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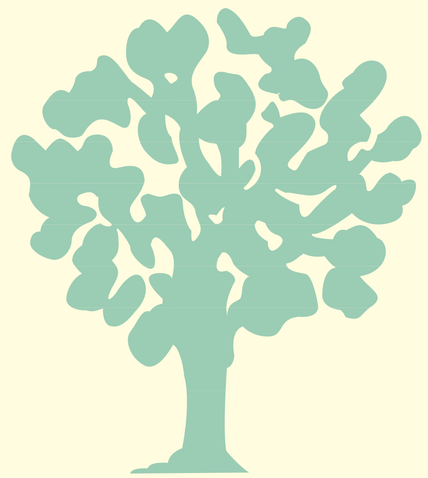
The Environment as a Production Input: A Tutorial

Jeffrey R. Vincent



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South Asian Network for Development and Environmental Economics

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The Environment as a Production Input: A Tutorial

Jeffrey R. Vincent

1. Introduction

Production is often affected by environmental conditions. For example, rice harvests might be damaged by infiltration of saline water from neighboring shrimp farms, fish catch might be reduced by the loss of spawning grounds when mangroves are cut down, and timber harvests might fall as a result of damage from acid rain and other forms of air pollution. In all these cases, environmental quality is acting as a nonmarket, or unpriced, production input. Damage to the environment reduces the supply of this input, and as a result production falls. Conversely, programs to improve environmental quality can benefit environmentally sensitive forms of production by raising the supply of such inputs. These production-related benefits can be among the most important benefits generated by environmental improvements. This is especially likely to be the case in developing countries, where sectors such as agriculture, forestry, and fishing typically account for a larger share of overall economic activity than in developed countries.

In principle, the valuation of changes in environmental quality that affect production is straightforward: one needs to estimate the change in profit caused by the environmental change. There are several ways to estimate the change in profit, however, and one might not have the data necessary to use all of them. Understanding the relationships among the different approaches, and the conditions under which they are valid, is thus important. Moreover, sometimes one might only be able to estimate a component of the change in profit, such as the change in revenue or the change in cost. A question that arises is whether partial measures such as these can be used to value environmental changes.

This tutorial covers these types of issues. Its purpose is to review the relationships among three key functions in production economics—production functions, cost functions, and profit functions—and to review how they can be used to value changes in environmental quality. The tutorial is in two parts. The first part (sections 2-6) is purely conceptual. It illustrates the relationships among the three functions by referring to a specific type of production function, the Cobb-Douglas function. This is the most common production function used in applied economic analysis. The first part of the tutorial begins by reviewing how a production function can be used to value changes in environmental quality (section 2), and then it reviews how cost and profit functions can be derived from a production function and used to perform the same valuation (sections 3-4). Use of a production function is typically called the primal approach, while use of cost and profit functions is typically called the dual approach.

Although the first part of the tutorial refers to the specific case of Cobb-Douglas technology, the intention is to provide intuition about fundamental points that are generally relevant, not points that are unique to that technology. Issues that arise if certain assumptions made in the first part of the tutorial do not hold are discussed at the end of first part (section 6), as are implications for empirical work (section 5). The first part of the tutorial should thus prepare one to read more advanced material on production economics, such as Chambers (1988), Just and Pope (2001), and Mundlak (2001), and more advanced material on the valuation of the environment as a production input, such as Point (1995), Huang and Smith (1998), Freeman (2003, Ch. 9), and McConnell and Bockstael (2005). It does not attempt to cover all the topics in these sources or to provide as rigorous derivations.

The second part of the tutorial (section 7) is empirical. It demonstrates how to use the econometrics program Stata to estimate production and profit functions for rice production and to use these functions to value changes in water availability. The data for this demonstration are a subset of the data used by Subhrendu Pattanayak and Randall Kramer in their article, "Worth of watersheds" (*Environment and Development Economics* 6:123-146, 2001).[1] Pattanayak and Kramer analyzed the impact of a reforestation program in Indonesia on the production of rice and coffee on farms located downstream of the reforested area. Reforestation can affect the infiltration of rainfall into the ground, which in turn affects the seepage of groundwater into rivers. This seepage, or baseflow, is an important source of water during the dry season. Although Pattanayak and Kramer analyzed the impact of changes in baseflow on both rice and coffee, the Stata demonstration in this tutorial refers to impacts on just rice and utilizes data from just a subset of the farms that the authors analyzed.

2. Production function

The focus in this section and sections 3-5 is on a private firm that produces a single output, which it sells in a competitive market. To produce this output, the firm uses a single, variable input, which it buys in a competitive market. So, both the output produced by the firm and the input that it uses are priced, and the prices are not affected by the firm's supply of the output or its demand for the input. We consider a simple static setting and ignore dynamic issues related to risk or fixed inputs (investment, depreciation). The implication of these various assumptions is that the change in the profit of the firm equals the welfare impact on the owner of the firm, as it gives the change in the owner's income.

In addition to the priced input that is under the control of the owner, output is affected by environmental quality, which is a public good that is beyond the control of the owner. Implicitly, therefore, environmental quality represents a second production input. The question to be answered is, "How does profit change if environmental quality changes?" We will review, in order, how to answer this question from three vantage points: the production function (this section), the cost function (section 3), and the profit function (section 4). To make a connection to the empirical analysis in the second part of the tutorial, one can think of the firm as a farm, the output as rice, the priced input as farm labor, and environmental quality as baseflow.

2.1 Variables and assumptions

A production function is a technical relationship that relates physical quantities of outputs to physical quantities of inputs. For a firm that produces a single output q, uses a single variable input x, and is affected by environmental quality E, the Cobb-Douglas production function is

$$q = ax^bE^r,$$

where

[1] This data set can be downloaded from http://www.sandeeonline.org

q = output
x = variable input
E = environmental quality
α, β, γ = parameters.

The variables q, x, and E are all assumed to be positive (> 0). As can be seen, the Cobb-Douglas production function indicates that the two inputs interact in a multiplicative way and are both essential to production: if either $x = 0$ or $E = 0$, then $q = 0$ too. We will assume that production is "well-behaved" in the senses that $\alpha > 0$, which is necessary if q, x, and E are all positive, and $0 < \beta < 1$ and $0 < \gamma < 1$, which imply that production is increasing in both inputs but has diminishing returns.

Figure 1 depicts this production function for two levels of environmental quality, E_0 (lower quality) and E_1 (higher quality). The vertical axis shows level of output (q), while the horizontal axis shows the level of the variable input (x). The production function slopes upward because it is increasing in the variable input ($0 < \beta$), but its slope becomes smaller as the variable input increases, due to diminishing returns ($\beta < 1$). The fact that the slope is positive but diminishing can be verified by taking the first and second derivatives of the production function with respect to the variable input:

First derivative: $$\frac{\partial q}{\partial x} = \alpha\beta x^{\beta-1}E^{\gamma} > 0$$

Second derivative: $$\frac{\partial^2 q}{\partial x^2} = \alpha\beta(\beta-1)x^{\beta-2}E^{\gamma} < 0$$

All the terms in the first derivative are positive, so the derivative is positive too. This derivative gives the *marginal product* of x: the incremental output that is produced if one more unit of the variable input is used. All the terms in the second derivative are positive except one, $\beta - 1$, which is negative because $0 < \beta < 1$, and so the derivative is negative.

2.2 Deriving the input demand function

The production function with the higher level of environmental quality (E_1) is above the one for the lower level (E_0) because production is increasing in environmental quality ($0 < \gamma$): for a given level of the variable input, output is higher if environmental quality is higher. How does this change affect the firm's profit? To answer this question, we need to bring another function into the picture: the *input demand function* for the variable input. The input demand function gives the profit-maximizing level of x: the choice of x that maximizes the firm's profit for a given level of E. To derive this function, we first need to define the firm's profit, which equals the difference between the revenue from selling the output and the expenditure on the variable input. If we define

p = price of output
w = price of variable input,

then profit, π, is given by

$$p = pq - wx.$$

If we substitute the production function for q, then this expression becomes

$$\pi = p\left(\alpha x^{\beta}E^{\gamma}\right) - wx\ .$$

Profit-maximizing use of *x* occurs where the first derivative of this expression equals zero. The first derivative is

$$\frac{\partial \pi}{\partial x} = p\left(\alpha\beta x^{\beta-1}E^{\gamma}\right) - w\,.$$

If we equate the derivative to zero, then we obtain

$$w = p\alpha\beta x^{\beta-1}E^{\gamma}.$$

The right-hand side of this new expression is the *marginal value product* of the variable input: the price of output, *p*, multiplied by the marginal product of the input ($\partial q/\partial x$), $\alpha\beta x^{\beta-1}E^{\gamma}$. It gives the firm's marginal willingness to pay for the input and can be interpreted as the *inverse input demand function*. The expression thus says that the firm should use *x* up to the point where its marginal value product (demand) equals its price, *w* (supply). Using *x* beyond this point would generate additional revenue, but the incremental revenue would be less than the cost of the additional amount of *x*.

A slight rearrangement of this expression is convenient for graphical purposes:

$$\frac{w}{p} = \alpha\beta x^{\beta-1}E^{\gamma}\,.$$

This is the profit-maximizing condition expressed in physical terms instead of monetary terms. The right-hand side, which is the marginal product of the variable input, is the inverse input demand function expressed in physical terms. The left-hand side is also in physical terms because it is a price ratio, and the monetary units cancel out. For example, if the input price *w* is in rupees per day and the output price *p* is in rupees per kilogram, then the units of the price ratio *w*/*p* are kilograms per day. So, profit maximization occurs where the marginal product of the variable input equals the ratio of input price to output price.

If we solve the profit-maximizing condition for *x* instead of for the price ratio, then we obtain the input demand function in standard (not inverse) form:

$$x^{*} = \left(\frac{p\alpha\beta E^{\gamma}}{w}\right)^{\frac{1}{1-\beta}}.$$

We denote the level of the variable input by x^* to indicate that it is the profit-maximizing value. Note that the input demand function includes only prices (*p*, *w*), environmental quality (*E*), and parameters from the production function (α, β). It does not include the physical quantity of output (*q*). Given that $0 < \beta < 1$, the exponent is positive, and so the function is increasing in output price and environmental quality but decreasing in input price. Given that the output and input prices appear as a ratio, if both prices change by the same factor—for example, if *p* becomes λp and *w* becomes λw—then the optimal level of the input does not change. There is no "money illusion." In other words, the input demand function is homogeneous of degree 0 in prices. We will work with the inverse input demand function in the rest of this section but return to the input demand function in standard form when we analyze the profit function in section 4.

2.3 Change in profit, without and with input adjustment

Figure 2 has two panels. The top panel repeats Figure 1, and the bottom panel shows inverse demand for the variable input in physical terms (i.e., marginal product) at the lower level of environmental quality:

$$\frac{\partial q}{\partial x} = \alpha\beta x^{\beta-1}E_0^{\gamma}.$$

Profit-maximization occurs at x_0, where marginal product equals the price ratio (point a in the bottom panel) and output is at q_0 (point A in the top panel). This is the profit-maximizing combination of x and q, given that environmental quality is at E_0:

$$\pi^*_0 = pq_0 - wx_0.$$

If environmental quality improves from E_0 to E_1, then output at x_0 rises to $q_1|_{x0}$ (point A' in the top panel): output is now determined by the higher production function. (Read $q_1|_{x0}$ as "output when E is at E_1 but x is at x_0.") Profit rises too, to

$$\pi_1|_{x_0} = pq_1|_{x_0} - wx_0,$$

with the change in profit, $\pi^*_1|_{x0} - \pi^*_0$, thus being given by just the increase in revenue,

$$p(q_1|_{x_0} - q_0).$$

It is important to recognize that this expression does not equal the full change in profit that results from the environmental improvement. The expression fails to account for the fact that the environmental improvement causes not only the production function to shift but also the inverse input demand function. Figure 3 shows how the latter shifts in response to the environmental improvement, and it also shows the resulting impact on output. In the bottom panel, the inverse input demand function shifts upward when E_0 is replaced by E_1: the environmental improvement causes the marginal product of the variable input to rise. Profit-maximization now occurs at x_1 (point b in the bottom panel), which is greater than x_0, and so output rises to q_1 (point B in the top panel). Hence, after allowing for the adjustment in the variable input, maximum profit is given by

$$\pi^*_1 = pq_1 - wx_1.$$

This, not $\pi_1|_{x0}$, is the correct expression for maximum profit at E_1. The change in profit,

$$\pi^*_1 - \pi^*_0 = p(q_1 - q_0) - w(x_1 - x_0),$$

is now not just a change in revenue: it also includes the change in expenditure on the variable input. Although expenditure on the input rises by $w(x_1 - x_0)$, revenue rises by an even greater amount, $p(q_1 - q_1|_{x0})$, because the marginal product of the variable input is greater than the price ratio up to point b. Profit thus rises too: $\pi^*_1 > \pi^*_1|_{x0}$. If one calculates the increase in profit as just the increase in revenue at the initial level of the variable input (i.e., as $p(q_1|_{x0} - q_0)$, then one understates the benefit of improved environmental quality.

2.4 Magnitude of the change in profit

How big is the increase in profit, $\pi^*_1 - \pi^*_0$? This can be depicted in two ways. Both are shown in Figure 4. Compared to Figure 3, the upper panel of Figure 4 includes two additional line segments, which are tangent to the production functions at points x_0 and x_1.[2] The intercept of

[2] I am grateful to Subhrendu Pattanayak for suggesting the addition of these tangents.

each tangent shows the profit associated with the corresponding production point, expressed in physical units instead of money. From above, maximum profit at point A (i.e., for E_0) is given by

$$\pi^*_0 = pq_0 - wx_0,$$

which solved for q_0 yields

$$q_0 = \frac{\pi_0^*}{p} + \frac{w}{p}x_0 .$$

The equation for the tangent at point A is thus

$$q = \frac{\pi_0^*}{p} + \frac{w}{p}x .$$

Its slope, w/p, equals the slope of the production function. The slope of the production function is by definition the marginal product of the variable input, and so the tangency simply reflects the profit-maximizing condition,

$$\frac{w}{p} = \frac{\partial q}{\partial x} .$$

We can derive the equation for the tangent to the higher production function (i.e., the one with E_0) at point B by using the same logic:

$$q = \frac{\pi_1^*}{p} + \frac{w}{p}x .$$

The difference between the intercepts of the tangents,

$$\frac{\pi_1^*}{p} - \frac{\pi_0^*}{p},$$

gives the increase in profits in physical terms. The figure does not show the line passing through point A', which would cross the production function with E_1 instead of being tangent to it (because the profit-maximizing condition does not hold at A') and have an intercept between those of the two tangents (because profit at point A' is higher than at point A but lower than at point B).

In the bottom panel, the increase in profit is shown by the cross-hatched area between the two inverse input demand functions. The cross-hatched area equals the change in *consumer surplus for the variable input*, where the "consumer" is the firm. This is easily demonstrated. Consumer surplus is the area under an inverse input demand function and above the price line. The expression for this in the case of E_1 is

$$\int_0^{x_1}\left(\alpha\beta x^{\beta-1}E_1^{\gamma}\right)dx - \frac{w}{p}x_1 ,$$

which simplifies to

$$q_1 - \frac{w}{p}x_1 ,$$

which in turn is the same as profit in physical terms, $\frac{\pi_1^*}{p}$. Parallel analysis for the inverse input demand function that includes E_0 yields consumer surplus equal to $\frac{\pi_0^*}{p}$. The change in consumer surplus is thus exactly the same as the difference between the intercepts in the top panel, $\frac{\pi_1^*}{p} - \frac{\pi_0^*}{p}$.

3. Cost function

3.1 Definition and characteristics

A cost function is an economic relationship that relates the minimum cost of production to the quantity of output, the prices of variable inputs, and the quantities of fixed inputs, including environmental inputs. In the case we have been considering, the cost of production is just the firm's expenditure on the single variable input *x*:

$$C = wx \cdot$$

We seek to determine the quantity of *x* that minimizes *C* for a given level of output:

$$\min_x wx \quad \text{subject to} \quad q = \alpha x^{\beta} E^{\gamma} .$$

If *q* is given and *E* is not under the control of the firm, then there is only a single quantity of *x* that satisfies the "subject to" production constraint, and this quantity must necessarily equal the cost-minimizing value. We can determine this quantity by solving the constraint for *x*:

$$x = \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta}} .$$

This is the *conditional input demand function*. It is "conditional" because it depends on the quantity of output, *q*, unlike the input demand function derived in section 2.2, which depends only on exogenous variables (prices and environmental quality). If we denote the cost-minimizing quantity of *x* by x^*, then the cost function, $C^* = wx^*$, is given by

$$C^* = w\left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta}} .$$

Note that the cost function includes only the quantity of output (*q*), the price of the variable input (*w*), and environmental quality (*E*), along with the parameters from the production function. Written in implicit form, without any of the functional detail, the cost function is $C^*(q, w, E)$. Three important characteristics of the cost function are:

1. It is increasing in output: $\frac{\partial C^*}{\partial q} = \frac{w}{\beta}\left(\frac{1}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}}$, which is positive. An increase in output raises production cost.
2. It is increasing in the price of the variable input: $\frac{\partial C^*}{\partial w} = \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta}}$, which is positive. An increase in the price of the variable input raises production cost.
3. It is decreasing in environmental quality: $\frac{\partial C^*}{\partial E} = -\frac{\gamma w}{\beta}\left(\frac{q}{\alpha} \right)^{\frac{1}{\beta}} E^{\frac{-\gamma-\beta}{\beta}}$, which is negative. An increase in environmental quality reduces production cost.

Note in the second point that when we differentiate the cost function with respect to input price,

$$\frac{\partial C^*}{\partial w} = \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta}} = x .$$

This result is known as *Hotelling's lemma*.

3.2 Cost function for a production function with two variable inputs

The simplicity of the single variable input model obscures the role of minimization in deriving the cost function, as there is only one value of x that satisfies the production constraints. To make the mathematics of minimization more explicit, we need to analyze a production function with more than one variable input. Consider a Cobb-Douglas production function with two variable inputs, x_1 and x_2:

$$q = \alpha x_1^{\beta 1} x_2^{\beta 2} E^{\gamma}.$$

(The subscript 1 now refers to a type of input, not to the level of environmental quality.) The cost-minimization problem for this function is

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2 \quad \text{subject to} \quad q = \alpha x_1^{\beta_1} x_2^{\beta_2} E^{\gamma}.$$

To determine the cost-minimizing values of the two inputs, we first solve the production constraint for x_2,

$$x_2 = \left(\frac{q}{\alpha x_1^{\beta_1} E^{\gamma}} \right)^{\frac{1}{\beta_2}},$$

and then substitute this into the cost expression to obtain

$$w_1 x_1 + w_2 \left(\frac{q}{\alpha x_1^{\beta_1} E^{\gamma}} \right)^{\frac{1}{\beta_2}}.$$

Note that we have reduced the cost-minimization problem from two choice variables (x_1, x_2) to one (x_1). We can therefore determine the cost-minimizing value of x_1, x_1^*, by differentiating this expression with respect to x_1, setting the result equal to zero, and solving the resulting first-order condition for x_1. If we do this, then we obtain

$$x_1^* = \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta_1 + \beta_2}} \left(\frac{\beta_1 w_2}{\beta_2 w_1} \right)^{\frac{\beta_2}{\beta_1 + \beta_2}}.$$

This is the conditional input demand function for x_1. Unlike the conditional input demand function for x in the single input production function, this one includes input prices and not just the physical levels of output and environmental quality.

By symmetry, the corresponding conditional input demand function for x_2 is

$$x_2^* = \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta_1 + \beta_2}} \left(\frac{\beta_2 w_1}{\beta_1 w_2} \right)^{\frac{\beta_1}{\beta_1 + \beta_2}}.$$

The cost function, $w_1 x_1^* + w_2 x_2^*$, is therefore

$$C^* = w_1 \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta_1 + \beta_2}} \left(\frac{\beta_1 w_2}{\beta_2 w_1} \right)^{\frac{\beta_1}{\beta_1 + \beta_2}} + w_2 \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta_1 + \beta_2}} \left(\frac{\beta_2 w_1}{\beta_1 w_2} \right)^{\frac{\beta_2}{\beta_1 + \beta_2}}.$$

This resembles the cost function for the single input production function by including the quantity of output (q), prices of the variable inputs (w_1, w_2), and environmental quality (E), along with the parameters from the production function.

Hotelling's lemma still applies: if we differentiate the cost function with respect to w_1 (or w_2), then we obtain the conditional input demand function for x_1 (or x_2). Note that the input prices appear in the conditional input demand functions as ratios. The conditional input demand functions are thus homogeneous of degree 0 in prices: use of the inputs does not change if both input prices change by the same multiplicative factor. This condition holds trivially when there is just a single variable input because, as we've seen, the conditional input demand function in that case does not include input prices:

$$x = \left(\frac{q}{\alpha E^{\gamma}}\right)^{\frac{1}{\beta}}.$$

In contrast, if both input prices change by λ times, then cost changes by λ times too: the cost function is homogeneous of degree 1 in prices. This can be seen by considering the summary expression for the cost function,

$$C^{*}= w_1x_1^{*} + w_2x_2^{*}.$$

If both input prices change by λ times, then x_1^* and x_2^* do not change (because they are homogeneous of degree 0), but w_1 and w_2 become λw_1 and λw_2, and so cost becomes λC^*:

$$(\lambda w_1)x_1^{*} + (\lambda w_2)x_1^{*} = \lambda C^{*}$$

Analogous reasoning can be used to demonstrate that the cost function for the single variable input case is also homogeneous of degree 1 in prices. So, the cost function is not merely increasing in input prices; it increases proportionately.

3.3 Deriving the marginal cost function

Let's return to the cost function for the production function with a single variable input. The top panel of Figure 5 depicts the cost function at the initial (lower) level of environmental quality, E_0. The horizontal axis shows level of output (q), while the vertical axis shows the minimum cost of production (C^*). The cost function slopes upward because it is increasing in output, and its slope becomes steeper as output increases due to diminishing returns: $\beta < 1$ implies that $1/\beta$, the coefficient on q in the cost function, is greater than one, so cost increases exponentially as output rises.

Let us use the cost function to determine how an improvement in environmental quality affects the firm's profit. As in the case of the production function, we need to bring another function into the picture: the *marginal cost function*. This function gives the minimum cost of producing each incremental unit of q. Deriving it is easy, as we simply differentiate the cost function with respect to q. We did this already, when demonstrating that the cost function is increasing in output:

$$\frac{\partial C^{*}}{\partial q} = \frac{w}{\beta}\left(\frac{1}{\alpha E^{\gamma}}\right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}}$$

Marginal cost thus equals the slope of the cost function. This parallels the relationship between the inverse input demand function and the production function: as discussed earlier, the inverse input demand function is related to the marginal product of the input, which equals the slope of the production function.

The bottom panel of Figure 5 shows the marginal cost function. The function is upward-sloping, which reflects the fact that the slope of the cost function becomes progressively steeper as output increases. This can be demonstrated by differentiating the marginal cost function with respect to q:

$$\frac{\partial^2 C^*}{\partial q^2} = \frac{w}{\beta}\frac{1-\beta}{\beta}\left(\frac{1}{\alpha E^\gamma}\right)^{\frac{1}{\beta}} q^{\frac{1-2\beta}{\beta}} .$$

This expression is positive because $\beta < 1$ implies that $1 - \beta > 0$.

3.4 Change in profit, without and with output adjustment

In the case of the production function, we used the inverse input demand function to determine the profit-maximizing output level. Now, we use the marginal cost function to do this. We can use the implicit form of the cost function, $C^*(q, w, E)$, to rewrite the firm's profit,

$$\pi = pq - wx,$$

as

$$\pi = pq - C^*(q, w, E).$$

The profit-maximizing output level occurs where the first derivative of this expression with respect to q equals zero. The first derivative is

$$\frac{\partial \pi}{\partial q} = p - \frac{\partial C^*}{\partial q} .$$

If we equate the right-hand side to zero, then we obtain

$$p = \frac{\partial C^*}{\partial q} .$$

Profit-maximizing production occurs where output price equals marginal production cost. Written explicitly, this expression is

$$p = \frac{w}{\beta}\left(\frac{1}{aE^\gamma}\right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}} .$$

Figure 6 is the same as Figure 5 except it shows the profit-maximizing level of output, q_0, at the lower level of environmental quality. In the bottom panel, the firm should produce up to the point where output price equals marginal production cost (point a). Producing beyond this point would generate additional revenue, but the incremental amount would be less than the incremental production cost. In the top panel, this yields a total (minimized) cost of C_0^* (point A).

Figure 7 shows the impact on total cost if environmental quality improves from E_0 to E_1. The cost function shifts downward, because cost is decreasing in environmental quality. The cost of producing q_0 falls to $C_1 |_{q0}$ (point A' in the top panel). Profit thus rises, to

$$\pi_1 |_{q0} = pq_0 - C_1 |_{q0}$$

The change in profit if output is held constant at q_0, $\pi_1 |_{q0} - \pi^*_0$, is given by just the decrease in cost,

$$C^*_0 - C_1 |_{q0}$$

As in the case of the comparison of profit at points A and A' in Figure 2, this expression does not equal the full change in profit that results from the environmental improvement. It fails to account for the fact that the environmental improvement causes not only the cost function to shift but also the marginal cost function. As a result, it understates the increase in profit because it ignores the firm's output supply response. Figure 8 shows how the marginal cost function shifts in response to the environmental improvement (the bottom panel), and it also shows the resulting impact on output and total cost (the top panel). In the bottom panel, the marginal cost function shifts downward when E_0 is replaced by E_1: the environmental improvement causes marginal production cost to fall. Profit-maximization now occurs at q_1 (point b in the bottom panel), which is greater than q_0: output rises. Total cost is now C_1^* (point B in the top panel).

After allowing for the adjustment in output, profit is thus given by

$$\pi_1 = pq_1 - C^*_1,$$

and the change in profit,

$$\pi^*_1 - \pi^*_0 = p(q_1 - q_0) - (C^*_1 - C^*_0),$$

is not just a change in cost: it also includes a change in revenue. Given that $C = wx$, we can also write this as

$$\pi^*_1 - \pi^*_0 = p(q_1 - q_0) - w(x_1 - x_0).$$

This is exactly the same as the final expression for the change in profit in the case of the production function analysis. We have used two approaches to arrive at the same result.

3.5 Magnitude of the change in profit

As in Figure 4, the increase in profit can be depicted in two ways. Both are shown in Figure 9. Compared to Figure 8, the top panel of Figure 9 includes two additional tangents. The intercept of each tangent on the horizontal axis shows the profit associated with the corresponding production point, expressed in physical terms instead of money. From above, profit at point A (i.e., for E_0) is given by

$$\pi^*_0 = pq_0 - C_0 ,$$

which solved for q_0 yields

$$q_0 = \frac{\pi_0^*}{p} + \frac{1}{p} C_0^* .$$

The equation for the tangent at point A is thus

$$q = \frac{\pi_0^*}{p} + \frac{1}{p} C .$$

From the profit-maximizing condition, the inverse slope of the tangent, p, equals the slope of the cost function (= marginal production cost). We can derive the equation for the tangent to the lower cost function (i.e., the one with E_1) at point B by using the same logic:

$$q = \frac{\pi_1^*}{p} + \frac{1}{p} C .$$

The increase in profits is the difference between the intercepts on the horizontal axis,

$$\frac{\pi_1^*}{p} - \frac{\pi_0^*}{p} ,$$

which is the same result as in the top panel of Figure 4. The figure does not show the line passing through point A', which would cross the cost function for E_1 instead of being tangent to it (because the profit-maximizing condition does not hold at A') and have an intercept on the horizontal axis between those of the two tangents (because profit at point A' is higher than at point A but lower than at point B).

In the bottom panel, the increase in profit is shown by the cross-hatched area between the two marginal cost functions. The cross-hatched area equals the change in *producer surplus*. This is easily demonstrated. Producer surplus is the difference between total revenue and total variable cost, which in the figure is the area below the output price line and above the marginal cost function. The expression for this area in the case of E_1 is

$$pq_1 - \int_0^{q_1} \frac{w}{\beta} \left(\frac{1}{\alpha E_1^{\gamma}} \right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}} dq ,$$

which simplifies to

$$pq_1 - w \left(\frac{q_1}{\alpha E_1^{\gamma}} \right)^{\frac{1}{\beta}} ,$$

or simply $pq_1 - C_1^*$: profit at the higher level of environmental quality, π_1^*. Parallel analysis for the marginal cost function that includes E_0 yields producer surplus equal to profit at the lower level of environmental quality π_0^*. The change in producer surplus is thus exactly the same as the monetary change in profit, $\pi_1^* - \pi_0^*$.

