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This paper focuses on the function of international currencies as foreign exchange vehicles, which has the character of network externality. On January 1999, the Euro was introduced in some European countries (Euroland) where the functions of the Euro were limited as a currency. After July 2002, the Euro started functioning fully as a currency, and competition between the Euro and the US dollar, the dominant international currency, began. We present a currency competition model with decreasing transaction costs that reflect the character of network externality, to investigate competition between the Euro and the Dollar. We suggest that the impact of the introduction of the Euro is a determining factor in whether the Euro or the Dollar will emerge as the dominant international currency.

JEL Classification: F3, F4

Keywords: Currency Competition, the Euro, Foreign Exchange, Vehicle Currency

1. Introduction

In January 1999, a single-currency, the Euro, was introduced in 11 EU countries (Euroland), not as a means of exchange but as a means of denomination. As Euroland has almost the same sized economy as the United States, the emergence of a single-currency area accelerates the integration of money and financial markets in the EU. In addition, it strengthens trade and financial linkages

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between the EU and other areas in the world, promoting the possibility of the Euro as an international currency. The new international-currency Euro might compete with the US dollar, which has been the key currency in the world since World War Ⅱ.

The decline of the dollar as the key currency began with the transition toward a flexible exchange rate system in major countries. The key currency and the international currency have nine functions as shown in Table 1. After the evolution of a flexible exchange rate system, various currencies with the role of international currencies have evolved because of exchange rate risk. Because of the volatility of exchange rates, trading corporations tend to hedge risk by using their home currency, which has resulted in a declining role of the US dollar for denomination, vehicle and payments in the private sector. Moreover, investors prefer to diversify their portfolio internationally to hold portfolios denominated in different currencies. This results in the divergence of the portfolio role of international currencies. The dominant role of the US dollar in the Bretton Woods system has diminished. However, the US dollar as a foreign exchange vehicle currency plays a more important role in foreign exchange markets. The exchange vehicle currency mediates transactions among local currencies, whose transaction volume is small in the market. Even if a local currency is used as a trade vehicle currency, it is difficult to match the currency with the counterpart local currency in the foreign exchange market. Accordingly, home traders in the local-currency country conduct transactions using the counterpart currency through the temporal exchange of the international currency as the exchange vehicle currency. Diversification of the international uses of currency requires the exchange vehicle role of international currencies. The more uses of international currencies are diversified, the more important the exchange vehicle currency becomes in flexible exchange rate regimes. This paper focuses on the function of the foreign exchange vehicle by international currencies to clarify future competition between the Euro and the US dollar.

The increasing return model is applied to explain the characteristics of the network effect of the exchange vehicle role of international currency in foreign exchange markets. The network effect means that the more an international currency is used by foreign-exchange-use traders, the smaller the transaction fees for the international currency. To explain this, the decreasing cost model that applies the increasing return model is introduced. Recent studies by Krugman (1991), Matsuyama (1992) and Murphy, Shleifer and Vishy (1989) have attempted to derive multiple equilibriums in discussing both good economic development and bad economic development. Their studies research the role of expectation in future returns and the historically initial equilibrium. This paper applies the increasing return model to our non-linear decreasing transaction model to describe the network effect of the international currency. Krugman (1991) constructs a linear model with increasing returns to investigate the characteristics of local equilibrium. Krugman's model is expanded to a non-linear model in our model to investigate the possibility of the bifurcation of our dynamic system. Using our model, we can show the state of bifurcation, to explain the possibility of the dominant role of the Euro via the US dollar.

The remainder of this paper is organized as follows. Section 2 explains the function of an international currency. Section 3 introduces the model to explain the characteristics of the foreign exchange vehicle role of the international currency, and discusses the competition between the Euro and the US dollar. Section 4 summarizes, discussing the future role of the Euro.

2. The functions of an international currency

There are three ordinary functions of an international currency: 1) a unit of exchange, 2) a means of payment, and 3) a store of value. These three functions of currency are not independent. Any currency, or any medium of exchange, functions as a store of value that implies purchasing power. Money cannot work as a medium if the currency does not have the role of a unit of account. Therefore, currency must have these functions.

Table 1: Functions of an international currency

Functions	Private Sector	Foreign Exchange Markets	Official sector
Units of Account	Denomination		Anchor
Means of payments	Vehicle, Payments	Exchange Vehicle, Interventions	Official Payments
Store of Value	Portfolio allocation		Official reserves

Source) Krugman (1980) and author.

