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Beauty, Ethics and Numbers in Boethius' Quadrivial Treatises

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Abstract. The convergence of the Neoplatonic/Neopythagorean approach with the Aristotelian organization of the sciences is one of the most interesting features that characterizes the two influential mathematical treatises on *On Arithmetics* (*De institutione arithmetica*) and *On Music* (*De institutione musica*) by Severinus Boethius. Basing his reasoning on Nicomachus and Ptolemy, Boethius follows the philosophical tradition that had tried to reconcile Plato's and Aristotle's views. This attitude is examined in the present paper as regards Boethius' response concerning the relation between numbers, ethics and aesthetics. His view emerges as coming out of a rather complex construction, which assigns the ethical scope of mathematics in indicating to the human mind how to correct the ratios that realize the best relationship in movements of the soul and the body. More precisely, its ethical aim is to correct the specific form of movement of human beings, that is their actions, exemplified in the way in which mathematical ratios represent the forms of government and musical ratios evoke and heal psychophysical affections. More complex, on the other hand, is the relationship between mathematics and beauty. In clear antithesis to the position taken by Augustine on the beauty of the rhythmic patterns that better represent the beauty of unity, Boethius does not relate the mathematical ratios of the consonances to an esthetical judgment by making use of the category of beauty. For him, the physical world is totally immersed in changes and movements, and this cannot but impede things from expressing the stable unity, which is required for contemplating the beautiful.

Keywords. Boethius, ethics, aesthetics, medieval philosophy.

Within the framework of his syncretic and ambitious intellectual endeavor, the Late Roman scholar Severinus Boethius (died 524/525) deserved great importance to mathematics, which, in agreement with Aristotle, was classified by him as one of the three theoretical sciences. Boethius divided mathematics into the four disciplines of arithmetic, music, geometry and astronomy, named with the fortunate neologism *quadrivium*, the «four-fold path» to truth which allowed the mind to raise itself from the uncertainty of sense perceptions to the certainty of intelligible truth. Boethius was also the author of two highly influential treatises on the first and the second

quadrivial disciplines, *On Arithmetics* (*De institutione arithmetica*) and *On Music* (*De institutione musica*), which were grounded on the Neoplatonic/Neopythagorean approach to these sciences, largely mediated from his main sources, namely Nicomachus of Gerasa and Ptolemy. The convergence of this Neoplatonic background with the Aristotelian view of mathematical sciences is one of the most interesting features of these treatises, and the present paper examines how it affected Boethius's theory on the relation between numbers, ethics and esthetics. Such theory, by combining the Platonic and Aristotelean legacies within the framework of the late Roman educational system, will transmit to the Western culture a renewed ideal of scientific knowledge, that the medieval scholars will then, in turn, integrate within the new Christian idea of wisdom[1].

1. ARISTOTELIAN AND PLATONIC ATTITUDES TO THE BEAUTIFUL AND THE GOOD IN MATHEMATICS AND THEIR RECEPTION IN BOETHIUS

The ancient Greek concept of beauty was much broader than ours. "Beauty" included the "good" of thoughts and customs, and the compound *kalos k'agathos* expressed an aesthetic and ethical ideal. Both Plato and Aristotle shared this view, though offered a different approach to these notions, particularly in the case of their application to numbers, ratios and geometrical figures[2]. A relevant passage for the present enquiry is in *Metaphysics* 13.3, in the course of a long discussion contesting the Platonic and Pythagorean doctrine of mathematics. Here, Aristotle openly states that the sciences of numbers and figures are connected to the good and the beautiful. Yet, he remarks that these two notions are different, «for the former always implies conduct as its subject, while the beautiful is found also in motionless things». Second, he states that mathematical sciences do not mention the good and the beautiful openly, but consider only their effects, and, leaving aside the good, he adds that «the chief forms of beauty are order and symmetry and definiteness, which the mathematical sciences demonstrate in a special degree» and that the beautiful, being the causative principle of order and definiteness, must be treated by these sciences «as in some sense a cause» (*Metaphysics*, 13, 1078a31-b5; transl. Ross [1924]).

Plato had also associated mathematics with the good and the beautiful, yet he had adduced different reasons for their relationship to mathematical entities, and the difference between the two concepts was not highlighted by him. The topic is discussed in almost all his dialogues. In the *Philebus*, for instance, Socrates asserts that the highest beauty is that of straight lines and circles, for these «are not relatively beautiful, like other things, but eternally and absolutely beautiful» (*Philebus*, 64d-65a.). Here, goodness and beauty are intimately associated with measure and proportion (*symmetria*), which require a quantitative dimension. In the *Republic*, while discussing the musical education of the rulers, Socrates insists that mathematical harmonies should consider which numbers are concordant, which are not, and why they are so, and that this task «is useful if directed towards investigating the beautiful and the good,

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[1] This paper benefited from valuable comments of Stefano Perfetti and from the English revision of Janet Donovan. I am very grateful to both of them and remain the only responsible for any error. Hereafter, all references to the Latin text of the *De institutione arithmetica* are from the edition by Oosthout, Schilling [1999] and to the Latin text of *De institutione musica* are from the edition by Friedlein [1867]. The two works are referred to in footnotes as *Inst. arith.* and *Inst. mus.* while in the text as *On Arithmetics* and *On Music* respectively. The references to *De consolatione philosophiae* are from the edition by Moreschini [2005], referred to as *Cons. Phil.* For a detailed analysis on the origin of the quadrivium see Merlan [1975]: 88-95 and Guillaumin [1990].