4. Profit function

4.1 Definition

Like the cost function, the profit function is an economic relationship, not a technical relationship. It relates maximum attainable profit to output price (not output quantity, as in the cost function), the prices of variable inputs, and the quantities of fixed inputs, including environmental inputs. It is the solution to the problem,

$$\max_x \; pq - wx \text{ s.t. } q = \alpha x^{\beta} E^{\gamma} .$$

4.2 Deriving the output supply and profit functions

The unconditional profit-maximizing input demand function, derived in section 2.2, is

$$x^* = \left(\frac{p \alpha \beta E^{\gamma}}{w} \right)^{\frac{1}{1-\beta}} .$$

If we substitute this for x in the production function, then we obtain the profit-maximizing level of output:

$$q^* = \alpha\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{\beta}{1-\beta}} E^\gamma .$$

This expression is termed the *output supply function*. Recall that the profit-maximizing condition in the analysis of the cost function was that output price equals marginal production cost:

$$p = \frac{w}{\beta}\left(\frac{1}{\alpha E^\gamma}\right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}} .$$

If we solve this condition for q, then we obtain

$$q = \alpha\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{\beta}{1-\beta}} E^\gamma ,$$

which is just the output supply function. The output supply function and the marginal cost function are thus two versions of the same supply relationship. Indeed, marginal cost functions are often called "supply curves." Like the unconditional input demand function, the output supply function is homogeneous of degree 0 in prices, increasing in output price and environmental quality, and decreasing in input price.

We obtain the profit function by substituting the output supply function for q and the unconditional input demand function for x into the basic expression for profit, $\pi = pq - wx$:

$$\pi^* = p\left(\alpha\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{\beta}{1-\beta}} E^\gamma\right) - w\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{1}{1-\beta}} .$$

This is more complex than the cost function because it incorporates adjustments in both the input x and the output q, not just the former. Like the cost function, it is homogeneous of degree 1 in prices: profit increases by λ times if output price and input price both increase by λ times. Unlike the cost function, it is increasing in environmental quality: an improvement in environmental quality reduces cost but raises profit. This can be verified by differentiating the profit function with respect to E, $\partial\pi^*/\partial E$.

If we differentiate the profit function with respect to input price w and multiply the result by –1, then we obtain the unconditional input demand function,

$$-\frac{\partial\pi^*}{\partial w} = \left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{1}{1-\beta}} = x^* ,$$

while if we differentiate it with respect to output price p, then we obtain the output supply function,

$$\frac{\partial\pi^*}{\partial p} = \alpha\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{\beta}{1-\beta}} E^\gamma = q^* .$$

These results are known as Shephard's lemma, which is the analogue to Hotelling's lemma for the cost function.

In the analysis of the cost function, we repeated the corresponding derivations for the case of two variable inputs. We will not do so here, as the derivations above show explicitly how the profit function results from the solution to an optimization problem.

4.3 Change in profit

We plotted the production and cost functions against the physical variables x and q, respectively, and we needed to add a bottom panel to the figures to account for changes in these variables in response to the improvement in environmental quality. In contrast, we can plot the profit function against environmental quality E and directly read off of it the impact of the environmental improvement on profit.

Figure 10 illustrates this. For given prices, the environmental improvement results in movement along the profit function, not a shift in the function. The function slopes upward (profit is increasing in environmental quality), but the slope diminishes. The latter reflects the diminishing returns to environmental quality in production ($\gamma < 1$). The proof of this, which requires checking that the sign of the second derivative $\partial^2\pi^*/\partial E^2$ is negative, is left as an exercise to the reader. The improvement in environmental quality from E_0 to E_1 on the horizontal axis results in an increase in profit from π_0^* to π_1^* on the vertical axis. If we know the profit function, then we can value the environmental improvement in one step, unlike the two steps that are required if we use either the production function or the cost function.

5. Empirical implications

Although the preceding analysis has been purely theoretical, it contains a number of important lessons for empirical analysis. They can be summarized as follows.

5.1 Three types of individual functions—input demand, marginal cost (or output supply), and profit—can be used to estimate the change in profit resulting from an environmental change

As emphasized throughout the preceding sections, change in profit is the proper measure of the impact of an environmental change on a firm. Change in profit can be calculated by estimating and manipulating any of three individual functions:

(i) If the input demand function is estimated, then it can be used to calculate the change in consumer surplus between one level of environmental quality and another, and that change equals the change in profit.

(ii) If the marginal cost function is estimated, then it can be used to calculate the change in producer surplus between one level of environmental quality and another, and that change equals the change in profit. The same holds for the output supply function, which as we've seen is closely related to the marginal cost function.

(iii) If the profit function is estimated, then it can be used directly to calculate the change in profit between one level of environmental quality and another.

Our analysis assumed a single input and a single output. If there are multiple inputs or outputs, then sets of input demand or marginal cost (or output supply) functions must be used instead of individual ones. This point is elaborated in section 6.

5.2 Use of full information requires estimating a system of equations, not just a single one

Although the change in profit can be calculated using individual functions, each of the three approaches presented in sections 2-4—production function, cost function, profit function—involves a system of interrelated functions: a production function plus an input demand function, a cost function plus marginal cost and conditional input demand functions, and a profit function plus input demand and output supply functions. If one wishes to use full information related to any of these approaches, then one must estimate a system of equations instead of an individual equation. The estimation of a system of equations is demonstrated in section 7.

Compared to estimating an individual equation (i.e., an input demand, marginal cost or output supply, or profit function), estimating a system of equations is more data-intensive, but it can yield statistically more efficient results. The gain in statistical efficiency is usually smaller, however, if the number of observations is smaller or if variables that can be excluded when an individual equation is estimated contain relatively more measurement error. Estimating a system of equations is thus not always more desirable. If data are incomplete, then it might not even be possible. In that case, one must rely on the estimation of individual equations (Huang and Smith 1998).

5.3 Endogeneity can be a source of bias in estimating all three functions, but especially the production function

We wrote the Cobb-Douglas production function in section 2 as a deterministic relationship:

$$q = \alpha x^{\beta} E^{\gamma} .$$

In practice, this function is not known to the econometrician, who must instead estimate it. The standard estimation procedure for a Cobb-Douglas function is to gather data across a set of firms, take the natural logarithm of each side of the function, add a stochastic error term to it (to account for unobserved factors that affect output and for measurement error in the output data), and then use regression methods to estimate the resulting log-log equation,

$$1\text{n}\ q_i = b_0 + b_1\, 1\text{n}\ x_i + b_2\, 1\text{n}\ E_i + \varepsilon_i .$$

i denotes firm, and ε is the error term. The regression coefficients b_0, b_1, and b_2 provide estimates of ln α, β (not ln β), and γ (not ln γ), respectively.

If one uses ordinary least squares (OLS) to estimate this equation, then one likely obtains biased estimates of the regression coefficients. This is because the variable input, *x*, is an endogenous variable. Unlike *E*, it is chosen by firms. As a result, it is likely to be correlated with the error term ε. This is easiest to see by considering the conditional input demand function,

$$x = \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta}} .$$

Note that this function includes output, q. If some unobserved factor generates a shock ε that affects q through the production function, then x will be affected too through the conditional input demand function. The variable input in the production function is thus correlated with the error term in the production function. This correlation has long been known to lead to biased estimates of the coefficients in a production function (Hoch 1958).

To reduce this bias, one must use an estimator other than OLS, such as two-stage least squares. But successful application of two-stage least squares requires one or more *instrumental variables* that are valid and strong: variables that are highly correlated with the endogenous explanatory variable but are not correlated with the error term and are not included in the original equation (the production function in this case). Obtaining such variables can be difficult, and using instruments that are invalid or weak can create statistical problems that are worse than the endogeneity problem that one is trying to use them to solve (Murray 2006).

Endogeneity affects the cost and marginal cost functions, too. Recall that these functions are given by

$$C^* = w\left(\frac{q}{\alpha E^\gamma}\right)^{\frac{1}{\beta}}.$$

$$\frac{\partial C^*}{\partial q} = \frac{w}{\beta}\left(\frac{1}{\alpha E^\gamma}\right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}}.$$

Both include q as an explanatory variable, which is endogenous because the firm influences it through the choice of x. In the case of agriculture, the argument is sometimes made that output is only weakly endogenous with variable inputs, because the latter are applied toward the start of the growing season. The gap in time between the start of the season and harvest reduces the feedback from output shocks to input demand, it is argued. This argument should always be supported by additional evidence that the shocks do not in fact occur at points in the growing season when farmers respond to them through input adjustments, or, if they occur later in the season, that farmers do not respond to forecasts related to them.

If this argument does not hold, then one must again use instrumental variables in estimating these functions. A variable that is not included in the cost or marginal cost functions is output price, p. This is a promising instrument, as it is likely to be exogenous (more on this in a moment). But if one has data on output price, then one has the option of avoiding the cost-function approach altogether and using instead the profit-function approach, which is less prone to endogeneity bias. The profit function and the two functions associated with it, the input demand and output supply functions, do not include any choice variables on their right-hand sides:

$$\pi^* = p\left(\alpha\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{\beta}{1-\beta}} E^\gamma\right) - w\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{1}{1-\beta}}.$$

$$x^* = \left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{1}{1-\beta}}.$$

$$q^* = \alpha\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{\beta}{1-\beta}} E^\gamma.$$

The only explanatory variables are prices (*p*, *w*) and environmental quality (*E*). Assuming that *E* is determined by the actions of economic agents other than the affected firms (e.g., deforestation by households in upland areas affects baseflow received by farmers downstream), then it is clearly exogenous. If microdata are used to estimate these functions, then input and output prices are also likely to be exogenous. An exception is when one or more firms have market power, which is discussed in section 6. If aggregate industry-level data are used to estimate the profit function, then prices are unlikely to be exogenous, and one must again use instrumental variables to correct for the resulting bias. Obtaining valid instruments generally becomes more difficult as data become more aggregated.

Other problems can also occur even when microdata are used. For example, if the firms are located in the same region, then prices might not vary much across them, and this can preclude the estimation of coefficients on the price variables. If the purpose of the analysis is to measure the impact of a change in environmental quality, however, then this is not necessarily a problem. Mundlak (1996) also notes that firms often make decisions on the basis of expected prices, not the market prices observed by econometricians. He demonstrates that there can be a substantial loss of statistical efficiency if one uses market prices as proxies for expected prices when estimating a profit function, and he argues that this statistical inefficiency can be a more serious problem than the endogeneity bias associated with estimating a production function. The most serious problem is when one or more markets are missing and thus complete price data do not exist. This problem is also discussed in section 6.

5.4 Change in revenue is a biased measure of change in profit

If one estimates a production function, then one can use it to predict output with and without an environmental change. One can then predict the change in revenue by multiplying output price by the change in output (= output with the change – output without the change). There are two such predicted changes in revenue, one partial and one complete, depending on whether or not one also estimates the input demand function.

If one does not estimate the input demand function, then one predicts the change in revenue using only the production function. This corresponds to $p(q_1 \mid x_0 - q_0)$ in Figure 2. This is a partial change in revenue because it fails to account for adjustments to input use, which affect output. When environmental quality improves, this partial change in revenue understates the increase in profits, as discussed in section 2.3. When environmental quality deteriorates, the opposite is true: the partial measure of the loss in revenue overstates the loss in profits. This is illustrated in Figure 11, which looks just like Figure 3 except that the subscripts 0 and 1 have been reversed to indicate that the change is from better environmental quality to worse. The reduction in revenue associated with the drop from point A to point A' overstates the decrease in profits because it ignores the reduction in costs as input use falls from x_0 to x_1. Using a production function to estimate the negative impact of environmental degradation is commonly called the *damage function approach*. The fact that this approach tends to exaggerate damage estimates is unfortunately often overlooked.

If one also estimates the input demand function, then one can account for the input adjustment and thus predict the complete change in revenue. This corresponds to $p(q_1 - q_0)$ in Figures 3 and 11. The complete change in revenue overstates the increase in profits when environmental quality improves (Figure 3), because it fails to account for the cost of increased input use,

$w(x_1 - x_0)$. It also overstates the decrease in profits when environmental quality deteriorates (Figure 11), because it similarly fails to account for cost savings as the firm reduces input use. In the latter case (environmental deterioration), the complete change in revenue is more biased than the partial change. This can be seen easily in Figure 11, where the complete change in revenue is associated with the drop from point A to point B, which exceeds the partial reduction associated with the drop from point A to point A'.

Of course, if one estimates not only the production function but also the input demand function, then there is no reason to predict a change in revenue: one can instead predict the change in profit, which is (or should be) the objective of the analysis.[3] Use of the change in revenue as a proxy for the production impact of an environmental change is thus pertinent only when one estimates only the production function and thus predicts the partial change in revenue. The fact that the partial change in revenue is a biased measure of the change in profit does not mean it has no value for economic analysis. For example, suppose that the purpose of the analysis is to determine whether a prospective program to improve environmental quality is economically justified. If the predicted partial change in revenue exceeds the cost of the program, then one can be confident that the program is justified, because a conservative (downwardly biased) measure of the benefits has been used. By the same token, if the predicted partial change in revenue does not exceed the cost of the program, then one cannot say whether or not the program is justified: perhaps the predicted benefits would have exceed the program cost if the conceptually correct benefit measure, the change in profit, had been used instead of the partial change in revenue. The partial change in revenue can thus be used to construct one-sided benefit-cost tests.

5.5 Change in cost is a biased measure of change in profit

Analogous points can be made about the bias associated with using the change in cost as a proxy for the change in profit. When environmental quality improves but output is held at the initial level q_0, the resulting reduction in cost, $C^*_0 - C_1 \mid q_0$, understates the positive impact of the environmental improvement on the firm. This point was made in section 3.4. It is the mirror image of the downward bias that occurs when the variable input is held at the initial level x_0 and the change in revenue, from the production function, is used to measure the change in profit. When environmental quality deterioriates but output is held at the initial level, the bias is in the opposite direction: the increase in cost overstates the damage to the firm. This illustrated in Figure 12, which looks like Figure 8 except that the subscripts 0 and 1 have been reversed. The cross-hatched area in the bottom panel indicates the amount by which the increase in cost overstates the loss of producer surplus, which has the same shape as in Figure 9.

Figure 12 can also be used to illustrate the bias associated with using the *replacement-cost method* to value environmental damage. If a firm attempted to restore output to the initial level q_0, then it would incur costs equal to the area given by the approximately trapezoidal area $q_1ba'q_0$ in the bottom panel. This area exceeds the loss of producer surplus, and so the replacement cost overstates the loss of profit. The problem with the replacement-cost method is clear: only an irrational firm would attempt to restore output to the initial level q_0 after environmental quality has deteriorated, because the marginal cost of producing beyond the new profit-maximizing output level q_1 exceeds the marginal benefit, which is given by the output price p. Simply put, the replacement cost does not generate benefits of equivalent value.

[3] Auffhammer et al. (2006) did not do this because they lacked reliable data on some inputs and their prices.

The results in this section and section 5.4 illustrate the importance of accounting for adjustments firms make in response to environmental changes. The failure to account fully for these adjustments is the reason why partial or approximate measures of economic impacts, such as revenue-based damage costs or cost-based replacement costs, provide biased measures of welfare impacts.

6. Implications of relaxing key assumptions

6.1 Multiple firms

Our analysis assumed a single, price-taking firm. If the environmental change affects multiple firms but does not affect input or output prices, then the sum of changes in profits across firms, where the changes are calculated at fixed prices, is a valid measure of the welfare impact on the set of firms. If the environmental change affects a large share of the firms in an industry, however, then it probably affects prices too. For example, an environmental improvement that affects an entire industry would be expected to reduce output price, due to the increased supply of the output (assuming a downward-sloping demand function for the output), and to raise input price, due to the increased demand for the input (assuming an upward-sloping supply function for the input). Deterioration in environmental quality would be expected to have the opposite effects. The welfare impact on the set of firms is still given by the sum of profit changes across the firms, but now the latter must account for the price changes. Moreover, the price changes create additional welfare impacts on consumers of the output and suppliers of the input, which must be taken into account if the objective is to measure the overall social welfare impact (see Freeman 2003, pp. 276-279).

Just et al. (1982, Chs. 8-9)[4] deal with these sorts of aggregation issues. A sufficiently large environmental change—for example, global warming—could have economy-wide effects, in which case the impacts would need to be measured using a computable general equilibrium (CGE) model. General-equilibrium impacts of environmental changes, or policies to address them, can differ substantially from partial-equilibrium impacts (Hazilla and Kopp 1990). Bergman (2003) reviews the application of CGE models to environmental issues.

6.2 Noncompetitive markets

Issues similar to the ones just discussed occur if the firm is large and faces either the demand function for the output it produces or the supply function for the input it consumes. The firm then has market power and can earn above-normal profits by acting as a monopolist and forcing output prices up or a monopsonist and forcing input prices down. One must again account for such price changes when measuring the impact of the environmental change on the firm's profits (Just et al. 1982, Ch. 10; Freeman 2003, pp. 279-281). One should also be aware that a welfare gain (or loss) for the firm now does not necessarily equal the corresponding welfare gain (or loss) for society, given the distortions created by the firm's manipulation of market prices.

6.3 Market distortions

Market power is one source of distortions that can cause market prices to deviate from marginal benefits and costs measured in social terms. Such distortions can also result from taxes, subsidies,

[4] A new edition of this book was published recently (Just et al. 2004).

regulations, and environmental externalities other than the ones that are the focus of a particular analysis (Freeman 2003, pp. 281-283). When analyzing the impacts of an environmental change on producers, one must therefore be clear about whether the objective is to measure those impacts in private terms or social terms. If the objective is the former—that is, if the objective is to measure impacts at market prices—then one can ignore the distortions. The objective of economic analysis is usually to measure impacts in social terms, however, and in that case one must use shadow prices to adjust for the distortions. Belli et al. (2001) contains especially lucid explanations of shadow-pricing techniques for various distortions.

6.4 Missing markets and household production

Production in developing countries is often by households, as in the case of smallholder farms. If the household faces complete markets for inputs and outputs—that is, if it can buy as much of an input or sell as much of an output as it desires at the prevailing market prices—then, leaving aside the issue of risk preferences (discussed in the next point), the change in profit in the productive activity is the correct measure of the welfare impact of an environmental change that affects the household through that activity. The existence of complete markets makes production decisions *separable* from other household decisions, in particular its consumption decisions (including the labor-leisure tradeoff). One can then use a profit function for the productive activity to measure the welfare impact of the environmental change. This is obviously a very convenient situation for economic analysis: one does not need to worry about the characteristics of the household's utility function, which is inherently more difficult to measure than its productive activities and their profitability.

Unfortunately, markets are often missing for households in developing countries, especially in rural areas (de Janvry et al. 1991). For example, households might face restrictions on the amount of labor they can buy or sell. When markets are missing, the household's production decisions are no longer separable from its consumption decisions, and the monetary change in profit from its productive activities no longer provides a valid welfare measure. One must instead calculate the change in profit by using shadow or *virtual prices*, which account for nonmarket utility effects. Unlike market prices, virtual prices are endogenous to the household—they are not determined solely by external factors—and they are unobserved. It is possible to test for the completeness of markets and thereby determine whether adjustments using virtual prices are necessary. For an example related to the valuation of an environmental change, see Pattanayak and Kramer (2001).

6.5 Risk

Our analysis assumed that prices are known with perfect certainty and that the firm's owner is risk neutral. If prices are not known with perfect certainty when production decisions are made (e.g., as in the case of agriculture) but the risk neutrality assumption still holds, then impacts on the firm can be measured in terms of the *expected change in profit*. For example, a set of alternative price scenarios could be prepared, probabilities could be attached to each, the change in profit could be calculated for each, and then the expected change in profit could be calculated by multiplying the change in profit for each scenario by the corresponding probability and summing across the scenarios. Analogous procedures can be used if the magnitude of the environmental change is not known with perfect certainty.

The situation is more complex if the firm's owner is not risk neutral. Then, the owner's risk preferences must be taken into account. The expected change in profit no longer provides a valid measure of the impact of the environmental change on the owner's utility. In effect, the expected change in profit must be adjusted for risk premia (Just et al. 1982, Ch. 11).

6.6 Fixed inputs

Our analysis ignored fixed inputs. It included just a single input, which was a variable one. Fixed inputs are in fact a critical feature of the analysis of producer impacts of environmental changes. If there are no fixed inputs, then under standard assumptions, such as constant returns to scale and free entry and exit of firms, firms should not earn a profit in the sense of a payment over and above the costs of the inputs (including managerial effort) they employ. The existence of such profits should immediately attract new firms into the industry, which would result in the profits being competed away (driven to zero). Total revenue minus total costs, where the latter includes only variable costs, should equal zero at all times.

If production requires a fixed input that is owned by the firm, and if the input varies in quality across firms, then persistent differences in economic surpluses can exist across firms. A good example is agricultural land. Land of higher quality is more productive, which increases the economic surplus of the farm that owns it. The higher surplus simply reflects the greater return generated by the land: although total revenue minus total variable costs is positive, total revenue minus total costs, where the latter includes an implicit payment for the land, would again equal zero. There is a non-zero *quasi-rent* (total revenue minus total variable costs; producer surplus) but a zero economic profit (total revenue minus total costs). Indeed, if the farmer were a tenant who literally rented the land, then he would pay a rent equal to the quasi-rent and would consequently earn zero profits. The landowner would be the one who benefited economically from the land's higher quality. Because fixed costs are fixed, a change in quasi-rent equals a change in profit. The ownership of fixed inputs thus affects the distribution of the production impacts of environmental changes: whether the impacts appear as changes in the firm's profits, which occurs if the firm owns the fixed input, or the income of the owner of the fixed input, which occurs if the owner is different from the firm.

If there is no variation in the quality of the fixed input, then there should not be persistent differences in economic surpluses across firms as long as there is free entry and exit: that is, there should be zero profits in the long run. In the short run, however, environmental changes can affect quasi-rents. If environmental quality improves, then the existing firms in an industry earn above-normal returns during the transition period when new firms, attracted by the above-normal returns, are making the investments necessary to enter into production. These above-normal returns vanish once the new firms begin producing. Conversely, if environmental quality deteriorates, then the existing firms earn below-normal returns during the transitional period when they depreciate their fixed inputs and scale back production. The key point here is that the change in an affected firm's profits reflects a change in quasi-rents and is temporary, converging to zero as the level of fixed inputs across firms adjusts to a new competitive level at the new level of environmental quality.

The issues of imperfect knowledge about future prices or the future magnitude of an environmental change, discussed in the previous point, also affect a firm's investment decisions. These effects can be complex, especially when the environmental change is irreversible. See Mäler and Fisher (2005) for more details.

6.7 Multiple outputs

Introductory expositions of producer theory usually assume that a firm makes a single output. In fact, firms often make more than one output. A farm that grows several crops is a good example. The theory of production by multi-output firms is well-developed (Chambers 1988, Ch. 7), as is the theory of welfare measurement for such firms (Just et al. 1982, Appendix A). The most natural way to measure the welfare impacts of an environmental impact on a multi-output firm is to use a multi-output profit function. This is the approach used by Pattanayak and Kramer (2001) in their study of the impacts of changes in baseflow on Indonesian farms that produce rice and coffee. It is also possible to add up changes in producer surpluses across the set of outputs, or consumer surpluses across the set of inputs, that are affected by the environmental change. Certain technical requirements must be satisfied to do this, however. One must also take care to ensure that these changes are added correctly, as they are interrelated. For more details, see Huang and Smith (1998), Freeman (2003, pp. 267-276), and McConnell and Bockstael (2005, section 3.3).

6.8 Multiple inputs

Our assumption of a single non-environmental input was extreme and was done to simplify the graphical exposition of the three approaches (production, cost, and profit functions). Assuming that the firm makes a single output, there is little difference between the single-input and multi-input cases when using a profit or marginal cost function to measure the impacts of an environmental change. One simply must make sure that all the relevant input prices are included in the profit or marginal cost function and the other functions that are associated with it (output supply and input demand, or cost and conditional input demand), if those functions are also estimated. Additional complications arise when changes in consumer surpluses for inputs are used to measure the impacts. As in the case of multiple outputs, one must check some technical conditions and add up carefully (Huang and Smith 1998; McConnell and Bockstael 2005, section 3.4). The technical conditions are analogous to the conditions for *weak complementarity* identified originally by Mäler (1974) for using inputs, such as travel expenditures, to measure the benefits of environmental improvements to consumers, such as the availability of outdoor recreation sites. An input is weakly complementary to environmental quality if two conditions hold: (i) demand for the input increases when environmental quality improves, and (ii) a change in environmental quality has no impact on the affected party (the consumer or the firm) if demand for the input equals zero, which occurs when the price of the input exceeds the choke price.

6.9 Nonconvexities

Our analysis assumed that environmental quality enters production in a "well-behaved" manner. For example, we assumed that the production function is continuously differentiable (the derivatives $\partial q/\partial E$ and $\partial^2 q/\partial E^2$ exist) and concave ($\partial q/\partial E > 0$, $\partial^2 q/\partial E^2 < 0$) with respect to environmental quality. These assumptions were convenient ones, but they do not necessarily hold in reality. The production set could instead be nonconvex. A simple example is a threshold effect, such as catastrophic crop loss if the amount of rainfall is below a minimum level. Although a production, cost, or profit function that ignores such a threshold could provide accurate predictions as long as environmental quality stays within the well-behaved production region, it would likely provide very misleading ones if the threshold were crossed. Moreover, decisions that make a threshold more likely to be crossed have a cost, a loss of resilience, that is not reflected in normal accounting procedures (Mäler et al. 2007) and thus not in data on profits and costs. The economic analysis of nonconvex production systems is an active area of research. For a good introduction, see Dasgupta and Mäler (2004).

7. Using Stata to estimate a production function, a profit function, and a profit-function system

7.1 Overview and policy context

This example is a simplified version of the analysis of watershed values in Indonesia by Pattanayak and Kramer (2001). Because it has been simplified, the results generated by the example should not be taken as true values. Like the original analysis, this example involves the estimation of an agricultural profit function by using data from a 1996 survey of farm households in the Manggarai District on the island of Flores. One of the inputs in this function is an environmental service: baseflow. Baseflow refers to the seepage of groundwater into a region's waterways. It provides an indicator of the amount of soil moisture that is available for crops.

The policy context for Pattanayak and Kramer's study was a proposed reforestation program in a national park, Ruteng, which lies upstream of the district. Like many parks in developing countries, Ruteng had suffered from encroachment at the time of the study, and much of it had been cleared of forest. In response, the Government of Indonesia proposed a reforestation program to reestablish tree cover in the denuded area. Forest cover can affect baseflow in both positive and negative ways. To value the net change in baseflow, one needs to know how the change affects agricultural profits.

Pattanayak and Kramer used data from 487 households in estimating their profit function. This example uses data from just 92 households. The reason for the difference is that the profit function in this example includes just one crop, rice, whereas Pattanayak and Kramer's included two, rice and coffee. Although some farmers in the district specialized in a single crop in 1996, most grew both. The sample for this example includes households that grew predominantly rice, which are defined here as farms that earned at least 75 percent of their gross revenue from rice. Fifty-nine of these 92 households grew only rice.

Aside from excluding terms related to coffee, the profit function in this example is very similar to Pattanayak and Kramer's. Although the main objective of the example is to illustrate the estimation of a profit-function system—that is, the profit function along with an output supply function (for rice) and an input-demand function (for farm labor)—for the purpose of comparison the example also involves the estimation of the profit function on its own and a Cobb-Douglas production function.

Estimation is done using the econometrics program Stata, which is currently one of the most popular programs used by economists and other social scientists. The example is written in a way that assumes the user has installed Stata and is familiar with its basic commands. For information on ordering Stata, visit www.stata.com. An excellent online tutorial for using Stata can be found at http://www.ats.ucla.edu/STAT/stata/webbooks/reg/default.htm. Additional information on more specialized topics can be found at http://www.ats.ucla.edu/STAT/stata/.

7.2 Description of the data

Data for the example are in the Stata dataset, "Rice and baseflow.dta." The dataset contains the following 14 variables:

kues	Household ID number
desa	Village ID number
kecano	County ID number
profit	Annual farm profit
ppadi	Rice price per kilogram
plabor	Wage rate per day
padi	Padi output, in kilograms
labor	Labor inputs (household and hired combined), in days
farmsz	Farm size, in hectares
bftot	Annual baseflow in the village, in meters
irrih	Fraction of farm that is irrigated
slope	Average slope of the farm
hujan	Annual rainfall in the village, in meters

Some of the variables vary by households, while others vary by village. The dataset is complete, so we do not need to worry about missing values. The value of *labor* on one farm is listed as 0, which clearly cannot be correct, but we will leave this value as it is.

One unusual feature is that farm profit (*profit*) and prices (*ppadi, plabor*) are not expressed in monetary terms. Instead, they are expressed in kilograms of fertilizer. Pattanayak and Kramer transformed the original variables by dividing by the price of fertilizer. This is one way of putting the data in "real" terms (i.e., relative prices). Despite this, we will refer to these three variables as "monetary" variables.

