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Local asymptotic stability of a general equilibrium for an economy under imperfect competition ★

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Abstract

We prove that a general equilibrium for an economy under imperfect competition is locally asymptotically stable, in which imperfectively competitive firms can influence a price adjustment process.

Key words: General equilibrium, Imperfect competition, Local asymptotic stability.
JEL classification: C62, D43

1. Introduction

This paper studies an asymptotic stability of a general equilibrium for an economy under imperfect competition. The assumption of perfect competition that prices are adjusted by a fictive auctioneer may not be reasonable under imperfect competition in which firms can influence prices. Thus, we must define another adjustment process.

Several papers have analyzed distinct adjustment processes. First, in Negishi (1961), an excess demand in each imperfectly competitive market is always assumed to be zero. Hence, a dynamic process in the market is *a kind of a feed-back adjustment process* (Negishi, 1961, p. 199) by which firms derive the maximum profit. However, the assumption that the excess demand in imperfectly competitive markets is always cleared is strong. Second, Fisher (1970, 1972) and Allingham (1976) studied a price adjustment process conducted by individuals, not the auctioneer. The process is in a directly opposite position to a typical tâtonnement process. The supposition that every imperfectively competitive firm can change prices in its own way is also strict as well as that prices

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are completely adjusted by the auctioneer. Thus, we take a third position that the price adjustment process may be influenced by imperfectly competitive firms.

Because we suppose consumers act as price takers, we may derive a demand function by the standard methods. On the other hand, we hypothesize that each firm may influence a price of its product. Since actions of imperfectly competitive firms in the framework of general equilibrium has been atypically treated in previous studies, we must explain the behavior of firms in great detail. That explanation should be required in order to verify the stability of an equilibrium in the economy. We assume that as the firm increases or decreases output under a current price, it estimates a rate of change in price, which is based on expected demands for its product. The firm produces its profit-maximizing output based on an anticipated price for its product. A notable feature of our economic model is that every firm can impact on a price-adjustment process conducted by the fictive auctioneer. At the equilibrium, the firms' estimated prices correspond with current market prices, and excess demand in each market is cleared. Our purpose is to prove a local and global stability of the equilibrium for the economy under the influence of dominant firms.

The rest of the paper is organized as follows. In Section 2, we construct our economic model. We mention commodities and prices in Subsection 2. 1, discuss the consumption sector in Subsection 2. 2, and study about the behavior of imperfectly competitive firms in Subsection 2. 3. In Section 3, a general equilibrium for an economy under imperfect competition is defined, and the local asymptotic stability of the equilibrium for the economy is proved.

2. Construction of the model

2.1. *Commodities and prices*

We assume that there exist s total commodities, $s-1$ products, and one factor of production, the s-th commodity. We prepare the following sets for indices of commodities:

$I := \{1, \cdots, s\}$;
$J := \{1, \cdots, s-1\}$;
$I_{-j} := \{1, \cdots, j-1, j+1, \cdots, s\}$ for any $j \in J$;
$J_{-j} := \{1, \cdots, j-1, j+1, \cdots, s-1\}$ for any $j \in J$.

We postulate that the s-th commodity is provided by consumers only. This implies that the production factor is not produced by firms. We may assume that s is an odd number without loss of generality. Let p_j denote a price of the j-th commodity for any $j \in I$. And let p denote a price vector $(p_1, \cdots, p_s)$. We assume that every price vector belongs to the set $P := \{p \in R^s | p_j > 0$ for any $j \in I\}$.

2.2. *Consumption sector*

We assume that every consumer acts as a price taker. Thus, we may derive a demand function by the standard methods and retain specifications about a consumption sector to a minimum. We consider that consumers' total demand, $x(p) := (x_1(p), \cdots, x_s(p))$, is given. We set up the following assumption (A. 1) about the demand function.

(A. 1) (i) The total demand function $x : P \to R^s$ is of class C^2;
(ii) The function $x(p)$ is homogeneous of degree 0 for any $p \in P$; and
(iii) The condition $\frac{\partial x_i(p)}{\partial p_j} > 0$ holds for any i and j in I $(i \neq j)$.

These conditions are standard in analyses of a stability of a general equilibrium.

