

ANALOGUE COSMOLOGY IN A HADRONIC FLUID *

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Abstract. Analog gravity models of general relativity seem promising routes to providing laboratory tests of the foundation of quantum field theory in curved space-time. In contrast to general relativity, where geometry of a spacetime is determined by the Einstein equations, in analog models geometry and evolution of analog spacetime are determined by the equations of fluid mechanics. In this paper we study the analogue gravity model based on massless pions propagating in an expanding hadronic fluid. The analog expanding spacetime takes the form of an FRW universe, with the apparent and trapping horizons defined in the standard way.

1. Introduction

The expansion of hadronic matter that takes place immediately after a heavy ion collision has certain similarity with the cosmological expansion. We study the analogue geometry of the expanding hadronic fluid, using the formalism of relativistic acoustic geometry [1, 2, 3]. We show that the propagation of massless pions provides a geometric analog of expanding spacetime equivalent to an open ($k = -1$) FRW cosmology. Here, we study general conditions for the formation of a trapped region with the inner boundary as a marginally trapped surface.

Our approach is based on the linear sigma model combined with a boost invariant Bjorken type spherical expansion.¹ A Bjorken-type expansion is a simple and very useful hydrodynamic model that reflects the boost invariance of the deep inelastic scattering in high energy collisions.

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¹The assumption of spherical expansion is appropriate for e^+e^- collisions [4] as in this case the jets are produced with no directional preference.

To describe the effective geometry of the expanding hadronic fluid, we introduce the analogue gravity metric $G_{\mu\nu}$. The dynamics of massless pions is described by the equation of motion for three pion fields π^i and sigma meson field σ propagating in curved space-time. The equation of motion is equivalent to the d'Alembertian equation of motion for a massless scalar field propagating in a (3+1)-dimensional Lorentzian geometry

$$(1.1) \quad \frac{1}{\sqrt{-G}} \partial_\mu (\sqrt{-G} G^{\mu\nu}) \partial_\nu \pi + V(\sigma, \pi) \pi = 0,$$

where $G_{\mu\nu}$ is the analogue metric tensor and $V(\sigma, \pi)$ is a potential that describes effective interaction between the mesons.

2. Dynamics of the chiral fluid

In order to draw the analogy with cosmology now we consider a spherically symmetric Bjorken expansion of the chiral fluid which is invariant under radial boosts. In this model the radial three-velocity in radial coordinates $x^\mu = (t, r, \vartheta, \varphi)$ is a simple function $v = r/t$. Then the four-velocity is given by $u^\mu = (t/\tau, r/\tau, 0, 0)$, where $\tau = \sqrt{t^2 - r^2}$ is the proper time. With the substitution $t = \tau \cosh y$, $r = \tau \sinh y$ the radial velocity is expressed as $v = \tanh y$ and the four-velocity as $u^\mu = (\cosh y, \sinh y, 0, 0)$. This substitution may be regarded as a coordinate transformation from ordinary radial coordinates to new coordinates $(\tau, y, \vartheta, \varphi)$ in which the flat background metric takes the form

$$(2.1) \quad g_{\mu\nu} = \text{diag} (1, -\tau^2, -\tau^2 \sinh^2 y, -\tau^2 \sinh^2 y \sin^2 \theta),$$

and the velocity components become $u^\mu = (1, 0, 0, 0)$. Hence, the new coordinate frame is comoving. The metric corresponds to an FRW expanding cosmological model with cosmological scale $a = \tau$ and negative spatial curvature. We mapped the spatially flat Minkowski spacetime into an expanding FRW spacetime with cosmological scale $a = \tau$ and negative spatial curvature. The resulting flat spacetime with metric (2.1) is known in cosmology as the *Milne universe* [6].

The temperature of the expanding chiral fluid, to a good approximation, is proportional to τ^{-1} . This follows from the fact that the chiral matter is dominated by massless pions, and hence, the density of the fluid may be approximated by the density $\rho = (g\pi^2/30)T^4$ of an ideal massless boson gas [7]. Using this and the energy-momentum conservation one finds $T = c_0/\tau$ where the constant c_0 may be fixed from the phenomenology of high energy collisions.

3. Dynamics of pions

The dynamics of pions in the hadronic fluid can be described using a linear sigma model as an effective low energy model of strong interactions. The basic

model involves four scalar fields (three pions and a sigma meson) $\varphi \equiv (\sigma, \boldsymbol{\pi})$ which constitute the $(\frac{1}{2}, \frac{1}{2})$ representation of the chiral $SU(2) \times SU(2)$. In the chirally symmetric phase at temperatures above the chiral transition point the mesons are massive with equal masses. In the chirally broken phase the pions are massless and sigma meson acquires a nonzero mass proportional to the chiral condensate.

At temperatures below the chiral phase transition point the pions, although being massless, propagate slower than light [8, 9, 10] with a velocity approaching zero at the critical temperature. Hence, it is very likely that there exists a region where the flow velocity exceeds the pion velocity and the analogue trapped region may form.

