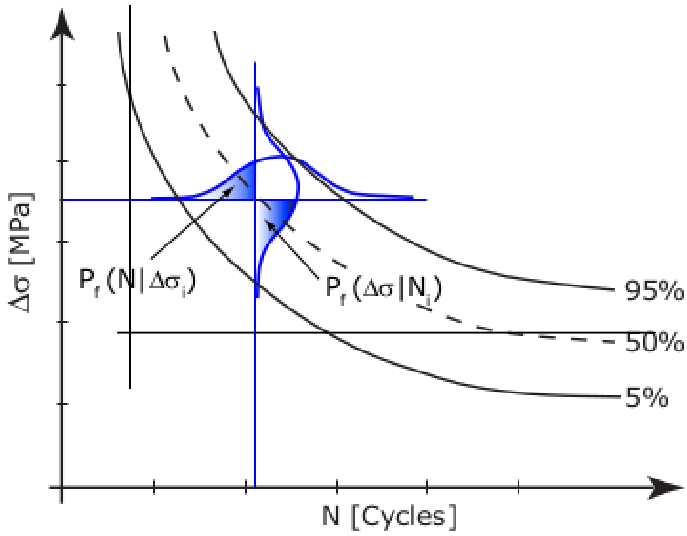
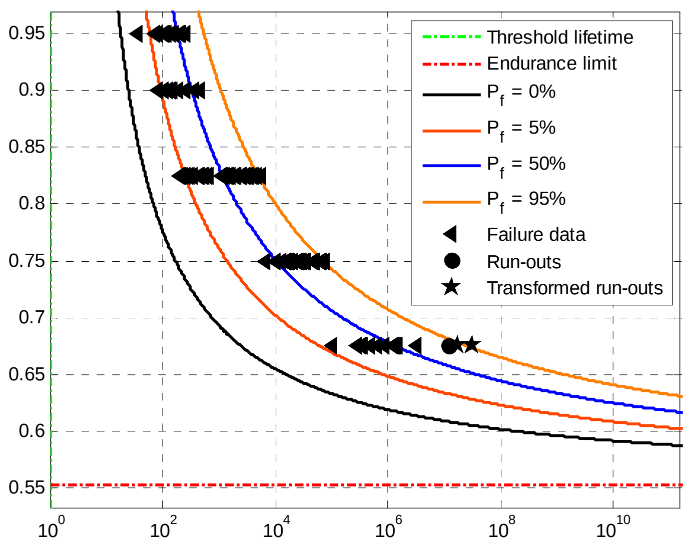


Figure 4: S-N field according to [6] illustrating the compatibility condition.

This means that any V value is associated to a percentile curves but also to a damage stage that may be unequivocally related to a probability of failure. In this way, an extension of lifetime prediction under varying load is achieved by identifying damage with the *V* value, and the *V* value with the probability of failure represented by its cumulative distribution function.

Now, the approach proposed consists in deriving the cdf for the Miner number, as resulting from the S-N field found and the stress history applied. With this aim, simultaneous calculation of the normalized variable *V* and the corresponding Miner number *M* at any stage of the loading history makes it possible to relate any Miner number along the damage process to a probability of failure, i.e. mapping of the Miner number into a cumulative distribution function [7].



ESTIMATED PARAMETERS				
B	C	β	δ	λ
0	-0.59	3.86	1.39	1.56

Figure 5: Weibull S-N field with estimated parameters derived from Holmen's constant stress range concrete tests using the probabilistic fatigue model from [6].

CALCULATION OF THE MINER NUMBER AND ITS PROBABILISTIC ANALYSIS ACCORDING TO THE APPROACH PROPOSED

In this Section, the Miner numbers obtained in the experimental program of Holmen [4] are estimated for the concrete specimens subject to pseudo-random load using the load collective described in the precedent Section as being representative of the real load to which the concrete specimen is subjected during each test. In fact, three different test series, see Tab. 4, were performed with small differences among the stress collectives applied, the influence of which is disregarded. Thereafter, these Miner number results are related to the normalized variable *V* and, subsequently, to probability of failure. Finally, this theoretical Miner number distribution will be compared with that obtained directly from the experimental results.