2.3. *Production sector*

A behavior of imperfectly competitive firms in a general equilibrium analyses is not entirely a matter of common knowledge. For this reason, we must study the behavior of firms in detail. For the sake of simplicity, we presume that each of $s-1$ firms produces one product by inputting $s-1$ types of production factors provided from the other firms and consumers. Thus, we may designate a firm producing the j-th commodity as the j-th firm for any $j \in J$. We suppose that: Each firm can recognize the demand for its product at a current price. However, at prices different from the prevailing price, the firm must estimate demands with which it should be faced. Thus, every imperfectly competitive firm subjectively anticipates demands for its product at each price different from the current price. The firm makes a prediction for a price based on the expected demands and maximizes its profit under a constraint of production technologies.

First, we discuss a production technology for each imperfectly competitive firm. Fix any $j \in J$. Let $w_j := (w_{j1}, \cdots, w_{jj-1}, w_{jj+1}, \cdots, w_{js})$ denote a vector of production factors and let $f_j : R^{s-1} \to R$ be a production function. Subsequent to this, we use f_j^m and f_j^{mn} as symbols denoting a derivative of the first and second order of f_j, that is, $f_j^m := \frac{\partial f_j(w_j)}{\partial w_{jm}}$ and $f_j^{mn} := \frac{\partial}{\partial w_{jn}} \frac{\partial f_j(w_j)}{\partial w_{jm}}$. Concerning production technologies, we set up the following assumption (A. 2).

(A. 2) For any $j \in J$: (i) $f_j : R^{s-1} \to R$ is of class C^2;
(ii) For any $m \in I_{-j}$, $f_j^m > 0$; and
(iii) For any $m \in I_{-j}$ and $n \in I_{-j}$ $(m \neq n)$, $f_j^{mm} < 0$ and $f_j^{mn} > 0$;
(iv) There is some $\bar{y}_j \in R$ satisfying $f_j(w_j) < \bar{y}_j < \infty$ for any $w_j \in R^{s-1}$ if $w_{js} < \infty$.

Items (i) and (ii) are standard. The condition (iii) is stronger than the strict-concavity of f_j, however, we need the condition to obtain the required result. The property (iv) means that every firm cannot produce its product infinitely if an input of the s-th production factor provided from consumers is finite. The supposition seems to be quite natural.

Then, we mention a price estimation for every firm. Fix any $j \in J$. Let $\varphi_j \in (-1, \infty)$ denote a rate of changes in price. Then, the j-th firm's subjective demand for its product is denoted as follows:

$$e_j(p_1, \cdots, (1+\varphi_j)p_j, \cdots, p_s).$$

This is abbreviately described by $e_j(\varphi_j, p)$. We set up the following assumption concerning the function $e_j : (-1, \infty) \times P \to R$:

(A. 3) For any $j \in J$, e_j is homogeneous of degree 0 with respect to $p \in P$.

This assumption means that an expected demand for each firm depends only on relative prices. That is, every firm is not possessed with the illusion that the expected demand changes by simultaneous changes of prices.

Suppose that a price p is fixed. Then we can define

$$\varphi_j : (-\infty,\, \bar{y}_j) \to (-1, \infty)$$

as a function associating φ_j to y_j such that $e_j(\varphi_j, p) = y_j$ holds for any $j \in J$. We set up the following assumption (A. 4) concerning the function φ_j.

(A. 4) For any $j \in J$: (i) The function $\varphi_j : (-\infty,\, \bar{y}_j) \to (-1, \infty)$ is linear;
(ii) The condition $\varphi_j'(y_j) < 0$ holds for any $y_j \in (-\infty,\, \bar{y}_j)$.

We need the condition (i) to guarantee the uniqueness of the maximum solution for each firm. The property (ii) implies that the j-th firm considers the possibility to raise (reduce) the price of its product by a decrease (increase) in production. If the firm inputs all production factors provided from consumers, the maximal output is $\bar{y}_j$ or less by (A. 2)(iv). Thus, it is reasonable that the firm considers $(1 + \varphi_j(\bar{y}_j))p_j > 0$. This is compatible with the supposition that the lower bound of the range of φ_j is -1.