The dynamics of mesons in a medium is described by a chirally symmetric Lagrangian of the form [9, 10, 11]

$$(3.1) \quad \mathcal{L} = \frac{1}{2}(a g^{\mu\nu} + b u^\mu u^\nu) \partial_\mu \varphi \partial_\nu \varphi - \frac{m_0^2}{2} \varphi^2 - \frac{\lambda}{4} (\varphi^2)^2,$$

where u_μ is the velocity of the fluid, and $g_{\mu\nu}$ is the background metric. The parameters a and b depend on the local temperature T and on the parameters of the model m_0 and λ and may be extracted from the pion self-energy at non-zero temperature. [11] At zero temperature the medium is absent in which case $a = 1$ and $b = 0$. Propagation of pions is governed by the equation of motion (1.1) with the analogue metric tensor given by

$$(3.2) \quad G_{\mu\nu} = \frac{a}{c_\pi} [g_{\mu\nu} - (1 - c_\pi^2) u_\mu u_\nu],$$

and the pion velocity squared $c_\pi^2 = a/(a+b)$. Hence, the pion field propagates in a (3+1)-dimensional effective geometry described by the metric $G_{\mu\nu}$. It is convenient to work in comoving coordinates $(\tau, y, \vartheta, \varphi)$ with background metric $g_{\mu\nu}$ defined by (2.1). In these coordinates the analogue metric tensor (3.2) is diagonal with components

$$(3.3) \quad G_{\mu\nu} = \frac{a}{c_\pi} \text{diag} (c_\pi^2, -\tau^2, -\tau^2 \sinh^2 y, -\tau^2 \sinh^2 y \sin^2 \theta),$$

where the parameters a and c_π are functions of the temperature T which in turn is a function of τ . In the following we assume that these functions are positive.

In contrast to [1], where it was assumed that both the background geometry and the flow were stationary, in an expanding fluid the flow is essentially time dependent. Hence, the acoustic geometry formalism must be adapted to a non-stationary space time.

4. Analogue horizons

For a relativistic flow in curved spacetime the apparent and trapping horizons may be defined in the same way as in general relativity.

The key element in the study of trapped surfaces is the expansion parameter ε_{\pm} of null geodesics. A two-dimensional surface S with spherical topology is called a *trapped surface* if the families of ingoing and outgoing null geodesics normal to the surface are both converging or both diverging. More precisely, the expansion parameters

$$(4.1) \quad \varepsilon_{\pm} = \nabla_{\mu} l^{\mu}_{\pm}$$

on a trapped surface S should satisfy $\varepsilon_{+}\varepsilon_{-} > 0$. A two-dimensional surface H is said to be *future inner marginally trapped* if the future directed null expansions on H satisfy the conditions: $\varepsilon_{+}|_H = 0$, $l^{\mu}_{-}\partial_{\mu}\varepsilon_{+}|_H > 0$ and $\varepsilon_{-}|_H < 0$. We shall refer to this surface as the *apparent horizon* since it is equivalent to the apparent horizon in cosmological context.

To define the analogue apparent horizon we need to examine the behaviour of radial null geodesics of the analogue metric (3.3) in which a and c_{π} are functions of τ . Using the geodesic equation $l^{\mu}\nabla_{\mu}l^{\nu} = 0$, from (4.1) we find the condition for the apparent horizon

$$(4.2) \quad \frac{1}{v} \pm \frac{\dot{\alpha}}{\beta} = 0$$

where $\alpha(\tau) = \tau\sqrt{a/c_{\pi}}$ and $\beta(\tau) = \sqrt{ac_{\pi}}$. This equation defines a hypersurface dubbed the *analogue trapping horizon* and its solution determines the location of the analogue apparent horizon r_H as a function of time. From (4.2) it follows that the region of spacetime $\tanh y \geq |\beta/\dot{\alpha}|$ is trapped. Specifically, it is future trapped if $\dot{\alpha} < 0$ and past trapped if $\dot{\alpha} > 0$.

Spacetime diagrams corresponding to the metric (3.3) are presented in Fig. 4.1 showing future directed radial null geodesics. The origin in the plots in both panels corresponds to the critical value τ_c at which c_{π} vanishes. At $\tau = \tau_{\max}$ we have $|\beta/\dot{\alpha}| = 1$ so the trapping horizon ends at the point $\tau = \tau_{\max}$, $y = \infty$.

We next examine the analogue Hubble rate, in particular its behavior in the neighborhood of the critical point. For the spacetime defined by the metric (3.3) the Hubble rate is given by $\mathcal{H} = \partial_{\tau}(\tau\sqrt{a/c_{\pi}})/(a\tau)$. We find that \mathcal{H} is negative for τ in the entire range $\tau_c \leq \tau \leq \tau_{\max}$ and scales as $\mathcal{H} \propto -(\tau - \tau_c)^{-1.17}$ as τ approaches τ_c . Hence, our cosmological model describes a shrinking FRW universe with a singularity at the critical point.