7.3 Estimating the production function

The Cobb-Douglas function is most common specification for an agricultural production function. It is a multiplicative function:

$$y = \beta_0 x_1^{\beta 1} x_2^{\beta 2} x_3^{\beta 3} x_4^{\beta 4} x_5^{\beta 5} x_6^{\beta 6}$$

y is harvest, and x_1, x_2, etc. are inputs. There are six inputs:

x_1	*labor*
x_2	*farmsz*
x_3	*bftot*
x_4	*irrih*
x_5	*slope*
x_6	*hujan*

Only the first one, *labor*, can be varied by farmers.

We can determine the marginal impact of baseflow on harvest by partially differentiating the production function with respect to x_3. After a bit of reorganization, the partial derivative yields

$$\frac{\partial y}{\partial x_3} = \beta_3 y / x_3 \ .$$

This is the marginal impact, or marginal product, in physical terms, such as kilograms of rice per meter of baseflow. To obtain the marginal impact in monetary terms, we need to multiply by the price of the crop, *p*:

$$p\frac{\partial y}{\partial x_3} = p\beta_3 y / x_3 .$$

This is just the standard marginal value product of an input. Once we have estimated the Cobb-Douglas production function, we can therefore easily calculate the marginal impact of baseflow.

To convert a Cobb-Douglas function to a form that can be estimated by using ordinary-least squares regression, we take the natural logarithm of each side:

$$1\text{n } (y) = \alpha + \beta_1 1\text{n } (x_1) + \beta_2 1\text{n } (x_2) + \beta_3 1\text{n } (x_3) + \beta_4 1\text{n } (x_4) + \beta_5 1\text{n } (x_5) + \beta_6 1\text{n } (x_6)$$

where $\alpha = \ln(\beta_0)$. The coefficient on $\ln(x_3)$, β_3, is the coefficient that we need for our marginal impact formula, along with data on harvest (y), baseflow (x_3), and crop price (p). The following Stata commands generate the logarithmic variables:

```
generate lpadi      =ln( padi )
generate llabor     =ln( labor)
generate lfarmsz    = ln( farmsz )
generate lbftot     =ln( bftot )
generate lhujan     =ln( hujan )
```

We do not convert *irrih* (x_4) and *slope* (x_5) to logarithms, because they can values of zero, whose logarithm does not exist. The actual production function estimated was thus

$$1\text{n } (y) = \alpha + \beta_1 1\text{n } (x_1) + \beta_2 1\text{n } (x_2) + \beta_3 1\text{n } (x_3) + \beta_4 x_4 + \beta_5 x_5 + \beta_6 1\text{n } (x_6).$$

To estimate the production function, we issue the following command:

```
regress  lpadi llabor lfarmsz lbftot lhujan irrih slope
```

We get the following results:

```
. regress  lpadi llabor lfarmsz lbftot lhujan irrih slope
```

Source	SS	df	MS
Model	41.5667717	6	6.92779528
Residual	60.0948891	84	.715415347
Total	101.661661	90	1.12957401

Number of obs	=	91
F(6, 84)	=	9.68
Prob > F	=	0.0000
R-squared	=	0.4089
Adj R-squared	=	0.3667
Root MSE	=	.84582

lpadi	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
llabor	.4105994	.1082056	3.79	0.000	.1954207	.625778
lfarmsz	.3862747	.0943555	4.09	0.000	.1986385	.5739109
lbftot	1.822436	.8337055	2.19	0.032	.1645211	3.480351
lhujan	2.145706	.9129661	2.35	0.021	.3301724	3.961239
irrih	1.074014	.3468321	3.10	0.003	.3843005	1.763728
slope	-.2249773	.088351	-2.55	0.013	-.4006729	-.0492816
_cons	3.12828	1.065881	2.93	0.004	1.008659	5.2479

The regression equation fits the data reasonably well considering that the data are cross-sectional ($R^2 = 0.41$). The coefficient on the logarithm of baseflow (*lbftot*) is positive and significantly different from zero at a 5-percent level: the *P*-value for this coefficient, 0.032, is less than 0.05, and its 95% confidence interval (0.165, 3.48) does not straddle zero.

The following command generates a variable, *marginal_impact_production*, that equals the marginal value product of baseflow for each household:

```
generate marginal_impact_production=ppadi*_b[lbftot]*padi/bftot
```

Note that this command refers to the value of the coefficient on *lbftot* that is stored in memory,

```
_b[lbftot]
```

instead of directly including the numerical value of the coefficient (i.e., 1.822436…). This leads to a more precise estimate of *marginal_impact_production* and enables us to reuse the command if we make certain changes to the regression, such as dropping some observations. We can obtain summary statistics for the marginal value product by issuing the command,

```
su  marginal_impact_production
```

which returns the following information:

```
su  marginal_impact_production
    Variable |   Obs       Mean      Std. Dev.      Min       Max
-------------+----------------------------------------------------------
 marginal_i~n |    92    2773.764    4885.  759   111,6826   29844.87
```

The marginal value product of baseflow has a mean of 2,774 and ranges from 112 to 29,845.

7.4 Estimating the profit function

We derived the profit function for the Cobb-Douglas production function in section 4. Economists typically do not posit a production function and then derive the profit function for it. They typically assume a *flexible functional form* for the profit function and analyze it instead of the specification that is unique to a particular production function. A flexible functional form is one that provides a good approximation to the actual function, regardless of the shape of the actual function, which is not directly observed by the econometrician. Flexible functional forms include interaction terms (variables multiplied by each other) and higher-order terms (variables raised to powers). Due to these characteristics, they have nonzero first and second derivatives.

We assume that the profit function has the following specification:

Profit function

$$\begin{aligned}\pi = {} & \beta_p p + \beta_w w + \beta_{pw} 2(pw)^{0.5} \\ & + \beta_{p2}(px_2) + \beta_{p3}(px_3) + \beta_{p4}(px_4) + \beta_{p5}(px_5) + \beta_{p6}(px_6) \\ & + \beta_{w2}(wx_2) + \beta_{w3}(wx_3) + \beta_{w4}(wx_4) + \beta_{w5}(wx_5) + \beta_{w6}(wx_6).\end{aligned}$$

π is profit (*profit*), and p is the price of the crop (*ppadi*). This specification assumes that there is only one priced variable input (*labor*) and that this input is input 1. w is the price of this input (*plabor*). Although the profit function includes the price of input 1, it does not include the quantity (x_1). As discussed in section 4, profit functions include output and input prices and the quantities of fixed inputs, but not the quantities of variable inputs.

The profit function includes 13 explanatory variables. The explanatory variables are in three groups:

(i) p, w, and twice the square root of their product (the first line)

(ii) the product of p with each of the 5 fixed or unpriced inputs (x_2, x_3, x_4, x_5, x_6; the second line)

(iii) the product of w with each of the 5 fixed or unpriced inputs.

For example, px_2 means p multiplied by x_2. p is *ppadi*, so if x_2 is *farmsz*, then px_2 = *ppadi ´ farmsz*. To estimate the profit function, we first need to construct these variables. The following Stata commands do this. This command generates the interaction term in group (i):

```
generate two_ppadixplabor_sqrt=2*(ppadi*plabor)^0.5
```

These commands generate the 5 variables in group (ii):

```
generate ppadixfarmsz=ppadi*farmsz
generate ppadixbftot=ppadi*bftot
generate ppadixirrih=ppadi* irrih
generate ppadixslope=ppadi* slope
generate ppadixhujan=ppadi* hujan
```

Finally, these commands generate the 5 variables in group (iii):

```
generate plaborxfarmsz=plabor*farmsz
generate plaborxbftot=plabor*bftot
generate plaborxirrih=plabor*irrih
generate plaborxslope=plabor*slope
generate plaborxhujan=plabor*hujan
```

With the variables created, we can estimate the profit function by issuing the following command:

```
regress profit ppadi plabor two_ppadixplabor_sqrt ppadixfarmsz ppadixbftot ppadixirrih ppadixslope ppadixhujan plaborxfarmsz plaborxbftot plaborxirrih plaborxslope plaborxhujan
```

We obtain the following results:

```
. reg profit ppadi plabor two_ppadixplabor_sqrt ppadixfarmsz ppadixbftot ppadixirrih ppadixslope ppadixhujan plaborxfarmsz plaborxbftot plaborxirrih plaborxslope plaborxhujan
```

Source	SS	df	MS
Model	328934107	13	25302623.6
Residual	278454364	78	3569927
Total	607388470	91	6674598.57

Number of obs	=	92
F(13, 78)	=	7.09
Prob > F	=	0.0000
R-squared	=	0.5416
Adj R-squared	=	0.4651
Root MSE	=	1889.4

profit	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
papadi	-9574.432	10439.65	-0.92	0.362	-30358.17	11209.31
plabor	4023.331	2362.436	1.70	0.093	-679.9186	8726.581
two_ppadix~t	-3951.722	3233.716	-1.22	0.225	-10389.56	2486.113
ppadixfarmsz	374.5587	513.9781	0.73	0.468	-648.6932	1397.811
ppadixixbftot	2218.165	5944.482	0.37	0.710	-9616.39	14052.72
ppadixirrih	3701.61	1607.209	2.30	0.024	501.903	6901.317
ppadixslope	581.5932	583.7256	1.00	0.322	-580.5152	1743.702
ppadixhujan	5903.578	2004.021	2.95	0.004	1913.879	9893.276
plaborxfar~z	-3.569244	92.82444	-0.04	0.969	-188.3685	181.23
plaborxbftot	200.0229	1092.98	0.18	0.855	-1975.933	2375.979
plaborxirrih	445.0603	332.5379	-1.34	0.185	-1107.092	216.9717
plaborxslope	-149.0845	107.256	-1.39	0.168	-362.6147	64.44572
plaborxhujan	-820.3523	392.6401	-2.09	0.040	-1602.039	-38.66591
-cons	-308.3875	699.5395	-0.44	0.661	-1701.064	1084.289

The profit function includes two variables involving baseflow, the interaction with price of rice (*ppadixbftot*) and the interaction with price of labor (*plaborxbftot*). Neither coefficient is significantly different from zero. From this, one might conclude that baseflow does not affect profit, but this would be a surprising conclusion in view of our earlier result that baseflow has a significant impact on production. It would also be an incorrect conclusion. The lack of significance of the baseflow variables in the profit function in fact results from our failure to use all the information that we have on rice production by the households. Specifically, we have not used the information on the quantity of output produced or the quantity of labor inputs used. To do this, we need to estimate not only the profit function but also the output supply and input demand functions.

7.5 Estimating the profit function system

The first step in estimating the profit function system is to determine the equations of the output supply and input demand functions. From section 4, we know that we can derive these functions by differentiating the profit function with respect to output price (*p*), which yields the output-supply function, and differentiating it with respect to input price (*w*), which yields the negative of the input-demand function. The resulting functions are:

Output-supply function ($\partial\pi / \partial p = y$)

$$y = \beta_p + \beta_{pw} (w/p)^{0.5} + \beta_{p2}(x_2) + \beta_{p3} (x_3) + \beta_{p4}(x_4) + \beta_{p5}(x_5) + \beta_{p6}(x_6)$$

Input-demand function ($-\partial\pi / \partial w = x_1$)

$$x_1 = -\beta_w - \beta_{pw} (p/w)^{0.5} - \beta_{w2}(x_2) - \beta_{w3} (x_3) - \beta_{w4}(x_4) - \beta_{w5}(x_5) - \beta_{w6}(x_6)$$

Note that coefficients from the profit function also appear in these two functions. For example, the coefficient β_{pw} shows up in all three, although it multiplies different versions of a variable involving *p* and *w* in each case. Given that the same coefficients appear in all three equations, we cannot estimate the equations separately. We need to estimate them jointly, as a single system. There are different ways to do this. We will use a technique called seemingly-unrelated regression, which is implemented in Stata by using the “reg3” command with the “sure” option.

Before we can estimate the system, we need to construct the variable $(w/p)^{0.5}$, which is in the output-supply function, and the variable $(p/w)^{0.5}$, which is in the input-demand function. We generate these variables by issuing the following pair of commands:

```
generate ppadi_plabor_sqrt=(ppadi/plabor)^0.5
generate plabor_ppadi_sqrt=(plabor/ppadi)^0.5
```

Next, we need to tell Stata which variables are in the three functions (profit, output supply, input demand). We do this by using the "global" command. For the profit function, we enter

```
global profit_function "(profit ppadi plabor  two_ppadixplabor_sqrt ppadixfarmsz ppadixbftot ppadixirrih ppadixslope ppadixhujan plaborxfarmsz plaborxbftot plaborxirrih plaborxslope plaborxhujan )"
```

This creates an equation named "profit_function" that has *profit* as the dependent variable and *ppadi*, *plabor*, etc. as explanatory variables. Note that all the explanatory variables are listed after *profit* inside the parentheses. Similarly, the following two commands create equations named "output_supply_function" and "input_demand_function":

```
global output_supply_function "(padi plabor_ppadi_sqrt farmsz bftot irrih slope hujan)"
global input_demand_function "(labor ppadi_plabor_sqrt farmsz bftot irrih slope hujan)"
```

Defining these functions is not enough; we also need to tell Stata that some of the coefficients are the same across the equations. The following commands create these cross-equation constraints:

```
constraint define 1 [profit]ppadi = [padi]_cons
constraint define 2 [profit]plabor = -[labor]_cons
constraint define 3 [profit]two_ppadixplabor_sqrt = [padi]plabor_ppadi_sqrt
constraint define 4 [profit]two_ppadixplabor_sqrt = -[labor]ppadi_plabor_sqrt
constraint define 5 [profit]ppadixfarmsz = [padi]farmsz
constraint define 6 [profit]plaborxfarmsz = -[labor]farmsz
constraint define 7 [profit]ppadixbftot = [padi]bftot
constraint define 8 [profit]plaborxbftot = -[labor]bftot
constraint define 9 [profit]ppadixirrih = [padi]irrih
constraint define 10 [profit]plaborxirrih = -[labor]irrih
constraint define 11 [profit]ppadixslope = [padi]slope
constraint define 12 [profit]plaborxslope = -[labor]slope
constraint define 13 [profit]ppadixhujan = [padi]hujan
constraint define 14 [profit]plaborxhujan = -[labor]hujan
```

Consider the first constraint. It says that the coefficient on *ppadi* in the equation with *profit* as the dependent variable (i.e., the profit function) equals the constant in the equation with *padi* as the dependent variable (i.e., the output supply function). This is correct: β_p is the coefficient on p in the equation for the profit function, and it is also the intercept in the equation for the output supply function. Now consider the second constraint, which says that the coefficient on *plabor* in the profit function equals the negative of the constant in the input demand function. This is again correct: β_w is the coefficient on w in the equation for the profit function, and $-\beta_w$ is the intercept in the equation for the input demand function.

With the variables all constructed, the equations defined, and the cross-equation coefficient constraints defined, we are ready to estimate the profit function system. We do so by entering the following command:

```
reg3 $profit_function $output_supply_function $input_demand_function, constraints(1 2 3 4
5 6 7 8 9 10 11 12 13 14) sure
```

"reg3" is the command to estimate a system of equations in Stata. Following it are the names of the equations, each preceded by a dollar sign ($), which is a symbol that informs Stata that the equations have been previously defined and stored under the indicated names. Next, the numbers of the constraints are given. Finally, the options for "reg3" are listed. Just one option is given, "sure", which tells Stata to calculate nonzero covariances across the three equations.

The command returns the results given on the next page. Coefficients for each equation are listed separately in the bottom half of the table, under the headings "profit", "padi", and "labor" (i.e., the name of the dependent variable in each equation). Note that the cross-equation constraints have worked: for example, the coefficient on *ppadi* in the profit function equals the intercept ("_cons") in the output supply function, while the coefficient on *plabor* in the profit function equals the negative of the intercept in the input demand function. Of the two variables in the profit function that involve baseflow, i.e., *ppadixbftot* and *plaborxbftot*, the former is now significantly different from zero. This indicates that baseflow has a significant impact on output supply, which is consistent with our results for the production function, but not input demand. Indeed, none of the variables in the labor demand function are significant, which probably reflects the fact that some of the households in our sample produce coffee in addition to rice. The labor variable refers to labor used for both crops, but we are treating it as referring to labor for just rice. By limiting the analysis to rice production, we have thus created a measurement error problem. As evidence that this explanation is correct, Pattanayak and Kramer obtained significant coefficient estimates for nearly all the variables in the labor demand equation in their two-output (rice and coffee) profit function system. One exception, however, is the coefficient on baseflow. So, both our simplified analysis of the partial sample of households and their more complete analysis of the full sample indicate that baseflow affects profit only through an impact on output supply, not through labor demand.

```
. reg3 $profit_function $output_supply_function $input_demand_function, constraints(1 2 3
4 5 6 7 8 9 10 11 12 13 14) sure;

Seemingly unrelated regression

Constraints:
 ( 1)  [profit]ppadi - [padi]_cons = 0
 ( 2)  [profit]plabor + [labor]_cons = 0
 ( 3)  [profit]two_ppadixplabor_sqrt - [padi]plabor_ppadi_sqrt = 0
 ( 4)  [profit]two_ppadixplabor_sqrt + [labor]ppadi_plabor_sqrt = 0
 ( 5)  [profit]ppadixfarmsz - [padi]farmsz = 0
 ( 6)  [profit]plaborxfarmsz + [labor]farmsz = 0
 ( 7)  [profit]ppadixbftot - [padi]bftot = 0
 ( 8)  [profit]plaborxbftot + [labor]bftot = 0
 ( 9)  [profit]ppadixirrih - [padi]irrih = 0
 (10)  [profit]plaborxirrih + [labor]irrih = 0
```

(11) [profit]ppadixslope - [padi]slope = 0
(12) [profit]plaborxslope + [labor]slope = 0
(13) [profit]ppadixhujan - [padi]hujan = 0
(14) [profit]plaborxhujan + [labor]hujan = 0

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
profit	92	13	1969.63	0.4124	244.10	0.0000
padi	92	6	1371.712	0.1656	74.43	0.0000
labor	92	6	62.53323	0.0918	5.19	0.5199

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
profit						
ppadi	-6611.184	1997.347	-3.31	0.001	-10525.91	-2696.457
plabor	-81.15006	238.7752	-0.34	0.734	-549.1408	386.8407
two_ppadix~t	-226.4383	167.9182	-1.35	0.177	-555.5519	102.6754
ppadixfarmsz	648.9635	139.0561	4.67	0.000	376.4185	921.5085
ppadixbftot	3786.079	1099.325	3.44	0.001	1631.441	5940.716
ppadixirrih	2120.583	391.3434	5.42	0.000	1353.564	2887.602
ppadixslope	-200.7666	121.1408	-1.66	0.097	-438.1982	36.66511
ppadixhujan	1558.642	526.0069	2.96	0.003	527.6873	2589.597
plaborxfar~z	-13.71369	13.43874	-1.02	0.308	-40.05313	12.62575
plaborxbftot	142.9762	114.9671	1.24	0.214	-82.35517	368.3076
plaborxirrih	42.10814	45.73739	0.92	0.357	-47.5355	131.7518
plaborxslope	-3.411727	11.82228	-0.29	0.773	-26.58296	19.75951
plaborxhujan	-9.251947	52.30382	-0.18	0.860	-111.7656	93.26166
_cons	239.9639	145.1855	1.65	0.098	-44.59454	524.5224
padi						
plabor_ppa~t	-226.4383	167.9182	-1.35	0.177	-555.5519	102.6754
farmsz	648.9635	139.0561	4.67	0.000	376.4185	921.5085
bftot	3786.079	1099.325	3.44	0.001	1631.441	5940.716
irrih	2120.583	391.3434	5.42	0.000	1353.564	2887.602
slope	-200.7666	121.1408	-1.66	0.097	-438.1982	36.66511
hujan	1558.642	526.0069	2.96	0.003	527.6873	2589.597
_cons	-6611.184	1997.347	-3.31	0.001	-10525.91	-2696.457
labor						
ppadi_plab~t	226.4383	167.9182	1.35	0.177	-102.6754	555.5519
farmsz	13.71369	13.43874	1.02	0.308	-12.62575	40.05313
bftot	-142.9762	114.9671	-1.24	0.214	-368.3076	82.35517
irrih	-42.10814	45.73739	-0.92	0.357	-131.7518	47.5355
slope	3.411727	11.82228	0.29	0.773	19.75951	26.58296
hujan	9.251947	52.30382	0.18	0.860	-93.26166	111.7656
_cons	81.15006	238.7752	0.34	0.734	-386.8407	549.1408

7.6 Calculating marginal and total impacts of the change in baseflow

As in the case of the production function, we can calculate the marginal value product of baseflow, which is the derivative of the profit function with respect to baseflow:

$$\frac{\partial \pi}{\partial x_3} = \beta_{p3} p + \beta_{w3} w \,.$$

The marginal value product is just the weighted sum of the regression coefficients on the two variables in the profit function that are related to baseflow (β_{p3} = coefficient on *ppadixbftot*, β_{w3} = coefficient on *plaborxbftot*), where the weights are the corresponding prices (*p* = *ppadi*, *w* = *plabor*). Since the second coefficient is not significant, the expression simplifies to

$$\frac{\partial \pi}{\partial x_3} = \beta_{p3} p \,.$$

We can use this expression to calculate the marginal value product for each household by entering the following command:

```
generate marginal_impact_system=ppadi*[profit]_b[ppadixbftot]
```

The phrase [profit]_b[ppadixbftot] tells Stata to use the stored value of the estimated coefficient on *ppadixbftot* from the profit function. We can obtain summary statistics by entering the command,

```
su  marginal_impact_system
```

which returns the following information:

```
. su  marginal_impact_system;

     Variable  |    Obs        Mean      Std. Dev.      Min        Max
 ------------------------------------------------------------------------
 marginal_i~n  |     92    4229.804     1988. 991   1893.039   13535.23
```

The mean marginal value product is 4,300, which is half again as large as the estimate based on the production function, 2,774. This is consistent with our expectation that the benefit of an environmental improvement based on a production function understates the actual benefit (see section 2).

The benefit of increased baseflow that we just calculated is a marginal benefit: the benefit of one additional unit. The proposed reforestation program in Ruteng would not uniformly change baseflow by one unit for all farms, however. According to Pattanayak and Kramer, the predicted changes, in percentage terms, were as follows:

Kecano (county)	Percent change in baseflow
1 (Borong)	15
2 (Elar)	-17
3 (Langke Rembong)	-25
4 (Pembantu Borong)	9
5 (Pembantu Elar)	36
6 (Pembantu Lambaleda)	-23
7 (Pembantu Ruteng)	-12
8 (Ruteng)	-5
9 (Satarmese)	9

These are changes for a program that would increase forest cover by 25 percent. As can be seen, the program was predicted to increase baseflow in less than half the counties.

Per the discussion in sections 2-4, to value the impacts of these changes on farm profits, we need to determine the difference between profits with the baseflow changes and profits without them, i.e. $\pi_1^* - \pi_0^*$. For the particular specification of the profit function used here, π_1^* and π_0^* are given by

$$\pi_1^* = \beta_p p + \beta_w w + \beta_{pw} 2(pw)^{0.5}$$
$$+ \beta_{p2}(px_2) + \beta_{p3}(px_3^1) + \beta_{p4}(px_4) + \beta_{p5}(px_5) + \beta_{p6}(px_6)$$
$$+ \beta_{w2}(wx_2) + \beta_{w3}(wx_3^1) + \beta_{w4}(wx_4) + \beta_{w5}(wx_5) + \beta_{w6}(wx_6)$$

$$\pi_0^* = \beta_p p + \beta_w w + \beta_{pw} 2(pw)^{0.5}$$
$$+ \beta_{p2}(px_2) + \beta_{p3}(px_3^0) + \beta_{p4}(px_4) + \beta_{p5}(px_5) + \beta_{p6}(px_6)$$
$$+ \beta_{w2}(wx_2) + \beta_{w3}(wx_3^0) + \beta_{w4}(wx_4) + \beta_{w5}(wx_5) + \beta_{w6}(wx_6)$$

where x^1_3 is baseflow with the change and x^0_3 is baseflow without it. The difference in profit is thus given by

$$\pi_1^* - \pi_0^* = \beta_{p3} p(x_3^1 - x_3^0) + \beta_{w3} w(x_3^1 - x_3^0),$$

which further simplifies to

$$\pi_1^* - \pi_0^* = \beta_{p3} p(x_3^1 - x_3^0),$$

after considering that the estimate of β_{w3} (i.e., the coefficient on *plaborxbftot* in the profit function) is not significant. The change in profit is thus given by the product of the coefficient on the interaction term for rice price and baseflow (*ppadixbftot*), rice price (*ppadi*), and the change in baseflow.

The change in baseflow here is in meters, not percent. So, as a first step we need to construct this variable. We do so as follows. First, we construct a variable, *bfchange_percent*, that gives the change in percent:

```
generate  bfchange_percent=0
replace bfchange_percent=15 if kecano==1
replace bfchange_percent=-17 if kecano==2
replace bfchange_percent=-25 if kecano==3
replace bfchange_percent=9 if kecano==4
replace bfchange_percent=36 if kecano==5
replace bfchange_percent=-23 if kecano==6
replace bfchange_percent=-12 if kecano==7
replace bfchange_percent=-5 if kecano==8
replace bfchange_percent=9 if kecano==9
```

Then, we multiply this new variable times baseflow (*bftot*) and divide by 100 to convert the percentages in *bfchange_percent* to decimals:

```
generate bfchange=bftot*bfchange_percent/100
```

We now have all the information necessary for determining the change in profit resulting from the predicted change in baseflow, which we do by using the following command to create a variable named *total_impact_system*:

```
generate total_impact_system=[profit]_b[ppadixbftot]*ppadi *bfchange
```

The mean value of this variable is 7.33, determined by using the "summarize" command in Stata,

```
su total_impact_system
```

which returns

```
. su total_impact_system
```

Variable	Obs	Mean	Std. Dev.	Min	Max
marginal_i ~ n	92	7.332027	696. 6726	-3058.286	982.8646

On average, the reforestation program thus has a positive impact on farm profits. We can gauge the magnitude of the impact by applying the summarize command to *profit*,

```
su profit
```

which returns

```
. su profit
```

Variable	Obs	Mean	Std. Dev.	Min	Max
marginal_i ~ n	92	1136.492	2583. 524	.4166667	15713.38

Compared to mean profit (1136), the impact of the reforestation program is not very large, less than 1 percent.

A summary of the Stata commands reviewed in this section is given on the following two pages. They are the final pages of the tutorial.

