A Tractable Extension of the Spokes Model with Demand Uncertainty: A Note

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Abstract. The spokes model is an oligopoly model with product differentiation recently developed by Chen and Riordan. For the purpose of representing markets of cultural goods such as books and cinemas, we propose a tractable extension of the spokes model by introducing uncertainty about consumer trend from the point of view of firms. In this extension, many features such as expected demand, price and expected profit of a firm remain the same as in the original model.
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1 Introduction

Spatial representation is one of the fundamental tools for theoretical analysis of oligopolistic competition. Following the seminal work of Hotelling (1929), scholars have studied and presented many spatial models. The spokes model, recently developed by Chen and Riordan (2007), is among these and possesses several interesting features. In this model, the shape of the space on which consumers are located resembles the spokes of a wheel. At the end of each spoke, a variety of a differentiated product is located and a firm to produce that variety is also located if it exists. The distance from each consumer to a variety represents her taste for that variety, and firms price their varieties to maximize their profits. The spokes model can deal with the situation where a firm have to compete equally with all other firms, whereas in the popular circle model, which is initiated by Salop (1979), firms only compete with nearby firms. Chen and Riordan (2007) consider the spokes model with symmetric firms and no demand uncertainty, and calculate de-

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mand, price, and profit of each firm in the equilibrium.[1] They also consider the number of firms when there is free entry to the market.[2]

Because of symmetric global competition among firms, the spokes model may be useful for studying cultural goods such as books, cinemas and video games.[3] One feature in a cultural good market is that some consumers have their own tastes and different valuations of each variety, and this is represented quite well in the original spokes model. Another important feature is the existence of trends among consumers. Usually, sales of some varieties boom and others slump.[4] It is usually quite difficult for firms to acquire knowledge about trends in the near future when producing varieties. In other words, there is uncertainty about consumer trends from the point of view of firms. While Chen and Riordan (2007) consider the deterministic case without any uncertainty, this note constructs a tractable extension of the spokes model with uncertainty and *ex-post* asymmetry in the demand of consumers, with consideration of application to cultural goods markets.[5]

Section 2 is preliminary. In Section 3, we first consider the simple and extreme case in which all consumers are strongly affected by a trend. We refer to this case as *the extreme bandwagon effect case*. After firms price their varieties, a variety is revealed to be the bestseller with equal probability and all consumers turn out to be on the spoke of the bestseller. Firms' pricing strategies are based on their expected profits. Then we can calculate that the expected demand for a variety is the same as the demand in the deterministic case by Chen and Riordan (2007). This implies that the price and the expected profit for a firm are also the same to those in Chen and Riordan (2007). In Section 4, we consider a more general setting. Usually, whereas some consumers follow a trend, others stick to their own tastes. Hence, we introduce a parameter that represents the proportion of consumers affected by the trend. This is an intermediate case between the extreme bandwagon case and the deterministic case analyzed by Chen and Riordan (2007). Given the argument in Section 3, expected demand, price, and expected profit of

[1] Another interesting feature of the spokes model is that when the valuation of consumers on varieties is relatively low, price increases in the number of firms. This feature is further discussed in Chen and Riordan (2008).

[2] Notable extensions of the spokes model include the following. Caminal and Granero (2012) and Granero (2013) consider the case that the number of firms are quite large and there are both multi-product and single-product firms. Reggiani (2014) considers an extension with firms' location choices and studies asymmetric equilibria. All these models do not incorporate uncertainty.

[3] Caminal (2010) studies translation activities of cultural goods by using an extended spokes model.

[4] There are many empirical studies of trends among consumers in cultural goods markets. Sorensen (2007) and Carare (2012) consider the effects of bestseller lists on trends in the markets of books and application software, respectively. Chevalier and Mayzlin (2006) consider the effect of consumer reviews on sales in the online book market.

[5] In Hotelling's duopoly framework, Meagher and Zauner (2004) study how demand uncertainty affects the location choices of firms.

a firm are the same as in the deterministic case.[6] If new entrants care only for that the expected profit is nonnegative, under free entry, the number of firms is also the same to that in Chen and Riordan (2007). Additionally, we discuss the free entry condition if firms care for *ex-post* nonnegative profit.

This note introduces demand uncertainty into the spokes model with consideration of application to cultural goods markets. Even though there are various ways of introducing consumer distributions and probabilities, the approach in this extension is simple and natural, and maintains the global symmetry among varieties, which is a key property that makes the original model interesting. In this extension, important properties such as expected demand, price, and expected profit remain the same as in the original model. Hence, one can apply most of the results in Chen and Riordan (2007) to the markets with demand uncertainty.