[2] For an overview on this vast topic, further bibliographical references and details on mathematics, beauty and ethics in Plato and Aristotle see, for instance, Charles-Saget [1982]; Burnyeat [2000]; Riegel [2014]; Mendell [2017]. On mathamatical science see also O'Meara [2005], for late antiquity, and Panti [2008] for the Middle Ages.

but useless if otherwise pursued» (*Republic*, 7, 531c.)[3]. Clearly, for Plato mathematical objects are beautiful and good in themselves in so far as they are eternal and unchangeable. Therefore, notwithstanding Plato's and Aristotle's agreement on the aesthetical and ethical implications of mathematics, the two ancient philosophers provided different reasons for justifying this connection, depending in turn on their different conceptions of mathematical objects: for Plato, they are eternally subsistent ideas, while for Aristotle, they are forms inherent to natural things and subject to change.

More importantly for the present study, Plato and Aristotle offered different explanation on how mathematical beauty and goodness are connected to the physical world and, from here, to number and figure. For Plato, the cosmos reflects the numerical order that, in the *Timeus,* is described as impressed by the Demiurge, who looked at ideal numbers to structure the soul of the world. Consequently, beauty and goodness pertain to natural things in as much as they reflect the eternal numerical ratios, in the likeness of which all things and beings are shaped. These ratios, in turn, correspond to «the non-sensible concords in the divine attunement» of the cosmos. This last feature is exemplified in musical intervals, namely in how those which sound "good" to the ears, i.e.[4] the consonances, correspond to simple mathematical ratios, the same as those that shape the soul of the world[5]. In the Aristotelian view, on the other hand, when the mind thinks about numbers, geometrical figures and similar "forms", or categories of substances, it separates them from reference to natural things. Therefore, to grasp beauty and goodness in numbers means: first, to abstract them from their physical instantiation and, second, to consider their properties as if they were actually separate. This mental attitude is specific also to those intermediate sciences between pure mathematics and physics, such as optics or harmonics, which do not deal with vision or sound in themselves, as physics does, or with lines and numbers in themselves, as mathematics does, but with lines and numbers as attributes of vision or sound respectively[6].

Besides, mathematics has also a different collocation in the Platonic and Aristotelian divisions of philosophy. In *Metaphysics* 11.7 Aristotle proposes a classification of the sciences that reserves an independent collocation to mathematics, assigned to the domain of the theoretical sciences together with, but separated from, physics and from first philosophy, or metaphysics, as we now call it. Mathematical sciences deal with the study of the "forms" mentally separated from matter, while physics accounts for material things subject to change and movement. For Plato, on the contrary, the rather imperfect knowledge that human beings can attain of mutable things can be reached only by means of the very mathematical sciences. These are the disciplines presented in the *Republic* as those which bring the mind to the higher realm of dialectic and offer the access to what participates of number but is not number itself, i.e. the natural entities. The educational path, consequently, starts with abstract calculus, then continues with geometry and solid geometry, i.e. the sciences of immutable forms, to astronomy, the science of perfect movement, up to harmonics, the mathematical ratios governing the universe (*Republic*, 7, 522c-531d, 537c-d).

These divergent yet fundamental and longlasting ideas on mathematics and on its relation to aesthetics and ethics formed the basis of all subsequent intellectual approaches to the same

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[3] See also, for instance, Charles-Saget [1982]: 14 ff.
[4] See Burnyeat [2000]: 53.
[5] Among the many studies devoted to this topic in Plato's philosophy, I have followed Burnyeat [2000].

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[6] *Metaphysics*, 13, 3, 1077b23-78a23, transl. Ross [1924]: «For just as the universal propositions of mathematics deal not with objects which exist separately, […] but with magnitudes and numbers, […] the mathematical sciences will not for that reason be sciences of sensibles – nor, on the other hand, of other things separate from sensibles. […] The same account may be given of harmonics and optics; for neither considers its objects qua sight or qua voice, but qua lines and numbers; but the latter are attributes proper to the former».

topics, and, at the turn of the fifth century of the Christian era, Boethius was well familiar with the rich philosophical and scientific tradition, mostly Neoplatonic, which had tried to interpret and, to a certain extent, mingle these same approaches in the light of the higher task of reconciling the disagreements between Plato and Aristotle. Boethius inherited the same syncretic attitude. As Chadwick remarks, his literary and educational program was «a faithful reflection of the assumptions and teaching practice of the contemporary Platonic schools of Athens and Alexandria», where «young students were introduced to arithmetic through Nicomachus of Gerasa, to harmony through Nicomachus and Ptolemy, to geometry through Euclid, to astronomy through Ptolemy's *Syntaxis* (the "Almagest")*;* then the *Organon* of Aristotle to which Porphyry's *Isagoge* is prefixed, […] finally up to metaphysical questions such as evil and providence or the supreme God at the apex of Platonic theology» (Chadwick [1981]: 21 ff.). Receptive of this unitary system of education, Boethius was conscious of the demanding task that its first inspirers, above all the Pythagorean Nicomachus of Gerasa and Claudius Ptolemy, had undertaken to incorporate their mathematical investigations as starting steps of the whole intellectual process. He inherited from them the same aspiration to make evident that «in mathematics lies the key to the rational organization of the universe» (Barker [1989]: 8).

For Boethius, however, the search for this key also passes through a further step: his ambitious plan of translating into Latin and expounding the authoritative texts of Aristotle and Plato, whose convergence – imperative for him – had to be carefully reconstructed and harmonized under the unity of a single philosophical wisdom[7]. Boethius, standing "on the shoulders" of those giants, added the educational system of the Roman culture, represented by Vitruvius and Cicero, to the revered

Greek tradition of thought, and opened the way to the new Christian search for wisdom, already pursued by Augustine[8]. His mathematical treatises are the first products of this outstanding mission, and his theory of the beautiful and the good in mathematics was deeply affected by this search of this cultural and philosophical unity.