7.7 Summary list of Stata commands

For generating variables in the production function:

```
generate lpadi=ln( padi )
generate llabor=ln( labor)
generate lfarmsz = ln( farmsz )
generate lbftot=ln( bftot )
generate lhujan=ln( hujan )
```

For estimating the production function:

```
regress  lpadi llabor lfarmsz lbftot lhujan irrih slope
```

For generating and summarizing the marginal impact of baseflow from the production function:

```
generate marginal_impact_production=ppadi*_b[lbftot]*padi/bftot
su  marginal_impact_production
```

For generating the variables in the profit function:

```
generate two_ppadixplabor_sqrt=2*(ppadi*plabor)^0.5
generate ppadixfarmsz=ppadi*farmsz
generate ppadixbftot=ppadi*bftot
generate ppadixirrih=ppadi* irrih
generate ppadixslope=ppadi* slope
generate ppadixhujan=ppadi* hujan
generate plaborxfarmsz=plabor*farmsz
generate plaborxbftot=plabor*bftot
generate plaborxirrih=plabor*irrih
generate plaborxslope=plabor*slope
generate plaborxhujan=plabor*hujan
```

For estimating the profit function:

```
regress profit ppadi plabor two_ppadixplabor_sqrt ppadixfarmsz ppadixbftot ppadixirrih
ppadixslope ppadixhujan plaborxfarmsz plaborxbftot plaborxirrih plaborxslope plaborxhujan
```

For generating additional variables in the output supply and input demand functions:

```
generate ppadi_plabor_sqrt=(ppadi/plabor)^0.5
generate plabor_ppadi_sqrt=(plabor/ppadi)^0.5
```

For defining the profit, output supply, and input demand functions:

```
global profit_function “(profit ppadi plabor  two_ppadixplabor_sqrt ppadixfarmsz ppadixbftot
ppadixirrih ppadixslope ppadixhujan plaborxfarmsz plaborxbftot plaborxirrih plaborxslope
plaborxhujan )”

global output_supply_function “(padi plabor_ppadi_sqrt farmsz bftot irrih slope hujan)”

global input_demand_function “(labor ppadi_plabor_sqrt farmsz bftot irrih slope hujan)”
```

For creating cross-equation coefficient constraints:

```
constraint define 1 [profit]ppadi = [padi]_cons
constraint define 2 [profit]plabor = -[labor]_cons
constraint define 3 [profit]two_ppadixplabor_sqrt = [padi]plabor_ppadi_sqrt
constraint define 4 [profit]two_ppadixplabor_sqrt = -[labor]ppadi_plabor_sqrt
constraint define 5 [profit]ppadixfarmsz = [padi]farmsz
constraint define 6 [profit]plaborxfarmsz = -[labor]farmsz
constraint define 7 [profit]ppadixbftot = [padi]bftot
constraint define 8 [profit]plaborxbftot = -[labor]bftot
constraint define 9 [profit]ppadixirrih = [padi]irrih
constraint define 10 [profit]plaborxirrih = -[labor]irrih
constraint define 11 [profit]ppadixslope = [padi]slope
```

```
constraint define 12 [profit]plaborxslope = -[labor]slope
constraint define 13 [profit]ppadixhujan = [padi]hujan
constraint define 14 [profit]plaborxhujan = -[labor]hujan
```

For estimating the profit function system:

```
reg3 $profit_function $output_supply_function $input_demand_function, constraints(1 2 3 4
5 6 7 8 9 10 11 12 13 14) sure
```

For generating and summarizing the marginal impact of baseflow from the profit function system:

```
generate marginal_impact_system=ppadi*[profit]_b[ppadixbftot]
su  marginal_impact_system
```

For generating a variable giving the percent change in baseflow:

```
generate  bfchange_percent=0
replace bfchange_percent=15 if kecano==1
replace bfchange_percent=-17 if kecano==2
replace bfchange_percent=-25 if kecano==3
replace bfchange_percent=9 if kecano==4
replace bfchange_percent=36 if kecano==5
replace bfchange_percent=-23 if kecano==6
replace bfchange_percent=-12 if kecano==7
replace bfchange_percent=-5 if kecano==8
replace bfchange_percent=9 if kecano==9
```

For generating the change in baseflow in absolute terms:

```
generate  bfchange=bftot*bfchange_percent/100
```

For generating and summarizing the total impact of baseflow from the profit function system:

```
generate total_impact_system=[profit]_b[ppadixbftot]*ppadi *bfchange
su total_impact_system
```

8. Acknowledgements

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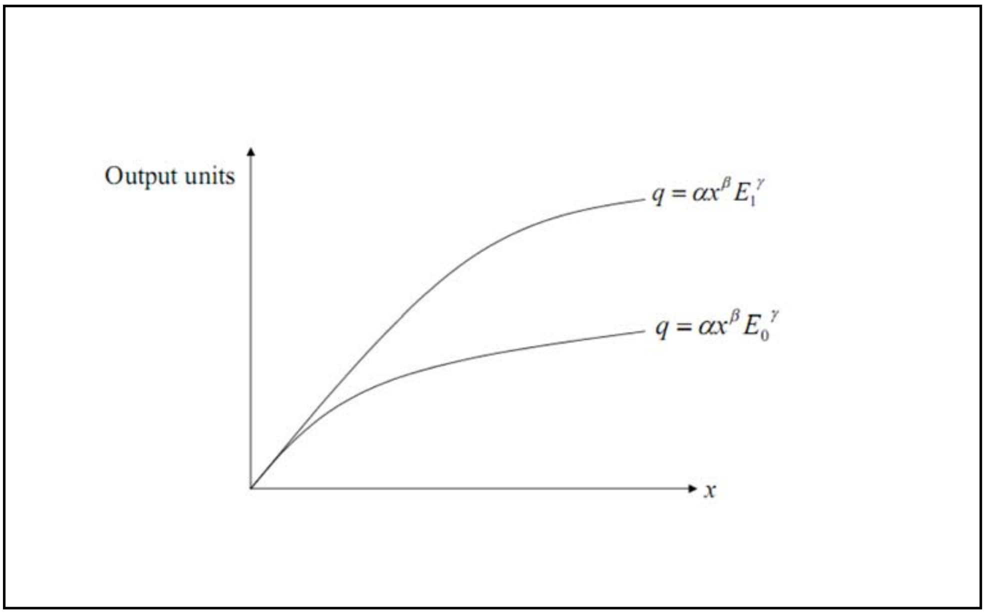
FIGURES

Figure 1:



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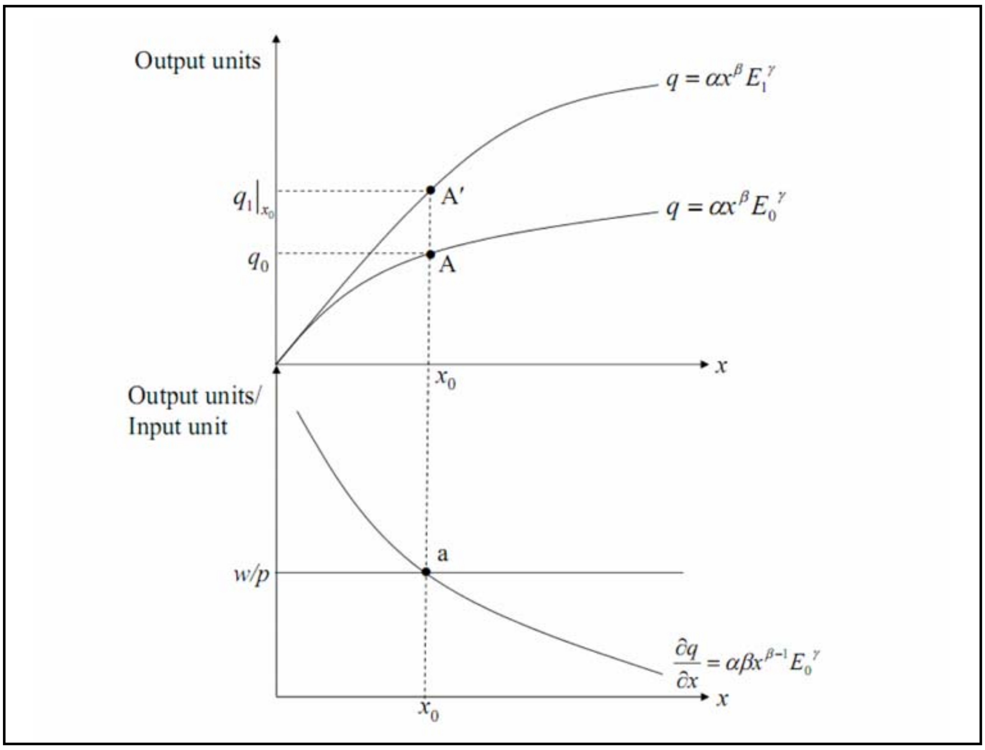

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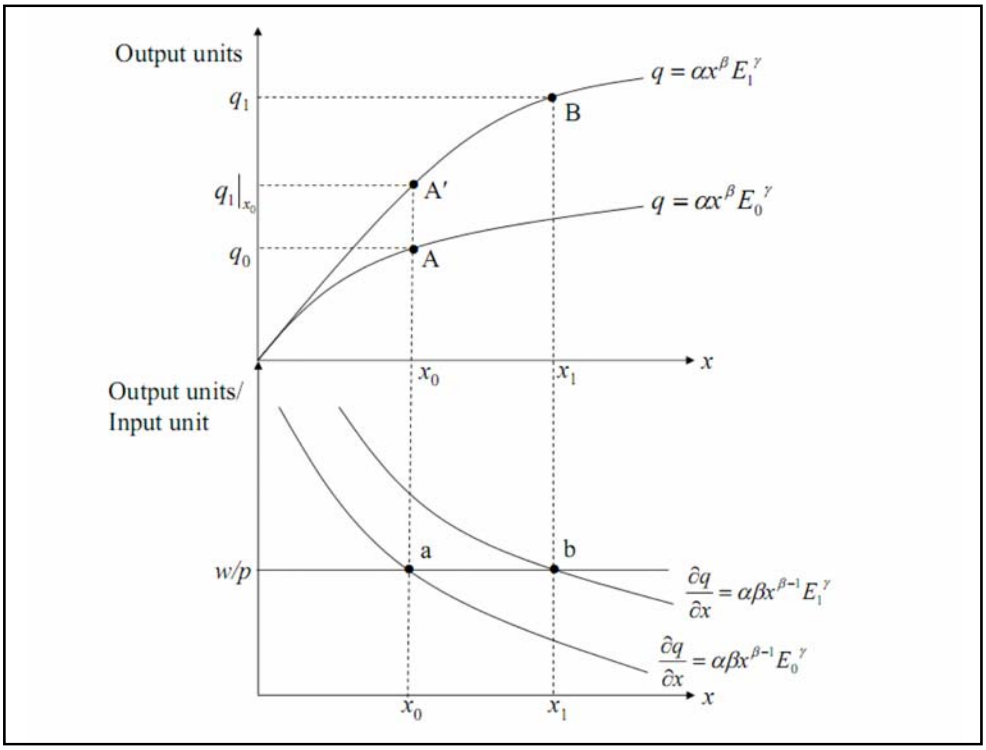

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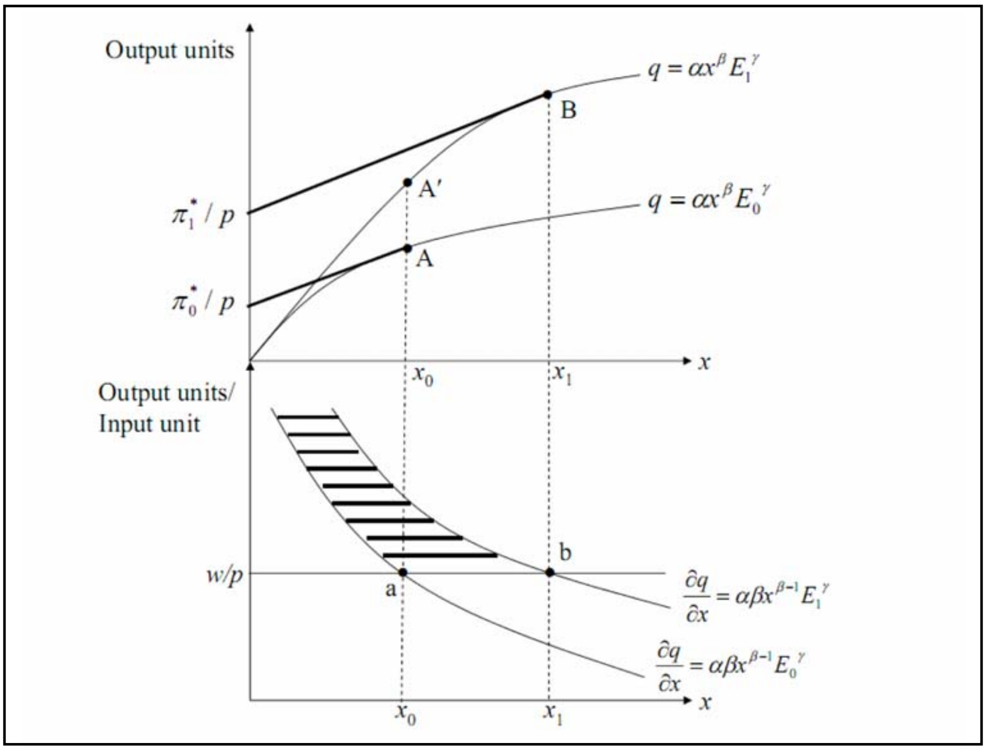

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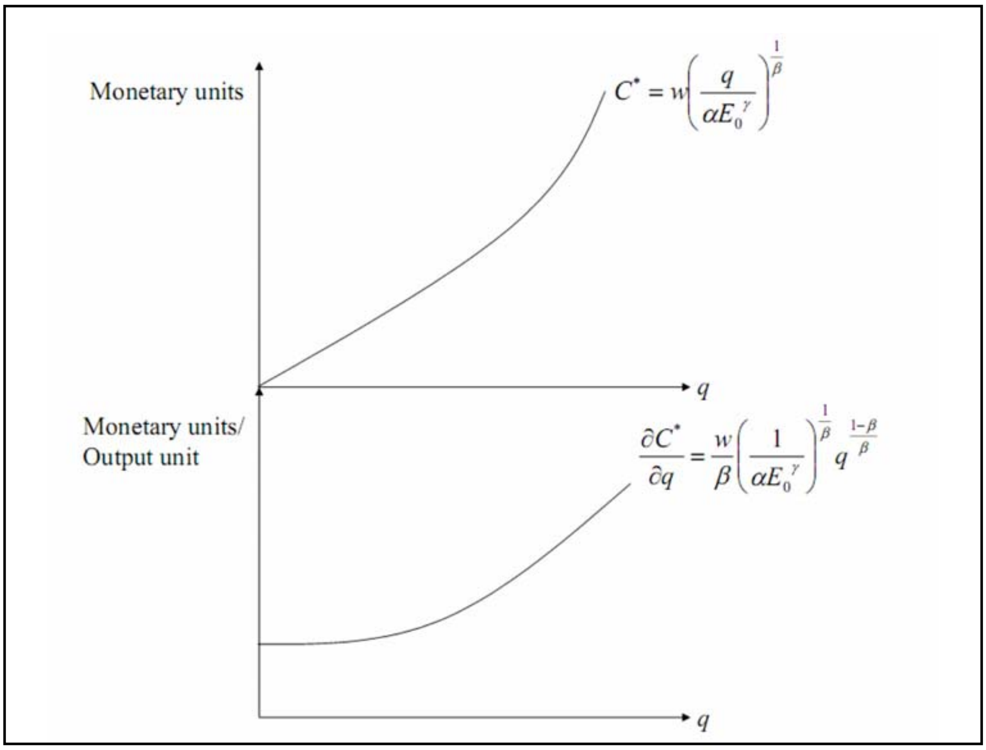

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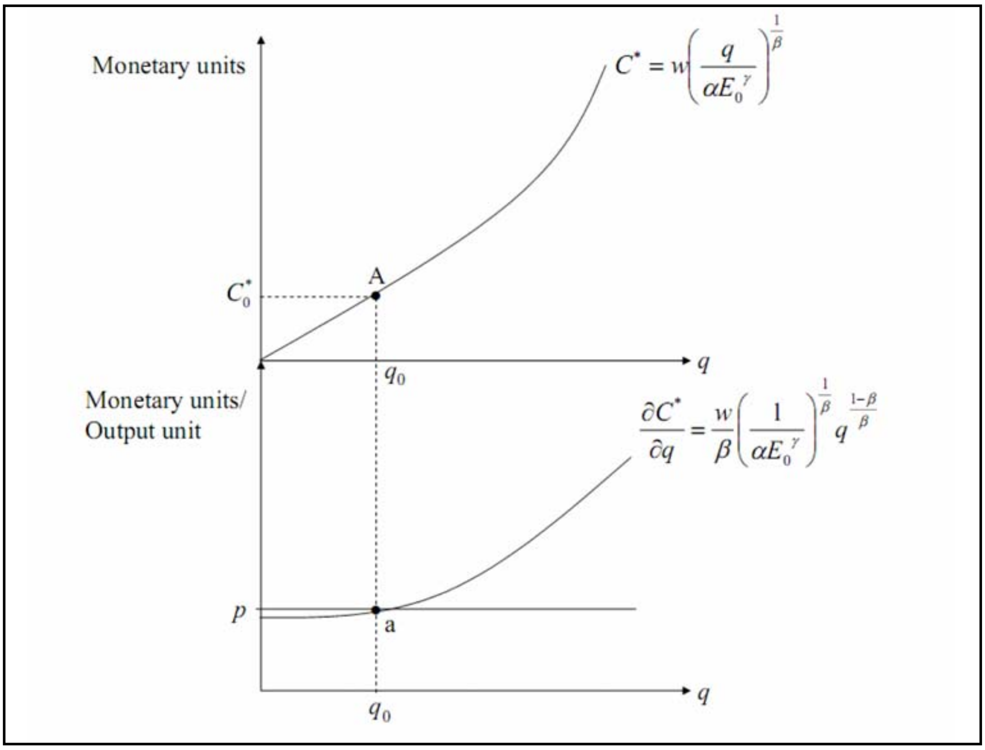

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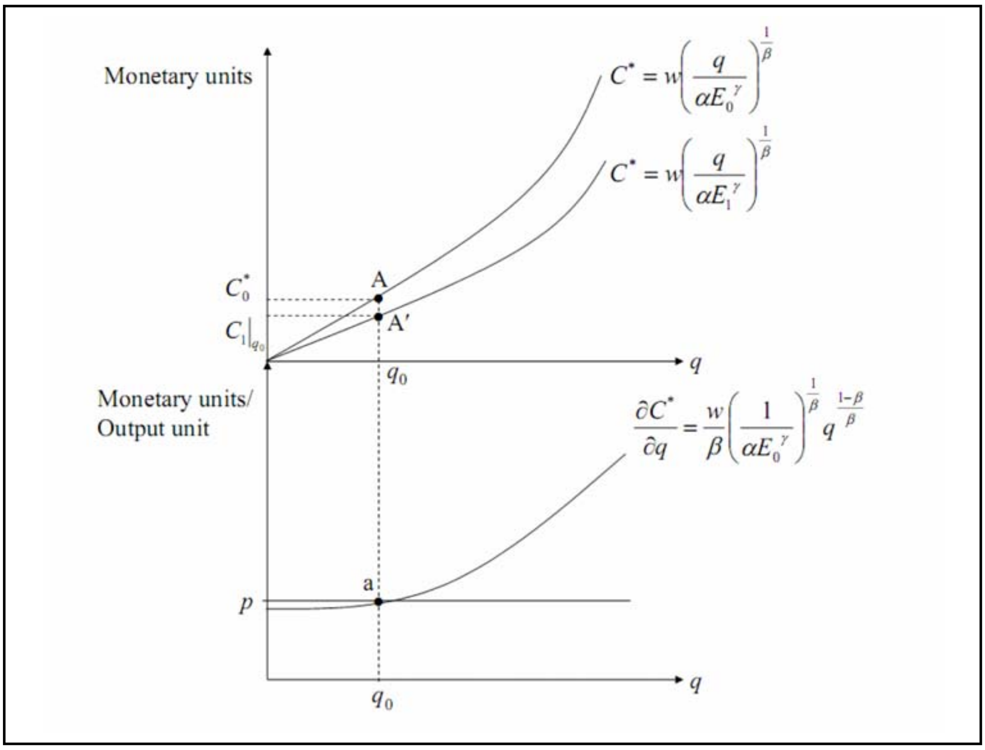

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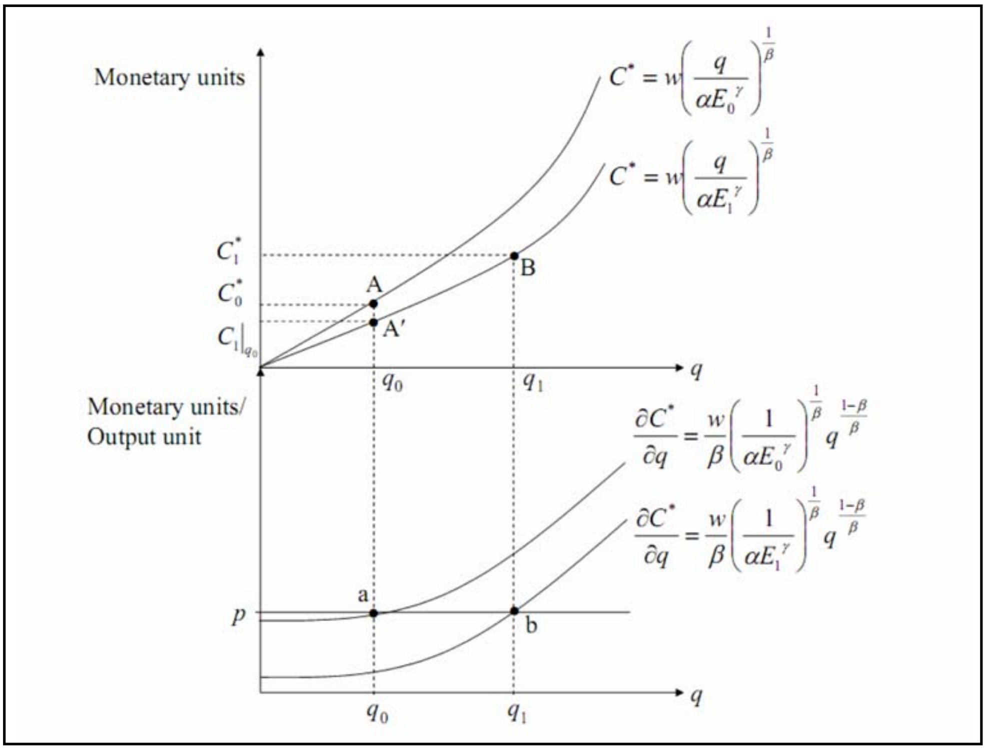

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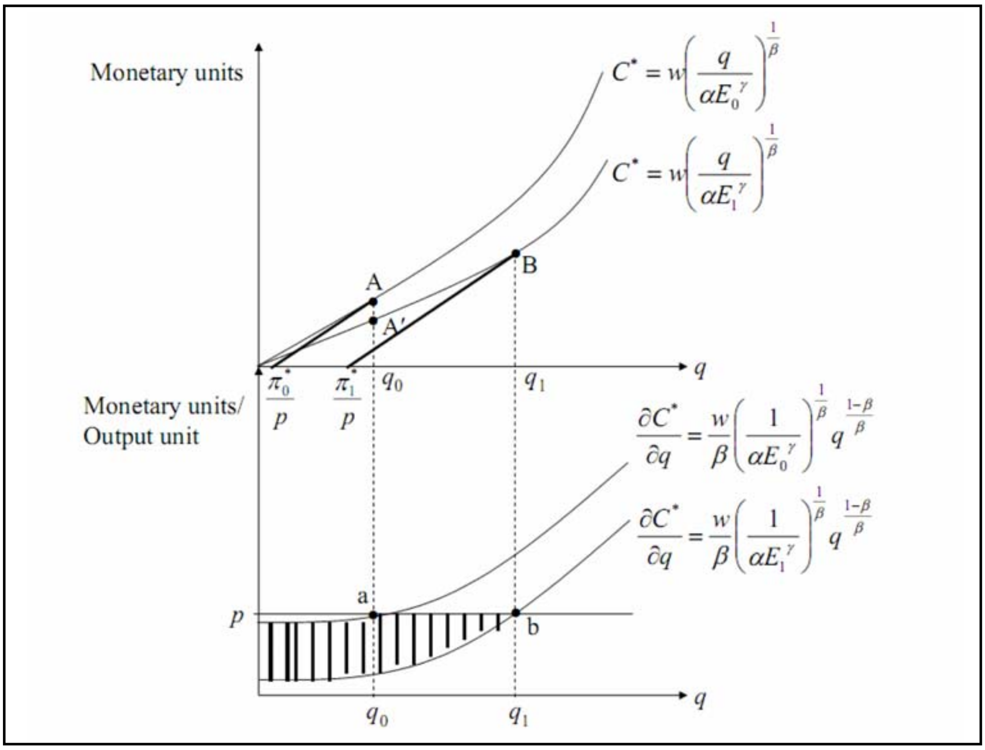

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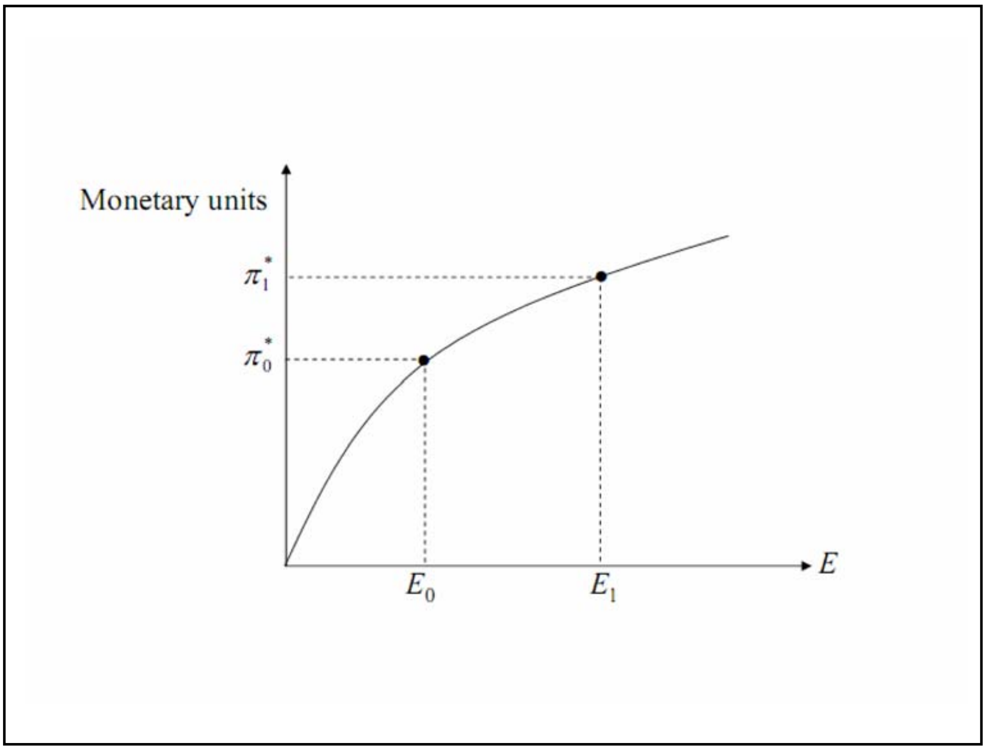

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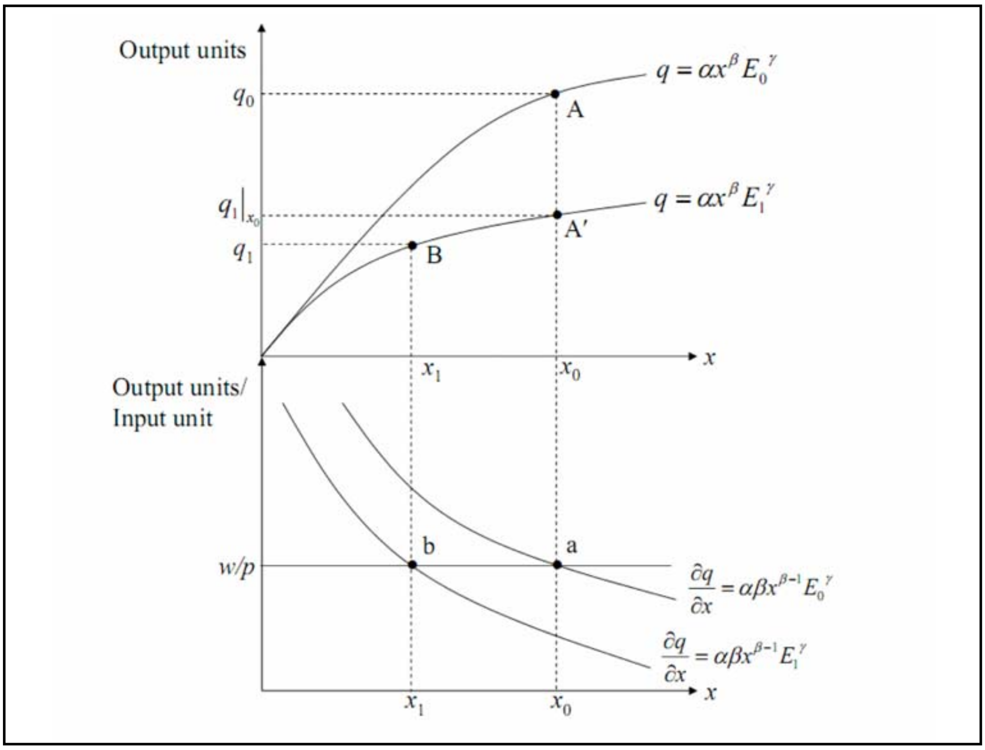

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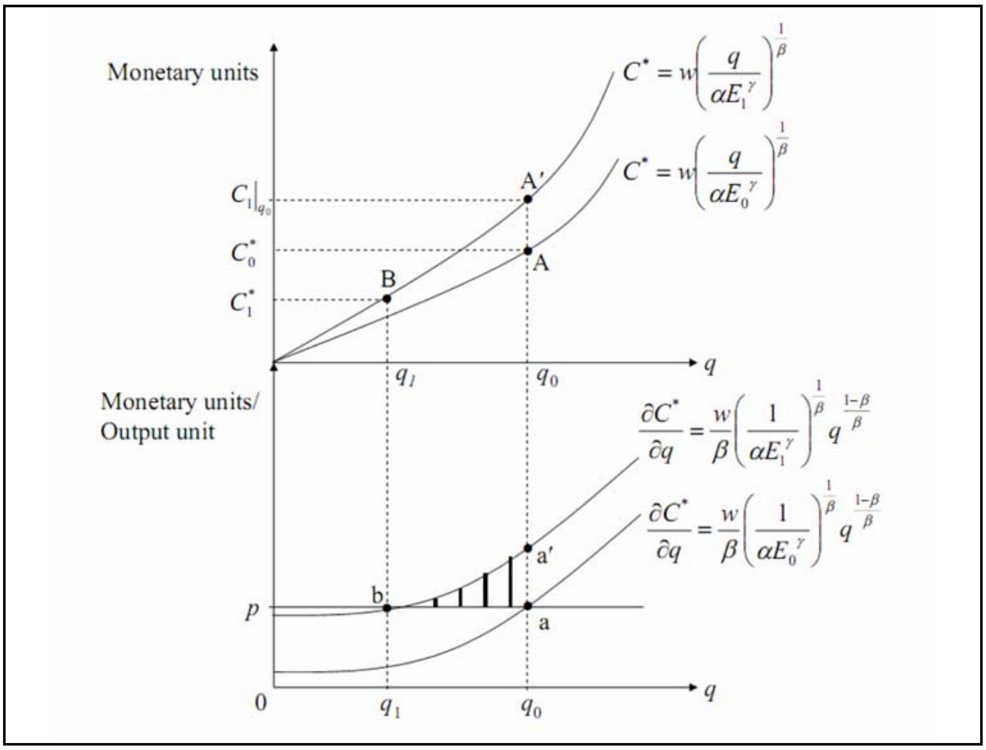

The Environment as a Production Input: A Tutorial

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TABLE OF CONTENTS

The Environment as a Production Input: A Tutorial

Jeffrey R. Vincent

1. Introduction

Production is often affected by environmental conditions. For example, rice harvests might be damaged by infiltration of saline water from neighboring shrimp farms, fish catch might be reduced by the loss of spawning grounds when mangroves are cut down, and timber harvests might fall as a result of damage from acid rain and other forms of air pollution. In all these cases, environmental quality is acting as a nonmarket, or unpriced, production input. Damage to the environment reduces the supply of this input, and as a result production falls. Conversely, programs to improve environmental quality can benefit environmentally sensitive forms of production by raising the supply of such inputs. These production-related benefits can be among the most important benefits generated by environmental improvements. This is especially likely to be the case in developing countries, where sectors such as agriculture, forestry, and fishing typically account for a larger share of overall economic activity than in developed countries.