As usual, we assume that every firm maximizes its profit:

$$\pi_j(p) := \max_{w_j} \big(1 + \varphi_j(f_j(w_j))\big) p_j f_j(w_j) - \sum_{m \in J_{-j}} p_m w_{jm}. \tag{1}$$

The first order condition of the problem (1) for any $j \in J$ is

$$p_j f_j^m \big(1 + \varphi_j(y_j) + \varphi_j'(y_j) y_j\big) - p_m = 0 \text{ for any } m \in I_{-j}. \tag{2}$$

For any $j \in J$, we define $\sigma_j(y_j) := \frac{y_j}{\varphi_j(y_j)} \varphi_j'(y_j)$. That is, $\frac{1}{\sigma_j(y_j)}$ is the elasticity of demand with respect to each output y_j. If we adopt the concept, the first order condition (2) may be rewritten as

$$p_j f_j^m \big\{1 + \varphi_j(y_j)\big(1 + \sigma_i(y_j)\big)\big\} - p_m = 0 \text{ for any } m \in J_{-i}. \tag{3}$$

Then, the second order condition of (1) is given by the negative definiteness of the Hessian matrix of the function $p_j f_j^m (1 + \varphi_j(f_j(w_j)) + \varphi_j'(y_j) f_j(w_j)) - p_m$. If we define η_j by

$$\eta_j := 1 + \varphi_j(y_j)(1 + \sigma_j(y_j)),$$

then the Hessian matrix of the function is

$$H_j := \Big(p_j \eta_j f_j^{mn} + 2p_j \varphi_j'(y_j) f_j^m f_j^n,\ m \in I_{-j},\ n \in I_{-j}\Big).$$

We establish the following lemma about the Hessian matrix.

Lemma. *For any $j \in J$, H_j is negative definite under* (A. 2) *and* (A. 4).

Proof. Fix any $j \in J$. Suppose that $|H_j^{(m)}|$ is the m-th order principal minor for any $m = 1, \cdots, s-1$. It should be noted that $|H_j|$ is written as $|H_j^{(s-1)}|$ for convenience' sake. Then, the negative definiteness of the matrix H_j is equivalent to the following property: $|H_j^{(1)}| < 0$, $|H_j^{(2)}| > 0$, $\cdots$, $|H_j^{(s-1)}| > 0$. Let F_j be the Hessian matrix

$$F_j := \Big(f_j^{mn};\ m \in I_{-j}, n \in I_{-j}\Big)$$

and denote the m-th order principal minor of F_j by $|F_j^{(m)}|$ for any $m = 1, \cdots, s-1$. Further, for any $n = 1, \cdots, s-1$, let $|G_j^{(n)}(m)|$ $(1 \leqq m \leqq n)$ be determinants defined as follows:

$$
\begin{aligned}
&\left|G_j^{(1)}(1)\right| := f_j^m, \\
&\left|G_j^{(2)}(1)\right| := \begin{vmatrix} f_j^m & f_j^{mn} \\ f_j^n & f_j^{nn} \end{vmatrix}, \quad \left|G_j^{(2)}(2)\right| := \begin{vmatrix} f_j^{mm} & f_j^m \\ f_j^{nm} & f_j^n \end{vmatrix}, \\
&\quad \vdots \\
&\left|G_j^{(s-1)}(m)\right| := \begin{vmatrix} f_j^{11} & \cdots & f_j^1 & \cdots & f_j^{1s} \\ \vdots & & \vdots & & \vdots \\ f_j^{s1} & \cdots & f_j^s & \cdots & f_j^{ss} \end{vmatrix}.
\end{aligned}
$$

It must be noted that $f_j^1 \cdots f_j^s$ is located in the m-th column. If we define a_j by

$$
a_j := \frac{2\varphi_j'(y_j)}{\eta_j},
$$

the negative definiteness of H_j is equivalent to the following condition (4) from the basic properties of the determinant:

$$
\begin{aligned}
&\left|H_j^{(1)}\right| = p_j\eta_j\left(\left|F_j^{(1)}\right| + a_j f_j^m \left|G_j^{(1)}(1)\right|\right) < 0, \\
&\left|H_j^{(2)}\right| = (p_j\eta_j)^2\left(\left|F_j^{(2)}\right| + a_j f_j^m \left|G_j^{(2)}(1)\right| + a_j f_j^n \left|G_j^{(2)}(2)\right|\right) > 0, \\
&\quad \vdots \\
&\left|H_j^{(s-1)}\right| = (p_j\eta_j)^{s-1}\left(\left|F_j^{(s-1)}\right| + a_j \sum_{m=1}^{s-1} f_j^m \left|G_j^{(s-1)}(m)\right|\right) > 0.
\end{aligned}
\tag{4}
$$