2. THE GOOD AND ETHICS IN BOETHIUS'S QUADRIVIAL TREATISES

In his prologue to *On Arithmetics*, in an elaborate dedication to his father-in-law and mentor Symmachus, Boethius manifests his intention to fulfil his enquiry into the first of the *matheseos disciplinae* by using Nicomachus' *Introduction to Arithmetics*. Following this source, he begins the subsequent *proemium* in the name of Pythagoras, whose methodological approach to the mathematical sciences was presented by Nicomachus as a convergence of Aristotelian logic and Platonic theology. Let us briefly summarize how Boethius explains Pythagoras' approach. To begin with, Nicomachus had called the four mathematical disciplines of arithmetic, music, geometry and astronomy as «methods (*methodoi*)» for proceeding upwards towards higher knowledge, and Boethius, accordingly, refers to them as «a fourfold path (*quadrivium*)», because through them «we bring a superior mind from the knowledge offered by the senses to the more certain things of the intellect»[9]. These sciences are therefore seen as four steps (*gradus*), which are ordered in the same sequence that Nicomachus had established: arithmetic, music, geometry, astronomy. Only in this order, in fact, does the mind gradually rise from sense perception to the certainty of intelligible truth. Boethius intends to follow the same path, and to offer a complete presentation of the "four

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[7] See Boethius, *Commentarium in librum Aristotelis Perihermeneias secundae editionis*, 2, Introduction, in *Documenta Catholica omnia* at: www.documentacatholicaomnia.eu.

[8] See Restani [2008] and [2004].

[9] *Inst. arith.* 1,1 and Nicomachus, *Introductio Arithmetica*, 1, 1.1. The critical edition of Boethius's text by Oosthout-Schilling [1999] gives references to all parallel passages. For Nicomachus as sources of Boethius' *On Music* see Bower [1978].

sisters". It seems that he really fulfilled the task, though only his *On Arithmetics* and *On Music* have survived and are known to us[10].

Let us come back to Pythagoras. For Boethius, still in line with Nicomachus, Pythagoras is the true wise scholar who incarnates this correct methodological approach precisely because he moved from sense perception, and, having discovered the numerical ratios which structure natural things, realized that they were the same as those which bind together soul and body in human beings and in the cosmos; hence the turn from perceptions to the higher truths of the mind occurs through mathematics. But how did Pythagoras realize this? The answer, according to Nicomachus and, consequently, to Boethius, is found in the Aristotelian logic, and specifically in the doctrine of categories. This is made explicit by Boethius in both *On Arithmetics* and *On Music*, which follow Nicomachus' *Introduction*. In *On music*, for instance, Boethius asserts:

Pythagoras was the first to call the study of wisdom "philosophy", which he held as knowledge and disciplinary study of what it is properly and truly said "to be" (esse). Moreover, he considered "to be" (esse) those things that neither increase by augmentation (intensione), neither decrease by diminution (deminutione), neither are altered by any chance occurrences. These things are forms, magnitudes, qualities, relations and all the others which, considered in themselves, are immutable, but, joined to natural bodies (corporibus), suffer radical change and are altered in many ways because of their relationship with a changeable thing. (Inst. musica, II, 1; My transl. on the basis of Bower-Palisca [1989]: 53)

This same sentence has an equivalent in *On Arithmetics*, and is a faithful translation from Nichomacus. For him, the «real» or «bodiless» things, which Boethius translates «to be (*esse*)», are exactly the unchanging objects of true knowl-

edge discovered by Pythagoras[11]. These are magnitudes, qualities, relations and so on; this means that they are not the Platonic forms or ideas of number or geometrical figures as such, but more specifically the «primordial forms of being», that is to say an intentional rielaboration in Neopythagoric perspective of the Aristotelian categories that define the prime category: the substance[12]. These "categories" are the indispensable premise for selecting the one that pertains to mathematics, namely the quantity, divided by Aristotle into continuous quantity, or magnitude, and discrete quantity, or multitude: magnitude is equivalent to geometrical figures and is continuous with its constitutive parts, as it is, for instance, in a tree or a stone; while multitude consists of many parts, as in a flock or a wood, and is equivalent to number (*Categories*, 6). Within this classification, the four disciplines reveal their order: arithmetic studies multitude, or number, in itself, music studies it in relation to something else (*ad aliquid*), geometry studies immovable magnitude, and astronomy movable.

The topic of ethics in mathematical sciences is grounded on these premises. Apparently, magnitude and multitude have nothing to do with the superior knowledge of the truth, and cannot introduce to the good, because it is evident that these

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[10] On the remarks by Cassiodorus about the quadrivial treatises of Boethius see, for instance, Chadwick [1981]: 70.

[11] For a detailed analysis of these passages and discussion on the "real things" as forms in Nicomachus see Helmig [2007].

[12] D'Onofrio [2001]: 22-25: «Una intenzionale rielaborazione in chiave neopitagorica dell'elenco delle categorie aristoteliche posteriori alla sostanza». The correspondence of the «real things»/*esse* is fully given in *On Arithmetics*, which follows verbatim Nicomachus' *Introduction*. They are, in the same order given by Nicomachus: qualities (*qualitates*), quantities (*quantitates*), configurations (*formae* which translates the Greek *schematismi*, namely geometrical figures), largeness (*magnitudines*), smallness (*parvitates*), equalities (*aequalitates*), relations (*habitudines*), actualities (*actus*) dispositions (*dispositiones*) places (*loca*), times (*tempora*) ecc. The Aristotelian categories are: substance, quantity, quality (qualification), relation, place (where), time (when), being-in-a-position, having, doing, being-affected. The latter is missing from the Nicomachean/Boethian re-elaboration.

two forms of the *esse* of quantity are not immutable or eternal themselves, being subject to change in natural things. Yet, if these *esse* are thought of as separate from bodies and things, they are, as Nicomachus and Boethius assert, «stable and unchangeable». This is precisely the nature of mathematical objects for Aristotle, as seen above, and this conception is eventually the key to understanding why magnitudes and multitudes are linked to morality. In their connection with natural things and beings, in fact, they convey the changes and movements of all things and, more importantly, in the case of human beings they express their actions through the definiteness of numbers and figures. Actions are conscious movements of human beings, and, as Aristotle asserts in the passage from *Metaphysics* 13 mentioned at the beginning of this paper, «the good pertains to actions alone». Hence, there is a link between human actions, magnitudes and multitudes. Boethius identifies such a link precisely in the notion of definiteness, and assumes that it pertains both to the good, which is always definite and lacking indeterminacy, and to the mathematical investigations, given that magnitudes and multitudes are forms of bodily definiteness[13]. Hence, mathematical investigations have an ethical significance.

To develop this last argument Boethius is mainly guided by Ptolemy, who, in his *Harmonics*, drew his classification and presentation of the mathematical disciplines from Aristotle's *Metaphysics*[14]. Ptolemy identifies harmony (*harmonia*) with a movement producing some «good» end, «such as good melody, good rhythm, good order and beauty», harmony being «the cause, which imposes the appropriate form on the underlying matter» (Ptolemy, *Harmonics*, III, 3; transl. Barker [1989]: 371). Goodness itself, therefore, falls under mathematical knowledge, and Boethius faithfully proclaims the same view:

Great certainty is the profit which comes from this science [i.e. arithmetics] if we do not ignore the fact that the definite goodness (bonitas definita), which is the object of science and is always subject to imitation and perceptible (imitabilis et perceptibilis), is the first nature (prima natura), which perpetually remains in the honor of its substance. On the contrary, the ugliness of evil is infinite (infinitum), not sustained by its foundations, but always modified by its nature. [...] This will clearly appear if we understand that all the genera of dissimilarities (inaequalitatis species) grew from the similarity, which is their foundation (ab aequalitatis crevisse primordiis). (Inst. arith., I, 32, 1-2; transl. adapted from Moreschini [2014]: 17-18)

The way in which arithmetics reveals and introduces the good is therefore to be found in how it makes the mind aware of what is stable in magnitude and multitude with reference to the peculiar movements of human beings, namely their actions. If these mathematical qualities are thought of as separate from human beings, they reveal the reason for their stability, i.e. the unity of «the first nature», which is the perfect equality to which they lean; if, on the other hand, they are thought of within human beings, these qualities, unstable as they are, may be reduced to equality by means of «imitation and perception», as the passage asserts. Therefore, the moral goal of mathematics is to induce such an imitation in what is deviated from its first nature. It is worth noting that while the insistence on the "definite" character of goodness in Nicomachus marks the contrast between the divine One and the indefinite dyad, which for the Pythagoreans symbolizes the evil as the beginning of multiplicity, in Boethius the goodness of unity is more clearly the "model" to look at for harmonizing the dissimilitude, which is constitutive of natural reality.

Boethius offers two examples of how mathematical sciences are connected to ethics. One pertains to arithmetics, the other to music. Both of them are not included in Nicomachus' tractate, but are inspired by Boethius' other main source, namely Ptolemy's *Harmonics*[15]. Ptolemy had

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[13] For instance in *Inst. mus.*, I, 3: «In quibus autem pluralitas differentiam facit, ea necesse est in quadam numerositate consistere».

[14] On Ptolemy as sources of Boethius' *On Music* see Panti [2017].

[15] See Panti [2017].

asserted that *harmonia* is to be identified as corresponding neither to something natural, nor to God, «but, clearly, to reason, which falls between the other causes mentioned (God and Nature) and joins with them in producing the good» (Ptolemy, *Harmonics,* III, 3, 92.16-24; transl. Barker [1989]: 371-372). Reason, eventually, is the special movement of human minds and is strictly linked to their actions, because human actions are – or should be – rational. In line with these principles, in *On Arithmetics* Boethius extends the notion of action to the political guide of civil life, and compares arithmetic, harmonic and geometric means (*medietates*) to oligarchic, aristocratic and democratic systems of government respectively:

> And thus, the arithmetic mean is compared to a state ruled by a few, because a greater proportion is in its smaller terms. [...] The harmonic mean is the state ruled by the best (optimates), because a greater proportion is found in the greater terms. In the same fashion a geometrical mean is of a state that is democratic (popularis) and equalized. For it is composed of an equal proportion of all, both in its greater and in its smaller terms, and among all there is a parity of mediation that preserves in proportion an equal right (aequum ius). (Inst. arith., II, 45; my translation)

Boethius suggests that these means were known to Pythagoras, Plato and Aristotle and that the central and highest of them is the harmonic (expressed by the proportion 3:4:6), the closest to unity given that the ratio of the differences between the first and the middle and between the middle and the last is equal to the ratio of the extremes, namely $(a-b):(b-c)=a:c$. This, in fact, corresponds to the government of the aristocrats, a clear reference to the Roman senatorial class to which Boethius himself belonged[16].