In principle, the valuation of changes in environmental quality that affect production is straightforward: one needs to estimate the change in profit caused by the environmental change. There are several ways to estimate the change in profit, however, and one might not have the data necessary to use all of them. Understanding the relationships among the different approaches, and the conditions under which they are valid, is thus important. Moreover, sometimes one might only be able to estimate a component of the change in profit, such as the change in revenue or the change in cost. A question that arises is whether partial measures such as these can be used to value environmental changes.

This tutorial covers these types of issues. Its purpose is to review the relationships among three key functions in production economics—production functions, cost functions, and profit functions—and to review how they can be used to value changes in environmental quality. The tutorial is in two parts. The first part (sections 2-6) is purely conceptual. It illustrates the relationships among the three functions by referring to a specific type of production function, the Cobb-Douglas function. This is the most common production function used in applied economic analysis. The first part of the tutorial begins by reviewing how a production function can be used to value changes in environmental quality (section 2), and then it reviews how cost and profit functions can be derived from a production function and used to perform the same valuation (sections 3-4). Use of a production function is typically called the primal approach, while use of cost and profit functions is typically called the dual approach.

Although the first part of the tutorial refers to the specific case of Cobb-Douglas technology, the intention is to provide intuition about fundamental points that are generally relevant, not points that are unique to that technology. Issues that arise if certain assumptions made in the first part of the tutorial do not hold are discussed at the end of first part (section 6), as are implications for empirical work (section 5). The first part of the tutorial should thus prepare one to read more advanced material on production economics, such as Chambers (1988), Just and Pope (2001), and Mundlak (2001), and more advanced material on the valuation of the environment as a production input, such as Point (1995), Huang and Smith (1998), Freeman (2003, Ch. 9), and McConnell and Bockstael (2005). It does not attempt to cover all the topics in these sources or to provide as rigorous derivations.

The second part of the tutorial (section 7) is empirical. It demonstrates how to use the econometrics program Stata to estimate production and profit functions for rice production and to use these functions to value changes in water availability. The data for this demonstration are a subset of the data used by Subhrendu Pattanayak and Randall Kramer in their article, "Worth of watersheds" (*Environment and Development Economics* 6:123-146, 2001).[1] Pattanayak and Kramer analyzed the impact of a reforestation program in Indonesia on the production of rice and coffee on farms located downstream of the reforested area. Reforestation can affect the infiltration of rainfall into the ground, which in turn affects the seepage of groundwater into rivers. This seepage, or baseflow, is an important source of water during the dry season. Although Pattanayak and Kramer analyzed the impact of changes in baseflow on both rice and coffee, the Stata demonstration in this tutorial refers to impacts on just rice and utilizes data from just a subset of the farms that the authors analyzed.

2. Production function

The focus in this section and sections 3-5 is on a private firm that produces a single output, which it sells in a competitive market. To produce this output, the firm uses a single, variable input, which it buys in a competitive market. So, both the output produced by the firm and the input that it uses are priced, and the prices are not affected by the firm's supply of the output or its demand for the input. We consider a simple static setting and ignore dynamic issues related to risk or fixed inputs (investment, depreciation). The implication of these various assumptions is that the change in the profit of the firm equals the welfare impact on the owner of the firm, as it gives the change in the owner's income.

In addition to the priced input that is under the control of the owner, output is affected by environmental quality, which is a public good that is beyond the control of the owner. Implicitly, therefore, environmental quality represents a second production input. The question to be answered is, "How does profit change if environmental quality changes?" We will review, in order, how to answer this question from three vantage points: the production function (this section), the cost function (section 3), and the profit function (section 4). To make a connection to the empirical analysis in the second part of the tutorial, one can think of the firm as a farm, the output as rice, the priced input as farm labor, and environmental quality as baseflow.

2.1 Variables and assumptions

A production function is a technical relationship that relates physical quantities of outputs to physical quantities of inputs. For a firm that produces a single output q, uses a single variable input x, and is affected by environmental quality E, the Cobb-Douglas production function is

$$q = ax^bE^r,$$

where

[1] This data set can be downloaded from http://www.sandeeonline.org

q = output
x = variable input
E = environmental quality
α, β, γ = parameters.

The variables q, x, and E are all assumed to be positive (> 0). As can be seen, the Cobb-Douglas production function indicates that the two inputs interact in a multiplicative way and are both essential to production: if either $x = 0$ or $E = 0$, then $q = 0$ too. We will assume that production is "well-behaved" in the senses that $\alpha > 0$, which is necessary if q, x, and E are all positive, and $0 < \beta < 1$ and $0 < \gamma < 1$, which imply that production is increasing in both inputs but has diminishing returns.

Figure 1 depicts this production function for two levels of environmental quality, E_0 (lower quality) and E_1 (higher quality). The vertical axis shows level of output (q), while the horizontal axis shows the level of the variable input (x). The production function slopes upward because it is increasing in the variable input ($0 < \beta$), but its slope becomes smaller as the variable input increases, due to diminishing returns ($\beta < 1$). The fact that the slope is positive but diminishing can be verified by taking the first and second derivatives of the production function with respect to the variable input:

First derivative: $$\frac{\partial q}{\partial x} = \alpha \beta x^{\beta-1} E^{\gamma} > 0$$

Second derivative: $$\frac{\partial^2 q}{\partial x^2} = \alpha \beta (\beta - 1) x^{\beta-2} E^{\gamma} < 0$$

All the terms in the first derivative are positive, so the derivative is positive too. This derivative gives the *marginal product* of x: the incremental output that is produced if one more unit of the variable input is used. All the terms in the second derivative are positive except one, $\beta - 1$, which is negative because $0 < \beta < 1$, and so the derivative is negative.

2.2 Deriving the input demand function

The production function with the higher level of environmental quality (E_1) is above the one for the lower level (E_0) because production is increasing in environmental quality ($0 < \gamma$): for a given level of the variable input, output is higher if environmental quality is higher. How does this change affect the firm's profit? To answer this question, we need to bring another function into the picture: the *input demand function* for the variable input. The input demand function gives the profit-maximizing level of x: the choice of x that maximizes the firm's profit for a given level of E. To derive this function, we first need to define the firm's profit, which equals the difference between the revenue from selling the output and the expenditure on the variable input. If we define

p = price of output
w = price of variable input,

then profit, π, is given by

$$p = pq - wx.$$

If we substitute the production function for q, then this expression becomes

$$\pi = p\left(\alpha x^{\beta} E^{\gamma}\right) - wx .$$

Profit-maximizing use of *x* occurs where the first derivative of this expression equals zero. The first derivative is

$$\frac{\partial \pi}{\partial x} = p\left(\alpha\beta x^{\beta-1}E^{\gamma}\right) - w.$$

If we equate the derivative to zero, then we obtain

$$w = p\alpha\beta x^{\beta-1}E^{\gamma}.$$

The right-hand side of this new expression is the *marginal value product* of the variable input: the price of output, *p*, multiplied by the marginal product of the input ($\partial q/\partial x$), $\alpha\beta x^{\beta-1}E^{\gamma}$. It gives the firm's marginal willingness to pay for the input and can be interpreted as the *inverse input demand function*. The expression thus says that the firm should use *x* up to the point where its marginal value product (demand) equals its price, *w* (supply). Using *x* beyond this point would generate additional revenue, but the incremental revenue would be less than the cost of the additional amount of *x*.

A slight rearrangement of this expression is convenient for graphical purposes:

$$\frac{w}{p} = \alpha\beta x^{\beta-1}E^{\gamma}.$$

This is the profit-maximizing condition expressed in physical terms instead of monetary terms. The right-hand side, which is the marginal product of the variable input, is the inverse input demand function expressed in physical terms. The left-hand side is also in physical terms because it is a price ratio, and the monetary units cancel out. For example, if the input price *w* is in rupees per day and the output price *p* is in rupees per kilogram, then the units of the price ratio *w*/*p* are kilograms per day. So, profit maximization occurs where the marginal product of the variable input equals the ratio of input price to output price.

If we solve the profit-maximizing condition for *x* instead of for the price ratio, then we obtain the input demand function in standard (not inverse) form:

$$x^{*} = \left(\frac{p\alpha\beta E^{\gamma}}{w}\right)^{\frac{1}{1-\beta}}.$$

We denote the level of the variable input by x^* to indicate that it is the profit-maximizing value. Note that the input demand function includes only prices (*p*, *w*), environmental quality (*E*), and parameters from the production function (α, β). It does not include the physical quantity of output (*q*). Given that $0 < \beta < 1$, the exponent is positive, and so the function is increasing in output price and environmental quality but decreasing in input price. Given that the output and input prices appear as a ratio, if both prices change by the same factor—for example, if *p* becomes λp and *w* becomes λw—then the optimal level of the input does not change. There is no "money illusion." In other words, the input demand function is homogeneous of degree 0 in prices. We will work with the inverse input demand function in the rest of this section but return to the input demand function in standard form when we analyze the profit function in section 4.

2.3 Change in profit, without and with input adjustment

Figure 2 has two panels. The top panel repeats Figure 1, and the bottom panel shows inverse demand for the variable input in physical terms (i.e., marginal product) at the lower level of environmental quality:

$$\frac{\partial q}{\partial x} = \alpha\beta x^{\beta-1} E_0^{\ \gamma}.$$

Profit-maximization occurs at x_0, where marginal product equals the price ratio (point a in the bottom panel) and output is at q_0 (point A in the top panel). This is the profit-maximizing combination of x and q, given that environmental quality is at E_0:

$$\pi^*_0 = pq_0 - wx_0.$$

If environmental quality improves from E_0 to E_1, then output at x_0 rises to $q_1 \mid_{x0}$ (point A' in the top panel): output is now determined by the higher production function. (Read $q_1 \mid_{x0}$ as "output when E is at E_1 but x is at x_0.") Profit rises too, to

$$\pi_1\big|_{x_0} = pq_1\big|_{x_0} - wx_0,$$

with the change in profit, $\pi^*_1 \mid_{x0} - \pi^*_0$, thus being given by just the increase in revenue,

$$p\left(q_1\big|_{x_0} - q_0\right).$$

It is important to recognize that this expression does not equal the full change in profit that results from the environmental improvement. The expression fails to account for the fact that the environmental improvement causes not only the production function to shift but also the inverse input demand function. Figure 3 shows how the latter shifts in response to the environmental improvement, and it also shows the resulting impact on output. In the bottom panel, the inverse input demand function shifts upward when E_0 is replaced by E_1: the environmental improvement causes the marginal product of the variable input to rise. Profit-maximization now occurs at x_1 (point b in the bottom panel), which is greater than x_0, and so output rises to q_1 (point B in the top panel). Hence, after allowing for the adjustment in the variable input, maximum profit is given by

$$\pi^*_1 = pq_1 - wx_1.$$

This, not $\pi_1 \mid_{x0}$, is the correct expression for maximum profit at E_1. The change in profit,

$$\pi^*_1 - \pi^*_0 = p(q_1 - q_0) - w(x_1 - x_0),$$

is now not just a change in revenue: it also includes the change in expenditure on the variable input. Although expenditure on the input rises by $w(x_1 - x_0)$, revenue rises by an even greater amount, $p(q_1 - q_1 \mid_{x0})$, because the marginal product of the variable input is greater than the price ratio up to point b. Profit thus rises too: $\pi^*_1 > \pi^*_1 \mid_{x0}$. If one calculates the increase in profit as just the increase in revenue at the initial level of the variable input (i.e., as $p(q_1 \mid_{x0} - q_0)$, then one understates the benefit of improved environmental quality.

2.4 Magnitude of the change in profit

How big is the increase in profit, $\pi^*_1 - \pi^*_0$? This can be depicted in two ways. Both are shown in Figure 4. Compared to Figure 3, the upper panel of Figure 4 includes two additional line segments, which are tangent to the production functions at points x_0 and x_1.[2] The intercept of

[2] I am grateful to Subhrendu Pattanayak for suggesting the addition of these tangents.

each tangent shows the profit associated with the corresponding production point, expressed in physical units instead of money. From above, maximum profit at point A (i.e., for E_0) is given by

$$\pi^*_0 = pq_0 - wx_0,$$

which solved for q_0 yields

$$q_0 = \frac{\pi_0^*}{p} + \frac{w}{p}x_0 .$$

The equation for the tangent at point A is thus

$$q = \frac{\pi_0^*}{p} + \frac{w}{p}x .$$

Its slope, w/p, equals the slope of the production function. The slope of the production function is by definition the marginal product of the variable input, and so the tangency simply reflects the profit-maximizing condition,

$$\frac{w}{p} = \frac{\partial q}{\partial x} .$$

We can derive the equation for the tangent to the higher production function (i.e., the one with E_0) at point B by using the same logic:

$$q = \frac{\pi_1^*}{p} + \frac{w}{p}x .$$

The difference between the intercepts of the tangents,

$$\frac{\pi_1^*}{p} - \frac{\pi_0^*}{p},$$

gives the increase in profits in physical terms. The figure does not show the line passing through point A', which would cross the production function with E_1 instead of being tangent to it (because the profit-maximizing condition does not hold at A') and have an intercept between those of the two tangents (because profit at point A' is higher than at point A but lower than at point B).

In the bottom panel, the increase in profit is shown by the cross-hatched area between the two inverse input demand functions. The cross-hatched area equals the change in *consumer surplus for the variable input*, where the "consumer" is the firm. This is easily demonstrated. Consumer surplus is the area under an inverse input demand function and above the price line. The expression for this in the case of E_1 is

$$\int_0^{x_1} \left(\alpha\beta x^{\beta-1} E_1^{\gamma}\right) dx - \frac{w}{p}x_1 ,$$

which simplifies to

$$q_1 - \frac{w}{p}x_1 ,$$

which in turn is the same as profit in physical terms, $\frac{\pi_1^*}{p}$. Parallel analysis for the inverse input demand function that includes E_0 yields consumer surplus equal to $\frac{\pi_0^*}{p}$. The change in consumer surplus is thus exactly the same as the difference between the intercepts in the top panel, $\frac{\pi_1^*}{p} - \frac{\pi_0^*}{p}$.

3. Cost function

3.1 Definition and characteristics

A cost function is an economic relationship that relates the minimum cost of production to the quantity of output, the prices of variable inputs, and the quantities of fixed inputs, including environmental inputs. In the case we have been considering, the cost of production is just the firm's expenditure on the single variable input *x*:

$$C = wx \cdot$$

We seek to determine the quantity of *x* that minimizes *C* for a given level of output:

$$\min_x wx \quad \text{subject to} \quad q = \alpha x^{\beta} E^{\gamma}.$$

If *q* is given and *E* is not under the control of the firm, then there is only a single quantity of *x* that satisfies the "subject to" production constraint, and this quantity must necessarily equal the cost-minimizing value. We can determine this quantity by solving the constraint for *x*:

$$x = \left(\frac{q}{\alpha E^{\gamma}}\right)^{\frac{1}{\beta}}.$$

This is the *conditional input demand function*. It is "conditional" because it depends on the quantity of output, *q*, unlike the input demand function derived in section 2.2, which depends only on exogenous variables (prices and environmental quality). If we denote the cost-minimizing quantity of *x* by x^*, then the cost function, $C^* = wx^*$, is given by

$$C^* = w\left(\frac{q}{\alpha E^{\gamma}}\right)^{\frac{1}{\beta}}.$$

Note that the cost function includes only the quantity of output (*q*), the price of the variable input (*w*), and environmental quality (*E*), along with the parameters from the production function. Written in implicit form, without any of the functional detail, the cost function is $C^*(q, w, E)$. Three important characteristics of the cost function are:

1. It is increasing in output: $\frac{\partial C^*}{\partial q} = \frac{w}{\beta}\left(\frac{1}{\alpha E^{\gamma}}\right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}}$, which is positive. An increase in output raises production cost.
2. It is increasing in the price of the variable input: $\frac{\partial C^*}{\partial w} = \left(\frac{q}{\alpha E^{\gamma}}\right)^{\frac{1}{\beta}}$, which is positive. An increase in the price of the variable input raises production cost.
3. It is decreasing in environmental quality: $\frac{\partial C^*}{\partial E} = -\frac{\gamma w}{\beta}\left(\frac{q}{\alpha}\right)^{\frac{1}{\beta}} E^{\frac{-\gamma-\beta}{\beta}}$, which is negative. An increase in environmental quality reduces production cost.

Note in the second point that when we differentiate the cost function with respect to input price,

$$\frac{\partial C^*}{\partial w} = \left(\frac{q}{\alpha E^{\gamma}}\right)^{\frac{1}{\beta}} = x.$$

This result is known as *Hotelling's lemma*.

3.2 Cost function for a production function with two variable inputs

The simplicity of the single variable input model obscures the role of minimization in deriving the cost function, as there is only one value of x that satisfies the production constraints. To make the mathematics of minimization more explicit, we need to analyze a production function with more than one variable input. Consider a Cobb-Douglas production function with two variable inputs, x_1 and x_2:

$$q = \alpha x_1^{\beta 1} x_2^{\beta 2} E^{\gamma}.$$

(The subscript 1 now refers to a type of input, not to the level of environmental quality.) The cost-minimization problem for this function is

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2 \quad \text{subject to} \quad q = \alpha x_1^{\beta_1} x_2^{\beta_2} E^{\gamma} .$$

To determine the cost-minimizing values of the two inputs, we first solve the production constraint for x_2,

$$x_2 = \left(\frac{q}{\alpha x_1^{\beta_1} E^{\gamma}} \right)^{\frac{1}{\beta_2}},$$

and then substitute this into the cost expression to obtain

$$w_1 x_1 + w_2 \left(\frac{q}{\alpha x_1^{\beta_1} E^{\gamma}} \right)^{\frac{1}{\beta_2}}.$$

Note that we have reduced the cost-minimization problem from two choice variables (x_1, x_2) to one (x_1). We can therefore determine the cost-minimizing value of x_1, x_1^*, by differentiating this expression with respect to x_1, setting the result equal to zero, and solving the resulting first-order condition for x_1. If we do this, then we obtain

$$x_1^* = \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta_1 + \beta_2}} \left(\frac{\beta_1 w_2}{\beta_2 w_1} \right)^{\frac{\beta_2}{\beta_1 + \beta_2}} .$$

This is the conditional input demand function for x_1. Unlike the conditional input demand function for x in the single input production function, this one includes input prices and not just the physical levels of output and environmental quality.

By symmetry, the corresponding conditional input demand function for x_2 is

$$x_2^* = \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta_1 + \beta_2}} \left(\frac{\beta_2 w_1}{\beta_1 w_2} \right)^{\frac{\beta_1}{\beta_1 + \beta_2}} .$$

The cost function, $w_1 x_1^* + w_2 x_2^*$, is therefore

$$C^* = w_1 \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta_1 + \beta_2}} \left(\frac{\beta_1 w_2}{\beta_2 w_1} \right)^{\frac{\beta_1}{\beta_1 + \beta_2}} + w_2 \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta_1 + \beta_2}} \left(\frac{\beta_2 w_1}{\beta_1 w_2} \right)^{\frac{\beta_2}{\beta_1 + \beta_2}} .$$

This resembles the cost function for the single input production function by including the quantity of output (q), prices of the variable inputs (w_1, w_2), and environmental quality (E), along with the parameters from the production function.

Hotelling's lemma still applies: if we differentiate the cost function with respect to w_1 (or w_2), then we obtain the conditional input demand function for x_1 (or x_2). Note that the input prices appear in the conditional input demand functions as ratios. The conditional input demand functions are thus homogeneous of degree 0 in prices: use of the inputs does not change if both input prices change by the same multiplicative factor. This condition holds trivially when there is just a single variable input because, as we've seen, the conditional input demand function in that case does not include input prices:

$$x = \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta}}.$$

In contrast, if both input prices change by λ times, then cost changes by λ times too: the cost function is homogeneous of degree 1 in prices. This can be seen by considering the summary expression for the cost function,

$$C^{*} = w_1 x_1^{*} + w_2 x_2^{*}.$$

If both input prices change by λ times, then x_1^* and x_2^* do not change (because they are homogeneous of degree 0), but w_1 and w_2 become λw_1 and λw_2, and so cost becomes λC^*:

$$(\lambda w_1)x_1^{*} + (\lambda w_2)x_1^{*} = \lambda C^{*}$$

Analogous reasoning can be used to demonstrate that the cost function for the single variable input case is also homogeneous of degree 1 in prices. So, the cost function is not merely increasing in input prices; it increases proportionately.

3.3 Deriving the marginal cost function

Let's return to the cost function for the production function with a single variable input. The top panel of Figure 5 depicts the cost function at the initial (lower) level of environmental quality, E_0. The horizontal axis shows level of output (q), while the vertical axis shows the minimum cost of production (C^*). The cost function slopes upward because it is increasing in output, and its slope becomes steeper as output increases due to diminishing returns: $\beta < 1$ implies that $1/\beta$, the coefficient on q in the cost function, is greater than one, so cost increases exponentially as output rises.

Let us use the cost function to determine how an improvement in environmental quality affects the firm's profit. As in the case of the production function, we need to bring another function into the picture: the *marginal cost function*. This function gives the minimum cost of producing each incremental unit of q. Deriving it is easy, as we simply differentiate the cost function with respect to q. We did this already, when demonstrating that the cost function is increasing in output:

$$\frac{\partial C^{*}}{\partial q} = \frac{w}{\beta} \left(\frac{1}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}}$$

Marginal cost thus equals the slope of the cost function. This parallels the relationship between the inverse input demand function and the production function: as discussed earlier, the inverse input demand function is related to the marginal product of the input, which equals the slope of the production function.

The bottom panel of Figure 5 shows the marginal cost function. The function is upward-sloping, which reflects the fact that the slope of the cost function becomes progressively steeper as output increases. This can be demonstrated by differentiating the marginal cost function with respect to q.

$$\frac{\partial^2 C^*}{\partial q^2} = \frac{w}{\beta}\frac{1-\beta}{\beta}\left(\frac{1}{\alpha E^\gamma}\right)^{\frac{1}{\beta}} q^{\frac{1-2\beta}{\beta}} .$$

This expression is positive because $\beta < 1$ implies that $1 - \beta > 0$.

3.4 Change in profit, without and with output adjustment

In the case of the production function, we used the inverse input demand function to determine the profit-maximizing output level. Now, we use the marginal cost function to do this. We can use the implicit form of the cost function, $C^*(q, w, E)$, to rewrite the firm's profit,

$$\pi = pq - wx,$$

as

$$\pi = pq - C^*(q, w, E).$$

The profit-maximizing output level occurs where the first derivative of this expression with respect to q equals zero. The first derivative is

$$\frac{\partial \pi}{\partial q} = p - \frac{\partial C^*}{\partial q} .$$

If we equate the right-hand side to zero, then we obtain

$$p = \frac{\partial C^*}{\partial q} .$$

Profit-maximizing production occurs where output price equals marginal production cost. Written explicitly, this expression is

$$p = \frac{w}{\beta}\left(\frac{1}{aE^\gamma}\right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}} .$$

Figure 6 is the same as Figure 5 except it shows the profit-maximizing level of output, q_0, at the lower level of environmental quality. In the bottom panel, the firm should produce up to the point where output price equals marginal production cost (point a). Producing beyond this point would generate additional revenue, but the incremental amount would be less than the incremental production cost. In the top panel, this yields a total (minimized) cost of C_0^* (point A).

Figure 7 shows the impact on total cost if environmental quality improves from E_0 to E_1. The cost function shifts downward, because cost is decreasing in environmental quality. The cost of producing q_0 falls to $C_1 |_{q0}$ (point A' in the top panel). Profit thus rises, to

$$\pi_1 |_{q0} = pq_0 - C_1 |_{q0}$$

The change in profit if output is held constant at q_0, $\pi_1 |_{q0} - \pi^*_0$, is given by just the decrease in cost,

$$C^*_0 - C_1 |_{q0}$$

As in the case of the comparison of profit at points A and A' in Figure 2, this expression does not equal the full change in profit that results from the environmental improvement. It fails to account for the fact that the environmental improvement causes not only the cost function to shift but also the marginal cost function. As a result, it understates the increase in profit because it ignores the firm's output supply response. Figure 8 shows how the marginal cost function shifts in response to the environmental improvement (the bottom panel), and it also shows the resulting impact on output and total cost (the top panel). In the bottom panel, the marginal cost function shifts downward when E_0 is replaced by E_1: the environmental improvement causes marginal production cost to fall. Profit-maximization now occurs at q_1 (point b in the bottom panel), which is greater than q_0: output rises. Total cost is now C_1^* (point B in the top panel).

After allowing for the adjustment in output, profit is thus given by

$$\pi^*_1 = pq_1 - C^*_{1,}$$

and the change in profit,

$$\pi^*_1 - \pi^*_0 = p(q_1 - q_0) - (C^*_1 - C^*_0)_,$$

is not just a change in cost: it also includes a change in revenue. Given that $C = wx$, we can also write this as

$$\pi^*_1 - \pi^*_0 = p(q_1 - q_0) - w(x_1 - x_0)_.$$

This is exactly the same as the final expression for the change in profit in the case of the production function analysis. We have used two approaches to arrive at the same result.

3.5 Magnitude of the change in profit

As in Figure 4, the increase in profit can be depicted in two ways. Both are shown in Figure 9. Compared to Figure 8, the top panel of Figure 9 includes two additional tangents. The intercept of each tangent on the horizontal axis shows the profit associated with the corresponding production point, expressed in physical terms instead of money. From above, profit at point A (i.e., for E_0) is given by

$$\pi^*_0 = pq_0 - C_0 \ ,$$

which solved for q_0 yields

$$q_0 = \frac{\pi_0^*}{p} + \frac{1}{p}C_0^* .$$

The equation for the tangent at point A is thus

$$q = \frac{\pi_0^*}{p} + \frac{1}{p}C .$$

From the profit-maximizing condition, the inverse slope of the tangent, p, equals the slope of the cost function (= marginal production cost). We can derive the equation for the tangent to the lower cost function (i.e., the one with E_1) at point B by using the same logic:

$$q = \frac{\pi_1^*}{p} + \frac{1}{p}C\,.$$

The increase in profits is the difference between the intercepts on the horizontal axis,

$$\frac{\pi_1^*}{p} - \frac{\pi_0^*}{p}\,,$$

which is the same result as in the top panel of Figure 4. The figure does not show the line passing through point A', which would cross the cost function for E_1 instead of being tangent to it (because the profit-maximizing condition does not hold at A') and have an intercept on the horizontal axis between those of the two tangents (because profit at point A' is higher than at point A but lower than at point B).