2 Preliminaries

Most notations follow Chen and Riordan (2007). There are $i = 1, 2, \ldots, N$ potential varieties of a differentiated product. In the geometric representation, there are N spokes (or lines). Each spoke is denoted as ℓ_i, the length of each spoke is $1/2$ and the end of all spokes are connected at the center. The other end point of ℓ_i, at which variety i is located, is called the origin of the spoke.

There are $j = 1, 2, \ldots, n$ firms with $2 \leq n \leq N$. Firm j is also located at the origin of ℓ_j and produces variety j. Thus variety $j = 1, 2, \ldots, n$ is actually produced and available for consumers, while variety $i = n+1, \ldots, N$ is not produced and unavailable for consumers.[7] The constant marginal cost of production is normalized to zero.

Consumers are distributed over spokes. The total mass of consumers is assumed to be 1. The form of the distribution is later discussed. Consumer (ℓ_i, x_i) is at a point on ℓ_i with the distance x_i from the origin of ℓ_i where i is located. Variety i is first preferred by consumer (ℓ_i, x_i), and her valuation of this variety is $v - x_i$. v is the same for all consumers. Her second preferred variety $k \neq i$ is randomly chosen by nature with probability $1/(N-1)$, and her valuation of variety k is $v-(1-x_i)$. Note that $1-x_i$ is the distance from (ℓ_i, x_i) to k. Also note that each of varieties i and k could be available or unavailable. Her valuation on any variety other than i and k is zero. Given a price profile $(p_1, \ldots, p_n)$ and consumer (ℓ_i, x_i)'s first and second preferred varieties i and k, she purchases i if (i) i is available and $v - x_i - p_i > 0$ and (ii) $v-x_i-p_i \geq v-(1-x_i)-p_k$ or k is unavailable, and she purchases k if (i) k is available and $v-(1-x_i)-p_k > 0$ and (ii) $v-(1-x_i)-p_k > v-x_i-p_i$

[6] Orbacha and Einavb (2007) discuss several reasons why the uniform price rule is adapted in the cinema industry.

[7] See Figure 1 in Chen and Riordan (2007) for a graphical illustration.

or i is unavailable. Otherwise, she purchases nothing.

The timing of the revelation of information is as follows. (1) In the *ex-ante* stage, firms simultaneously price their varieties. At this point, firms do not know the distribution of consumers. They only know the probability of the occurrence of each consumer distribution. On this basis, firms maximize their expected profits. (2) In the *ex-post* stage, the distribution of consumers is revealed to firms, and the demand for each variety can be calculated. The forms of the distributions of consumers distributions are discussed in Sections 3 and 4.

In terms of game theory, players are firms, and their strategies are the prices of their varieties. We focus on symmetric pure strategy Nash equilibria.

3 The extreme bandwagon effect case

First, consider the extreme case in which all consumers are under a bandwagon effect. In the *ex-ante* stage, firms only know that with probability $1/N$, each variety will be the bestseller (or winner). In the *ex-post* stage, a variety i becomes the bestseller and all consumers turn out to be uniformly distributed over spoke ℓ_i. From the point of view of firms, in the *ex-ante* stage, consumers are distributed uniformly over all spokes. In the *ex-post* stage, because of the trend, all consumers on spokes other than ℓ_i move $1/2$ length toward a variety i.

Firm j prices the variety based on the expected profit. Let variety b be the bestseller revealed in the *ex-post* stage. From firm j's point of view, there are essentially three potential *ex-post* situations. *Case 1* is that the bestseller b is produced by firm j, *i.e.*, $b = j$. This occurs with probability $1/N$. *Case 2* is that the bestseller b is not j and is available. This case occurs with probability $(n-1)/N$. *Case 3* is that the bestseller b is not available. This case occurs with probability $(N-n)/N$. Let q_j^1 denote the *ex-post* demand for variety j in *Case 1* and q_j^3 denote that in *Case 3*. In *Case 2*, the *ex-post* demand for variety j varies depending on the price of the bestseller p_b. Thus let q_{jb}^2 be the *ex-post* demand for variety j when firm b produces the bestseller. We make calculations for each case.

Case 1. q_j^1 can be calculated as follows.

$$\begin{aligned} q_j^1 =& 2 \cdot \Big(\frac{1}{N-1} \cdot \sum_{k \in \{1,\ldots,n\} \setminus \{j\}} \max\{\min\{\frac{1}{2} + \frac{p_k - p_j}{2}, \frac{1}{2}\}, 0\} \\ &+ \frac{N-n}{N-1} \cdot \max\{\min\{v - p_j, \frac{1}{2}\}, 0\}\Big) \end{aligned}$$