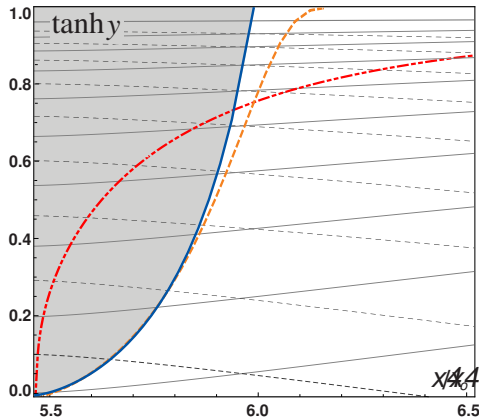


FIG. 4.1: Spacetime diagram of outgoing (full line) and ingoing (dashed line) radial null geodesics in (τ, y) coordinates. The shaded area represents the evolution of the trapped region. The trapping horizon is represented by the full bold line with the endpoint at $\tau = \tau_{\max} = 6.0182\tau_0$. The dashed and dash-dotted bold lines represent the evolution of the analogue and naive Hubble horizons, respectively.

5. Surface gravity and analogue Hawking effect

Next we study the Hawking effect associated with the analogue apparent horizon. The surface gravity κ of a Killing horizon can be defined by

$$(5.1) \quad \xi^\nu \nabla_\nu \xi_\mu = \kappa \xi_\mu,$$

evaluated on the horizon. If the geometry were stationary, the analogue apparent horizon would coincide with the analogue event horizon at the hypersurface defined by $v = c_\pi$.

In the case of non-stationary spacetime, the apparent horizon is neither Killing nor null. The definition of surface gravity in this case is not unique [12] and several ideas have been put forward how to generalize the definition of surface gravity for the apparent horizon [13, 14, 15, 16]. We adopt the prescription of [13] which, we believe, is most suitable for spherical symmetry. This prescription involves the so called Kodama vector K^μ [17] which generalizes the concept of the time translation Killing vector to non-stationary spacetimes. The Kodama vector we define as [14, 18]

$$(5.2) \quad K^\alpha = k \epsilon^{\alpha\beta} n_\beta \text{ for } \alpha = 0, 1; \quad K^i = 0 \text{ for } i = 2, 3,$$

where $\epsilon^{\alpha\beta}$ is the covariant two-dimensional Levi-Civita tensor in the space normal to the surface of spherical symmetry and n_α is a vector normal to that surface. The normalization factor k has to be adjusted so that K^μ coincides with the time

translation Killing vector ξ^μ for a stationary geometry. In analogy with (5.1) the surface gravity κ is defined by [13, 19]

$$(5.3) \quad K^\alpha \nabla_{[\alpha} K_{\beta]} = \kappa K_\beta,$$

where the quantities should be evaluated on the trapping horizon. Using this definition we find [20]

$$(5.4) \quad \kappa = \frac{c_\pi}{2\tau} \frac{1 + 2c_\pi v(1-v) - (2+c_\pi)v^3}{\gamma v(1+c_\pi v)^2} + \frac{\ddot{a}}{2\beta} \frac{v}{\gamma(1+c_\pi v)^2}$$

where it is understood that the right-hand side is evaluated on the trapping horizon.

In the limiting case when the quantities a , and c_π are constants, the apparent horizon is determined by the condition $v = c_\pi$ and the expression for κ reduces to $\kappa = 1/2t = \sqrt{1 - c_\pi^2}/2\tau$. Hence, the analogue surface gravity is finite for any physical value of c_π and is maximal when $c_\pi = 0$. However, with $c_\pi = 0$ the horizon degenerates to a point located at the origin $r = 0$. The temperature

$$(5.5) \quad T_H = \frac{\kappa}{2\pi}$$

is the analogue Hawking temperature of thermal pions emitted at the apparent horizon as perceived by an observer at infinity. Since the background geometry is flat, this temperature equals the locally measured Hawking temperature at the horizon. As we move along the trapping horizon the radius of the apparent horizon increases and the Hawking temperature decreases rapidly with τ . Hence, there is a correlation between T_H and the local fluid temperature T , which is related to τ .

In contrast to the usual general relativistic Hawking effect, where the Hawking temperature is much smaller than the temperature of the background, the analogue horizon temperature is of the order or even larger than the local temperature of the fluid. The Hawking temperature correlates with the local temperature of the fluid at the apparent horizon and diverges at the critical point [21].

6. Conclusion

Formation of an analogue apparent horizon in an expanding hadronic fluid is similar to the formation of a black hole in a gravitational collapse although the role of an outer trapped surface is exchanged with that of an inner trapped surface. Unlike a black hole in general relativity, the formation of which is indicated by the existence of an outer marginally trapped surface, the formation of an analogue black (or white) hole in an expanding fluid is indicated by the existence of a future or past inner marginally trapped surface.

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