Since the total number of cycles, but not the real pseudo-random loading sequence, applied during Holmen's varying loading tests [4] is provided, only an estimation of the real loading history can be achieved as the number of replications of the basic stress block necessary to accomplish the total number of cycles. As a result, the value of the Miner number obtained for the different tests using the substitutive basic stress block appraisal differs from that given by Holmen, see Tab. 4, which displays the Miner numbers as directly overtaken from Holmen and those estimated by equating the total number of cycles resulting for repeated application of the basic stress block. A median error of about 10% is observed, which seems to be acceptable for this study as the calculated Miner numbers represents an underestimation of the real ones. A practical coincidence between both Miner number families may be enforced by adequately determination of the number of replications, with a possible fraction of the last replication, irrespective of the total number of cycles considered, i.e. alternative to the values contained in Tab. 3.

Once the Miner number resulting for any of the real experimental tests is found as a result of the application of the prescribed number of replications of the basic stress blocks, we proceed to establish the probability of failure associated with them. First, the S-N field is evaluated from the constant stress range tests using the ProFatigue code, see Fig. 5. This provides the model parameters and accordingly, the cdf of the normalizing variable $V=(\log N-B)\ (\log \Delta\sigma-C)$ thus relating *V* values to probability of failure, see Fig. 6.

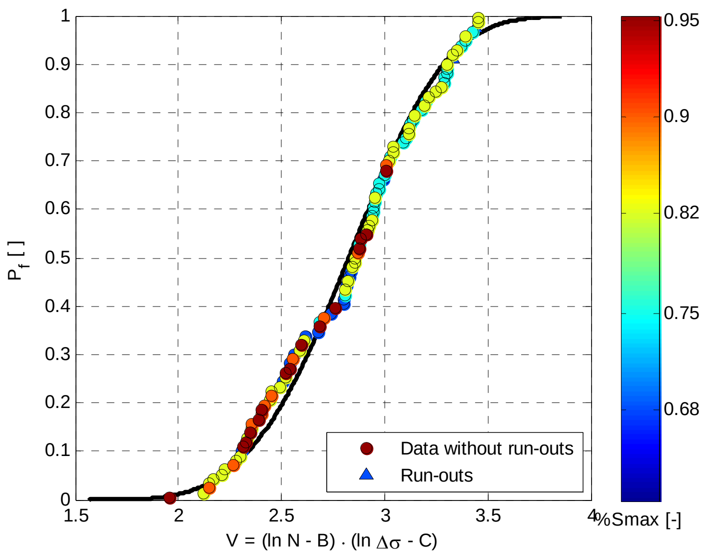


Figure 6: Experimental Weibull cumulative distribution functions for the normalized variable *V* obtained from the fatigue results under constant stress range tests of Holmen [4] using the probabilistic fatigue model of Castillo and Fernández-Canteli [6].

For each test, an experimental Miner number M_i has been obtained from the particular stress history "*i*" applied during the test consisting in a number of replications of the basic stress block according to Tab. 3. The same stress history provides the corresponding value of the normalized variable V_i for such a test, the probability of failure related to which is given by the cdf of the normalized variable *V*. In this way, the same probability of failure obtained for the V_i value is assigned to the corresponding value of the experimental Miner number M_i. Thus, an unequivocally correspondence between the experimental *V* values and those of the Miner number *M* is established, allowing the cdf predicted for the experimental Miner results to be established by simple mapping of the Miner numbers obtained for the experimental test failures into probabilities of failure, see Fig. 7. Accordingly, an approach for probabilistic lifetime prediction is established by means of the normalized variable *V* that improves the reliability provided by the conventional Miner rule.