In the above expression, 2 is the density of consumers on ℓ_j. The first term in the bracket represents the size of consumers whose second preferred varieties are available. The proportion of consumers whose second preferred varieties

is k is $1/(N-1)$. Note that the marginal consumer who is indifferent between j and k considers that

$$v - p_j - x_j = v - p_k - (1 - x_j) \iff x_j = \frac{1}{2} + \frac{p_k - p_j}{2}$$

as long as she is certainly on ℓ_j, *i.e.*, $x_j \in [0, 1/2]$. If $1/2 + (p_k - p_j)/2 > 1/2$, all consumers on ℓ_j demand variety j. If $1/2 + (p_k - p_j)/2 < 0$, all consumers on ℓ_j demand variety k. The second term in the bracket calculates the demand of consumers whose second preferred varieties are not available and $(N - n)/(N - 1)$ is the proportion of those consumers. The marginal consumer who is indifferent between purchasing j and nothing considers that

$$v - p_j - x_j = 0 \iff x_j = v - p_j$$

as long as she is on ℓ_j.
Case 2. q^2_{jb} can be calculated as follows.

$$q^2_{jb} = 2 \cdot \frac{1}{N-1} \cdot \max\{\min\{\frac{p_b - p_j}{2}, \frac{1}{2}\}, 0\}$$

2 in the above expression is the density of consumers on ℓ_b. $1/(N-1)$ is the proportion of consumers whose second preferred variety is j. The condition for the marginal consumer who is indifferent between b and j is that as long as she is on ℓ_b,

$$v - p_b - x_b = v - p_j - (1 - x_b) \iff x_b = \frac{1}{2} + \frac{p_j - p_b}{2}$$

Thus the length of the part of ℓ_b on which there exist consumers who prefers j to b is that $1/2 - x_b = (p_b - p_j)/2$.
Case 3. q^3_j can be calculated as follows.

$$q^3_j = 2 \cdot \frac{1}{N-1} \cdot \max\{\min\{v - p_j - \frac{1}{2}, \frac{1}{2}\}, 0\}$$

2 in the above expression is the density of consumers on ℓ_b. $1/(N-1)$ is the proportion of consumers who consider variety j as their second preferred one. The condition for the marginal consumer who is indifferent between purchasing j and nothing is that as long as she is on ℓ_b,

$$v - p_j - (1 - x_b) = 0 \iff x_b = -v + p_j + 1$$

Thus the length of the part of ℓ_b on which there exist consumers who prefers j to nothing is that $1/2 - x_b = v - p_j - 1/2$.

Now let us focus on the *ex-ante* stage and calculate expected demand q_j for firm j.

$$q_j = \frac{1}{N} \cdot q^1_j + \sum_{b \in \{1,\ldots,n\} \setminus \{j\}} \frac{1}{N} \cdot q^2_{jb} + \frac{N-n}{N} \cdot q^3_j$$

$$=2\cdot\frac{1}{N}\cdot\frac{1}{N-1}\cdot\sum_{k\in\{1,\ldots,n\}\setminus\{j\}}\max\{\min\{\frac{1}{2}+\frac{p_k-p_j}{2},1\},0\}$$
$$+2\cdot\frac{1}{N}\cdot\frac{N-n}{N-1}\cdot\max\{\min\{v-p_j,1\},0\}$$

By comparing this expression with Chen and Riordan (2007), it turns out that q_j *is actually equivalent to the exact demand for firm* j *in the deterministic case.*

Chen and Riordan (2007) calculate the symmetric equilibrium price by a firm in the deterministic case under the condition that $1 \leq v \leq 2(N-1)/(n-1)+(2N-n-1)/\{2(N-n)\}$.[8] Even in the current model, firm j maximizes its expected profit $p_j \cdot q_j$ in the *ex-ante* stage, which is the same as in Chen and Riordan (2007). Thus price and expected profit of a firm can be calculated as in Chen and Riordan (2007). In summary, we have the following proposition.

Proposition 1. *In the extreme bandwagon case, expected demand, price, and expected profit of a firm in the symmetric equilibrium are the same as those in the deterministic case of Chen and Riordan (2007).*

According to Proposition 1 of Chen and Riordan (2007), the price by a firm in the symmetric equilibrium p^* depends on four regions of v as follows.

$$p^*=\begin{cases}\dfrac{2N-n-1}{n-1} & \text{if } \dfrac{2(N-1)}{n-1}<v\leq\dfrac{2(N-1)}{n-1}+\dfrac{2N-n-1}{2(N-n)} \text{ (Region 1)}\\ v-1 & \text{if } 2<v\leq\dfrac{2(N-1)}{n-1} \text{ (Region 2)}\\ \dfrac{2(N-n)v+(n-1)}{4N-3n-1} & \text{if } \dfrac{1}{2}+\dfrac{N-1}{2N-n-1}<v\leq 2 \text{ (Region 3)}\\ v-\dfrac{1}{2} & \text{if } 1<v\leq\dfrac{1}{2}+\dfrac{N-1}{2N-n-1} \text{ (Region 4)}\end{cases}$$