In the bottom panel, the increase in profit is shown by the cross-hatched area between the two marginal cost functions. The cross-hatched area equals the change in *producer surplus*. This is easily demonstrated. Producer surplus is the difference between total revenue and total variable cost, which in the figure is the area below the output price line and above the marginal cost function. The expression for this area in the case of E_1 is

$$pq_1 - \int_0^{q_1} \frac{w}{\beta}\left(\frac{1}{\alpha E_1^{\,\gamma}}\right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}}\, dq\,,$$

which simplifies to

$$pq_1 - w\left(\frac{q_1}{\alpha E_1^{\,\gamma}}\right)^{\frac{1}{\beta}},$$

or simply $pq_1 - C_1^*$: profit at the higher level of environmental quality, π_1^*. Parallel analysis for the marginal cost function that includes E_0 yields producer surplus equal to profit at the lower level of environmental quality π_0^*. The change in producer surplus is thus exactly the same as the monetary change in profit, $\pi_1^* - \pi_0^*$.

4. Profit function

4.1 Definition

Like the cost function, the profit function is an economic relationship, not a technical relationship. It relates maximum attainable profit to output price (not output quantity, as in the cost function), the prices of variable inputs, and the quantities of fixed inputs, including environmental inputs. It is the solution to the problem,

$$\max_x \; pq - wx \text{ s.t. } q = \alpha x^{\beta} E^{\gamma}.$$

4.2 Deriving the output supply and profit functions

The unconditional profit-maximizing input demand function, derived in section 2.2, is

$$x^* = \left(\frac{p\alpha\beta E^{\gamma}}{w}\right)^{\frac{1}{1-\beta}}.$$

If we substitute this for x in the production function, then we obtain the profit-maximizing level of output:

$$q^* = \alpha\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{\beta}{1-\beta}} E^\gamma .$$

This expression is termed the *output supply function*. Recall that the profit-maximizing condition in the analysis of the cost function was that output price equals marginal production cost:

$$p = \frac{w}{\beta}\left(\frac{1}{\alpha E^\gamma}\right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}} .$$

If we solve this condition for q, then we obtain

$$q = \alpha\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{\beta}{1-\beta}} E^\gamma ,$$

which is just the output supply function. The output supply function and the marginal cost function are thus two versions of the same supply relationship. Indeed, marginal cost functions are often called "supply curves." Like the unconditional input demand function, the output supply function is homogeneous of degree 0 in prices, increasing in output price and environmental quality, and decreasing in input price.

We obtain the profit function by substituting the output supply function for q and the unconditional input demand function for x into the basic expression for profit, $\pi = pq - wx$:

$$\pi^* = p\left(\alpha\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{\beta}{1-\beta}} E^\gamma\right) - w\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{1}{1-\beta}} .$$

This is more complex than the cost function because it incorporates adjustments in both the input x and the output q, not just the former. Like the cost function, it is homogeneous of degree 1 in prices: profit increases by λ times if output price and input price both increase by λ times. Unlike the cost function, it is increasing in environmental quality: an improvement in environmental quality reduces cost but raises profit. This can be verified by differentiating the profit function with respect to E, $\partial\pi^*/\partial E$.

If we differentiate the profit function with respect to input price w and multiply the result by –1, then we obtain the unconditional input demand function,

$$-\frac{\partial\pi^*}{\partial w} = \left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{1}{1-\beta}} = x^* ,$$

while if we differentiate it with respect to output price p, then we obtain the output supply function,

$$\frac{\partial\pi^*}{\partial p} = \alpha\left(\frac{p\alpha\beta E^\gamma}{w}\right)^{\frac{\beta}{1-\beta}} E^\gamma = q^* .$$

These results are known as Shephard's lemma, which is the analogue to Hotelling's lemma for the cost function.

In the analysis of the cost function, we repeated the corresponding derivations for the case of two variable inputs. We will not do so here, as the derivations above show explicitly how the profit function results from the solution to an optimization problem.

4.3 Change in profit

We plotted the production and cost functions against the physical variables *x* and *q*, respectively, and we needed to add a bottom panel to the figures to account for changes in these variables in response to the improvement in environmental quality. In contrast, we can plot the profit function against environmental quality *E* and directly read off of it the impact of the environmental improvement on profit.

Figure 10 illustrates this. For given prices, the environmental improvement results in movement along the profit function, not a shift in the function. The function slopes upward (profit is increasing in environmental quality), but the slope diminishes. The latter reflects the diminishing returns to environmental quality in production ($\gamma < 1$). The proof of this, which requires checking that the sign of the second derivative $\partial^2\pi^*/\partial E^2$ is negative, is left as an exercise to the reader. The improvement in environmental quality from E_0 to E_1 on the horizontal axis results in an increase in profit from π_0^* to π_1^* on the vertical axis. If we know the profit function, then we can value the environmental improvement in one step, unlike the two steps that are required if we use either the production function or the cost function.

5. Empirical implications

Although the preceding analysis has been purely theoretical, it contains a number of important lessons for empirical analysis. They can be summarized as follows.

5.1 Three types of individual functions—input demand, marginal cost (or output supply), and profit—can be used to estimate the change in profit resulting from an environmental change

As emphasized throughout the preceding sections, change in profit is the proper measure of the impact of an environmental change on a firm. Change in profit can be calculated by estimating and manipulating any of three individual functions:

(i) If the input demand function is estimated, then it can be used to calculate the change in consumer surplus between one level of environmental quality and another, and that change equals the change in profit.

(ii) If the marginal cost function is estimated, then it can be used to calculate the change in producer surplus between one level of environmental quality and another, and that change equals the change in profit. The same holds for the output supply function, which as we've seen is closely related to the marginal cost function.

(iii) If the profit function is estimated, then it can be used directly to calculate the change in profit between one level of environmental quality and another.

Our analysis assumed a single input and a single output. If there are multiple inputs or outputs, then sets of input demand or marginal cost (or output supply) functions must be used instead of individual ones. This point is elaborated in section 6.

5.2 Use of full information requires estimating a system of equations, not just a single one

Although the change in profit can be calculated using individual functions, each of the three approaches presented in sections 2-4—production function, cost function, profit function—involves a system of interrelated functions: a production function plus an input demand function, a cost function plus marginal cost and conditional input demand functions, and a profit function plus input demand and output supply functions. If one wishes to use full information related to any of these approaches, then one must estimate a system of equations instead of an individual equation. The estimation of a system of equations is demonstrated in section 7.

Compared to estimating an individual equation (i.e., an input demand, marginal cost or output supply, or profit function), estimating a system of equations is more data-intensive, but it can yield statistically more efficient results. The gain in statistical efficiency is usually smaller, however, if the number of observations is smaller or if variables that can be excluded when an individual equation is estimated contain relatively more measurement error. Estimating a system of equations is thus not always more desirable. If data are incomplete, then it might not even be possible. In that case, one must rely on the estimation of individual equations (Huang and Smith 1998).

5.3 Endogeneity can be a source of bias in estimating all three functions, but especially the production function

We wrote the Cobb-Douglas production function in section 2 as a deterministic relationship:

$$q = \alpha x^{\beta} E^{\gamma} .$$

In practice, this function is not known to the econometrician, who must instead estimate it. The standard estimation procedure for a Cobb-Douglas function is to gather data across a set of firms, take the natural logarithm of each side of the function, add a stochastic error term to it (to account for unobserved factors that affect output and for measurement error in the output data), and then use regression methods to estimate the resulting log-log equation,

$$1\text{n}\, q_i = b_0 + b_1\, 1\text{n}\, x_i + b_2\, 1\text{n}\, E_i + \varepsilon_i .$$

i denotes firm, and ε is the error term. The regression coefficients b_0, b_1, and b_2 provide estimates of ln α, β (not ln β), and γ (not ln γ), respectively.

If one uses ordinary least squares (OLS) to estimate this equation, then one likely obtains biased estimates of the regression coefficients. This is because the variable input, x, is an endogenous variable. Unlike E, it is chosen by firms. As a result, it is likely to be correlated with the error term ε. This is easiest to see by considering the conditional input demand function,

$$x = \left(\frac{q}{\alpha E^{\gamma}} \right)^{\frac{1}{\beta}} .$$

Note that this function includes output, q. If some unobserved factor generates a shock ε that affects q through the production function, then x will be affected too through the conditional input demand function. The variable input in the production function is thus correlated with the error term in the production function. This correlation has long been known to lead to biased estimates of the coefficients in a production function (Hoch 1958).

To reduce this bias, one must use an estimator other than OLS, such as two-stage least squares. But successful application of two-stage least squares requires one or more *instrumental variables* that are valid and strong: variables that are highly correlated with the endogenous explanatory variable but are not correlated with the error term and are not included in the original equation (the production function in this case). Obtaining such variables can be difficult, and using instruments that are invalid or weak can create statistical problems that are worse than the endogeneity problem that one is trying to use them to solve (Murray 2006).

Endogeneity affects the cost and marginal cost functions, too. Recall that these functions are given by

$$C^* = w\left(\frac{q}{\alpha E^{\gamma}}\right)^{\frac{1}{\beta}}.$$

$$\frac{\partial C^*}{\partial q} = \frac{w}{\beta}\left(\frac{1}{\alpha E^{\gamma}}\right)^{\frac{1}{\beta}} q^{\frac{1-\beta}{\beta}}.$$

Both include q as an explanatory variable, which is endogenous because the firm influences it through the choice of x. In the case of agriculture, the argument is sometimes made that output is only weakly endogenous with variable inputs, because the latter are applied toward the start of the growing season. The gap in time between the start of the season and harvest reduces the feedback from output shocks to input demand, it is argued. This argument should always be supported by additional evidence that the shocks do not in fact occur at points in the growing season when farmers respond to them through input adjustments, or, if they occur later in the season, that farmers do not respond to forecasts related to them.

If this argument does not hold, then one must again use instrumental variables in estimating these functions. A variable that is not included in the cost or marginal cost functions is output price, p. This is a promising instrument, as it is likely to be exogenous (more on this in a moment). But if one has data on output price, then one has the option of avoiding the cost-function approach altogether and using instead the profit-function approach, which is less prone to endogeneity bias. The profit function and the two functions associated with it, the input demand and output supply functions, do not include any choice variables on their right-hand sides:

$$\pi^* = p\left(\alpha\left(\frac{p\alpha\beta E^{\gamma}}{w}\right)^{\frac{\beta}{1-\beta}} E^{\gamma}\right) - w\left(\frac{p\alpha\beta E^{\gamma}}{w}\right)^{\frac{1}{1-\beta}}.$$

$$x^* = \left(\frac{p\alpha\beta E^{\gamma}}{w}\right)^{\frac{1}{1-\beta}}.$$

$$q^* = \alpha\left(\frac{p\alpha\beta E^{\gamma}}{w}\right)^{\frac{\beta}{1-\beta}} E^{\gamma}.$$

The only explanatory variables are prices (p, w) and environmental quality (E). Assuming that E is determined by the actions of economic agents other than the affected firms (e.g., deforestation by households in upland areas affects baseflow received by farmers downstream), then it is clearly exogenous. If microdata are used to estimate these functions, then input and output prices are also likely to be exogenous. An exception is when one or more firms have market power, which is discussed in section 6. If aggregate industry-level data are used to estimate the profit function, then prices are unlikely to be exogenous, and one must again use instrumental variables to correct for the resulting bias. Obtaining valid instruments generally becomes more difficult as data become more aggregated.

Other problems can also occur even when microdata are used. For example, if the firms are located in the same region, then prices might not vary much across them, and this can preclude the estimation of coefficients on the price variables. If the purpose of the analysis is to measure the impact of a change in environmental quality, however, then this is not necessarily a problem. Mundlak (1996) also notes that firms often make decisions on the basis of expected prices, not the market prices observed by econometricians. He demonstrates that there can be a substantial loss of statistical efficiency if one uses market prices as proxies for expected prices when estimating a profit function, and he argues that this statistical inefficiency can be a more serious problem than the endogeneity bias associated with estimating a production function. The most serious problem is when one or more markets are missing and thus complete price data do not exist. This problem is also discussed in section 6.

5.4 Change in revenue is a biased measure of change in profit

If one estimates a production function, then one can use it to predict output with and without an environmental change. One can then predict the change in revenue by multiplying output price by the change in output (= output with the change – output without the change). There are two such predicted changes in revenue, one partial and one complete, depending on whether or not one also estimates the input demand function.

If one does not estimate the input demand function, then one predicts the change in revenue using only the production function. This corresponds to $p(q_1 | x_0 - q_0)$ in Figure 2. This is a partial change in revenue because it fails to account for adjustments to input use, which affect output. When environmental quality improves, this partial change in revenue understates the increase in profits, as discussed in section 2.3. When environmental quality deteriorates, the opposite is true: the partial measure of the loss in revenue overstates the loss in profits. This is illustrated in Figure 11, which looks just like Figure 3 except that the subscripts 0 and 1 have been reversed to indicate that the change is from better environmental quality to worse. The reduction in revenue associated with the drop from point A to point A' overstates the decrease in profits because it ignores the reduction in costs as input use falls from x_0 to x_1. Using a production function to estimate the negative impact of environmental degradation is commonly called the *damage function approach*. The fact that this approach tends to exaggerate damage estimates is unfortunately often overlooked.

If one also estimates the input demand function, then one can account for the input adjustment and thus predict the complete change in revenue. This corresponds to $p(q_1 - q_0)$ in Figures 3 and 11. The complete change in revenue overstates the increase in profits when environmental quality improves (Figure 3), because it fails to account for the cost of increased input use,

$w(x_1 - x_0)$. It also overstates the decrease in profits when environmental quality deteriorates (Figure 11), because it similarly fails to account for cost savings as the firm reduces input use. In the latter case (environmental deterioration), the complete change in revenue is more biased than the partial change. This can be seen easily in Figure 11, where the complete change in revenue is associated with the drop from point A to point B, which exceeds the partial reduction associated with the drop from point A to point A'.

Of course, if one estimates not only the production function but also the input demand function, then there is no reason to predict a change in revenue: one can instead predict the change in profit, which is (or should be) the objective of the analysis.[3] Use of the change in revenue as a proxy for the production impact of an environmental change is thus pertinent only when one estimates only the production function and thus predicts the partial change in revenue. The fact that the partial change in revenue is a biased measure of the change in profit does not mean it has no value for economic analysis. For example, suppose that the purpose of the analysis is to determine whether a prospective program to improve environmental quality is economically justified. If the predicted partial change in revenue exceeds the cost of the program, then one can be confident that the program is justified, because a conservative (downwardly biased) measure of the benefits has been used. By the same token, if the predicted partial change in revenue does not exceed the cost of the program, then one cannot say whether or not the program is justified: perhaps the predicted benefits would have exceed the program cost if the conceptually correct benefit measure, the change in profit, had been used instead of the partial change in revenue. The partial change in revenue can thus be used to construct one-sided benefit-cost tests.

5.5 Change in cost is a biased measure of change in profit

Analogous points can be made about the bias associated with using the change in cost as a proxy for the change in profit. When environmental quality improves but output is held at the initial level q_0, the resulting reduction in cost, $C^*_0 - C_1 \mid q_0$, understates the positive impact of the environmental improvement on the firm. This point was made in section 3.4. It is the mirror image of the downward bias that occurs when the variable input is held at the initial level x_0 and the change in revenue, from the production function, is used to measure the change in profit. When environmental quality deterioriates but output is held at the initial level, the bias is in the opposite direction: the increase in cost overstates the damage to the firm. This illustrated in Figure 12, which looks like Figure 8 except that the subscripts 0 and 1 have been reversed. The cross-hatched area in the bottom panel indicates the amount by which the increase in cost overstates the loss of producer surplus, which has the same shape as in Figure 9.

Figure 12 can also be used to illustrate the bias associated with using the *replacement-cost method* to value environmental damage. If a firm attempted to restore output to the initial level q_0, then it would incur costs equal to the area given by the approximately trapezoidal area $q_1ba'q_0$ in the bottom panel. This area exceeds the loss of producer surplus, and so the replacement cost overstates the loss of profit. The problem with the replacement-cost method is clear: only an irrational firm would attempt to restore output to the initial level q_0 after environmental quality has deteriorated, because the marginal cost of producing beyond the new profit-maximizing output level q_1 exceeds the marginal benefit, which is given by the output price p. Simply put, the replacement cost does not generate benefits of equivalent value.

[3] Auffhammer et al. (2006) did not do this because they lacked reliable data on some inputs and their prices.

The results in this section and section 5.4 illustrate the importance of accounting for adjustments firms make in response to environmental changes. The failure to account fully for these adjustments is the reason why partial or approximate measures of economic impacts, such as revenue-based damage costs or cost-based replacement costs, provide biased measures of welfare impacts.

6. Implications of relaxing key assumptions

6.1 Multiple firms

Our analysis assumed a single, price-taking firm. If the environmental change affects multiple firms but does not affect input or output prices, then the sum of changes in profits across firms, where the changes are calculated at fixed prices, is a valid measure of the welfare impact on the set of firms. If the environmental change affects a large share of the firms in an industry, however, then it probably affects prices too. For example, an environmental improvement that affects an entire industry would be expected to reduce output price, due to the increased supply of the output (assuming a downward-sloping demand function for the output), and to raise input price, due to the increased demand for the input (assuming an upward-sloping supply function for the input). Deterioration in environmental quality would be expected to have the opposite effects. The welfare impact on the set of firms is still given by the sum of profit changes across the firms, but now the latter must account for the price changes. Moreover, the price changes create additional welfare impacts on consumers of the output and suppliers of the input, which must be taken into account if the objective is to measure the overall social welfare impact (see Freeman 2003, pp. 276-279).

Just et al. (1982, Chs. 8-9)[4] deal with these sorts of aggregation issues. A sufficiently large environmental change—for example, global warming—could have economy-wide effects, in which case the impacts would need to be measured using a computable general equilibrium (CGE) model. General-equilibrium impacts of environmental changes, or policies to address them, can differ substantially from partial-equilibrium impacts (Hazilla and Kopp 1990). Bergman (2003) reviews the application of CGE models to environmental issues.

6.2 Noncompetitive markets

Issues similar to the ones just discussed occur if the firm is large and faces either the demand function for the output it produces or the supply function for the input it consumes. The firm then has market power and can earn above-normal profits by acting as a monopolist and forcing output prices up or a monopsonist and forcing input prices down. One must again account for such price changes when measuring the impact of the environmental change on the firm's profits (Just et al. 1982, Ch. 10; Freeman 2003, pp. 279-281). One should also be aware that a welfare gain (or loss) for the firm now does not necessarily equal the corresponding welfare gain (or loss) for society, given the distortions created by the firm's manipulation of market prices.

6.3 Market distortions

Market power is one source of distortions that can cause market prices to deviate from marginal benefits and costs measured in social terms. Such distortions can also result from taxes, subsidies,

[4] A new edition of this book was published recently (Just et al. 2004).

regulations, and environmental externalities other than the ones that are the focus of a particular analysis (Freeman 2003, pp. 281-283). When analyzing the impacts of an environmental change on producers, one must therefore be clear about whether the objective is to measure those impacts in private terms or social terms. If the objective is the former—that is, if the objective is to measure impacts at market prices—then one can ignore the distortions. The objective of economic analysis is usually to measure impacts in social terms, however, and in that case one must use shadow prices to adjust for the distortions. Belli et al. (2001) contains especially lucid explanations of shadow-pricing techniques for various distortions.

6.4 Missing markets and household production

Production in developing countries is often by households, as in the case of smallholder farms. If the household faces complete markets for inputs and outputs—that is, if it can buy as much of an input or sell as much of an output as it desires at the prevailing market prices—then, leaving aside the issue of risk preferences (discussed in the next point), the change in profit in the productive activity is the correct measure of the welfare impact of an environmental change that affects the household through that activity. The existence of complete markets makes production decisions *separable* from other household decisions, in particular its consumption decisions (including the labor-leisure tradeoff). One can then use a profit function for the productive activity to measure the welfare impact of the environmental change. This is obviously a very convenient situation for economic analysis: one does not need to worry about the characteristics of the household's utility function, which is inherently more difficult to measure than its productive activities and their profitability.

Unfortunately, markets are often missing for households in developing countries, especially in rural areas (de Janvry et al. 1991). For example, households might face restrictions on the amount of labor they can buy or sell. When markets are missing, the household's production decisions are no longer separable from its consumption decisions, and the monetary change in profit from its productive activities no longer provides a valid welfare measure. One must instead calculate the change in profit by using shadow or *virtual prices*, which account for nonmarket utility effects. Unlike market prices, virtual prices are endogenous to the household—they are not determined solely by external factors—and they are unobserved. It is possible to test for the completeness of markets and thereby determine whether adjustments using virtual prices are necessary. For an example related to the valuation of an environmental change, see Pattanayak and Kramer (2001).

6.5 Risk

Our analysis assumed that prices are known with perfect certainty and that the firm's owner is risk neutral. If prices are not known with perfect certainty when production decisions are made (e.g., as in the case of agriculture) but the risk neutrality assumption still holds, then impacts on the firm can be measured in terms of the *expected change in profit*. For example, a set of alternative price scenarios could be prepared, probabilities could be attached to each, the change in profit could be calculated for each, and then the expected change in profit could be calculated by multiplying the change in profit for each scenario by the corresponding probability and summing across the scenarios. Analogous procedures can be used if the magnitude of the environmental change is not known with perfect certainty.

The situation is more complex if the firm's owner is not risk neutral. Then, the owner's risk preferences must be taken into account. The expected change in profit no longer provides a valid measure of the impact of the environmental change on the owner's utility. In effect, the expected change in profit must be adjusted for risk premia (Just et al. 1982, Ch. 11).

6.6 Fixed inputs

Our analysis ignored fixed inputs. It included just a single input, which was a variable one. Fixed inputs are in fact a critical feature of the analysis of producer impacts of environmental changes. If there are no fixed inputs, then under standard assumptions, such as constant returns to scale and free entry and exit of firms, firms should not earn a profit in the sense of a payment over and above the costs of the inputs (including managerial effort) they employ. The existence of such profits should immediately attract new firms into the industry, which would result in the profits being competed away (driven to zero). Total revenue minus total costs, where the latter includes only variable costs, should equal zero at all times.

If production requires a fixed input that is owned by the firm, and if the input varies in quality across firms, then persistent differences in economic surpluses can exist across firms. A good example is agricultural land. Land of higher quality is more productive, which increases the economic surplus of the farm that owns it. The higher surplus simply reflects the greater return generated by the land: although total revenue minus total variable costs is positive, total revenue minus total costs, where the latter includes an implicit payment for the land, would again equal zero. There is a non-zero *quasi-rent* (total revenue minus total variable costs; producer surplus) but a zero economic profit (total revenue minus total costs). Indeed, if the farmer were a tenant who literally rented the land, then he would pay a rent equal to the quasi-rent and would consequently earn zero profits. The landowner would be the one who benefited economically from the land's higher quality. Because fixed costs are fixed, a change in quasi-rent equals a change in profit. The ownership of fixed inputs thus affects the distribution of the production impacts of environmental changes: whether the impacts appear as changes in the firm's profits, which occurs if the firm owns the fixed input, or the income of the owner of the fixed input, which occurs if the owner is different from the firm.

If there is no variation in the quality of the fixed input, then there should not be persistent differences in economic surpluses across firms as long as there is free entry and exit: that is, there should be zero profits in the long run. In the short run, however, environmental changes can affect quasi-rents. If environmental quality improves, then the existing firms in an industry earn above-normal returns during the transition period when new firms, attracted by the above-normal returns, are making the investments necessary to enter into production. These above-normal returns vanish once the new firms begin producing. Conversely, if environmental quality deteriorates, then the existing firms earn below-normal returns during the transitional period when they depreciate their fixed inputs and scale back production. The key point here is that the change in an affected firm's profits reflects a change in quasi-rents and is temporary, converging to zero as the level of fixed inputs across firms adjusts to a new competitive level at the new level of environmental quality.

The issues of imperfect knowledge about future prices or the future magnitude of an environmental change, discussed in the previous point, also affect a firm's investment decisions. These effects can be complex, especially when the environmental change is irreversible. See Mäler and Fisher (2005) for more details.

6.7 Multiple outputs

Introductory expositions of producer theory usually assume that a firm makes a single output. In fact, firms often make more than one output. A farm that grows several crops is a good example. The theory of production by multi-output firms is well-developed (Chambers 1988, Ch. 7), as is the theory of welfare measurement for such firms (Just et al. 1982, Appendix A). The most natural way to measure the welfare impacts of an environmental impact on a multi-output firm is to use a multi-output profit function. This is the approach used by Pattanayak and Kramer (2001) in their study of the impacts of changes in baseflow on Indonesian farms that produce rice and coffee. It is also possible to add up changes in producer surpluses across the set of outputs, or consumer surpluses across the set of inputs, that are affected by the environmental change. Certain technical requirements must be satisfied to do this, however. One must also take care to ensure that these changes are added correctly, as they are interrelated. For more details, see Huang and Smith (1998), Freeman (2003, pp. 267-276), and McConnell and Bockstael (2005, section 3.3).

6.8 Multiple inputs

Our assumption of a single non-environmental input was extreme and was done to simplify the graphical exposition of the three approaches (production, cost, and profit functions). Assuming that the firm makes a single output, there is little difference between the single-input and multi-input cases when using a profit or marginal cost function to measure the impacts of an environmental change. One simply must make sure that all the relevant input prices are included in the profit or marginal cost function and the other functions that are associated with it (output supply and input demand, or cost and conditional input demand), if those functions are also estimated. Additional complications arise when changes in consumer surpluses for inputs are used to measure the impacts. As in the case of multiple outputs, one must check some technical conditions and add up carefully (Huang and Smith 1998; McConnell and Bockstael 2005, section 3.4). The technical conditions are analogous to the conditions for *weak complementarity* identified originally by Mäler (1974) for using inputs, such as travel expenditures, to measure the benefits of environmental improvements to consumers, such as the availability of outdoor recreation sites. An input is weakly complementary to environmental quality if two conditions hold: (i) demand for the input increases when environmental quality improves, and (ii) a change in environmental quality has no impact on the affected party (the consumer or the firm) if demand for the input equals zero, which occurs when the price of the input exceeds the choke price.

6.9 Nonconvexities

Our analysis assumed that environmental quality enters production in a “well-behaved” manner. For example, we assumed that the production function is continuously differentiable (the derivatives $\partial q/\partial E$ and $\partial^2 q/\partial E^2$ exist) and concave ($\partial q/\partial E > 0$, $\partial^2 q/\partial E^2 < 0$) with respect to environmental quality. These assumptions were convenient ones, but they do not necessarily hold in reality. The production set could instead be nonconvex. A simple example is a threshold effect, such as catastrophic crop loss if the amount of rainfall is below a minimum level. Although a production, cost, or profit function that ignores such a threshold could provide accurate predictions as long as environmental quality stays within the well-behaved production region, it would likely provide very misleading ones if the threshold were crossed. Moreover, decisions that make a threshold more likely to be crossed have a cost, a loss of resilience, that is not reflected in normal accounting procedures (Mäler et al. 2007) and thus not in data on profits and costs. The economic analysis of nonconvex production systems is an active area of research. For a good introduction, see Dasgupta and Mäler (2004).

7. Using Stata to estimate a production function, a profit function, and a profit-function system

7.1 Overview and policy context

This example is a simplified version of the analysis of watershed values in Indonesia by Pattanayak and Kramer (2001). Because it has been simplified, the results generated by the example should not be taken as true values. Like the original analysis, this example involves the estimation of an agricultural profit function by using data from a 1996 survey of farm households in the Manggarai District on the island of Flores. One of the inputs in this function is an environmental service: baseflow. Baseflow refers to the seepage of groundwater into a region's waterways. It provides an indicator of the amount of soil moisture that is available for crops.

The policy context for Pattanayak and Kramer's study was a proposed reforestation program in a national park, Ruteng, which lies upstream of the district. Like many parks in developing countries, Ruteng had suffered from encroachment at the time of the study, and much of it had been cleared of forest. In response, the Government of Indonesia proposed a reforestation program to reestablish tree cover in the denuded area. Forest cover can affect baseflow in both positive and negative ways. To value the net change in baseflow, one needs to know how the change affects agricultural profits.

Pattanayak and Kramer used data from 487 households in estimating their profit function. This example uses data from just 92 households. The reason for the difference is that the profit function in this example includes just one crop, rice, whereas Pattanayak and Kramer's included two, rice and coffee. Although some farmers in the district specialized in a single crop in 1996, most grew both. The sample for this example includes households that grew predominantly rice, which are defined here as farms that earned at least 75 percent of their gross revenue from rice. Fifty-nine of these 92 households grew only rice.

Aside from excluding terms related to coffee, the profit function in this example is very similar to Pattanayak and Kramer's. Although the main objective of the example is to illustrate the estimation of a profit-function system—that is, the profit function along with an output supply function (for rice) and an input-demand function (for farm labor)—for the purpose of comparison the example also involves the estimation of the profit function on its own and a Cobb-Douglas production function.

Estimation is done using the econometrics program Stata, which is currently one of the most popular programs used by economists and other social scientists. The example is written in a way that assumes the user has installed Stata and is familiar with its basic commands. For information on ordering Stata, visit www.stata.com. An excellent online tutorial for using Stata can be found at http://www.ats.ucla.edu/STAT/stata/webbooks/reg/default.htm. Additional information on more specialized topics can be found at http://www.ats.ucla.edu/STAT/stata/.