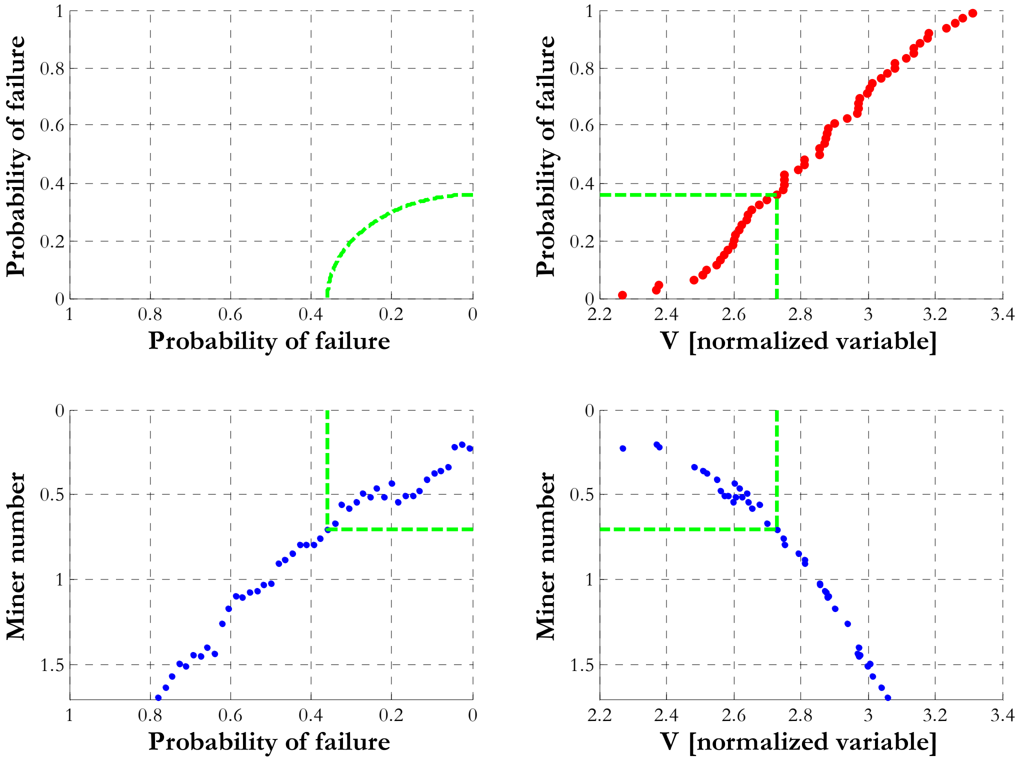
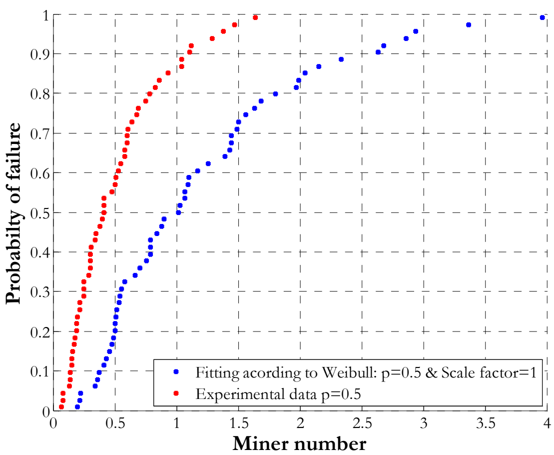
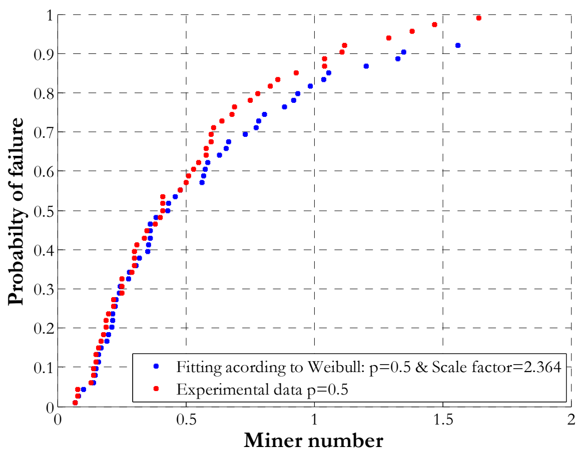


Figure 7: From the experimental Weibull cumulative distribution function for the normalized variable *V* (upper-right) and the established relation between *V* and *M* (down-right), the experimental cdf of the Miner number (down-left) is found for the fatigue of Holmen [5].

Now, the cumulative distribution function for the Miner number sample obtained from the results of the experimental program with varying stress range tests of Holmen is determined by applying any plotting point position rule. The Miner number values are ranged in increasing order and the corresponding probabilities of failure calculated. The resulting cdf is compared with that predicted from the normalized variable V distribution, as shown in Fig. 8.



(a)



(b)

Figure 8: a) Cumulative distribution functions for the Miner number fitted, respectively, from the experimental results and from the approach proposed based on the normalized variable V assuming a Weibull distribution and b) correspondence among those distributions when a scale correction is applied to the experimental results.

The precedent procedure is summarized as follows:

a) From Holmen's results, the experimental probabilistic S-N field is evaluated, in this case, using the ProFatigue program based on the Weibull model proposed by Castillo and Fernández-Canteli [6].
b) The load collective is defined from standards or other regulations related to the particular type of the structure considered.
c) A histogram is derived from the stress collective in a sufficient number of stress steps to guarantee accurateness in the calculations. In this case, the results provided by Holmen [4] are considered. Possible truncation and omission of some levels of the histogram are undertaken in order to approximate the histogram to the practical load distribution.
d) Since the real load sequence applied to the specimens is unknown, though the peak distribution applied during the test is defined, an approximation is assumed by considering a proportional reduction of the original load collective. In this case, a basic loading block is defined as 1/750 times of the original load histogram.
e) The Miner number obtained for any of the real tests is calculated. Herewith, the factual pseudo-random load history applied by Holmen's tests is not exactly known but can be approximated by means of the basic stress block derived from the load collective. A comparison is made between the Miner number calculated using repetitions of until the total number of cycles corresponds to that given by Holmen and the Miner number provided by Holmen. Differences aro und 10% are found these being considered acceptable.
f) For any test, the necessary repetitions of the basic stress block are evaluated as those giving the same total number of cycles to failure found by Holmen.
g) For any test, the normalized variable $V= (\log N-B) (\log \Delta\sigma-C)$ is calculated for the test stress history up to failure owning to the particular Miner number obtained for that test by replications of the basic stress block.
h) Since the cdf for V is defined according to the probabilistic fatigue model, and the correspondence between V and the Miner number M is established, it is possible to derive the cdf for the Miner number values, that is, it is possible to relate any value of the Miner number to the corresponding probability of failure. The immediate relation of the Miner number and probability is established.
i) The cdf of the experimental Miner number is calculated and compared with the theoretical one. A simple correction of the scale parameter lead to good agreement for fatigue lifetime prediction.

DISCUSSION OF THE RESULTS

Fig. 6 displays the cumulative distribution function corresponding to the experimental Miner numbers obtained from the test program of Holmen and the predicted cdf of the Miner number derived from the normalized variable V, which on its turn is calculated from the initial S-N field for constant stress range tests of Holmen. Since V belongs to a three parameter Weibull distribution family according to the probabilistic model of Castillo and Fernández-Canteli it is expected that the Miner number also belongs also to a three parameter Weibull family, as stated in [9], The location parameter of the M distribution $\lambda(M)$, i.e. the threshold M value below which the probability of failure is zero, is defined as that value of M associated with the location parameter for V, $\lambda(V)$. Account given of the small value of $\lambda(M)$, nullity of $\lambda(M)=0$ can be accepted, which implies the Miner number being described by a two-parameter Weibull distribution. Under this assumption, the parameter estimates of both M distributions, i.e. the experimental and the theoretical ones, are, respectively, β=1.436, δ=0.568 for the experimental Miner number and $\beta=1.515$, $\delta=1.341$ for the predicted one, i.e., for the theoretical one, confirming a reasonable coincidence between the shape parameters in both distributions but also with the shape parameter found for the S-N field. This allows us to assume "a priori" the value of the location parameter as known in the parameter estimation for the Miner number. On the contrary, a significant difference arises between both Miner scale parameters that, unfortunately stays on the unsafe side pointing out the necessity of introducing a correction if a safe lifetime prediction is intended. A scale parameter ratio $\delta_{mod} / \delta_{orig} =2.364$ is found, which can be used supposed an arbitrary correction for the moment being, see Fig. 7. In such a case, a practical coincidence is achieved between both probability distributions, experimental and analytical. It follows that the consequence of applying the Miner rule, which evidently cannot be accepted as a scientific law to reproduce faithfully the damage process, may be corrected by assuming a size effect that requires extrapolation of the theoretical distribution, as derived from the normalized variable, to a longer scale. The value of this correction must be checked for different materials and different load spectra so that further investigation is needed. Anyway, the modification of the scale parameter, being simple though not yet fully understood, opens a new way for predicting lifetime under pseudo-random varying loading. Assuming a Gumbel instead a Weibull distribution does not affect very much the calculations but obviates the question of the lower threshold value due to the existence of the Gumbel function practically from the first cycle. Other advantages, as reduction of the number of parameters and avoiding to accept nil probability of failure in legal cases, are also explained in [10].

MINER AS ALLEGED LINEAR CUMULATIVE DAMAGE HYPOTHESIS

The Miner number is generally accepted as representing a stage of damage resulting from a "linear" progression of damage accumulation. Nonetheless, the probabilistic conception of the S-N field permits us to reject this conventional cliché as being a gratuitous and wrong assertion. In fact, we have shown above how M can be related to probability of failure, as a measure of damage progression, by means of the normalized variable V considered in the fatigue approach proposed by Castillo and Fernández-Canteli [6] and the same can be achieved using as an alternative, in principle in better consonance to the real logarithmic scale characterizing lifetime problems, to the conventional the so called logarithmic Miner, denoted LM defined as:

$$LM = \sum \log n_i / \log N_i \quad (2)$$

which according to a parallel interpretation as that applied in the case of the conventional Miner number would be labelled as "logarithmic cumulative damage hypothesis" and, expectantly, lead to a fully different lifetime prediction.
After applying the same load spectra as in the Miner number case, totally different LM values are observed to those for conventional M, as expected. Notwithstanding, the same probabilities of failure are found for the reciprocal M and LM values in both cases. This proves that the probability of failure, as a measure of damage, happens to be independent of the model adopted (conventional or logarithmic Miner rule) if an adequate mapping of the measure of cumulative damage is adopted into the probability of failure is established, thus proving by extension, the inconsistency of the conventional belief, which denotes "linear" the damage progression for the conventional Miner number.

CONCLUSIONS

The main conclusions derived from the present work are:

- A probabilistic definition of the S-N field is necessary for the adequate probabilistic evaluation of the Miner number.
- A statistical interpretation of the Miner is possible, without practically maintaining the simplicity of its calculation in the conventional approach allowing an increase of reliability in the lifetime prediction of structural and mechanical components.
- The Miner number statistical distribution happens to be Weibull, as stated in former literature of the authors, whereas the prediction for the Miner number in a probabilistic way needs to be modified by introducing a scale correction.
- The statement that the Miner rule responds to a "linear cumulative damage hypothesis" is gratuitous and wrong.
- Other fatigue programs under variable loading with other materials should be considered in order to confirm the properties of the Miner distribution as exposed here.

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