On the other hand, there are six functions of an international currency classified by users, the private sector, the official sector, and traders in foreign exchange markets. Table 1 summarizes the functions of an international currency. In the private sector, the international currency is used as a denomination currency, vehicle currency, payment currency and investment currency for portfolio allocation. The denomination currency is used when private agents write an invoice to denominate prices in trading goods and services with foreign countries. In addition, agents use the currency in domestic trading within high-inflation countries. The latter case is called "currency substitution." The vehicle currency is used for trading with foreign agents as a payment currency for trading settlements. Furthermore, private agents use the international currency when they prefer a global portfolio allocation. If capital movement is liberalized, it is possible for private agents to invest in foreign assets and currencies. The international currency for portfolio allocation is one such asset. In foreign exchange markets (Forex markets), the trader uses the international currency as a foreign exchange vehicle, as mentioned above. In addition, in foreign exchange markets, official agents use the international currencies to intervene in markets for the stability of exchange rates. This is called the "intervention currency."

The official sector, the central bank, treasury, and international organizations, use the moneys internationally for anchor setting parity, for intervention to influence exchange rates, and for international reserve. If a country wants to peg its currency to another currency, it needs to indicate the anchor (or central) rate of parity, whose role is that of an international currency for official bodies.

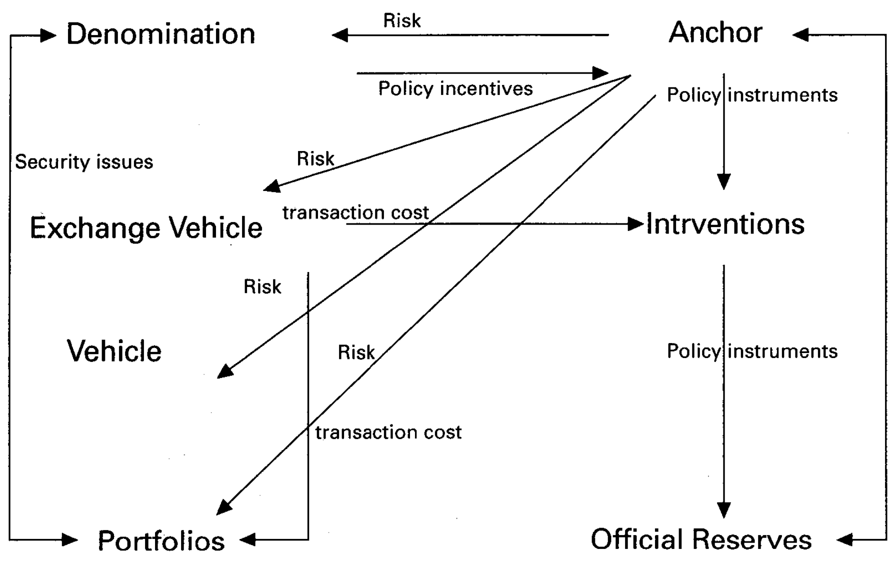
In official use, international currencies may also serve for official payments. For example, international currencies are used for transactions by the official sector and for grants among various official sectors. Furthermore, international reserves held by officials are usually international currencies because they need to retain intervention funds in the Forex markets. In addition, because the official sector holds a reserve to settle balance of payment deficits, it holds assets or currencies denominated in international currencies.

International currencies do not always have these six functions. Only the key currency, such as the US dollar at present, plays all of these roles. However, the US dollar is not always dominant in its international currency role. As mentioned above, private agents in industrialized countries prefer to use the home currency against exchange risk and for transaction costs. This results in the decline in the private use of the US dollar as the denomination currency and the vehicle currency. Meanwhile, because both private and official agents intend to diversify their reserves for exchange reasons or due to other risks, the use of the US dollar for portfolio allocation and official reserves has declined.

Furthermore, these functions are not independent, but synergetic. As Bénassy-Quere, Mojon, and Schor (1998) pointed out, there is synergy between the functions of international currencies. These synergetic effects appear through certain channels.

1) Transaction costs: the transaction costs of international currencies imply a bid-ask spread. Transaction costs become lower if the market for the foreign exchange vehicle is large and deep. In addition, large exchange volatility raises the costs, as Hartman (1997) pointed out. Monetary authorities tend to use the same currency for intervention. Meanwhile, private investors prefer to hold assets denominated in the currency.
2) Security issues: the supply of securities depends on the denomination function. If securities in a particular currency are easily available, the use of that currency expands as a vehicle and port-

Figure 1: Synergies between international currency functions



Source) Bénassy-Quere, Agnes, Benoit Mojon, and Armand-Denis Schor (1998).

folio currency.

3) Policy incentives: the monetary authorities have the incentive to peg their currency to an international currency if a large share of trade and capital flow is denominated in that currency.
4) Policy instruments: monetary authorities need to hold a particular currency to intervene in their currency to defend the peg when an international currency is used as an anchor.
5) Risk: incentives to denominate trade and capital flow in a particular currency increase if that currency is used as a peg, because the exchange rate volatility risk is lower.