Suppose y_j is the optimum level of output. Then, the condition, $|\sigma_j(y_j)| < 1$, should hold from the first order condition (3), and thus $\eta_j > 0$. From the result and (A. 4), $a_j < 0$ is also true. Accordingly, the first relation of (4) may be obtained by the assumption (A. 2). The second relation $|H_j^{(2)}| > 0$, that is, $|F_j^{(2)}| > 0$, $|G_j^{(2)}(1)| < 0$ and $|G_j^{(2)}(2)| < 0$, also holds by the same suppositions. Then, we verify that the property $|H_j^{(3)}| < 0$ is true under the previous result $|H_j^{(2)}| > 0$. A cofactor expansion of $|G_j^{(3)}(1)|$ along the first row is given by

$$
\left|G_j^{(3)}(1)\right| = f_j^m \begin{vmatrix} f_j^{nn} & f_j^{nq} \\ f_j^{qn} & f_j^{qq} \end{vmatrix} - f_j^{mn} \begin{vmatrix} f_j^n & f_j^{nq} \\ f_j^q & f_j^{qq} \end{vmatrix} + f_j^{mq} \begin{vmatrix} f_j^n & f_j^{nn} \\ f_j^q & f_j^{qn} \end{vmatrix}.
$$

We rearrange columns of each minor determinant of $|G_j^{(3)}(1)|$ so that f_j^{nn} and f_j^{qq} are diagonal elements. Needless to say, its sign must change. Then, $|G_j^{(3)}(1)|$ is equivalent to the following condition:

$$\left|G_j^{(3)}(1)\right| = f_j^{m}\left|F_j^{(2)}\right| - f_j^{mn}\left|G_j^{(2)}(1)\right| - f_j^{mq}\left|G_j^{(2)}(2)\right|.$$

It follows from the above results that $|F_j^{(2)}| > 0$, $|G_j^{(2)}(1)| < 0$ and $|G_j^{(2)}(2)| < 0$, and (A. 4) that $|G_j^{(3)}(1)| > 0$. Moreover, by the same method, it is clear that $|G_j^{(3)}(2)| > 0$ and $|G_j^{(3)}(3)| > 0$ are obtained. Therefore, since both $\eta_j > 0$ and $a_j < 0$ hold, the condition $|H_j^{(3)}| < 0$ is concluded under (A. 2).

Now, we set up the assumption of induction $|H_j^{(s-2)}| < 0$. The condition means that $|F_j^{(s-2)}| < 0$ and $|G_j^{(s-2)}(1)| > 0, \cdots, |G_j^{(s-2)}(s-2)| > 0$ hold. We will use the same method as the above argument. We consider a cofactor expansion of $|G_j^{(s-1)}(1)|$ along the first row. Further, we rearrange columns of each minor determinant of $|G_j^{(s-1)}(1)|$ so that f_j^{nn} and f_j^{qq} are diagonal elements. Suppose that $r_j(t, t')$ denotes a positive integer indicating the minimum number of rearrangements with respect to each (t, t')-minor determinant of $|G_j^{(s-2)}(1)|$. Then, the cofactor expansion of $|G_j^{(s-1)}(1)|$ along the first row is given by

$$\left|G_j^{(s-1)}(1)\right| = (-1)^{1+1+r(1,1)} f_j^{1}\left|F_j^{(s-2)}\right| + \sum_{n=2}^{s-2}(-1)^{1+n+r(1,n)} f_j^{1n}\left|G_j^{(s-2)}(n)\right|.$$

It is clear that $r(1,1) = 0$, $r(1,2) = 0$, $r(1,3) = 1, \cdots, r(1, s-2) = s-4$. By supposition, s is an odd number. Therefore, the condition $|G_j^{(s-1)}(1)| < 0$ holds under the assumption of induction and (A. 2). Furthermore, the conditions $|G_j^{(s-1)}(2)| < 0, \cdots, |G_j^{(s-1)}(s-1)| < 0$ are obtained by the same method. As a result, the condition $|H_j^{(s-1)}| > 0$ is true by (A. 2) since the conditions $\eta_j > 0$ and $a_j < 0$ hold. □