Yet, if the morality of mathematics were only a matter of comparison with human actions, it would not be such a fundamental issue. In *On music*, Boethius reveals more consistently and openly the ethical relevance of numbers, and particularly of mathematical ratios. Musical knowledge, in fact, concerns numbers and proportions, which are the quantity/*esse* not only of sounds, but also of what links together the body and the soul of human beings and converges with their composition (*animae et corporis status eisdem quodammodo proportionibus videatur esse compositus*) (*Inst. mus.* I, 1, 186, 9-13). Therefore, as the ear is affected by sound or the eye by a visible form, in the same way the judgment of the mind is affected by numbers or by continuous quantity (*Inst. mus.* I, 32, 222, 14-18)[17]. The ratio of pitches is what the *musicus* studies by means of speculation, but Boethius separates music from the other disciplines of the quadrivium by assigning to it alone a clear *influence* on morality. He says that this happens precisely because humans are «harmonic beings». Here, the famous theme of the Platonic *ethos* of music is openly evoked:

> There happen to be four mathematical disciplines, the other three share with music the task of searching for truth; but music is associated not only with speculation but with morality as well. For nothing is more characteristic of human nature than to be soothed by pleasant modes or disturbed by their opposites. (Inst. mus. I, 1, 179, 20-180, 10; transl. Bower-Palisca [1989]: 2)[18]

In other words, while the other mathematical disciplines describe the way in which multitude and magnitude realize their best relations and indicate, consequently, how the same ratios are the best dispositions in natural things and in human actions (as in the abovementioned case of the best forms of government), music *changes* human nature, and affects directly the relations between the soul and the body. Music, in fact, is a therapy for curing the diseases of the mind and of the

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[16] Moreschini [2014]: 21 refers to this passage as an obscure example, but Boethius' goal is clear at the light of the moral character of arithmetics and of its link with human actions.

[17] For the translation of these passages see Bower-Palisca [1989]: 49.

[18] See also the passage from Ptolemy's *Harmonics* quoted at above.

body, as it re-introduces the right ratios in what is deprived of harmony; yet, music can also be pernicious, if it transmits the altered ratios that cause bodily disharmony and psychological alterations. In agreement with Plato, music is really a powerful instrument for ethics and politics[19].

3. THE HIDDEN AESTHETICS OF NUMBERS

If we turn to the relation between mathematics and the beautiful in Boethius' quadrivial treatises, we will hardly find the same clear conception that relates mathematics to ethics. Differently from Augustine, who had developed in his dialogue *On music* a very detailed analysis of human perception of beauty as represented by rhythm, which, in turn, is the expression of numerical ratios[20], Boethius never mentions the beautiful or the beauty in his mathematical treatises. What pleases the ears is named in various ways by him, such as consonance (*consonantia*), sweetness (*dulcedo*), agreement (*concordia*)[21], pleasure and the relative adjectives, but never as beauty (*pulchritudo*) or beautiful (*pulchrum*). This is striking, mainly if compared with the Augustinian abundant and explicit references to beauty in number, rhythm and music. Augustine's aesthetics is clearly and openly grounded on numbers. For him, «the beautiful pleases through number» for a reason that Boethius would not but share: the analysis of our perception of the beautiful in musical rhythm leads to ratios that are ultimately rooted in equality[22].

Augustine, however, was not the only source which Boethius might have looked at for associating numbers and beauty. Ptolemy, who, as seen above, is also an important source for the Boethian conception of mathematical morality, openly

addressed the topic of beauty as an even higher task to which the harmonic investigation leads human beings. As has been rightly remarked, for Ptolemy «the task of harmonics is to explicate the mathematical foundations of systems whose beauty and excellence is evident to the ear, not those of some other, purely theoretical constructions. It seeks, in fact, to show that it is on rationally coherent mathematical patterns of order that the perceived beauty of real music rests»[23]. Ptolemy, in fact, asserts:

> *This sort of power [i.e. the harmonic power which grasps the differences between sounds] employs as its instruments and servants the highest and most marvelous of the senses, sight and hearing, which, of all the senses, are most closely tied to the ruling principle, and which are the only senses that assess their objects not only by the standard of pleasure but also, much more importantly, by that of beauty. (Ptolemy, Harm. III, 3, 93.11-15; transl. Barker [1989]: 372)*

If this human power to grasp sounds is so indissolubly linked with the beautiful, and both Augustine and Ptolemy had openly affirmed it, why, then, does Boethius not say the same thing in his quadrivial treatises? It is risky to speculate on an argument *ex silentio*, yet it is possible to propose an answer if we keep in mind what Aristotle asserts in *Metaphysics*, book 13, in the passage mentioned at the beginning of this paper: the good and the beautiful are different, for the former always implies conduct as its subject, while the beautiful is found also in motionless things. Boethius is clearly conscious of this difference. Music never lacks the "movement". Beauty, therefore, is not involved in music itself as is the good, which is always implied by movement, specifically by that of human actions,actions. Beauty is something something to which music conducts only when the appropriate movement towards unity is realized, and the ethical scope is reached. This is, however, a condition which cannot be gained stably with musical practice and skill, in the same way in which, for instance, music can be fruit-

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[19] Cf. *Inst. mus.*, I, 1.

[20] See Bettetini [1994]: 177-195.

[21] For instance, *Inst. mus.*, II, 10: «Pythagoras hoc modo repperit, qua proportione sibimet haec sonorum concordia iungeretur»; ibid. I, 1: «verum etiam delectetur saepius, si dulces coaptatique modi sint».

[22] For a detailed analysis of this theme in Augustine see Bettetini [1994] and Hentschel [2011].