Table 2: Statistics on the present international use of the main currencies in 1996

Market Share	USD	Yen	DM	Others*
Denomination of trade	47.6	4.8	15.3	18.2
Forex turnover	83.3	23.6	37.1	32.8
International bond issues	37.8	17.7	15.6	8.8
World portfolio	39.8	11.5	15.6	21.3
LDCs debt	50.0	18.1	16.0	0.1
Official reserves	56.4	7.1	13.7	12.1

*Other currencies include Pound Sterling, French Franc, Netherlands Guilder, ecu.
Source) Bénassy-Quere, Agnès, Benoît Mojon, and Armand-Denis Schor (1998)

Of these channels, this paper focuses on transaction costs because the costs depend on transaction volume, reflecting security issues, policy instruments, policy incentives, and risk. Transaction costs can be measured as bid-ask spreads in foreign exchange markets. The spread reflects not only the liquidity of each currency, but also the volatility of the exchange rates. Hartman (1998b) shows that surprises in daily turnover are related to increased volatility[1].

Table 2 shows statistics on the current use of the main currencies in 1996. From this table, the US dollar has the top share in international use, but does not have the dominant share except for the Forex turnover. Therefore, here, the US dollar as a key currency is sustained by its role as the foreign exchange vehicle. This paper focuses on this role of the foreign exchange vehicle. The reason that the exchange vehicle currency is important is that this function dominates the foreign exchange market. In particular, the currency that functions as a vehicle of trade between small-volume currencies in Forex markets emerges as the international currency. This logic is as same as for the evolution of money. Alogoskoufis et al. (1997) use Hartman's estimates to compute bid-ask spreads with zero volatility, which is shown in Table 3 by spot foreign exchange transaction

[1] As exchange rate volatility increases with turnover, the impact of a larger turnover on transaction costs is generally ambiguous.

costs. The differences in transaction costs due to differences in liquidity is so small for a ten thousand dollar transaction, but the actual difference is such a large amount because the amount of daily transactions in foreign exchange markets is enormous. Therefore, these differences are very important for traders who deal in certain currencies.

Table 3: Spot foreign exchange transaction costs
($ for a 10,000 transaction, assuming zero volatility)

$/DM	4.06
DM/Yen	4.37
$/Yen	4.16
FF/$	4.61
£/$	4.27

Source) Alogoskoufis, Portes and Rey (1997)

If the role of the US dollar as a foreign exchange vehicle declines, another international currency may emerge as a dominant currency in the Forex market. We investigate the shift from the US dollar to the Euro as a key currency. In the next section, a model with a decreasing cost function is developed.

3. Currency competition model

3-1) Framework of our model

We constructed a model of an exchange vehicle currency among foreign exchange traders for the following reasons. First, the use of an international currency as an exchange vehicle currency dominates the foreign exchange market, while the share of a trade vehicle currency is small in the market. This situation is due to a series of exchange control liberalization moves in developed and developing countries. Second, Krugman (1980) suggests that exchange vehicle currencies are the most important factor influencing the character of an international currency, which has a network effect including increasing returns. However, Krugman's paper does not show a dynamic model of an international currency. Thus, he does not explain sufficiently why a currency becomes an international currency.

Consider two relevant currencies. These are the main currencies used for trading in foreign exchange markets, and are assumed to be the Euro and the US dollar. Here, we introduce currency speculators in a third country, who maximize their returns in each period. As developed below, 1 denotes the currency or assets denominated in the Euro, and 2, those denominated in the US dollar. The third country's currencies are not international currencies, so traders here use foreign currencies for international transactions. They decide which currency to invest in inter-temporally without considering interest rate differentials. This assumption is based on Goodhart (1988), who interviewed many interbank traders and suggested they play an open spot position with little attention to interest rates. Since traders who trade in currencies daily often reverse the volume within a day, this assumption is plausible. Here, δ_j denotes the interest differentials between the interest rate in Country j and the home country.

Since traders face volatility of exchange rates between international currencies and the home currency, they anticipate exchange rates. ε_j represents the expected fluctuation in exchange rates. While ε_j generally depends on the volume of trade in the foreign exchange market, we assume ε_j is constant at first. This assumption is relaxed later.