For any $j \in J$, we denote a factor demand obtained as a solution of (2) as follows; $w_j(p) := (w_{j1}(p), \cdots, w_{jj-1}(p), w_{jj+1}(p), \cdots, w_{js}(p))$. And the supply function of the j-th product is defined by $y_j(p) := f_j(w_j(p))$. Thus, the excess demand function for each commodity is defined as follows:

$$z_j(p) := x_j(p) + \sum_{n \in J_{-j}} w_{nj}(p) - y_j(p) \text{ for any} j \in J \text{ and}$$
$$z_s(p) := x_s(p) + \sum_{j \in J} w_{js}(p) - \bar{x}_s$$

where $\bar{x}_s$ is the total initial endowment for consumers.

3. Theorem on a stability of an equilibrium

We define an equilibrium for an economy under imperfect competition as a state that the price estimated by each firm is consistent with the market price and that the excess demand in each market is cleared. The equilibrium is precisely defined as a list of a price and an allocation satisfying the conditions: $\varphi_j(y_j(p^*)) = 0$ for any $j \in J$; and $z_j(p) = 0$ for any $j \in I$. We verify that the equilibrium is locally asymptotically stable.

Since the supply function y_j is homogeneous of degree 0 for any $j \in J$ by the assumption (A. 2), we can normalize prices under (A. 1)(ii). Subsequent to this, for any $j \in J$, suppose that p_j is a relative price p_j/p_s of the j-th commodity to the s-th numéraire commodity and that p denotes a relative price vector $(p_1, \cdots, p_{s-1})$. If $z_j(p) = 0$ for any $j \in J$, then $z_s(p) = 0$ is concluded by Walras' law. Hence, we consider an adjustment process for excess demands of $s-1$ products except the s-th production factor:

$$\frac{dp_j(t)}{dt} = \varphi_j^*(p) + z_j(p) \text{ for any } j \in J \tag{5}$$

where $\varphi_j^*(p) := \varphi_j(y_j(p))$.

Now, we are in a position to prove the local asymptotic stability of the equilibrium for the economy.

Theorem. *Under the assusmptions* (A. 1) – (A. 4)*, the equilibrium for an economy under imperfect competition is locally asymptotically stable.*

Proof. Suppose that $J(p^*)$ denotes the square matrix of $s-1$ order defined by

$$J(p^*) := \left(\frac{\partial \varphi_j^*(p^*)}{\partial p_{j'}} + \frac{\partial z_j(p^*)}{\partial p_{j'}}, \quad j \in J,\ j' \in J \right).$$

Then, since the conditions $\varphi_j^*(p^*) = 0$ and $z_j(p^*) = 0$ hold at the equilibrium, we may approximate the above dynamical system (5) at the neighborhood of p^* as follows:

$$\frac{dp(t)}{dt} = J(p^*)(p - p^*). \tag{6}$$

If the real part of eigenvalues of $J(p^*)$ are negative, the required result may be obtained. Therefore, we must show that the matrix is negative definite, which is equal to the following two conditions:

$$\frac{\partial \varphi_j^*(p^*)}{\partial p_j} + \frac{\partial z_j(p^*)}{\partial p_j} < 0 \text{ for any } j \in J \text{ and} \tag{7}$$

$$\frac{\partial \varphi_j^*(p^*)}{\partial p_{j'}} + \frac{\partial z_j(p^*)}{\partial p_{j'}} > 0 \text{ for any } j \in J \text{ and } j' \in J_{-j}. \tag{8}$$

The proof of the condition (7): Suppose that the following (9) and (10) hold:

$$\frac{\partial w_{jm}(p^*)}{\partial p_j} > 0 \text{ for any } j \in J \text{ and } m \in J_{-j}, \text{ and} \tag{9}$$

$$\frac{\partial w_{nj}(p^*)}{\partial p_j} < 0 \text{ for any } n \in J \text{ and } j \in J_{-n}. \tag{10}$$

Then, it follows from (9) and (A. 2) that

$$\frac{\partial y_j(p^*)}{\partial p_j} := \frac{\partial f_j(w_j(p^*))}{\partial p_j} = \sum_{m \in J_{-j}} f_j^m \frac{\partial w_{jm}(p^*)}{\partial p_j} > 0. \tag{11}$$

Thus, the following condition also holds under the assumption (A. 4):

$$\frac{\partial \varphi_j^*(p^*)}{\partial p_j} := \varphi_j'(y_j)\frac{\partial y_j(p^*)}{\partial p_j} < 0. \tag{12}$$

Since the condition $\frac{\partial x_j(p^*)}{\partial p_j} < 0$ holds by Euler's formula under (A. 1), it follows from (10) and (11) that

$$\frac{\partial z_j(p^*)}{\partial p_j} := \frac{\partial x_j(p^*)}{\partial p_j} + \sum_{n \in J_{-j}} \frac{\partial w_{nj}(p^*)}{\partial p_j} - \frac{\partial y_j(p^*)}{\partial p_j} < 0. \tag{13}$$

From (12) and (13), the condition (7) is true and every diagonal element of $J(p^*)$ is negative. Thus, it remains to show that (9) and (10) are true.

The condition $p_j \frac{\partial f_j(w_j(p))}{\partial w_{jm}}\big(1 + \varphi_j(f_j(w_j(p))) + \varphi_j'(y_j)f_j(w_j(p)\big) = p_m$ always holds for any $j \in J$ and $m \in I_{-j}$ by (2). We obtain the following condition by differentiating both sides of the equation with respect to p_j:

$$H_j^{(s-1)} \begin{pmatrix} \dfrac{\partial w_{j1}(p^*)}{\partial p_j} \\ \vdots \\ \dfrac{\partial w_{js}(p^*)}{\partial p_j} \end{pmatrix} = \begin{pmatrix} -f_j^1 \eta_j \\ \vdots \\ -f_j^s \eta_j \end{pmatrix}.$$

Suppose that $\hat{H}_j^{(s-1)}(m)$ is a matrix substituted the m-th column of $H_j^{(s-1)}$ for the vector $(-f_j^1\eta_j, \cdots, -f_j^s\eta_j)^T$, then by Cramer's rule,

$$\frac{\partial w_{jm}(p^*)}{\partial p_j} = \frac{1}{\big|H_j^{(s-1)}\big|}\big|\hat{H}_j^{(s-1)}(m)\big|.$$

Since the determinant $|\hat{H}_j^{(s-1)}(m)|$ is equal to $-p_j^{s-2}\eta^{s-1}|G_j^{(s-1)}(m)|$, we obtain the required condition (9) from Lemma, as follows:

$$\frac{\partial w_{jm}(p^*)}{\partial p_j} = \frac{1}{\big|H_j^{(s-1)}\big|}\big(-p_j^{s-2}\eta^{s-1}\big|G_j^{(s-1)}(m)\big|\big) > 0.$$

By the same method used in the above proof of (9), we can show that the condition (10) is true. The condition $p_n\frac{\partial f_n(w_n(p))}{\partial w_{nm}}\big(1+\varphi_j(f_n(w_n(p)))+\varphi_n'(y_n)f_n(w_n(p)\big) = p_m$ holds by the first order condition (2) for any $n \in J$ and $m \in I_{-n}$. It follows from differentiating both sides of the equation with respect to p_j that

$$H_n^{(s-1)} \begin{pmatrix} \dfrac{\partial w_{n1}(p^*)}{\partial p_j} \\ \vdots \\ \dfrac{\partial w_{nj}(p^*)}{\partial p_j} \\ \vdots \\ \dfrac{\partial w_{ns}(p^*)}{\partial p_j} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

where 1 is in the j-th row. Suppose that $\hat{H}_n^{(s-1)}(j)$ is a matrix substituted the j-th column of $H_n^{(s-1)}$ for the vector $(0, \cdots, 1, \cdots, 0)^T$. Then, it follows from Cramer's rule that

$$\frac{\partial w_{nj}(p^*)}{\partial p_j} = \frac{1}{\left|H_n^{(s-1)}\right|}\left|\hat{H}_n^{(s-1)}(j)\right|.$$

It is clear that $|\hat{H}_n^{(s-1)}(m)|$ is equal to $|H_n^{(s-2)}|$. Thus, we obtain the required condition (10) from Lemma, as follows:

$$\frac{\partial w_{nj}(p^*)}{\partial p_j} = \frac{1}{\left|H_n^{(s-1)}\right|}\left|H_n^{(s-2)}\right| < 0.$$

The condition (8) may be verified by the same method used in the proof of (7). □

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