7.2 Description of the data

Data for the example are in the Stata dataset, "Rice and baseflow.dta." The dataset contains the following 14 variables:

kues	Household ID number
desa	Village ID number
kecano	County ID number
profit	Annual farm profit
ppadi	Rice price per kilogram
plabor	Wage rate per day
padi	Padi output, in kilograms
labor	Labor inputs (household and hired combined), in days
farmsz	Farm size, in hectares
bftot	Annual baseflow in the village, in meters
irrih	Fraction of farm that is irrigated
slope	Average slope of the farm
hujan	Annual rainfall in the village, in meters

Some of the variables vary by households, while others vary by village. The dataset is complete, so we do not need to worry about missing values. The value of *labor* on one farm is listed as 0, which clearly cannot be correct, but we will leave this value as it is.

One unusual feature is that farm profit (*profit*) and prices (*ppadi*, *plabor*) are not expressed in monetary terms. Instead, they are expressed in kilograms of fertilizer. Pattanayak and Kramer transformed the original variables by dividing by the price of fertilizer. This is one way of putting the data in "real" terms (i.e., relative prices). Despite this, we will refer to these three variables as "monetary" variables.

7.3 Estimating the production function

The Cobb-Douglas function is most common specification for an agricultural production function. It is a multiplicative function:

$$y = \beta_0 x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3} x_4^{\beta_4} x_5^{\beta_5} x_6^{\beta_6}$$

y is harvest, and x_1, x_2, etc. are inputs. There are six inputs:

x_1	*labor*
x_2	*farmsz*
x_3	*bftot*
x_4	*irrih*
x_5	*slope*
x_6	*hujan*

Only the first one, *labor*, can be varied by farmers.

We can determine the marginal impact of baseflow on harvest by partially differentiating the production function with respect to x_3. After a bit of reorganization, the partial derivative yields

$$\frac{\partial y}{\partial x_3} = \beta_3 y / x_3 \ .$$

This is the marginal impact, or marginal product, in physical terms, such as kilograms of rice per meter of baseflow. To obtain the marginal impact in monetary terms, we need to multiply by the price of the crop, *p*:

$$p\frac{\partial y}{\partial x_3} = p\beta_3 y / x_3 .$$

This is just the standard marginal value product of an input. Once we have estimated the Cobb-Douglas production function, we can therefore easily calculate the marginal impact of baseflow.

To convert a Cobb-Douglas function to a form that can be estimated by using ordinary-least squares regression, we take the natural logarithm of each side:

$$1\text{n } (y) = \alpha + \beta_1 1\text{n } (x_1) + \beta_2 1\text{n } (x_2) + \beta_3 1\text{n } (x_3) + \beta_4 1\text{n } (x_4) + \beta_5 1\text{n } (x_5) + \beta_6 1\text{n } (x_6)$$

where $\alpha = \ln(\beta_0)$. The coefficient on $\ln(x_3)$, β_3, is the coefficient that we need for our marginal impact formula, along with data on harvest (y), baseflow (x_3), and crop price (p). The following Stata commands generate the logarithmic variables:

```
generate lpadi      =ln( padi )
generate llabor     =ln( labor)
generate lfarmsz    = ln( farmsz )
generate lbftot     =ln( bftot )
generate lhujan     =ln( hujan )
```

We do not convert *irrih* (x_4) and *slope* (x_5) to logarithms, because they can values of zero, whose logarithm does not exist. The actual production function estimated was thus

$$1\text{n } (y) = \alpha + \beta_1 1\text{n } (x_1) + \beta_2 1\text{n } (x_2) + \beta_3 1\text{n } (x_3) + \beta_4 x_4 + \beta_5 x_5 + \beta_6 1\text{n } (x_6).$$

To estimate the production function, we issue the following command:

```
regress  lpadi llabor lfarmsz lbftot lhujan irrih slope
```

We get the following results:

```
. regress  lpadi llabor lfarmsz lbftot lhujan irrih slope
```

Source	SS	df	MS
Model	41.5667717	6	6.92779528
Residual	60.0948891	84	.715415347
Total	101.661661	90	1.12957401

Number of obs	=	91
F(6, 84)	=	9.68
Prob > F	=	0.0000
R-squared	=	0.4089
Adj R-squared	=	0.3667
Root MSE	=	.84582

lpadi	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
llabor	.4105994	.1082056	3.79	0.000	.1954207	.625778
lfarmsz	.3862747	.0943555	4.09	0.000	.1986385	.5739109
lbftot	1.822436	.8337055	2.19	0.032	.1645211	3.480351
lhujan	2.145706	.9129661	2.35	0.021	.3301724	3.961239
irrih	1.074014	.3468321	3.10	0.003	.3843005	1.763728
slope	-.2249773	.088351	-2.55	0.013	-.4006729	-.0492816
_cons	3.12828	1.065881	2.93	0.004	1.008659	5.2479

The regression equation fits the data reasonably well considering that the data are cross-sectional ($R^2 = 0.41$). The coefficient on the logarithm of baseflow (*lbftot*) is positive and significantly different from zero at a 5-percent level: the *P*-value for this coefficient, 0.032, is less than 0.05, and its 95% confidence interval (0.165, 3.48) does not straddle zero.

The following command generates a variable, *marginal_impact_production*, that equals the marginal value product of baseflow for each household:

```
generate marginal_impact_production=ppadi*_b[lbftot]*padi/bftot
```

Note that this command refers to the value of the coefficient on *lbftot* that is stored in memory,

```
_b[lbftot]
```

instead of directly including the numerical value of the coefficient (i.e., 1.822436…). This leads to a more precise estimate of *marginal_impact_production* and enables us to reuse the command if we make certain changes to the regression, such as dropping some observations. We can obtain summary statistics for the marginal value product by issuing the command,

```
su  marginal_impact_production
```

which returns the following information:

```
su  marginal_impact_production
```

Variable	Obs	Mean	Std. Dev.	Min	Max
marginal_i ~ n	92	2773.764	4885. 759	111,6826	29844.87

The marginal value product of baseflow has a mean of 2,774 and ranges from 112 to 29,845.

7.4 Estimating the profit function

We derived the profit function for the Cobb-Douglas production function in section 4. Economists typically do not posit a production function and then derive the profit function for it. They typically assume a *flexible functional form* for the profit function and analyze it instead of the specification that is unique to a particular production function. A flexible functional form is one that provides a good approximation to the actual function, regardless of the shape of the actual function, which is not directly observed by the econometrician. Flexible functional forms include interaction terms (variables multiplied by each other) and higher-order terms (variables raised to powers). Due to these characteristics, they have nonzero first and second derivatives.

We assume that the profit function has the following specification:

<u>Profit function</u>

$$\pi = \beta_p p + \beta_w w + \beta_{pw} 2(pw)^{0.5}$$
$$+ \beta_{p2}(px_2) + \beta_{p3}(px_3) + \beta_{p4}(px_4) + \beta_{p5}(px_5) + \beta_{p6}(px_6)$$
$$+ \beta_{w2}(wx_2) + \beta_{w3}(wx_3) + \beta_{w4}(wx_4) + \beta_{w5}(wx_5) + \beta_{w6}(wx_6).$$

π is profit (*profit*), and *p* is the price of the crop (*ppadi*). This specification assumes that there is only one priced variable input (*labor*) and that this input is input 1. *w* is the price of this input (*plabor*). Although the profit function includes the price of input 1, it does not include the quantity (x_1). As discussed in section 4, profit functions include output and input prices and the quantities of fixed inputs, but not the quantities of variable inputs.

The profit function includes 13 explanatory variables. The explanatory variables are in three groups:

(i) *p*, *w*, and twice the square root of their product (the first line)

(ii) the product of *p* with each of the 5 fixed or unpriced inputs (x_2, x_3, x_4, x_5, x_6; the second line)

(iii) the product of *w* with each of the 5 fixed or unpriced inputs.

For example, px_2 means *p* multiplied by x_2. *p* is *ppadi*, so if x_2 is *farmsz*, then px_2 = *ppadi ´ farmsz*. To estimate the profit function, we first need to construct these variables. The following Stata commands do this. This command generates the interaction term in group (i):

```
generate two_ppadixplabor_sqrt=2*(ppadi*plabor)^0.5
```

These commands generate the 5 variables in group (ii):

```
generate ppadixfarmsz=ppadi*farmsz
generate ppadixbftot=ppadi*bftot
generate ppadixirrih=ppadi* irrih
generate ppadixslope=ppadi* slope
generate ppadixhujan=ppadi* hujan
```

Finally, these commands generate the 5 variables in group (iii):

```
generate plaborxfarmsz=plabor*farmsz
generate plaborxbftot=plabor*bftot
generate plaborxirrih=plabor*irrih
generate plaborxslope=plabor*slope
generate plaborxhujan=plabor*hujan
```

With the variables created, we can estimate the profit function by issuing the following command:

```
regress profit ppadi plabor two_ppadixplabor_sqrt ppadixfarmsz ppadixbftot ppadixirrih
ppadixslope ppadixhujan plaborxfarmsz plaborxbftot plaborxirrih plaborxslope plaborxhujan
```

We obtain the following results:

```
. reg profit ppadi plabor two_ppadixplabor_sqrt ppadixfarmsz ppadixbftot ppadixirrih
ppadixslope ppadixhujan plaborxfarmsz plaborxbftot plaborxirrih plaborxslope plaborxhujan
```

Source	SS	df	MS
Model	328934107	13	25302623.6
Residual	278454364	78	3569927
Total	607388470	91	6674598.57

Number of obs	=	92
F(13, 78)	=	7.09
Prob > F	=	0.0000
R-squared	=	0.5416
Adj R-squared	=	0.4651
Root MSE	=	1889.4

profit	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
papadi	-9574.432	10439.65	-0.92	0.362	-30358.17	11209.31
plabor	4023.331	2362.436	1.70	0.093	-679.9186	8726.581
two_ppadix~t	-3951.722	3233.716	-1.22	0.225	-10389.56	2486.113
ppadixfarmsz	374.5587	513.9781	0.73	0.468	-648.6932	1397.811
ppadixixbftot	2218.165	5944.482	0.37	0.710	-9616.39	14052.72
ppadixirrih	3701.61	1607.209	2.30	0.024	501.903	6901.317
ppadixslope	581.5932	583.7256	1.00	0.322	-580.5152	1743.702
ppadixhujan	5903.578	2004.021	2.95	0.004	1913.879	9893.276
plaborxfar~z	-3.569244	92.82444	-0.04	0.969	-188.3685	181.23
plaborxbftot	200.0229	1092.98	0.18	0.855	-1975.933	2375.979
plaborxirrih	445.0603	332.5379	-1.34	0.185	-1107.092	216.9717
plaborxslope	-149.0845	107.256	-1.39	0.168	-362.6147	64.44572
plaborxhujan	-820.3523	392.6401	-2.09	0.040	-1602.039	-38.66591
-cons	-308.3875	699.5395	-0.44	0.661	-1701.064	1084.289

The profit function includes two variables involving baseflow, the interaction with price of rice (*ppadixbftot*) and the interaction with price of labor (*plaborxbftot*). Neither coefficient is significantly different from zero. From this, one might conclude that baseflow does not affect profit, but this would be a surprising conclusion in view of our earlier result that baseflow has a significant impact on production. It would also be an incorrect conclusion. The lack of significance of the baseflow variables in the profit function in fact results from our failure to use all the information that we have on rice production by the households. Specifically, we have not used the information on the quantity of output produced or the quantity of labor inputs used. To do this, we need to estimate not only the profit function but also the output supply and input demand functions.

7.5 Estimating the profit function system

The first step in estimating the profit function system is to determine the equations of the output supply and input demand functions. From section 4, we know that we can derive these functions by differentiating the profit function with respect to output price (*p*), which yields the output-supply function, and differentiating it with respect to input price (*w*), which yields the negative of the input-demand function. The resulting functions are:

<u>Output-supply function</u> $(\partial\pi / \partial p = y)$

$$y = \beta_p + \beta_{pw} (w/p)^{0.5} + \beta_{p2}(x_2) + \beta_{p3} (x_3) + \beta_{p4}(x_4) + \beta_{p5}(x_5) + \beta_{p6}(x_6)$$

<u>Input-demand function</u> $(-\partial\pi / \partial w = x_1)$

$$x_1 = -\beta_w - \beta_{pw} (p/w)^{0.5} - \beta_{w2}(x_2) - \beta_{w3} (x_3) - \beta_{w4}(x_4) - \beta_{w5}(x_5) - \beta_{w6}(x_6)$$

Note that coefficients from the profit function also appear in these two functions. For example, the coefficient β_{pw} shows up in all three, although it multiplies different versions of a variable involving *p* and *w* in each case. Given that the same coefficients appear in all three equations, we cannot estimate the equations separately. We need to estimate them jointly, as a single system. There are different ways to do this. We will use a technique called seemingly-unrelated regression, which is implemented in Stata by using the “reg3” command with the “sure” option.

Before we can estimate the system, we need to construct the variable $(w/p)^{0.5}$, which is in the output-supply function, and the variable $(p/w)^{0.5}$, which is in the input-demand function. We generate these variables by issuing the following pair of commands:

```
generate ppadi_plabor_sqrt=(ppadi/plabor)^0.5
generate plabor_ppadi_sqrt=(plabor/ppadi)^0.5
```

Next, we need to tell Stata which variables are in the three functions (profit, output supply, input demand). We do this by using the "global" command. For the profit function, we enter

```
global profit_function "(profit ppadi plabor  two_ppadixplabor_sqrt ppadixfarmsz ppadixbftot
ppadixirrih ppadixslope ppadixhujan plaborxfarmsz plaborxbftot plaborxirrih plaborxslope
plaborxhujan )"
```

This creates an equation named "profit_function" that has *profit* as the dependent variable and *ppadi*, *plabor*, etc. as explanatory variables. Note that all the explanatory variables are listed after *profit* inside the parentheses. Similarly, the following two commands create equations named "output_supply_function" and "input_demand_function":

```
global output_supply_function "(padi plabor_ppadi_sqrt farmsz bftot irrih slope hujan)"
global input_demand_function "(labor ppadi_plabor_sqrt farmsz bftot irrih slope hujan)"
```

Defining these functions is not enough; we also need to tell Stata that some of the coefficients are the same across the equations. The following commands create these cross-equation constraints:

```
constraint define 1 [profit]ppadi = [padi]_cons
constraint define 2 [profit]plabor = -[labor]_cons
constraint define 3 [profit]two_ppadixplabor_sqrt = [padi]plabor_ppadi_sqrt
constraint define 4 [profit]two_ppadixplabor_sqrt = -[labor]ppadi_plabor_sqrt
constraint define 5 [profit]ppadixfarmsz = [padi]farmsz
constraint define 6 [profit]plaborxfarmsz = -[labor]farmsz
constraint define 7 [profit]ppadixbftot = [padi]bftot
constraint define 8 [profit]plaborxbftot = -[labor]bftot
constraint define 9 [profit]ppadixirrih = [padi]irrih
constraint define 10 [profit]plaborxirrih = -[labor]irrih
constraint define 11 [profit]ppadixslope = [padi]slope
constraint define 12 [profit]plaborxslope = -[labor]slope
constraint define 13 [profit]ppadixhujan = [padi]hujan
constraint define 14 [profit]plaborxhujan = -[labor]hujan
```

Consider the first constraint. It says that the coefficient on *ppadi* in the equation with *profit* as the dependent variable (i.e., the profit function) equals the constant in the equation with *padi* as the dependent variable (i.e., the output supply function). This is correct: β_p is the coefficient on p in the equation for the profit function, and it is also the intercept in the equation for the output supply function. Now consider the second constraint, which says that the coefficient on *plabor* in the profit function equals the negative of the constant in the input demand function. This is again correct: β_w is the coefficient on w in the equation for the profit function, and $-\beta_w$ is the intercept in the equation for the input demand function.

With the variables all constructed, the equations defined, and the cross-equation coefficient constraints defined, we are ready to estimate the profit function system. We do so by entering the following command:

```
reg3 $profit_function $output_supply_function $input_demand_function, constraints(1 2 3 4
5 6 7 8 9 10 11 12 13 14) sure
```

"reg3" is the command to estimate a system of equations in Stata. Following it are the names of the equations, each preceded by a dollar sign ($), which is a symbol that informs Stata that the equations have been previously defined and stored under the indicated names. Next, the numbers of the constraints are given. Finally, the options for "reg3" are listed. Just one option is given, "sure", which tells Stata to calculate nonzero covariances across the three equations.

The command returns the results given on the next page. Coefficients for each equation are listed separately in the bottom half of the table, under the headings "profit", "padi", and "labor" (i.e., the name of the dependent variable in each equation). Note that the cross-equation constraints have worked: for example, the coefficient on *ppadi* in the profit function equals the intercept ("_cons") in the output supply function, while the coefficient on *plabor* in the profit function equals the negative of the intercept in the input demand function. Of the two variables in the profit function that involve baseflow, i.e., *ppadixbftot* and *plaborxbftot*, the former is now significantly different from zero. This indicates that baseflow has a significant impact on output supply, which is consistent with our results for the production function, but not input demand. Indeed, none of the variables in the labor demand function are significant, which probably reflects the fact that some of the households in our sample produce coffee in addition to rice. The labor variable refers to labor used for both crops, but we are treating it as referring to labor for just rice. By limiting the analysis to rice production, we have thus created a measurement error problem. As evidence that this explanation is correct, Pattanayak and Kramer obtained significant coefficient estimates for nearly all the variables in the labor demand equation in their two-output (rice and coffee) profit function system. One exception, however, is the coefficient on baseflow. So, both our simplified analysis of the partial sample of households and their more complete analysis of the full sample indicate that baseflow affects profit only through an impact on output supply, not through labor demand.

```
. reg3 $profit_function $output_supply_function $input_demand_function, constraints(1 2 3
4 5 6 7 8 9 10 11 12 13 14) sure;

Seemingly unrelated regression

Constraints:
 ( 1)  [profit]ppadi - [padi]_cons = 0
 ( 2)  [profit]plabor + [labor]_cons = 0
 ( 3)  [profit]two_ppadixplabor_sqrt - [padi]plabor_ppadi_sqrt = 0
 ( 4)  [profit]two_ppadixplabor_sqrt + [labor]ppadi_plabor_sqrt = 0
 ( 5)  [profit]ppadixfarmsz - [padi]farmsz = 0
 ( 6)  [profit]plaborxfarmsz + [labor]farmsz = 0
 ( 7)  [profit]ppadixbftot - [padi]bftot = 0
 ( 8)  [profit]plaborxbftot + [labor]bftot = 0
 ( 9)  [profit]ppadixirrih - [padi]irrih = 0
 (10)  [profit]plaborxirrih + [labor]irrih = 0
```

(11) [profit]ppadixslope - [padi]slope = 0
(12) [profit]plaborxslope + [labor]slope = 0
(13) [profit]ppadixhujan - [padi]hujan = 0
(14) [profit]plaborxhujan + [labor]hujan = 0

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
profit	92	13	1969.63	0.4124	244.10	0.0000
padi	92	6	1371.712	0.1656	74.43	0.0000
labor	92	6	62.53323	0.0918	5.19	0.5199

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
profit						
ppadi	-6611.184	1997.347	-3.31	0.001	-10525.91	-2696.457
plabor	-81.15006	238.7752	-0.34	0.734	-549.1408	386.8407
two_ppadix~t	-226.4383	167.9182	-1.35	0.177	-555.5519	102.6754
ppadixfarmsz	648.9635	139.0561	4.67	0.000	376.4185	921.5085
ppadixbftot	3786.079	1099.325	3.44	0.001	1631.441	5940.716
ppadixirrih	2120.583	391.3434	5.42	0.000	1353.564	2887.602
ppadixslope	-200.7666	121.1408	-1.66	0.097	-438.1982	36.66511
ppadixhujan	1558.642	526.0069	2.96	0.003	527.6873	2589.597
plaborxfar~z	-13.71369	13.43874	-1.02	0.308	-40.05313	12.62575
plaborxbftot	142.9762	114.9671	1.24	0.214	-82.35517	368.3076
plaborxirrih	42.10814	45.73739	0.92	0.357	-47.5355	131.7518
plaborxslope	-3.411727	11.82228	-0.29	0.773	-26.58296	19.75951
plaborxhujan	-9.251947	52.30382	-0.18	0.860	-111.7656	93.26166
_cons	239.9639	145.1855	1.65	0.098	-44.59454	524.5224
padi						
plabor_ppa~t	-226.4383	167.9182	-1.35	0.177	-555.5519	102.6754
farmsz	648.9635	139.0561	4.67	0.000	376.4185	921.5085
bftot	3786.079	1099.325	3.44	0.001	1631.441	5940.716
irrih	2120.583	391.3434	5.42	0.000	1353.564	2887.602
slope	-200.7666	121.1408	-1.66	0.097	-438.1982	36.66511
hujan	1558.642	526.0069	2.96	0.003	527.6873	2589.597
_cons	-6611.184	1997.347	-3.31	0.001	-10525.91	-2696.457
labor						
ppadi_plab~t	226.4383	167.9182	1.35	0.177	-102.6754	555.5519
farmsz	13.71369	13.43874	1.02	0.308	-12.62575	40.05313
bftot	-142.9762	114.9671	-1.24	0.214	-368.3076	82.35517
irrih	-42.10814	45.73739	-0.92	0.357	-131.7518	47.5355
slope	3.411727	11.82228	0.29	0.773	19.75951	26.58296
hujan	9.251947	52.30382	0.18	0.860	-93.26166	111.7656
_cons	81.15006	238.7752	0.34	0.734	-386.8407	549.1408

7.6 Calculating marginal and total impacts of the change in baseflow

As in the case of the production function, we can calculate the marginal value product of baseflow, which is the derivative of the profit function with respect to baseflow:

$$\frac{\partial \pi}{\partial x_3} = \beta_{p3} p + \beta_{w3} w .$$

The marginal value product is just the weighted sum of the regression coefficients on the two variables in the profit function that are related to baseflow (β_{p3} = coefficient on *ppadixbftot*, β_{w3} = coefficient on *plaborxbftot*), where the weights are the corresponding prices (p = *ppadi*, w = *plabor*). Since the second coefficient is not significant, the expression simplifies to

$$\frac{\partial \pi}{\partial x_3} = \beta_{p3} p .$$

We can use this expression to calculate the marginal value product for each household by entering the following command:

```
generate marginal_impact_system=ppadi*[profit]_b[ppadixbftot]
```

The phrase [profit]_b[ppadixbftot] tells Stata to use the stored value of the estimated coefficient on *ppadixbftot* from the profit function. We can obtain summary statistics by entering the command,

```
su  marginal_impact_system
```

which returns the following information:

```
. su  marginal_impact_system;

       Variable |     Obs        Mean      Std.  Dev.       Min        Max
----------------+---------------------------------------------------------
 marginal_i ~ n |      92    4229.804     1988.  991   1893.039   13535.23
```

The mean marginal value product is 4,300, which is half again as large as the estimate based on the production function, 2,774. This is consistent with our expectation that the benefit of an environmental improvement based on a production function understates the actual benefit (see section 2).

The benefit of increased baseflow that we just calculated is a marginal benefit: the benefit of one additional unit. The proposed reforestation program in Ruteng would not uniformly change baseflow by one unit for all farms, however. According to Pattanayak and Kramer, the predicted changes, in percentage terms, were as follows:

Kecano (county)	Percent change in baseflow
1 (Borong)	15
2 (Elar)	-17
3 (Langke Rembong)	-25
4 (Pembantu Borong)	9
5 (Pembantu Elar)	36
6 (Pembantu Lambaleda)	-23
7 (Pembantu Ruteng)	-12
8 (Ruteng)	-5
9 (Satarmese)	9

These are changes for a program that would increase forest cover by 25 percent. As can be seen, the program was predicted to increase baseflow in less than half the counties.

Per the discussion in sections 2-4, to value the impacts of these changes on farm profits, we need to determine the difference between profits with the baseflow changes and profits without them, i.e. $\pi_1^* - \pi_0^*$. For the particular specification of the profit function used here, π_1^* and π_0^* are given by

$$\begin{aligned}\pi_1^* &= \beta_p p + \beta_w w + \beta_{pw} 2(pw)^{0.5} \\ &+ \beta_{p2}(px_2) + \beta_{p3}(px_3^1) + \beta_{p4}(px_4) + \beta_{p5}(px_5) + \beta_{p6}(px_6) \\ &+ \beta_{w2}(wx_2) + \beta_{w3}(wx_3^1) + \beta_{w4}(wx_4) + \beta_{w5}(wx_5) + \beta_{w6}(wx_6)\end{aligned}$$

$$\begin{aligned}\pi_0^* &= \beta_p p + \beta_w w + \beta_{pw} 2(pw)^{0.5} \\ &+ \beta_{p2}(px_2) + \beta_{p3}(px_3^0) + \beta_{p4}(px_4) + \beta_{p5}(px_5) + \beta_{p6}(px_6) \\ &+ \beta_{w2}(wx_2) + \beta_{w3}(wx_3^0) + \beta_{w4}(wx_4) + \beta_{w5}(wx_5) + \beta_{w6}(wx_6)\end{aligned}$$

where x^1_3 is baseflow with the change and x^0_3 is baseflow without it. The difference in profit is thus given by

$$\pi_1^* - \pi_0^* = \beta_{p3} p(x_3^1 - x_3^0) + \beta_{w3} w(x_3^1 - x_3^0),$$

which further simplifies to

$$\pi_1^* - \pi_0^* = \beta_{p3} p(x_3^1 - x_3^0),$$

after considering that the estimate of β_{w3} (i.e., the coefficient on *plaborxbftot* in the profit function) is not significant. The change in profit is thus given by the product of the coefficient on the interaction term for rice price and baseflow (*ppadixbftot*), rice price (*ppadi*), and the change in baseflow.

The change in baseflow here is in meters, not percent. So, as a first step we need to construct this variable. We do so as follows. First, we construct a variable, *bfchange_percent*, that gives the change in percent:

```
generate  bfchange_percent=0
replace bfchange_percent=15 if kecano==1
replace bfchange_percent=-17 if kecano==2
replace bfchange_percent=-25 if kecano==3
replace bfchange_percent=9 if kecano==4
replace bfchange_percent=36 if kecano==5
replace bfchange_percent=-23 if kecano==6
replace bfchange_percent=-12 if kecano==7
replace bfchange_percent=-5 if kecano==8
replace bfchange_percent=9 if kecano==9
```

Then, we multiply this new variable times baseflow (*bftot*) and divide by 100 to convert the percentages in *bfchange_percent* to decimals:

```
generate bfchange=bftot*bfchange_percent/100
```

We now have all the information necessary for determining the change in profit resulting from the predicted change in baseflow, which we do by using the following command to create a variable named *total_impact_system*:

```
generate total_impact_system=[profit]_b[ppadixbftot]*ppadi *bfchange
```

The mean value of this variable is 7.33, determined by using the "summarize" command in Stata,

```
su total_impact_system
```

which returns

```
. su total_impact_system
```

Variable	Obs	Mean	Std. Dev.	Min	Max
marginal_i ~ n	92	7.332027	696. 6726	-3058.286	982.8646

On average, the reforestation program thus has a positive impact on farm profits. We can gauge the magnitude of the impact by applying the summarize command to *profit*,

```
su profit
```

which returns

```
. su profit
```

Variable	Obs	Mean	Std. Dev.	Min	Max
marginal_i ~ n	92	1136.492	2583. 524	.4166667	15713.38

Compared to mean profit (1136), the impact of the reforestation program is not very large, less than 1 percent.

A summary of the Stata commands reviewed in this section is given on the following two pages. They are the final pages of the tutorial.