All trade is assumed to have a fixed transaction fee ratio of c_j (j=1, 2), which is the decreasing function of the total assets balance, B_j (j=1, 2) denominated by each currency. In the real market, there are differentials between the bid rate and the ask rate, so-called bid-ask spreads, which are thought to be reflected in the search costs of each currency trade. Since search costs depend on the convenience of the meeting currency that the trader intends to buy and sell, the greater the volume of the transaction increases, the lower the transaction fee. That is, the transaction cost has the character of economy of scale. This results in a network effect of the use of international currency. The flows, not only of transactions in the foreign exchange market, but also of potential transactions, affect the costs, which are asset positions. We can then identify two cases, that is, c_j , which is the increasing function of B_j (Case 1) and a decreasing function (Case 2). However, each trader deals with c_j exogenously because the economy of scale is externality. The cost function is as follows:

$$C_j = c_j(B_j), \quad j=1,2 \tag{1}$$
$$c'_j = \frac{\partial c_j}{\partial B_j} > 0, \quad c''_j = \frac{\partial^2 c_j}{\partial B_j{}^2}.$$

The sign c''_j is identified by two cases: $c''_j>0$ is Case 1, $c''_j<0$ is Case 2.
Here, α is assumed to be the parameter representing the technology needed for transactions using the Euro. The cost function can be written as

$$C_1 = a\, c_1(B_1)\,. \tag{2}$$

We assume each trader also faces adjustment cost ϕ, depending on investment volume N. Adjustments of the asset position are assumed to be increasingly expensive. Function ϕ is the following increasing function:

$$\phi = \phi(N)\,. \tag{3}$$

Here, we assume $\phi' = \frac{\partial \phi}{\partial N} > 0, \quad \phi'' = \frac{\partial \phi''}{\partial N^2} > 0.$

The number of traders in the third country is assumed to be constant. For simplicity, traders' investment volumes are supposed to be instantaneous, constrained constants. When each trader determines whether to invest in assets denominated in the Euro or the US dollar, he or she faces the following constraint:

$$n_t = n_{1,t} + n_{2,t},$$

where n_t denotes the constraint of each speculator's total investment in period t, n_1, investment in assets denominated in the Euro, n_2, investment in assets in the US dollar. The total investments in this whole economy are defined as N in each period, which is assumed to be constant for simplicity. In other words, we assume a given accumulation rate of total assets. Investment constraint in this whole economy is as follows:

$$N = N_1 + N_2, \tag{4}$$

where N_1 denotes investments in Euro assets, and N_2, investments in US dollar assets. The stock constraint of each trader is as follows:

$$b = b_1 + b_2,$$

where b denotes the total holding assets of each speculator, b_1, the holding Euro assets, and b_2, the holding US dollar assets. The aggregated stock constraint of the whole economy is as follows:

$$B = B_1 + B_2, \tag{5}$$

where B denotes aggregate assets in this economy, B_1, aggregate Euro assets, and B_2, aggregate US dollar assets.

Let an individual speculator's profit function be represented by π as follows:

$$\pi = (\varepsilon_1 + \delta_1) b_1 + (\varepsilon_2 + \delta_2) b_2 - c_1(B_1) n_1 - c_2(B_2) n_2 - \phi(N_1 + N_2).$$

We can aggregate each speculator's profit function to obtain the profit function of this whole economy:

$$\Pi = (\varepsilon_1 + \delta_1) B_1 + (\varepsilon_2 + \delta_2) B_2 - \alpha c_1(B_1) N_1 - c_2(B_2) N_2 - \phi(N_1 + N_2). \tag{6}$$

Π denotes the aggregate profit function. The change of assets denominated by Currency 1 is as follows:

$$\dot{B}_1 = N_1. \tag{7}$$

Each trader maximizes inter-temporal profit subject to changes of assets denominated by the Euro. The traders' maximization problem is as follows:

$$\max \int_t^{\infty} \Pi_t e^{-p(s-t)} ds$$
$$\text{s.t. } \dot{B}_1 = N_1. \qquad (8)$$

Each speculator in each period determines the volume of investment in Euro assets or US dollar assets, while he or she regards the transaction cost function as given.

We can define the current Hamiltonian value as follows:

$$H=(\varepsilon_1+\delta_1)B_1+(\varepsilon_2+\delta_2)B_2-\alpha c_1(B_1)N_1-c_2(B_2)N_2-\phi(N_1)+\lambda N_1, \qquad (9)$$

where λ denotes the co-state variable. The necessary condition is as follows:

$$\frac{\partial H}{\partial N}=0, \qquad (10)$$

$$\dot{\lambda}=p\lambda-\frac{\partial H}{\partial B_1}. \qquad (11)$$

The transversality condition is as follows:

$$\lim_{t\to\infty}\lambda_t B_{1,t}\exp(-pt)=0. \qquad (12)$$

From (10),

$$-\alpha c_1(B_1)N_1-c_2(B_2)N_2-\phi'_{N1}+\lambda=0,$$

which can be written by a form of inverse function as follows:

$$N=N(\lambda,B_1), \qquad (13)$$

where our assumptions are denoted by: $\frac{dN}{d\lambda}=\phi''>0$, $\frac{dN}{dB_1}=-\frac{\alpha c_1'+c_2'}{\phi''}$.
From (13), Equation (11) yields

$$\dot{\lambda}=p\lambda-\left(\Omega-\alpha c_1'(B_1)N_1+c_2'(B_2)N_2\right), \qquad (14)$$

where $\Omega=(\varepsilon_1-\varepsilon_2)+(\delta_1-\delta_2)=$*const.* from our assumption. With attention to (12), Equation (14) can be solved as follows:

$$\lambda=\int_t^{\infty}\left(\Omega+\alpha c_1'(B_1)N_1-c_2'(B_2)N_2\right)\exp((-p)(s-t))ds, \tag{15}$$

where λ represents a current value of a series of differences in expected gain and transaction cost between two currencies. This suggests determinants of investment are current and future gains from both assets and transaction costs. In other words, the difference in future transaction costs depends on other speculators' expectations for future transaction costs due to externality.

3-2) Increasing transaction costs

A simple way to explore how speculations on assets denominated in the Euro are determined is to take a linear approximation of the two-equation system in a steady state. The result is

$$\begin{pmatrix}\dot{B}_1\\ \dot{\lambda}\end{pmatrix}=\begin{pmatrix}A_1 & \phi''_{\lambda}\\ A_2 & p\end{pmatrix}\begin{pmatrix}B_1-\bar{B}\\ \lambda-\bar{\lambda}\end{pmatrix}, \tag{16}$$

where we define $A_1\frac{\partial N}{\partial B_1}=<0,\ A_2=\alpha c_1''N_1+c''(\bar{N}-N_1)+\alpha c_1'N'_{1B1}+c_2N'_{1B1}$.

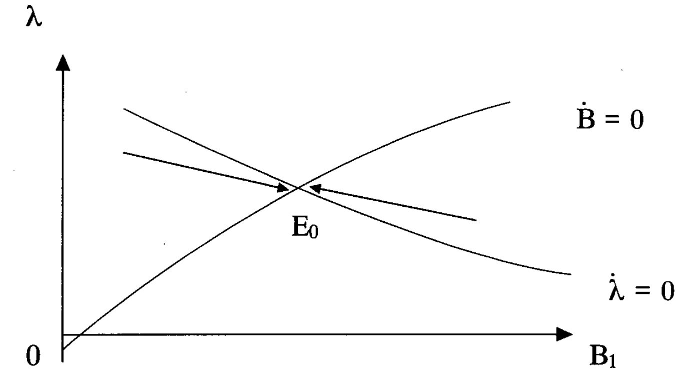
A_1 is negative, but A_2 is ambiguous. Here, we assume A_2 is positive, that is, the effect of increasing costs is greater than the effect of the first order of c.

The determinant of the above matrix is

det$=A_1\rho-A_2\phi''_{\lambda}$.

det is negative. Since the determinant in this case is negative, the characteristic root of the matrix is real and of opposite sign. Thus, this dynamic system has positive root $\theta+$, and negative root $\theta-$. The steady state is the saddle point. Figure 2 shows a phase diagram of Case 1 with a unique stable arm. Equilibrium is shown as E_0.

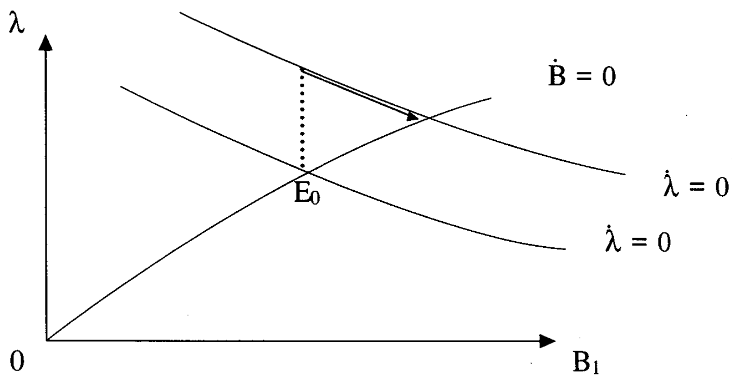
Figure 2: Phase diagram in the case of increasing function of transaction cost (1)



We investigate the effect of introducing the Euro. As banks in the US and Europe invest funds for new dealing systems, and have constructed Euroclear for convenient transaction, transaction cost is expected to decrease. Here, the essential difference between the ecu and the Euro is convenience.