Based on the price p^*, we can calculate the *ex-post* demand for a firm in the symmetric equilibrium. Let $q^{\#*}$ be the *ex-post* demand in *Case #*. In *Case 1*, *i.e.*, when the producing variety becomes the bestseller, $q^{1*}=1$. In *Case 2*, *i.e.*, when the bestseller is available and produced by another firm, $q_j^{2*}=0$. In *Case 3*, *i.e.*, when the bestseller is not available,

$$q^{3*}=\begin{cases}1/(N-1) & \text{(Region 1)}\\ 1/(N-1) & \text{(Region 2)}\\ \dfrac{4Nv-4N-2nv+n-2v+3}{(N-1)(4N-3n-1)} & \text{(Region 3)}\\ 0 & \text{(Region 4)}\end{cases}$$

[8] This is the condition for the existence of oligopolistic competition and the symmetric pure strategy Nash equilibrium.

Note that in Region 3, a certain proportion of consumers do not purchase the variety. The *ex-post* profit for a firm in *Case #* is then easily calculated as $p^* \cdot q^{\#*}$. In the *ex-post* sense, there are both cases: One in which all consumers buy a variety and one in which no one buys anything, even though the expected total demand in the market is the same as the total demand in the deterministic case of Chen and Riordan (2007).

4 The degree of uncertainty from the point of view of firms

In the previous section, we considered the extreme case of the bandwagon effect on consumers. In this section, we consider a more natural and general setting. In a market of cultural goods, there are often two types of consumers: those possessing own tastes and unaffected by others, and those affected by a trend.

Let $r \in [0, 1]$ be the proportion of consumers who are affected by a trend. In the *ex-ante* stage, from the point of view of firms, all consumers are uniformly distributed over all spokes. In the *ex-post* stage, variety i is revealed to be the bestseller with probability $1/N$ and r of consumers turn out to be uniformly distributed over ℓ_i. $r - 1$ of consumers, who are not affected by the trend, continue to be uniformly distributed over all N spokes in the *ex-post* stage. r may also be thought of as *the degree of uncertainty* from the point of view of firms. Note that if $r = 1$, it is the extreme bandwagon case discussed in Section 3. If $r = 0$, it is the deterministic case discussed in Chen and Riordan (2007). Whether $r = 0$ or $r = 1$, the firms' expected demand is the same.[9] Hence, we can easily obtain the following proposition.

Proposition 2. *For any degree of uncertainty, expected demand, price, and expected profit of a firm in the symmetric equilibrium are the same as those in the deterministic case in Chen and Riordan (2007).*

Chen and Riordan (2007) calculate the number of firms as a free entry equilibrium. Given a fixed entry cost $f > 0$, the equilibrium number of firms $n^* \in [0, N]$ satisfies $\pi^*(n^*) \geq f \geq \pi^*(n^* + 1)$ if $n^* < N$ and $\pi^*(n^*) \geq f$ if $n^* = N$, where $\pi^*(n^*)$ is the equilibrium profit in the deterministic case when the number of firms is n^*.

In this extension, the same condition for the entry of firms holds as long as a firm cares for whether the expected profit is larger than zero. This is quite a usual case.

[9] It is obvious that there exist alternative ways to formulate distributions and their probabilities with maintaining the same expected demand as long as a certain symmetry of distributions holds.

There may also be an industry with limited funds and firms that need to be careful to avoid bankruptcy if a trend goes toward a variety of another firm or unavailable one. The industry of minor genre book publishers is an example. As suggested in Section 3, in the worst case, a firm j has zero demand from consumers under the bandwagon effect in the *ex-post* stage and hence obtains zero profit from them. Therefore, if firms take the *ex-post* nonnegative profit condition into account, then the number of firms in the free entry equilibrium should satisfy that

$$(1-r)\cdot\pi^*(n^*) \geq f \geq (1-r)\cdot\pi^*(n^*+1) \iff \pi^*(n^*) \geq \frac{f}{1-r} \geq \pi^*(n^*+1)$$

if $n^* < N$, and $\pi^*(n^*) \geq f/(1-r)$ if $n^* = N$. The argument about the number of firms in the free-entry equilibrium in Section 3 of Chen and Riordan (2007) can be applied to this case by replacing f in them with $f/(1-r)$. According to Proposition 2 of Chen and Riordan (2007), if v is relatively high, *i.e.*, in Region 1 or 2, n^* essentially decreases in f in the deterministic case.[10] Thus we obtain that if v is relatively high and r increases, essentially the number of firms n^* may decrease. In other words, if firms care about *ex-post* nonnegative profits, the degree of uncertainty may reduce the number of varieties available to consumers.

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