7.7 Summary list of Stata commands

For generating variables in the production function:

```
generate lpadi=ln( padi )
generate llabor=ln( labor)
generate lfarmsz = ln( farmsz )
generate lbftot=ln( bftot )
generate lhujan=ln( hujan )
```

For estimating the production function:

```
regress  lpadi llabor lfarmsz lbftot lhujan irrih slope
```

For generating and summarizing the marginal impact of baseflow from the production function:

```
generate marginal_impact_production=ppadi*_b[lbftot]*padi/bftot
su  marginal_impact_production
```

For generating the variables in the profit function:

```
generate two_ppadixplabor_sqrt=2*(ppadi*plabor)^0.5
generate ppadixfarmsz=ppadi*farmsz
generate ppadixbftot=ppadi*bftot
generate ppadixirrih=ppadi* irrih
generate ppadixslope=ppadi* slope
generate ppadixhujan=ppadi* hujan
generate plaborxfarmsz=plabor*farmsz
generate plaborxbftot=plabor*bftot
generate plaborxirrih=plabor*irrih
generate plaborxslope=plabor*slope
generate plaborxhujan=plabor*hujan
```

For estimating the profit function:

```
regress profit ppadi plabor two_ppadixplabor_sqrt ppadixfarmsz ppadixbftot ppadixirrih
ppadixslope ppadixhujan plaborxfarmsz plaborxbftot plaborxirrih plaborxslope plaborxhujan
```

For generating additional variables in the output supply and input demand functions:

```
generate ppadi_plabor_sqrt=(ppadi/plabor)^0.5
generate plabor_ppadi_sqrt=(plabor/ppadi)^0.5
```

For defining the profit, output supply, and input demand functions:

```
global profit_function "(profit ppadi plabor  two_ppadixplabor_sqrt ppadixfarmsz ppadixbftot
ppadixirrih ppadixslope ppadixhujan plaborxfarmsz plaborxbftot plaborxirrih plaborxslope
plaborxhujan )"

global output_supply_function "(padi plabor_ppadi_sqrt farmsz bftot irrih slope hujan)"

global input_demand_function "(labor ppadi_plabor_sqrt farmsz bftot irrih slope hujan)"
```

For creating cross-equation coefficient constraints:

```
constraint define 1 [profit]ppadi = [padi]_cons
constraint define 2 [profit]plabor = -[labor]_cons
constraint define 3 [profit]two_ppadixplabor_sqrt = [padi]plabor_ppadi_sqrt
constraint define 4 [profit]two_ppadixplabor_sqrt = -[labor]ppadi_plabor_sqrt
constraint define 5 [profit]ppadixfarmsz = [padi]farmsz
constraint define 6 [profit]plaborxfarmsz = -[labor]farmsz
constraint define 7 [profit]ppadixbftot = [padi]bftot
constraint define 8 [profit]plaborxbftot = -[labor]bftot
constraint define 9 [profit]ppadixirrih = [padi]irrih
constraint define 10 [profit]plaborxirrih = -[labor]irrih
constraint define 11 [profit]ppadixslope = [padi]slope
```

```
constraint define 12 [profit]plaborxslope = -[labor]slope
constraint define 13 [profit]ppadixhujan = [padi]hujan
constraint define 14 [profit]plaborxhujan = -[labor]hujan
```

For estimating the profit function system:

```
reg3 $profit_function $output_supply_function $input_demand_function, constraints(1 2 3 4
5 6 7 8 9 10 11 12 13 14) sure
```

For generating and summarizing the marginal impact of baseflow from the profit function system:

```
generate marginal_impact_system=ppadi*[profit]_b[ppadixbftot]
su  marginal_impact_system
```

For generating a variable giving the percent change in baseflow:

```
generate  bfchange_percent=0
replace bfchange_percent=15 if kecano==1
replace bfchange_percent=-17 if kecano==2
replace bfchange_percent=-25 if kecano==3
replace bfchange_percent=9 if kecano==4
replace bfchange_percent=36 if kecano==5
replace bfchange_percent=-23 if kecano==6
replace bfchange_percent=-12 if kecano==7
replace bfchange_percent=-5 if kecano==8
replace bfchange_percent=9 if kecano==9
```

For generating the change in baseflow in absolute terms:

```
generate  bfchange=bftot*bfchange_percent/100
```

For generating and summarizing the total impact of baseflow from the profit function system:

```
generate total_impact_system=[profit]_b[ppadixbftot]*ppadi *bfchange
su total_impact_system
```

8. Acknowledgements

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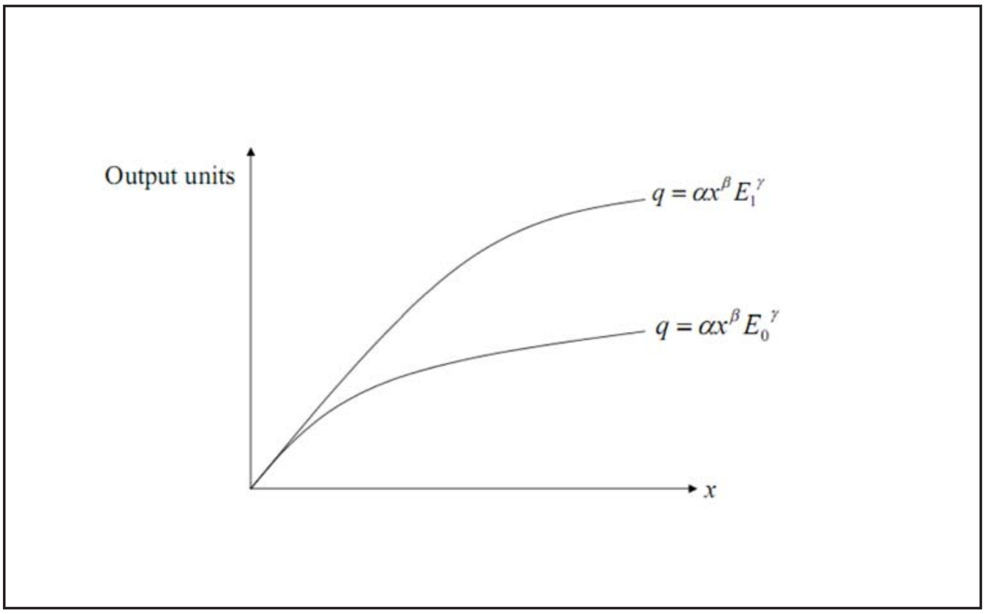
FIGURES

Figure 1:



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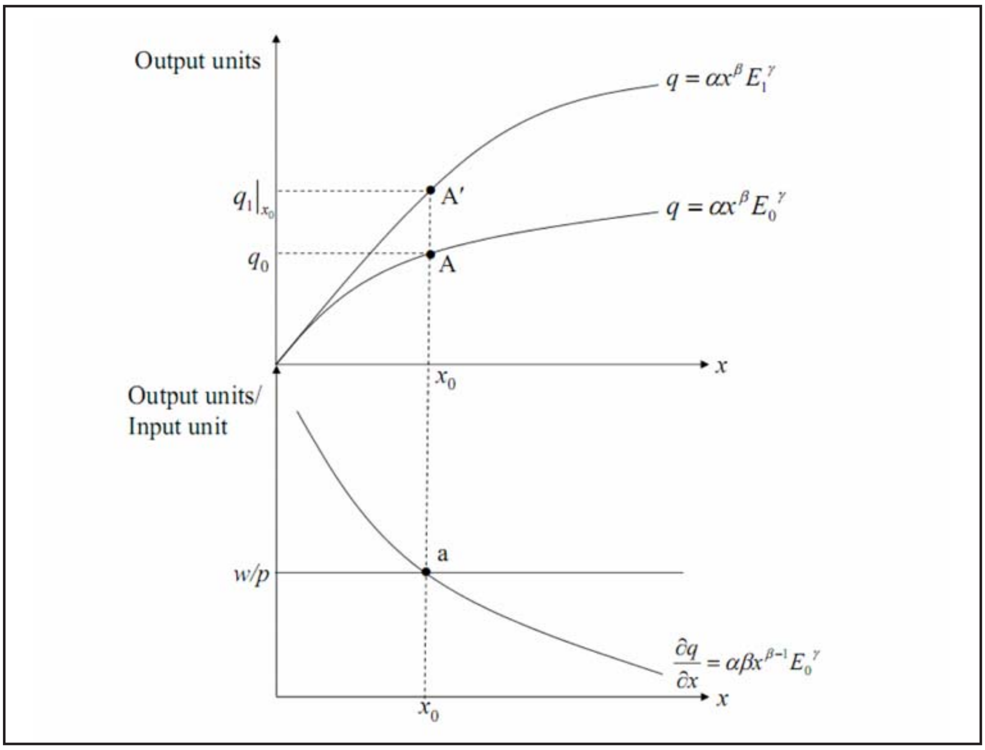

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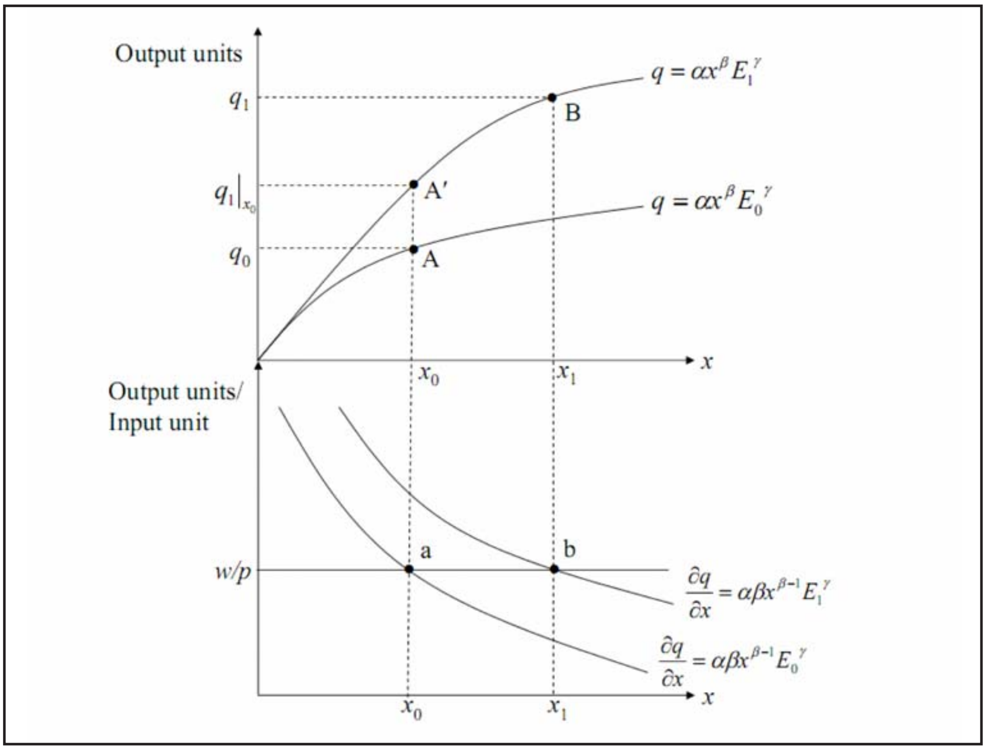

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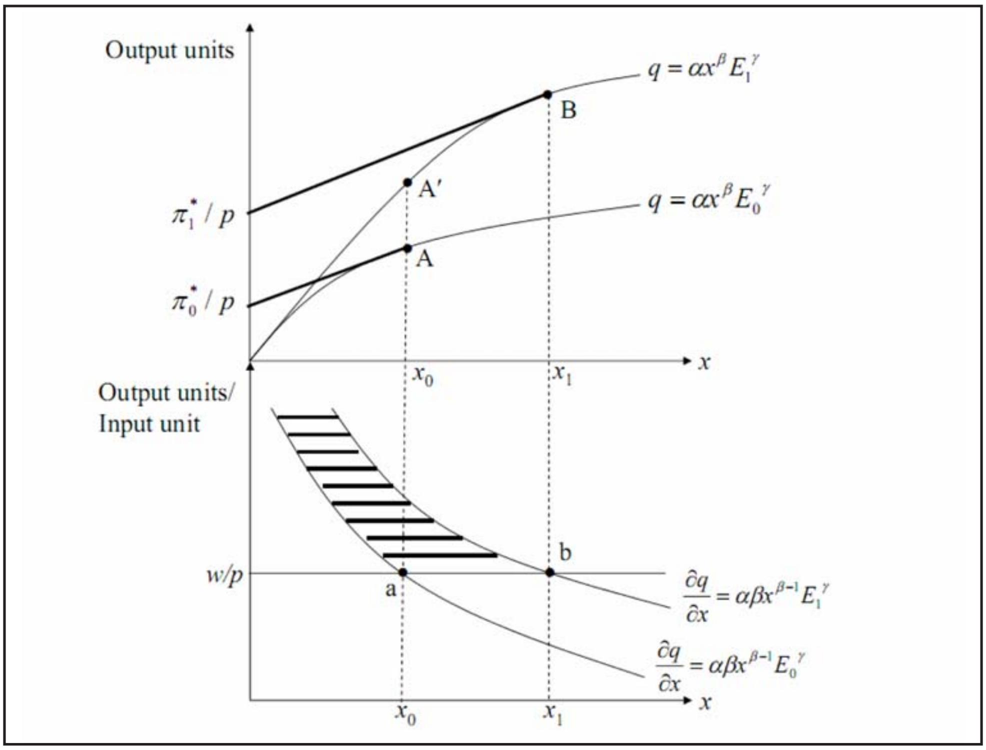

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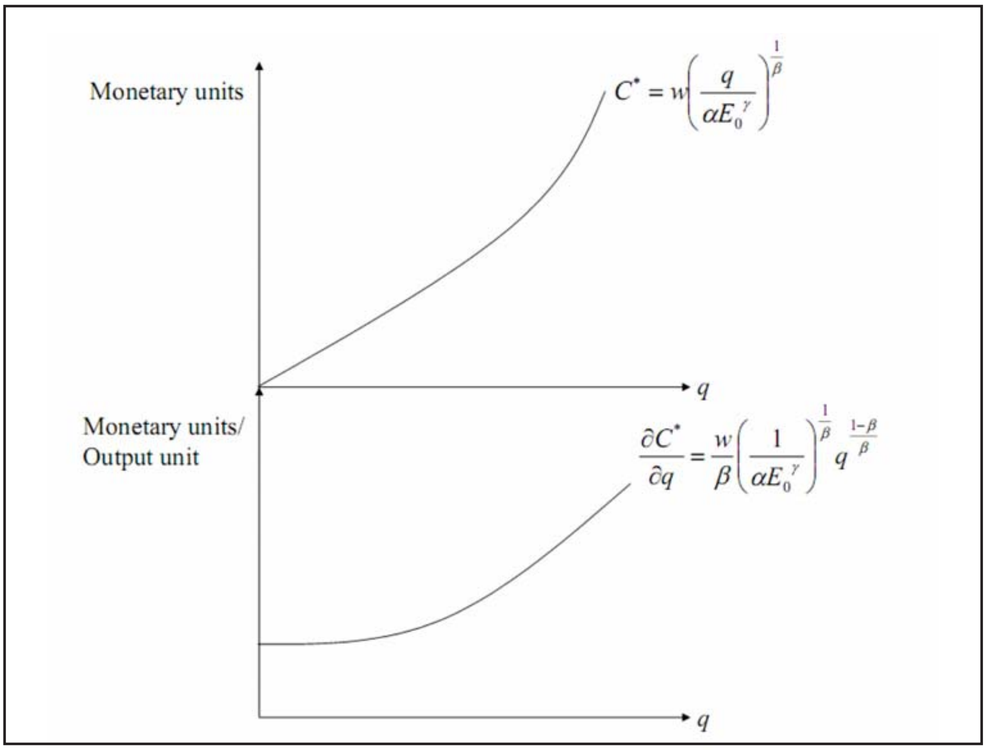

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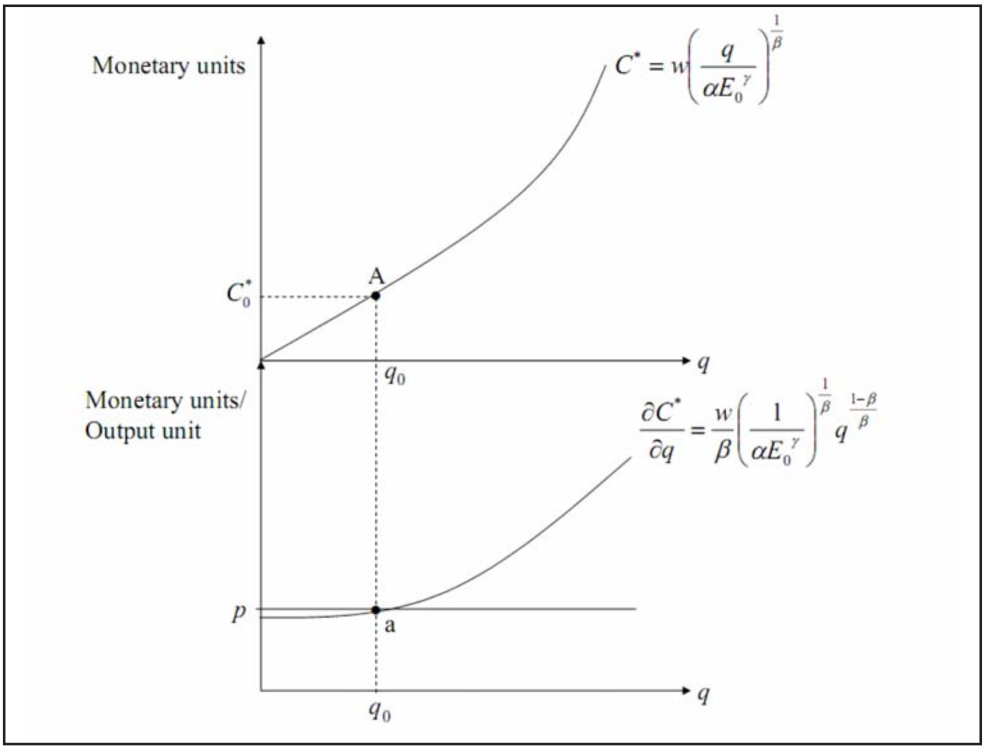

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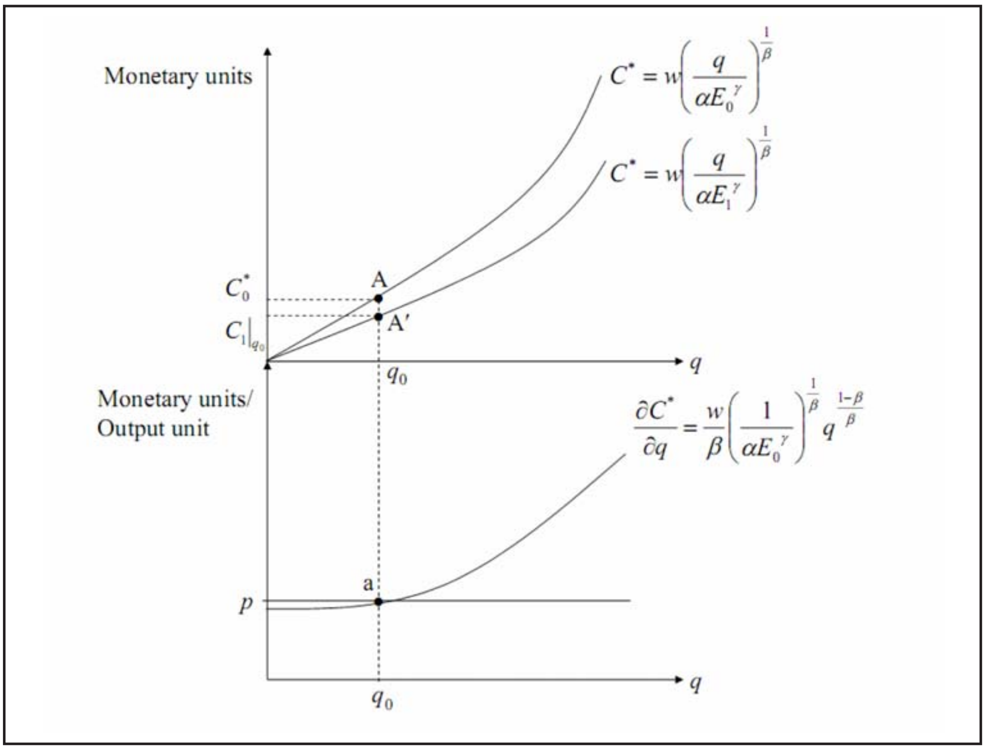

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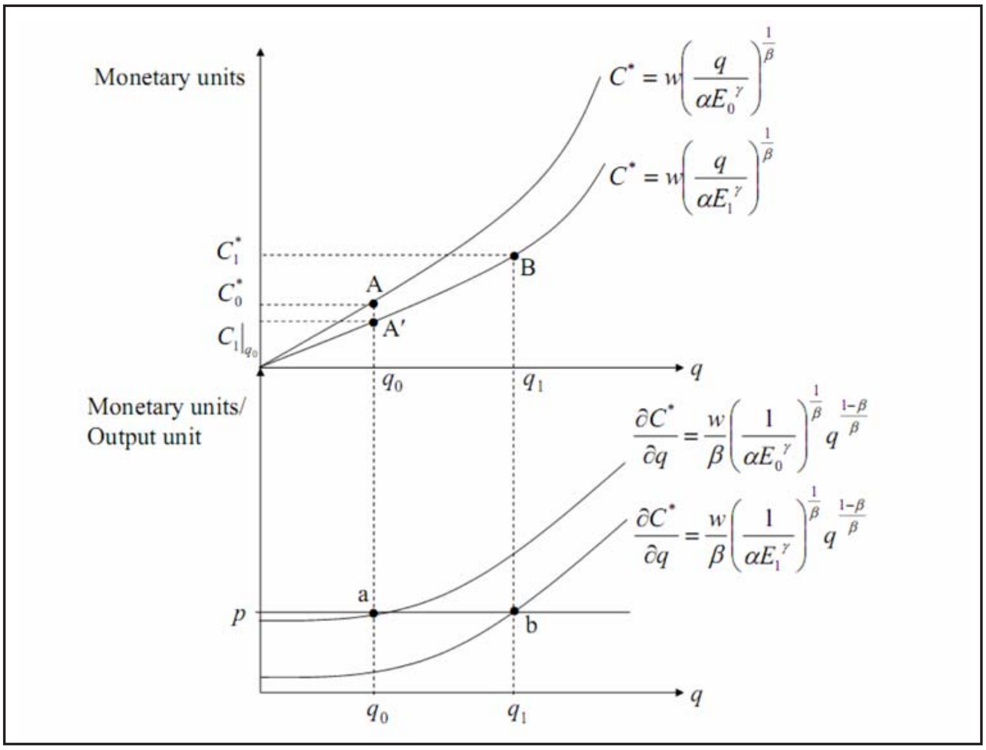

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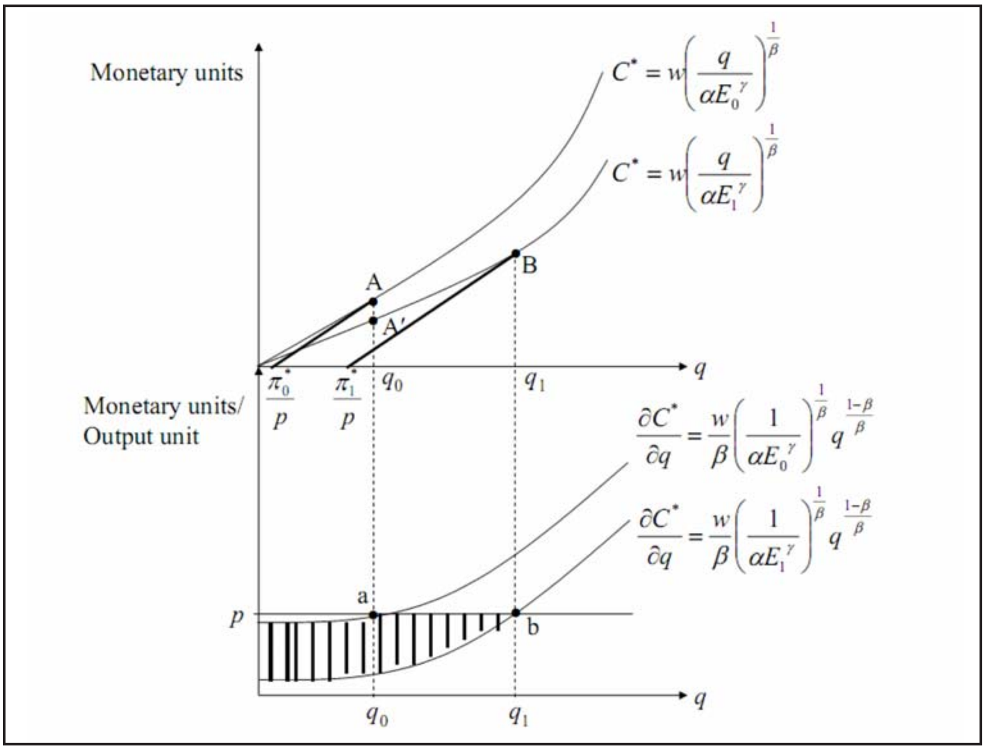

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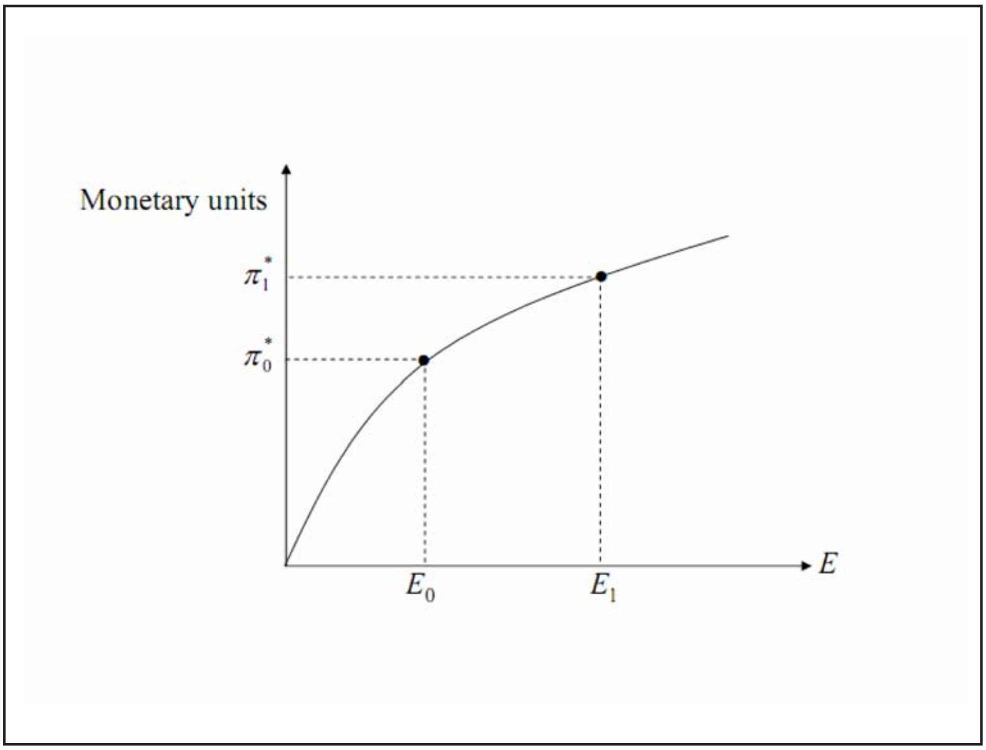

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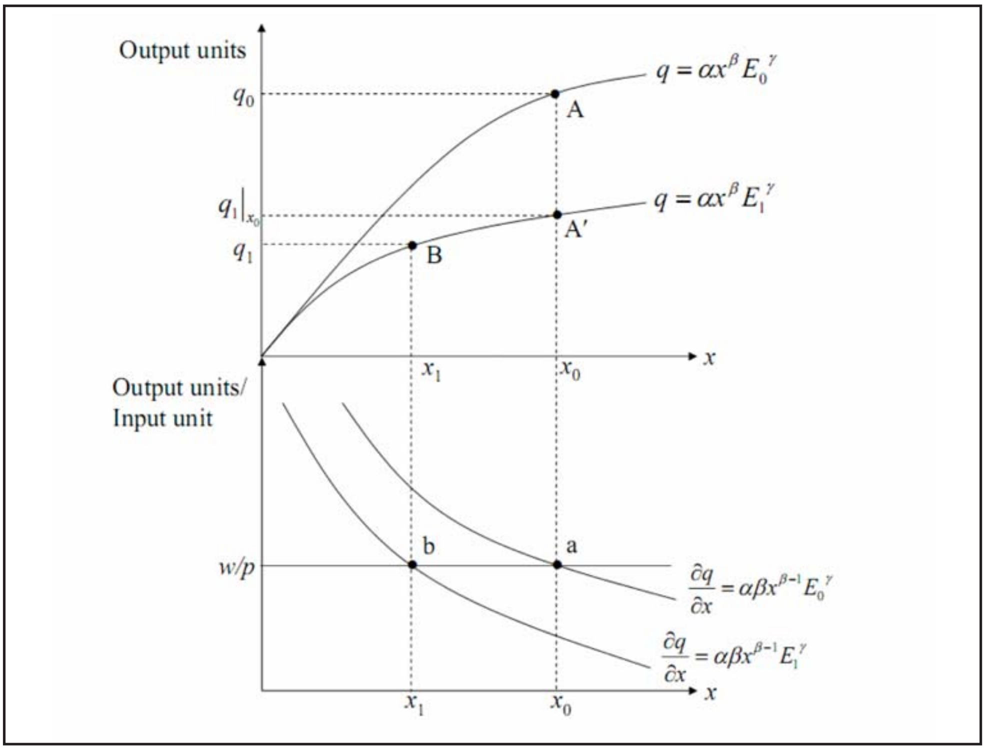

Figure 12: