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VORTEX SOLITONS AT THE INTERFACE BETWEEN TWO PHOTONIC LATTICES[†]

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Abstract. *Using numerical analysis we demonstrate the existence of vortex solitons at the interface separating two different photonic lattices. We consider the conditions for the existence of discrete vortex states at such interface and also study their stability. A novel type of interface vortex solitons with five lobes is observed. Also different topological charges and phase structures of such solutions are studied, as well as influence of different lattice intensities. Other observed solutions are in the form of discrete solitons with six lobes. For lower beam powers such solutions are stable during propagation, but for higher beam powers they oscillate during propagation in a way indicating the exchange of power between neighboring lobes, or show dynamical instabilities.*

Key words: *vortex, photonic lattice, surface soliton*

1. INTRODUCTION

One of the goals of modern nonlinear optics is the development of the ultimate fast, all-optical device in which light can be used to control light. Self-trapped, self-guided light beams, optical spatial solitons that do not spread because of diffraction when they propagate in a nonlinear bulk medium, are considered information-carrying units, and the process of all-optical switching can be associated with the evolution of different types of spatial optical solitons and the interactions between them.

Optical surface waves are a special type of localized waves existing at the interface between two media with different optical properties. They attract great attention with their possible application in surface sensing and probing, and have been the subject of intense study in diverse areas of physics [1]. Such surface waves were observed to exist in a variety of systems: between metal and a linear dielectric medium (plasmon waves) [2] at the boundary of semi-infinite periodic multilayer dielectric media [3], in Kerr media [4], waveguide arrays [5], metamaterials [6], optical amplifiers [7] etc.

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Special attention has been devoted to the study of nonlinear optical surface waves, owing to the fact that the nonlinear response of materials makes possible the dynamic control of surface localization. The interplay of periodicity and nonlinearity can facilitate the formation of different types of surface modes localized at and near the surface, and a series of theoretical [8–13] and subsequent experimental [14–17] investigations have demonstrated nonlinearity-induced light localization at the interface and the formation of the so-called discrete surface solitons.

There has been a renewed interest in optical beams carrying angular momentum–vortex solitons–in many branches of science, including plasmas, Bose-Einstein condensates, superfluids, and nonlinear optics [18, 19]. Vortex solitons are self-localized nonlinear waves that possess a phase singularity with a total phase accumulation of 2πTC for a closed circuit around the singularity. The integer number TC is the vorticity or topological charge of the vortex, and its sign defines the direction of the phase circulation. Nonlinear periodic systems such as photonic lattices can stabilize optical vortices in the form of stable discrete vortex solitons [20, 21]. Recently, some examples of surface vortex solitons have been observed, at the boundaries of photonic lattices [22, 23], or at the interface between two optical lattices with the same geometry but with different refractive index [24].

In this paper, we extend this analysis to the case of vortex solitons supported by different interfaces separating square and hexagonal photonic lattices [25, 26]. We study a more general case and investigate vortex solitons at the interface separating two lattices of different symmetries. In particular, we determine the conditions for the existence of discrete vortex states at such interface and also study their stability. We found novel types of interface vortex solitons with five lobes and considered also different topological charges and phase structures of such solutions. The existence domains of interface vortex solitons as well as the regions of stability are observed. Also, influence of different lattice intensities on such vortex states is studied.

2. MODELING OF VORTEX PROPAGATION AT LATTICE INTERFACES

The propagation of vortex beams at the interface separating square and hexagonal photonic lattice, is described using the scaled nonlinear Schrödinger equation for the optical electric field amplitude A [27]:

$$i\partial_z A+\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)A+\Gamma\frac{|A|^2+V}{1+|A|^2+V}A=0 \qquad (1)$$

where x, y and z are the transverse and longitudinal coordinates normalized to the characteristic beam width and diffraction length, Γ is the dimensionless strength of the nonlinearity, and $V(x,y)$ is the transverse lattice potential, given as a sum of square and hexagonal potentials $V(x,y)=V_s(x,y)+V_h(x,y)$, with the peak intensities V_{0s} and V_{0h}, respectively. A vortex beams, positioned at the corresponding interface lattice sites, are launched into the lattice, perpendicular to the input crystal face (see Fig. 1(a) and Fig. 4(a)).

First, we investigate the existence of vortex solitonic solutions. The above equation suggests their existence in the form $A=a(x,y)exp(i\mu z)$, where $a(x,y)=|a(x,y)|\exp[i\varphi(x,y)]$

is a complex-valued function, $\varphi(x; y)$ is the phase distribution, and μ is the propagation constant. After substitution of the solitonic solution form in Eq. (1), it transforms into:

$$-\mu A+(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})a+\Gamma a\frac{|a|^2+V}{1+|a|^2+V}=0 \quad (2)$$

The solitonic solutions can be found from Eq. (2) by using the modified Petviashvili's iteration method [28, 29]. We determine different classes of vortex surface solitons by launching vortex beams whose rings are covering lattice sites near the interface separating two photonic lattices. Vortex beams with different topological charges are used as input. In this paper, we analyze two different classes of interface vortex solitons: discrete solitons consisting of five and six lobes.

Next, to investigate the stability of such solutions, we use interface surface solitons as input beams in Eq. (1). Numerical procedure is based on the fast-Fourier transform split-step numerical algorithm.

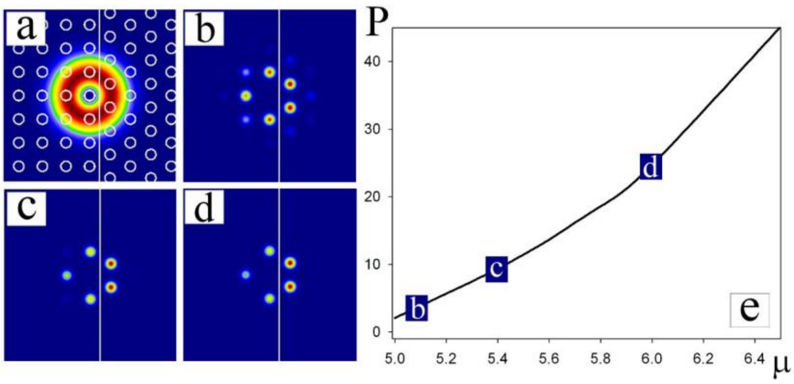

Fig. 1 Surface vortex solitons with five lobes. Input vortex beam is shown at the lattice interface (a), with the layout of the lattice beams indicated by open circles. (e) Power diagram for the existence of five-lobe surface vortex solitons. The corresponding intensity distributions for different vortex solitons are presented in (b), (c) and (d); the lines depict the lattice interface. Parameters: $\Gamma = 11$, the lattice peak intensities $V_{0s}=V_{0h}=2.5$, vortex topological charge TC=1.

3. INTERFACE DISCRETE VORTEX SOLITIONS

We start with searching for spatially localized vortex soliton solutions at the interface with the same lattice intensities ($V_{0s}=V_{0h}$). It is well known that the lattice induces confinement of the filaments approximately at the location of the incident vortex ring and the surrounding lattice sites. First, we choose the input ring vortex beam to cover the lattice sites adjacent to the square lattice part of the interface (Fig. 1(a)). The corresponding power diagram is presented in Fig. 1(e). The beam power for vortex solitons is given by the formula: $P=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|a|^2\,dxdy$. The characteristic outcomes in the form of five-lobe solution are shown in Fig. 1 (b), (c) and (d). Increasing the values of propagation constant μ, there are observed the solutions with higher intensity lobes in the hexagonal lattice part of the interface. Five-lobe vortex solitons with lower values of P are stable during propagation for short propagation distances, but asymmetric solutions for higher P show oscillations or dynamical instabilities.

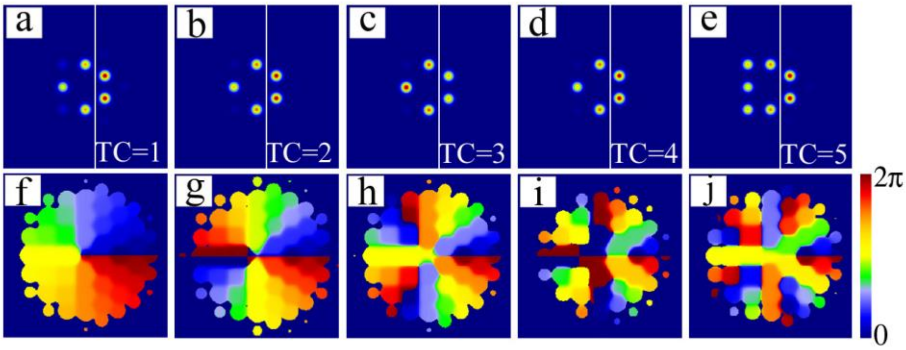

Fig. 2 Five-lobe surface vortex solitons with different topological charges: intensity distributions (the first row) and corresponding phase distributions (the second row). Parameters: $\mu=5.2$, other parameters are as in Fig.1.

Next, we choose the same input ring vortex beam as in Fig. 1(a) but with different topological charges TC. Figure 2 presents the five different kinds of discrete vortex solutions at the interface separating square and hexagonal lattice. The asymmetry of the vortex soliton depends on the input topological charge, and it is more pronounced for the vortex with TC=5. