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Existence of general equilibria for an economy under imperfect competition

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The aim of the paper is to establish the existence theorem on general equilibria for an economy under imperfect competition.

Keyword: General equilibria, Imperfect competitions.

1. Introduction

In this paper, we study general equilibria for economies under imperfect competition. In standard general equilibrium analyses, as typified in Debreu (1959), Hildenbrand (1974) and Aliprantis et al (1990), perfect competition is always adopted. It is the supposition that there exists a large number of participants in markets and they are so negligible as not to influence on every price. In many real markets, however, it is unimaginable that competitions are perfect. Therefore, we emphasize the importance of analyzing economies under imperfect competition.

Analyses of the economies have steadily made progress, nevertheless many problems still remain to be solved in this research area. One of significant themes is to construct models which are possible us to analyze the economies in intertemporal settings. We attack the theme by the standard way of extending economic models confined to a static situation, which were presented by Negishi (1961), the chapter 6 in Arrow and Hahn (1971), Gabszewicz and Vial (1972), Fitzroy (1974), Marschak and Selten (1974), Laffont and Laroque (1976) and Cornwall (1977).

The above contributions have been extended to some directions: At first, in the relation of increasing returns to scale, a monopolistically competitive economy was studied by Silvestre (1977). At second, in the relation of commodity differentiation, general equilibria for the economies were studied by

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Hart (1979), Hart (1985), Pascoa (1993) and Suzuki (2000). At last, the economies were analyzed by Benassy (1988) in the relevant to the concept of general equilibria with rigid prices. Together with our investigation, they are also the important subjects which we must take into consideration when we discuss genaral equilibria in the context of imperfect competition.

Imperfectly competitive firms have been typically classified into two groups according to their strategies in partial equilibrium analyses. One is the firms applying price policies, and the other is the firms adopting production plans as strategies. We also study these two types of the firms. Besides the traditional classification, the firms were assorted by Hart (1985) into two categories according to notions on the ground of price setting behavior, which are referred to as a subjective demand approach and an objective demand approach. The hypothesis in the former is that the imperfectly competitive firms conjecture demands for their products and the firms set prices along the notional demand curves. The supposition was adopted by Negishi (1961) and it was herited by Silvestre (1977) and Suzuki (2000). The supposition in the latter is that the firms fully recognize the demands and thus they can set the prices of their products on the basis of the actual demands functions. This approach was chosen by many of the previous literatures. We take another approach. We consider that the imperfectly competitive firm has a subjective probability which assigns a measure of feasibility for each event on its production set. We give the term feasibility to the property that production plans are just equal to the total demands. Since it is natural that the firm confronts with uncertainty on the outcomes, the supposition is justifiable. We further suppose that the firm estimates the mean production plan on the basis of the probability and that as usual, the firm choose the best price strategy giving the maximum value of the production plans. It should be noted that the expected production plan is not nacessarily feasible in actual.

In our economic model, there exist finitely many consumers and firms. And there exist $\ell + 1$ commodity streams on a compact time interval. As usual, every consumer chooses a consumption stream on the time interval as a price taker. One of these streams is a production factor and the others are products. The consumers have no dealings with each other. On the other hand, every firm behaves as a price setter in contrast to consumers. The firm selects a production plan on the time interval. One of these streams is a production factor provided from the consumers and the others are products supplied to them. The firms also do not business with each other. We consider such a simple transaction. Under these hypotheses, we establish the existence theorem on general equilibria for the economy under imperfect competition

The rest of the paper is organized as follows. In Section 2, a behavior of consumers is discussed. In Section 3, a behavior of imperfectly competitive firms applying strategies of prices is analyzed. Section 4 defines a monopolistically competitive economy and an equilibrium concept for the economy. The existence of equilibria for the economy is proved.

2. Consumers

We suppose that there exist finitely many consumers in our model. As we mentioned in Introduction, the consumers select consumption streams on a time interval. One of the consumption streams is a production factor and the others are products. Every consumer offers an initial endowment as a production factor to firms and he/she consumes products provided from the firms. As usual, the consumer behaves as a price taker.

Let $I := \{1, \cdots, n\}$ be the set of all names of consumers. Suppose that for any $j \in \{0, 1, \cdots, \ell\}$, $C_j(T)$ is the space of all continuous functions $x_j : T \to R$ on a nonempty, compact subset T of R, which has the topology $\mathscr{O}_j$ induced by the norm $\|x_j\| := \sup\{|x_j(t)| t \in T\}$. Then, we consider $C_0(T) \times C_1(T) \times \cdots \times C_\ell(T)$ as the commodity space, which has the product topology $\mathscr{O} := \mathscr{O}_0 \times \mathscr{O}_1 \times \cdots \times \mathscr{O}_\ell$. We write the space as $C(T)$ abbreviately. For any $i \in I$, let $X_i \subset C(T)$ be the set of all possible consumption plans, $\succsim_i$ be a preference relation defined on X_i and $e_i \in C(T)$ be an initial endowment. We set up the following assumptions on those concepts.

(A. 1) For any $i \in I$, (i) X_i is nonempty, convex and compact in $\mathscr{O}$;
(ii) $\succsim_i$ is reflexive, complete, transitive, and locally nonsatisted;
(iii) $\succsim_i$ is locally nonsatisted;
(iv) $\{x' \in X_i | x' \succsim_i x\}$ and $\{x' \in X_i | x \succsim_i x'\}$ are closed in $\mathscr{O}$ for any $x \in X_i$;
(v) $\{x' \in X_i | x' \succsim_i x\}$ is convex for any $x \in X_i$;
(vi) $e_i = (e_{i0}, e_{i1}, \cdots, e_{i\ell})$ satisfies $0 < e_{i0} < \infty$ and $e_{i1} = \cdots = e_{i\ell} = 0$.

The norm compactness of the consumption set X_i in (A. 1)(i) is indeed strong, however, the assumption must be set up to obtain positive results. The condition (vi) means that consumers have only the 0-th commodity stream as an initial endowment, which is used by firms as a production factor. We consider that the other commodities are products provided by the firms. The others in (A. 1) are already familiar.

Let $\mathscr{T}$ be the σ-algebra of Borel subsets of the set T. For any $j \in \{1, \cdots, \ell\}$, suppose that $ca_j(\mathscr{T})$ is the set of all countably additive signed measure with the norm $\|p_j\| := \sup\{\sum_{m=1}^{n} |p_j(T_m)| \,|\{T_1, \cdots, T_n\}$ is a partition of $T\}$, then we may consider the set P_j of all prices as follows:

$$P_j := \{p_j \in ca_j(\mathscr{T}) | \|p_j\| < \infty \text{ and } p_j(E) \geqq 0 \text{ for any } E \in \mathscr{T}\}. \tag{1}$$

For any $j \in \{1, \cdots, \ell\}$, we suppose that $ca_j(\mathscr{T})$ is endowed with the weak* topology denoted by $\sigma(ca_j(\mathscr{T}), C_j(T))$ and that $ca(\mathscr{T}) := ca_1(\mathscr{T}) \times \cdots \times ca_\ell(\mathscr{T})$ has the product topology $\sigma(ca_1(\mathscr{T}), C_1(T)) \times \cdots \times \sigma(ca_\ell(\mathscr{T}), C_\ell(T))$, which is denoted by $\sigma(ca(\mathscr{T}), C(T))$ abbreviately. We assume that the 0-th commodity is the numéraire whose price p_0 is 1 and therefore $p_j(t)/p_0(t) = p_j(t)$ for any $j \in \{1, \cdots, \ell\}$ and $t \in T$. Thus, we may define the price space P as follows:

$$P := P_1 \times \cdots \times P_\ell. \tag{2}$$

To proceed to analyze a behavior of consumers, we preliminarily provide some concepts related to productions. We suppose that there exist ℓ firms. For any $j \in \{1, \cdots, \ell\}$, let Y_j be the set of all possible production plans $y_j := (\eta_j^0, \eta_j)$ for the j-th firm, in which each of η_j^0 and η_j represents an input and an output. We assume that the set is a nonempty subset of $(C_0(T) \times C_j(T), \mathscr{O}_0 \times \mathscr{O}_j)$. Further, we define the set Y of all total production plans y as a subset of $(C(T), \mathscr{O})$. As we will mention afterward, the definition of Y is only delicate. For any $i \in I$ and $j \in \{1, \cdots, \ell\}$, let θ_{ij} be a share of consumer i to a profit $\eta_j^0 + \langle \eta_j, p_j \rangle$ of the j-th firm. All these concepts will be in detail explained in the following section. As usual, suppose that a budget set $B_i(y,p)$ for each consumer is defined by $\{x_i \in X_i | x_{i0} + \sum_{j=1}^{\ell} \langle x_{ij}, p_j \rangle \leqq e_{i0} + \max\{0, \sum_{j=1}^{\ell} \theta_{ij}(\eta_j^0 + \langle \eta_j, p_j \rangle)\}$, then an individual demand correspondence $X_i^* : Y \times P \twoheadrightarrow C(T)$ is defined as follows:

$$X_i^*(y,p) := \{x_i^* \in B_i(y,p) | \forall x_i \in B_i(y,p),\, x_i^* \succsim_i x_i\}. \tag{3}$$

LEMMA 1: *For any $i \in I$, the correspondence X_i^* has nonempty and convex values, and the graph $\{(y, p, x_i) \in C(T) \times ca(\mathscr{T}) \times C(T) | x_i \in X_i^*(y,p)\}$ is closed in $\mathscr{O} \times \sigma(ca(\mathscr{T}), C(T)) \times \mathscr{O}$.*

PROOF Since the budget set is nonempty and compact in $\mathscr{O}$ under (A. 1)(i) and (vi), it is clear by (A. 1)(ii) and (iv) that X^* has nonempty values. The convex valuedness is also clear by (A. 1)(i) and (v). Fix any $j \in \{1, \cdots, \ell\}$. Suppose that $Y_j \times P_j$ has the topology $\mathscr{O}_j \times \sigma(ca_j(\mathscr{T}), C_j(T))$. Then, since the map $(y_j, p_j) \mapsto \eta_j^0 + \langle \eta_j, p_j \rangle$ is continuous by Corollary 6. 47 in Aliprantis and Border (1994, p. 260), it may be proved that B_i is continuous under (A. 1)(vi) for any $i \in I$. Hence, the condition is verified by the standard method. □

3. The imperfectly competitive firms

We suppose that there exist ℓ imperfectly competitive firms in our model. For the sake of simplicity, we presume that every firm produces one commodity by using one homogeneous production factor provided from comsumers and that the firm supplies its product to the comsumers. That is, the firms do not business with each other. We further suppose that each firm's product is differentiated from the other firm's products. And thus, every firm may have an ability to manipulate a price of its product as a monopolistic competitor.

Let $J := \{1, \cdots, \ell\}$ be the set of all names of firms. For any $j \in J$, let Y_j denote the set of all possible production plans $y_j := (\eta_j^0, \eta_j)$. Each of η_j^0 and η_j represents an input and an output. We consider that the production plans are streams over a finite time horizon. We set up the following assumptions on the production set.

(A. 2) For any $j \in J$: (i) Y_j is a subset of $C_0(T) \times C_j(T)$ and $0 \in Y_j$; (ii) Y_j is convex, and compact in the product topology $\mathscr{O}_0 \times \mathscr{O}_j$.

The first harf of (A. 2)(i) means that each firm chooses simple production plans as we stated above. The second harf of the assumption is standard, which means that the firm need not necessarily be in operation. We must assume (A. 2)(ii) to obtain a positive result.

As we stated in Introduction, we hypothesize that every firm has a subjective probability assigning a measure of feasibility for each event on the production set. Let M_j be the set of all the probabilities on $\mathscr{B}(C_0(T)) \times \mathscr{B}(C_j(T)) \cap Y_j$ for any $j \in J$. The subjective probability should be naturally influenced by economic environments, that is, total consumption plans, total production plans and all prices. Let X be the set of all the total consumption streams defined by

$$X := X_1 + \cdots + X_n. \tag{4}$$

A typical element of the set is denoted by x. And let Y be the set of all the total production plans defined by the sum of Y_j^0:

$$Y := Y_1^0 + \cdots + Y_\ell^0, \tag{5}$$

in which $Y_j^0 := \{(\eta_j^0, 0, \cdots, \eta_j, \cdots, 0) \in C(T) | y_j := (\eta_j^0, \eta_j) \in Y_j\}$. Therefore, we may define the set Ω of all variables representing the economic environments as follows:

$$\Omega := X \times Y \times P. \tag{6}$$

We write an element (x, y, p) of Ω by ω. Although we describe the subjective probability by $\mu_j(\cdot\, | p_j, \omega)$ for technical reasons, it should be noted that $\mu_j(\cdot\, | p_j, \omega)$ does not depend on x, y_j and p_j in ω. We set up the following (A. 3) on the firm's subjective probabilities.

(A. 3) For any $j \in J$, suppose that $ca_j(\mathscr{T}) \times C(T) \times C(T) \times ca(\mathscr{T})$ has the product topology $\sigma(ca_j(\mathscr{T}), C_j(T)) \times \mathscr{O} \times \mathscr{O} \times \sigma(ca(\mathscr{T}), C(T))$ and M_j is endowed with the weak* topology, then $(p_j, \omega) \mapsto \mu_j(\cdot\, | p_j, \omega)$ is continuous.

From Theorem 2. 1 in Billingsley (1999, p. 16), the assumption (i) is equal to that as $(p_j^n, \omega^n) \to (p_j, \omega)$, then $|\mu_j(E | p_j^n, \omega^n) - \mu_j(E | p_j, \omega)| \to 0$ for any $E \in \mathscr{B}(C_0(T)) \times \mathscr{B}(C_j(T)) \cap Y_j$. That is, the function $(p_j, \omega) \mapsto \mu_j(E | p_j, \omega)$ is continuous for any $E \in \mathscr{B}(C_0(T)) \times \mathscr{B}(C_j(T)) \cap Y_j$.

Using the subjective probabilities, we may define a production plan for the j-th firm as follows:

$$\int_{Y_j} y_j \mu_j(y_j | p_j, \omega) := \Big(\int_{Y_j} \eta_j^0 \mu_j(y_j | p_j, \omega), \int_{Y_j} \eta_j \mu_j(y_j | p_j, \omega) \Big). \tag{7}$$

LEMMA 2: *For any* $j \in J$, $(p_j, \omega) \mapsto \int_{Y_j} y_j \mu_j(y_j | p_j, \omega)$ *is continuous.*

PROOF Fix any $j \in J$ throughout the proof. It is enough to be shown that one coordinate function $(p_j, \omega) \mapsto \int_{Y_j} \eta_j \mu_j(y_j | p_j, \omega)$ is continuous since the other may be proved by the same method. Let $f : Y_j \to C_j(T)$ be a function associating η_j to $y_j = (\eta_j^0, \eta_j)$. Since the function $y_j \mapsto \langle f(y_j), p_j \rangle$ is continuous for any $p_j \in P_j$ and hence it is also measurable, and $f(Y_j) = \{\eta_j | y_j \in Y_j\}$ is a separable subspace of $C_j(T)$, it is verified by Theorem 2 in

Diestel and Uhl (1977, p. 42) that the function f is strongly measurable, that is, there exists a sequence of simple functions $\varphi^m : Y_j \to C_j(T)$ satisfying the condition that as $m \to \infty$ then $\|f(y_j) - \varphi^m(y_j)\| \to 0$ for almost every $y_j \in Y_j$. The standard represntation of φ^m is given as follows:

$$\varphi^m(y_j) := \sum_{h=1}^{m} \chi_{E_h}(y_j)\eta_j^h. \tag{8}$$

We now consider the following inequality for any $m \in N$:

$$\begin{aligned}
&\Big\|\int_{Y_j} f(y_j)d\mu_j(y_j|p_j,\omega) - \int_{Y_j} f(y_j)d\mu_j(y_j|p_j^n,\omega^n)\Big\| \\
&\quad \leqq \Big\|\int_{Y_j} f(y_j)d\mu_j(y_j|p_j,\omega) - \int_{Y_j} \varphi^m(y_j)d\mu_j(y_j|p_j,\omega)\Big\| \\
&\quad + \Big\|\int_{Y_j} \varphi^m(y_j)d\mu_j(y_j|p_j,\omega) - \int_{Y_j} \varphi^m(y_j)d\mu_j(y_j|p_j^n,\omega^n)\Big\| \\
&\quad + \Big\|\int_{Y_j} \varphi^m(y_j)d\mu_j(y_j|p_j^n,\omega^n) - \int_{Y_j} f(y_j)d\mu_j(y_j|p_j^n,\omega^n)\Big\|.
\end{aligned} \tag{9}$$

We must verify that each term in the right side of (9) converges to 0 as $m \to \infty$ and $n \to \infty$.

On the first term: By the definition of the function φ^m for any $m \in N$ and the basic property of integral $\| \int_{Y_j} f(y_j)d\mu_j(y_j|p_j,\omega) - \int_{Y_j} \varphi^m(y_j)d\mu_j(y_j|p_j,\omega)\| \leqq \int_{Y_j} \|f(y_j) - \varphi^m(y_j)\|d\mu_j(y_j|p_j,\omega)$, as $m \to \infty$,

$$\Big\|\int_{Y_j} f(y_j)d\mu_j(y_j|p_j,\omega) - \int_{Y_j} \varphi^m(y_j)d\mu_j(y_j|p_j,\omega)\Big\| \to 0. \tag{10}$$

On the second term: It is clear that the following relation holds for any $m \in N$:

$$\begin{aligned}
&\Big\|\int_{Y_j}\varphi^m(y_j)d\mu_j(y_j|p_j,\omega)-\int_{Y_j}\varphi^m(y_j)d\mu_j(y_j|p_j^n,\omega^n)\Big\| \\
&=\Big\|\int_{Y_j}\sum_{h=1}^{m}\chi_{E_h}(y_j)\eta_j^h d\mu_j(y_j|p_j,\omega) \\
&\qquad -\int_{Y_j}\sum_{h=1}^{m}\chi_{E_h}(y_j)\eta_j^h d\mu_j(y_j|p_j^n,\omega^n)\Big\| \\
&=\Big\|\sum_{h=1}^{m}\Big(\mu_j(E_h|p_j,\omega)-\mu_j(E_h|p_j^n,\omega^n)\Big)\eta_j^h\Big\|.
\end{aligned} \tag{11}$$

That converges to 0 as $n \to \infty$ by the assumption (A. 3) and Theorem 2. 1 in Billingsley (1999, p. 16). Thus, the required condition is obtained.

On the last term: Fix any $m \in N$ before the closing sentence in the paragraph. Suppose that $g^m : Y_j \to R$ is a function defined by $g^m(y_j) := \|f(y_j) - \varphi^m(y_j)\|$, which is Lebesgue measurable. Then, from Theorem B in Halmos (1950, p. 85), there exists a simple function $\phi^h : Y_j \to R$ such that the sequence $\{\phi^h\}_{h=1}^{\infty}$ is increasing and $h \to \infty \Rightarrow \phi^h(y_j) \to g^m(y_j)$ for alomost every $y_j \in Y_j$. Indeed, for any $h \in N$ and $r \in \{1, 2, \cdots, 2^h h\}$, suppose that E_r^h and E^h are defined by $E_r^h := \{y_j \in Y_j | 2^{-h}(r-1) \leqq g^m(y_j) < 2^{-h}r\}$ and $E^h := \{y_j \in Y_j | g^m(y_j) \geqq h\}$, then ϕ^h is constructed as $\phi^h(y_j) := \sum_{r=1}^{h} 2^{-h}(r-1)\chi_{E_r^h}(y_j) + h\chi_{E^h}(y_j)$. Hence, it is shown by (A. 3) and Theorem 2. 1 in Billingsley (1999, p. 16) that for any $h \in N$, $n \to \infty \Rightarrow |\int_{Y_j}\phi^h(y_j)d\mu_j(y_j|p_j^n,\omega^n) - \int_{Y_j}\phi^h(\eta_j)d\mu_j(y_j|p_j,\omega)| \to 0$. And the condition $h \to \infty \Rightarrow |\int_{Y_j}\phi^h(y_j)d\mu_j(y_j|p_j,\omega) - \int_{Y_j}g^m(y_j)d\mu_j(y_j|p_j,\omega)| \to 0$ follows from Theorem D in Halmos (1950, p. 110). Accordingly, as $h \to \infty$ and $n \to \infty$, the property $|\int_{Y_j}\phi^h(y_j)d\mu_j(y_j|p_j^n,\omega^n) - \int_{Y_j}g^m(y_j)d\mu_j(y_j|p_j,\omega)| \to 0$ is obtained. Since $\psi^h(y_j) \leqq g^m(y_j)$ for any $h \in N$ and almost every $y_j \in Y_j$, $\int_{Y_j}\phi^h(y_j)d\mu_j(y_j|p_j^n,\omega^n) \leqq \int_{Y_j}g^m(y_j)d\mu_j(y_j|p_j^n,\omega^n)$. Hence, from the above conditions, $|\int_{Y_j}g^m(y_j)d\mu_j(y_j|p_j^n,\omega^n) - \int_{Y_j}g^m(y_j)d\mu_j(y_j|p_j,\omega)| \to 0$ as $n \to \infty$. Thus, by the definition of g^m, as $n \to \infty$,

$$\left|\int_{Y_j} \|f(y_j) - \varphi^m(y_j)\| d\mu_j(y_j | p_j^n, \omega^n) - \int_{Y_j} \|f(y_j) - \varphi^m(y_j)\| d\mu_j(y_j | p_j, \omega)\right| \to 0. \tag{12}$$

Since the second term $\int_{Y_j} \|f(y_j) - \varphi^m(y_j)\| d\mu_j(y_j | p_j, \omega) \to 0$ as $m \to \infty$, the required result is obtained. □

Now that the properties of the function $(p_j, \omega) \mapsto \int_{Y_j} y_j \mu_j(y_j | p_j, \omega)$ have been discussed, we may proceed to investigate a profit function of a firm. For any $j \in J$, the profit function $\pi_j : P_j \times \Omega \to R$ of the j-th firm is defined as follows:

$$\pi_j(p_j, \omega) := \max\left\{ \int_{Y_j} \eta_j^0 d\mu_j(y_j | p_j, \omega) + \left\langle \int_{Y_j} \eta_j d\mu_j(y_j | p_j, \omega), \, p_j \right\rangle \middle| p_j \in P_j \right\}. \tag{13}$$

REMARK The strategy set P_j for the j-th firm is bounded by the definition (1). We must suppose that the set has a sufficiently wide range so that the firm can choose the prices guaranteeing positive profits. This also means that potential firms are excluded, which have no option but to choose the strategy p_j with $\|p_j\| = \infty$ to obtain the maximum profit $\pi_j(p_j, \omega) = 0$.

Further, the best strategy correspondence $P_j^* : \Omega \twoheadrightarrow P_j$ of the j-th firm is defined as follows:

$$P_j^*(\omega) := \left\{ p_j^* \in P_j \middle| \pi_j(p_j^*, \omega) = \int_{Y_j} \eta_j^0 d\mu_j(y_j | p_j^*, \omega) + \left\langle \int_{Y_j} \eta_j d\mu_j(y_j | p_j^*, \omega), \, p_j^* \right\rangle \right\}. \tag{14}$$

Before we investigate propertiesof the profit function and the best strategy correspondence, we set up the following assumption.

(A. 4) For any $j \in J$ and $\omega \in \Omega$, the function $p_j \mapsto \int_{Y_j} \eta_j^0 d\mu_j(y_j | p_j, \omega) + \langle \int_{Y_j} \eta_j d\mu_j(y_j | p_j, \omega), \, p_j \rangle$ is quasiconcave.

This may be construed as follows: Firms suppose that a profit obtained by choosing moderate prices is greater than that gained by selecting excessive prices. Thus, the firms hesitate to choose 'small profit and quick return' policies under low prices or production plans aimed at high returns under high prices. The supposition is indeed strict, however, it may be justified if the firms have a tendency to adopt affordable prices.

LEMMA 3: *For any $j \in J$, the correspondence P_j^* has nonempty, convex values and a closed graph.*

PROOF : For any $j \in J$, P_j is a $\sigma(ca_j(\mathscr{T}), C_j(T))$-compact subset of $ca_j(\mathscr{T})$, for example, by Theorem 6.25 in Aliprantis and Border (1994, p. 250). Therefore, under (A.1)(i) and (A.2)(ii), Ω is compact in $\mathscr{O} \times \mathscr{O} \times \sigma(ca(\mathscr{B}(T)), C(T))$. Since the function π_j is continuous by Lemma 2 and Corollary 6.47 in Aliprantis and Border (1994, p. 260), the nonempty-valuedness of the correspondence is obvious. The convex-valuedness is also clear by (A.4). The closedness of the graph may be shown by the standard method under the result of Lemma 2. □

4. The existence theorems

(D.1)
$$\mathscr{E} := \Big(\langle C_1(T), ca_1(\mathscr{T})\rangle, \cdots, \langle C_\ell(T), ca_\ell(\mathscr{T})\rangle, \\ \{(X_i, \succsim_i, e_i)\}_{i\in I}, \{(Y_j, \mu_j)\}_{j\in J}, P, \theta\Big).$$

In the definition, P is treated as a component of the economy $\mathscr{E}$ since P is the strategy sets of the firms. We define an equilibrium for the economy $\mathscr{E}$ as follows:

(D.2) An equilibrium for the economy $\mathscr{E}$ under imperfect competition is the list $(x^*, \int_{Y_1} y_1 d\mu_1(y_1|p_1^*, \omega^*), \cdots, \int_{Y_\ell} y_\ell d\mu_\ell(y_\ell|p_\ell^*, \omega^*), p^*)$ fulfilling the next conditions:

(i) For any $i \in I$, $x_i^* \in B_i(y^*, p^*)$ satisfies $(\forall x_i \in B_i(y^*, p^*), x_i^* \succsim_i x_i)$;
(ii) For any $j \in J$, $p_j^* \in P_j$ satisfies the condition $\pi_j(y_j, \omega) = \max\{\int_{Y_j} \eta_j^0 d\mu_j(y_j|p_j^*, \omega^*) + \langle \int_{Y_j} \eta_j d\mu_j(y_j|p_j^*, \omega^*), p_j^*\rangle\}$;
(iii)$\sum_{i\in I} x_{i0}^* - \sum_{i\in I} e_{i0} - \sum_{j=1}^{\ell} \int_{Y_j} \eta_j^0 d\mu_j(y_j|p_j^*, \omega^*) \leqq \varepsilon$ and for any $j \in \{1, \cdots, \ell\}$, $\sum_{i\in I} x_{ij}^* - \int_{Y_j} \eta_j d\mu_j(y_j|p_j^*, \omega^*) \leqq \varepsilon$.

We set up the last assumption (A. 5).

(A. 5) Suppose that $\omega := (x, y, p)$ is a state satisfying the following conditions; an element of x is $\sum_{i\in I} x^*_{ij}(y,p)$ for any $j \in \{1, \cdots, \ell\}$, and an element of y is $\int_{Y_j} \eta_j d\mu_j(y_j|p_j,\omega)$ for any $j \in \{1, \cdots, \ell\}$. Then, for any $j \in J$,

$$\sum_{i\in I} x^*_{ij}(y,p) = \int_{Y_j} \eta_j d\mu_j(y_j|p_j,\omega). \tag{15}$$

THEOREM *Under the assumptions* (A. 1) – (A. 5), *there exist the equilibria for the economy $\mathscr{E}$ under imperfect competition.*

PROOF: Let $X^* : \Omega \twoheadrightarrow X$ be a correspondence defined by:

$$X^*(\omega) := X^*_1(y,p) + \cdots + X^*_n(y,p). \tag{16}$$

Let $Y^* : \Omega \twoheadrightarrow Y$ be a correspondence defined by:

$$Y^*(\omega) := \left\{ \left(\sum_{j=1}^{\ell} \int_{Y_j} \eta^0_j d\mu_j(y_j|p_j,\omega), \int_{Y_1} \eta_1 d\mu_1(y_1|p_1,\omega), \cdots, \int_{Y_\ell} \eta_\ell d\mu_\ell(y_\ell|p_\ell,\omega) \right) \middle| \forall j \in \{1, \cdots, \ell\}, \, p_j \in P^*_j(\omega) \right\}. \tag{17}$$

And let $P^* : \Omega \twoheadrightarrow P$ be a correspondence defined by:

$$P^*(\omega) := P^*_1(\omega) \times \cdots \times P^*_\ell(\omega). \tag{18}$$

Further, we define the correspondence $F : \Omega \twoheadrightarrow \Omega$ as follows:

$$F(\omega) := X^*(\omega) \times Y^*(\omega) \times P^*(\omega). \tag{19}$$

It is shown by Lemma 1, 2 and 3 that the correspondence F has nonempty, convex values and a closed graph. Thus, it is proved by the Fan-Glicksberg fixed point theorem (Fan (1952) and Glicksberg (1952)) that F has fixed points $\omega \in F(\omega)$.

At the fixed point ω, it is clear that the conditions (i) and (ii) in the definition (D. 2) are satisfied.

From the assumption (A. 5) and Walras' law, the condition (iii) in (D. 2) is also true.

It should be noted that an element $p_j \in P_j$ may be identified with $\hat{p}_j$ in the set $L_1(T, \mathscr{T}, p_j)$ of all integrable functions on T whose pairing is given by $\langle \hat{p}_j, x_j(i, \cdot) \rangle := \int_T \hat{p}_j(t) x_j(i,t) dp_j(t)$. Therefore, $\hat{p}_j$ represents prices on the time interval, which has a natural economic interpretation.

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