At first, this economy is at E_0. If parameter α is permanently reduced at the introduction of the Euro, the line of $\dot{\lambda}=0$ moves to upper $\dot{\lambda}=0$ in Figure 1. Equilibrium E_0 jumps to E_1, moving to a new equilibrium. The permanent decrease in α increases λ, which is the current value of a series of difference of transaction costs between two currencies. This results in the inflow of funds to Euro assets. However, inflow to Euro assets decreases because the transaction costs of the Euro are increasing. Figure 3 shows that the line of $\dot{\lambda}=0$ shifts upward, and the equilibrium jumps from E_0 to E_1, moving to E_2 due to a permanent decrease in α. A permanent decrease causes a decrease in the current value of the difference in transaction costs between the Euro and the US dollar. This results in capital flow from the US dollar to the Euro. However, the flow soon terminates in the economy due to increasing costs. The increase in the share of the Euro is limited even after α decreases.

Figure 3: Phase diagram in the case of increasing function of transaction cost (2)



3-3) Decreasing transaction costs

Here, transaction cost c is a decreasing function of B Thus, A_2 is negative as shown above. Trace (tr.) and determinant (det) of matrix (16) are as follows:

$$tr = A_1 + \rho = -\phi''^{-1}\left(\alpha c_1' + c_2'\right) + \rho\,, \tag{17}$$

$$\det = A_1\rho - A_2\phi''_\lambda$$

$$= -\left(\phi''_\lambda\left(\alpha c_1' + c_2'\right)\right)\left(\rho - \phi''_\lambda\left(\alpha c_1' + c_2'\right)\right) - \phi''_\lambda\left(\alpha c_1'' N + c_2''(\bar{N} - N_1)\right) \tag{18}$$

Both trace and determinant are ambiguous a priori. Here, we suppose *trace* is positive. That is,

$$\rho\phi''_\lambda - \left(\alpha c_1' + c_2'\right) > 0\,, \tag{19}$$

is assumed[2]. This inequality implies that ρ and ϕ''_λ are sufficiently large, or that technological innovation α is great. We can now distinguish between certain cases below, according to the parameters of the determinant because the determinant is ambiguous a priori.

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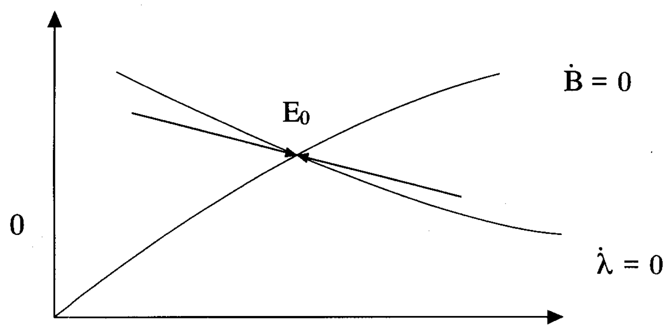
[2] The negative case is ingnored here because the system is stable.

Figure 4: Phase diagram in the case of decreasing function of transaction cost (1)



3-3-1) Case 1

We assume the following inequality:

$$\left|-(\alpha c_1' + c_2')\left(\rho - \phi''_{\lambda}(\alpha c_1' + c_2')\right)\right| < \left|\left(\alpha c_1'' N_1 + c_2''(\bar{N} - N_1)\right)\right|. \qquad (20)$$

Inequality (20) means that the degree of the decreasing cost of transactions is smaller. In Case 1, the network effect of an international currency as the Forex vehicle is not strong.

In Case 1, Equilibriums E_1 and E_2 are saddle points because the determinant of each equilibrium is negative. Because det is negative in this case, the equilibrium is saddle point with a unique stable arm, which we explored in the previous section. A phase diagram is shown as Figure 4 as in the previous section.

3-3-2) Case 2

In Case 2 of the parameters of the determinant, the following inequality is assumed:

$$\left|-(\alpha c_1' + c_2')\left(\rho - \phi''_{\lambda}(\alpha c_1' + c_2')\right)\right| < \left|\left(\alpha c_1'' N_1 + c_2''(\bar{N} - N_1)\right)\right|. \qquad (21)$$

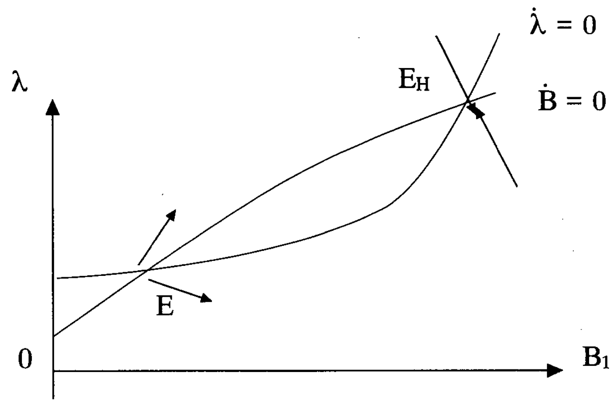
Inequality (20) means that the degree of the decreasing cost of transactions is larger. In Case 1, the network effect of an international currency as the Forex vehicle is strong.

Figure 5 shows the phase diagram of $\dot{\lambda}$=0 and $\dot{B}_1$=0 The lines of $\dot{\lambda}$ =0 and $\dot{B}_1$=0 are at the right upper slope with two equilibriums, E_L

and E_H. E_H is where the share of the Euro is highest in the equilibrium. In addition, E_H is the saddle point. This is because the determinant of the matrix of the coefficients of (16) is negative since c_I'' is smaller for the bigger B_I in our assumption. The share in E_L is lower than that in E_H. The determinant of the matrix of the coefficients in E_L is positive since c_I'' is bigger because of the relatively smaller B_I. Thus, this point is the source. Point 0 is also equilibrium where the share is zero. The determinant of the matrix of the coefficients at zero point is positive as mentioned above.

Thus, little use of the Euro can lead to greater use in Case 3 due to the expectations of the speculator. When the Euro was introduced, the frequency of use was lower than that of the US dollar. However, the share of the Euro could be higher if expectation of the use of the Euro evolves in the near future to a higher frequency than the US dollar.

Figure 5: Phase diagram in the case of decreasing function of transaction cost (2)



3-4) The effect of endogenously expected exchange rates in the dynamic system

In the previous section, expected fluctuations in exchange rates are assumed to be constant. However, this assumption does not reflect the actual situation in foreign exchange markets among leading countries, where flows of buying and selling determine exchange rates. Accordingly, we assume that this third country is not small, and we assume the endogenity of fluctuations in exchange rates as follows:

$$\varepsilon_1 = \eta_1 B_1,$$
$$\varepsilon_2 = \eta_2 B_2, \quad \eta_1, \eta_2 > 0. \tag{22}$$

Expected fluctuations in exchange rates between the Euro or the US dollar and the home currency depend on the net demand transaction volumes of the Euro or the US dollar. If the volume increases, the expected rate depreciates toward the currency. So, Ω is not constant, but $\Omega = \eta_1 B_1 - \eta_2 B_2 + \delta_1 - \delta_2$.

The trace and determinant of the matrix of the coefficients of a new dynamic system can be shown as follows:

$$tr. = A_1 + \rho = -\phi''^{-1}\left(\alpha c_1' + c_2'\right) + \rho, \tag{23}$$

$$\det = A_1\rho - A_2\phi''_\lambda.$$
$$= -\left(\phi''_\lambda\left(\alpha c_1' + c_2'\right)\right)\left(\rho - \phi''_\lambda\left(\alpha c_1' + c_2'\right)\right) - \phi''_\lambda\left(\alpha c_1'' N + c_2''(\bar{N} - N_1)\right) - \phi''_\lambda(\eta_2 + \eta_1). \tag{24}$$

From (24), η_1 and η_2 have a negative effect on the determinant, that is, they have a stabilizing effect with sufficiently large η_1 and η_2 whether the equilibrium is the saddle or the source. If the equilibrium is an unstable source, the unstable system might become a system with unique stable arm because of sufficiently large η_1 and η_2.

3-5) Possibility of bifurcation and the dynamics of currency competition

We now investigate the possibility of bifurcation of the trajectory in our system of decreasing transaction costs. After the Euro was introduced, the question of whether the Euro or the US dollar would be globally dominant in foreign exchange markets arose. Our exploration that focused on local analysis in the previous section is not sufficient for a global analysis. We use the Hopf theorem to investigate this. Our analysis focuses on the size of α , which means the innovation of transaction systems for the Euro.

When Equations (7) and (14) have imaginary characteristic roots, and when the real part of these characteristic roots is increasing for α decreasing, $\alpha = \alpha_0$ is the bifurcation point. When some $\bar{\alpha}$ is larger or lower than α_0, there is a closed trajectory for (λ, B_1). If $\bar{\alpha} < \alpha$, singulars on system α^* is a stable spiral. If $\bar{\alpha} < \alpha$, singulars on this system α^* are unstable spirals, and the trajectory approaches the closed trajectory around these singulars.

Now, suppose that the characteristic roots around the equilibrium of Equations (7) and (14) are imaginary when $\alpha = \alpha_0$. From (17),

$$tr. = A_1 + \rho = -\phi''_\lambda\left(\alpha_0 c_1' + c_2'\right) + \rho = 0, \tag{25}$$

$$tr^2. - 4\det = \alpha_0\phi''_\lambda\left(c_1''N_1 - c_1'\right) + \left(\rho - \phi''_\lambda c_2'\right)\rho + \phi''_\lambda c_2''\left(\bar{N} - N_1\right) < 0. \tag{26}$$

From (25) and (26), the range of α_0 is made clear as follows:

$$\frac{\rho - \phi''_\lambda}{c_1'} = \alpha_0 < -\frac{\left(\rho - \phi''_\lambda c_2'\right)\rho + \phi''_\lambda c_2''\left(\bar{N} - N_1\right)}{\left(c_1''N_1 - c_1'\right)\phi''_\lambda} \tag{27}$$

If α_0 is within the range above, the system has a closed trajectory[3] because the real part of the characteristic roots increase with increasing α. $\alpha = \alpha_0$ is the bifurcation point in this system. If there is a closed trajectory on some α ($>\alpha_0$), the equilibrium is source. In addition, the trajectory is a stable, limited cycle. The trajectory recurs from the neighborhood of the equilibrium approaching the closed trajectory. If α becomes larger than α_0, this change in α cause bifurcation by breaking the closed trajectory.

From this analysis, a cycle of shifting between the Euro and the US dollar as the key currency will occur if the impact introducing the Euro is within the range (27). If the value of α is larger than α_0, the Euro dominates the dollar in foreign exchange markets. The competition depends on the value of α .

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[3] Because $tr. = \rho - \phi''_\lambda\left(\alpha c_1' + c_2'\right)$ is assumed positive and $\alpha c_1' - c_2'$ is assumed positive, $\rho - \phi''_\lambda$ is positive.

4. Implications for the future of the Euro according to our model

This paper investigates the conditions under which the Euro dominate the US dollar in Forex markets. When the Euro was introduced, technical innovation in dealing in the Euro made transaction costs much lower due to network effects. We focus on the foreign exchange vehicle function of an international currency because this factor is more important than others. Even if there was no technical innovation in dealing in the Euro, the increasing transaction volumes in the Euro can be regarded as technical progress.

Table 4: Sizes of economies in major currencies areas

	GDP Base				Trade Base			
	Euro	USdollar	Yen	others	Euro	USdollar	Yen	others
Industrialized Countries	33.4	30.8	17.1	0.3	45.8	18.8	8.1	0.5
EU15	31.5	0.0	0.0	0.0	43.0	0.0	0.0	0.0
EU11	25.1	0.0	0.0	0.0	34.5	0.0	0.0	0.0
United States	0.0	27.5	0.0	0.0	0.0	14.2	0.0	0.0
Japan	0.0	0.0	17.0	0.0	0.0	0.0	8.0	0.0
Others	1.9	3.3	0.1	0.3	2.8	4.6	0.1	0.5
Developing Countries	1.3	15.7	0.4	1.1	2.4	22.1	1.1	1.4
Africa	0.5	0.7	0.0	0.2	0.7	1.3	0.0	0.4
Asia	0.3	7.5	0.3	0.2	1.3	14.6	1.0	0.2
Europe	0.1	0.8	0.0	0.1	0.2	0.9	0.0	0.6
Middle East Asia	0.0	1.0	0.0	0.1	0.0	1.7	0.0	0.2
Latin America	0.4	5.7	0.1	0.0	0.2	3.7	0.1	0.0
Total	34.7	46.4	17.5	1.4	48.1	40.9	9.2	1.8

Source) Calculation from the Appendixes in Kawai and Akiyama (1998).

Kawai and Akiyama (1998) calculate the economy sizes of major currency areas using the GDP base, and the trade base, which is the summation of exports and imports. The results are shown in Table 4. From the GDP base in this table, relative economy size of the US dollar is 46.4% in the world, the Euro, 34.7%, and the Yen, 17.5%. Developing countries peg their currencies to the US dollar to stabilize their currency exchange rates.

The relative size of economies on a trade basis is 40.9 for US dollars , 48.1% for Euros, and 9.2% for Yen. In this calculation, the Euro is relatively larger because it includes trade among EU 15 countries. However, the share of trade in the Euro by third countries is not large, so the share in US dollars might be de facto larger.

The US dollar may maintain its present position even after the introduction of the Euro for a while because of the hysteresis effect. However, if the Euroland expands with the addition of Central and Eastern Europe, it will result in higher frequency of use of the Euro as the denomination or vehicle currency. If Asian countries or others prefer to hold assets denominated in the Euro, the Euro will become a more important currency as a portfolio currency, which will result in reduced transaction cost and increased convenience for use of the Euro. Thus the Euro has the potential to emerge as the new key international currency in place of the US dollar.

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