[23] Barker [1989]: 271.

fully employed for curing illness, for relaxing the body, for exciting the soul and for the other functions related to guide the human behavior and to correct the different kinds of human characters. Boethius is clear in expressing this idea, and underlines that people have different reactions to music according to their characters, ages, attitudes: people «who are rougher delight in the rather uncultivated modes», while people «who are more gentle delight in more moderate modes» (*Inst. mus*. I, 2: 180-182). In musical melodies, the response of pleasure is dependent upon the attitude of each human being. Therefore, musical pleasure is strictly connected to what is good, because music acts upon the character and mental disposition. A moderate and "temperate" music corrects the lascivious dispositions or the excited characters, while the lascivious musical modes corrupt the good dispositions of character, and alter the right psychological and physical balance, so that what pleases is no more oriented to what is good (*Inst. mus*. I, 2: 184-187). For Boethius, however, it is not possible to associate in the same way what pleases in music with what is beautiful.

Yet, the complex relation of music to beauty has also a more hidden reason. Music is movement, hence the convergence to unity is something that is never *firmly* realized, given that music is the movement of sounds, and sounds are movements themselves. There is, therefore, only one kind of music that realizes by itself the stability of unity required to generate really "the beautiful". This is the music of the Platonic soul of the world. This is not a physical music, neither does it immediately refer to the music of the spheres, but is that overall perfect harmony of the animated principle that governs the world, as Boethius remarks in referring to the *Timeus*:

> What Plato rightfully said can likewise be understood: the soul of the universe was joined together according to musical concord. For when we hear what is properly and harmoniously united in sound in conjunction with that which is harmoniously coupled and joined together within us and are attracted to it, then we recognize that we ourselves are put together

> in its likeness, for likeness attracts, whereas unlikeness disgusts and repels. (Inst. mus. I, 1, 179.20-180.10; transl. Bower-Palisca [1989]: 2)

The supreme harmony of the world soul is what makes the universe participate in beauty, which, in itself, is not in the physical world, but in the idea of the cosmos in God's mind. Human reason is what can envisage true beauty, because reason is like a shadow of the divine mind. For Boethius, mathematical sciences are therefore the fundamental steps for disclosing beauty, which – in agreement with Aristotle – is «a cause» and is «in motionless things», and therefore – in agreement with the Christianized Platonism – it is in the Creator's mind. This implicit conclusion is explained in the last work by Boethius, the *Consolation of Philosophy*, written in about 523, while awaiting execution. The *Consolation* is an impressive dialogue between Boethius and Philosophy and portrays the progressive rising of the human mind from complaint and misery to the understanding that providence orders all things for the best, human beings included. Despite appearances, Philosophy proclaims that changing fortune does not rule the world, and that the good is not oppressed[24]. After having realized that the true good is the very source of happiness, Boethius and Philosophy go through each of the supposed goods, which include fame, honor, wealth and also beauty. As proved for all these earthly goods, also the search for physical beauty and pleasure is an illusion, because beauty is quickly lost in bodies subject to illness and corruption. And even something which is not subject to change, such as a bright and colorful jewel that pleases for its radiance, does not possess the real beauty. A key-passage from book 3 of the *Consolation* reveals that the precious stone, as well as all other earthly things, has nothing but a shadow of the higher beauty, the knowledge of which is reserved only for rational creatures:

> I mean, what is there <in the jewel>, lacking the soul's self-motion and indivisibility, that can rightly

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[24] See, *inter alia*, Chadwick [1981]; Moreschini [2014]; Donato [2013].

seem beautiful to a rational nature that has a soul? Even if, by their Creator's efforts and by their own play of colors, they draw to themselves something of the Beautiful at the farthest removed, they are all the same placed far below your own superiority and are in no way deserving of your amazement. (Cons. Phil. II, 5, 8-10; transl. adapted from Relihan [2001]: 35)

The key concept that reveals the true nature of beauty is in the mention of self-motion and indivisibility as attributes pertaining to the soul alone, which has a greater dignity than non-living things. And, among living beings, only the rational soul has the superiority needed to grasp what is really beautiful; and only rational beings can be really beautiful. Human nature, however, is beautiful not for its bodily attractiveness, or for goods such as fame, or «popular honors», because they do not possess «the beauty that belongs to real rank» (*Cons. Phil.* III, 4, 8), as is made clear in the following passage:

Contemplate the vastness, the mightiness, and the swiftness of the heavens and now, at last, stop admiring worthless things. And make no mistake: The heavens are less to be admired for these things than for the reason by which they are governed. And the fair sheen of beauty – how fugitive it is, how swift, more fleeting than the ever-changing flowers of spring! For if, as Aristotle says, people had the eyes of Lynceus, so that their vision could penetrate any barriers, wouldn't the body of Alcibiades, proverbially so beautiful on its surface, seem utterly shameful after they had peered at the guts within? And so, it is not your nature that makes you seem beautiful, but only the weakness of the eyes of those who look upon you. (Cons. Phil. III, 8, 8-11, transl. adapted from Relihan [2001]: 65-66)

Beauty is a matter of reason alone. It is, in Boethius' view, the very reason that governs all things, namely the divine reason. At the core of book 3, the celebrated hymn *O qui perpetua*, inspired from Plato's *Timaeus* and from Proclus' commentary on it and reminiscent of the ancient Neoplatonic hymns on cosmic theology, eventually addresses the beautiful to its proper subject, the

world as a unity in the mind of the Creator, who is the supreme beauty:

You, who governs the world by everlasting reason (perpetua ratione) [...] and, by resting unmoved, put all things in motion, no external causes demanded you to imitate the work (fingere opus) of the mutable matter, but the form of the true good planted within you (insita), totally deprived of envy. You lead all things out from their supreme exemplar, you, the most beautiful, by carrying in your own mind the beautiful world (pulchrum pulcherrimus ipse mundum mente gerens), and by forming it [the world] in conform likeness (similique in imagine formans), and also by commanding that perfect parts make it perfect, you tie (ligas) the elements with numbers ... and making the soul, which moves all things, as mean of a threefold nature, you spread (resolvis) it in equal members (per consona membra). (Cons. Phil. III, 9, transl. adapted from Relihan [2001]: 71-72)

The Creator is perfect unity, stable immobility, and supreme reason. The perpetual possession of these three elements characterizes God as supreme beauty (*pulcherrimus*), to whom the form of the true good belongs. Hence, the world in God's mind is beautiful (*pulcher*), because it is the supreme exemplar (the Platonic *idea*) of the physical world[25]. God is moved to create through his own goodness, and in so doing, he forms the cosmos as an *image* of the beautiful ideal, the latter being the immobile cause of motion of all movable things, and the perfect unity to which the physical plurality tends (*Timaeus*, 29e-30b). Finally, the hymn evokes the Platonic mathematics of the cosmos by asserting, still in line with the *Timaeus*, that God binds the world's elements together through numbers, and by keeping the equilibrium, namely the perfect ratio, of the elemental qualities of cold and hot, dry and wet. So, he also gives a threefold structure to the soul of the world, set in the middle of the cosmos to move all things[26]. Notwithstanding its perfect mathematical structure, the physical world is not

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[25] See Chadwick [1981]: 233-234.

[26] *Timaeus*, 31b-37a; *Laws* 896e; *Phaedrus* 245c.

called beautiful, as is the ideal world. For Boethius, the passage to the physical plurality, materiality and movement implies the rupture of the perfect unity and stability that characterize the ideal world in the divine mind. The real cosmos cannot but evoke sparse and sporadic glimpses of such exemplar beauty, and these glimpses progressively extend and become more "visible" as the creatures grow in the scale of dignity, up to the rational beings, the closest to the divine rationality.

4. CONCLUSIONS

This paper has evidenced the complexity of the Boethian notion of the good and the beautiful in mathematics, which emerges from his extant works on arithmetic and music. Basing his reasoning on Nicomachus's and Ptolemy's attempts at reconciling Plato's and Aristotle's views, the Roman philosopher saw in the mathematical sciences, i.e. the quadrivium, a path to philosophical wisdom, consisting in the passage from sensible knowledge to the highest achievements of the intellect, that is, the knowledge of what is true and stable. This passage is made possible thanks to the way in which mathematical sciences address the mind to the *esse*, namely the continuous and discrete quantities, which are stable and unchanging, if they are *mathematically* thought of as separate from the sensible things, while they are subject to change in their *physical* pertaining to sensible things. We have seen that the possibility to grasp these quantities in their stable ratios is exactly what makes the human mind realize that unity is the perfect ratio and supreme stability to which all other ratios tend.

Thus, the ethical aim of mathematics is to indicate to the human mind how to *correct* the ratios in physical things, and particularly that of the balance of the body and the soul, so as to realize the best relationship in movements and more precisely in the specific form of movement of the rational beings, that is in actions which direct us to unity and perfection. This is exemplified in *On Arithmetics*, in which Boethius represents specific

mathematical ratios as the most perfect forms of government. Yet, the ethical scope of mathematics is elaborated more at length in *On Music*, by presenting the ratios involved in musical consonances and harmonic patterns as having a specific effect in moving the human soul to emotions, actions, and to good or bad dispositions of the mind, namely to psychological and physical affections that music can both evoke and heal. Again, the therapy is the restoring of the perfect balance, the closest to unity.

More complex, on the other hand, is the Boethian relationship between mathematics and beauty. In clear antithesis to the position taken by Augustine on the beauty of the rhythmic patterns that better represent the beauty of unity, Boethius does not relate the mathematical ratios of the consonances to an esthetical judgment by making use of the category of beauty. For him, the physical world is totally immersed in changes and movements, and this cannot but impede things from expressing the *stable* unity, which is required for contemplating the beautiful. Music, in particular, is in itself movement, so that while it naturally expresses its ethical connection, which – as seen above – concerns human movements and actions, it cannot in the same way be linked to the beautiful.

Are there other mathematical sciences connected to beauty? Perhaps geometry, being the discipline of the immobile magnitude, or astronomy, whose object of study is the perpetual circular motion that displays in time the stable mathematical structure of the soul of the world; but, unfortunately, the treatises by Boethius devoted to these disciplines have not reached us. What can be said with regard to the absence of the use of the category of beauty in music is that it strongly affected the early medieval theory of music, deeply inspired by Boethius[27]. An inspec-

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[27] On this dependance, Bower-Palisca [1989]: XIX-XX assert: «When a tradition of independent musical treatises began in the ninth century, Boethius's treatise became the unique source for the thorough mathematical underpinning of Western musical theory. It is ironic that this work intended as an approach to logic and philosophy would essentially shape the most illiberal of the liberal

tion in the corpus of writings on music theory collected in the remarkable website *TML*. *Thesaurus Musicarum Latinarum* makes it clear that, apart from Augustine, the words *pulcher* and *pulchritudo* and their declinations are absent from musical considerations up to the writings of the eleventh century, when the theory of music started to develop autonomously from the Boethian theory and terminology. Gradually during that century, we can notice the reintroduction of this "missing word" to indicate the musical aggregates that are more agreeable to the ears[28]. The musical beautiful has definitively moved from the heavens to the earth.

REFERENCES

Barker, A., 1989: *Greek Musical Writings, vol. 2: Harmonic and Acoustic Theory*, Cambridge University Press, Cambridge.

Bettetini, M., 1994: *La misura delle cose. Struttura e modelli dell'universo secondo Agostino d'Ippona*, Rusconi, Milano.

Bower, C.M., 1978: *Boethius and Nicomachus: An Essay Concerning the Sources of the De institutione musica*, "Vivarium" 16, pp. 1-45.

Burnyeat, M.F., 2000: *Plato on Why Mathematics is Good for the Soul*, "Proceedings of the British Academy" 103, pp. 1-81.

Chadwick, H., 1981: *Boethius, The Consolations of Music, Logic, Theology, and Philosophy*, Oxford University Press, Oxford.

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arts». See also Pizzani [2002]. On the transmission of *On Arithmetics* see, *inter alia*, Masi [1983]. See also Panti [2008].

[28] *Thesaurum Musicarum Latinarum* at http://boethius.music.indiana.edu/tml/. This remarkable website offers the entire corpus of Latin treatises concerning music theory, from the 3rd to the 17th century. The search of *pulch** offers all occurrences of the declination of *pulcher* and *pulchritudo* for the selected centuries. The search has given 14 matches for the 3rd to 8th century, mainly in Augustine and Calcidius, while, for the following centuries, the term starts to be used with reference to musical melodies gradually, mainly from the 11th century, particularly from Guido d'Arezzo onwards.

Charles-Saget, A., 1982: *L'architecture du divin. Mathématique et philosophie chez Plotin et Proclus*, Les Belles Lettres, Paris.

D'Onofrio, G., 2001: *La scala ricamata. La philosophiae divisio di Severino Boezio, tra essere e conoscere*, in D'Onofrio, G. (ed.), *La Divisione della Filosofia e le sue Ragioni. Lettura di testi medievali (VI-XIII secolo)*, Avagliano, Cava De' Tirreni, pp. 11-63.

Donato, A., 2013: *Boethius' Consolation of Philosophy as a Product of Late Antiquity*, A&C Black, Bloomsbury.

Fournier, M., 2008: *Boethius and the Consolation of the Quadrivium*, "Medievalia et Humanistica" n.s. 34, pp. 1-21.

Friedlein, G., 1867: *Anicii Manlii Severini Boethii […] De institutione musica*, Teubner, Leipzig.

Guillaumin, J.-Y., 1990: *Le terme quadrivium de Boèce et ses aspects moraux*, "L'antiquité classique" 59, pp. 139-148.

Helmig, C., 2007: *The Relationship between Forms and Numbers in Nicomachus' Introduction to Arithmetic* in Bonazzi, M., Lévy, C., Steel, C. (eds.), *A Platonic Pythagoras. Platonism and Pythagoreanism in the Imperial Age*, Brepols, Turnhout, pp. 127-146.

Hentschel, F., 2011: *The Sensuous Music Aesthetics of the Middle Ages: The Cases of Augustine, Jacques de Liège and Guido of Arezzo*, "Plainsong and Medieval Music" 20, pp. 1-29 (on line at: https://doi.org/10.1017/S0961137110000173).

Masi, M., 1983: *Boethian Number Theory: A Translation of the De Institutione Arithmetica*, Rodopi, Amsterdam.

Mendell, H., 2017: *Aristotle and Mathematics*, in *The Stanford Encyclopedia of Philosophy*. https://plato.stanford.edu/archives/spr2017/entries/aristotle-mathematics/

Merlan, P., 1975: *From Platonism to Neoplatonism* (3rd edition), Martinus Nijhoff Publishers, The Hague.

Moreschini, C. (eds)., 2005: *Boethius, De consolatione philosophiae. Opuscula theologica*, Saur, München.

Moreschini, C., 2014: *A Christian in Toga: Boethi-*

us: Interpreter of Antiquity and Christian Theologian, Vandenheock & Ruprecht, Gottingen-Bristol.

O'Meara, D.J., 2005: *The Music of Philosophy in Late Antiquity*, in Sharples, R.W. (ed.), *Philosophy and the Sciences in Antiquity*, Ashgate, London, pp. 131-155.

Oosthout, H., Schilling, J. (eds.), 1999: *Anicii Manlii Severini Boethii De arithmetica, CCSL 94A*, Brepols, Turnhout.

Bower, C.M., Palisca, C. (eds.), 1989: *Boethius. Fundamentals of Music*, Yale University Press, New Haven.

Panti, C., 2008: *Filosofia della musica. Tarda Antichità e Medioevo*, Carocci, Roma.

Panti, C., 2017: *Boethius and Ptolemy on harmony, harmonics and human music*, "Micrologus" 25, pp. 3-35.

Pizzani, U., 2002: *The Transmission of Ancient Music Theory into the Middle Ages*, in Christyensen, T. (ed.), *The Cambridge History of Western Music Theory*, Cambridge University Press, Cambridge, pp. 136-167.

Restani, D., 2004: *Musica per governare. Alessandro, Adriano, Teoderico*, Longo, Ravenna.

Restani., D., 2008: *La musica humana e Boezio: ipotesi sulla formazione di un concetto*, "Philomusica on-line" 7, pp. 19-25.

Relihan, J.C. (ed.), 2001: *Boethius, Consolation of philosophy. Translated, with introduction and notes*, Hackett, Indianapolis-Cambridge.

Riegel, N., 2014: *Goodness and Beauty in Plato*, "Archai" 12, pp. 143-154.

Ross, W.D. (ed.), 1924: *Aristotle's Metaphysics, a Revised Text with Introduction and Commentary*, 2 vols., Oxford University Press, Oxford.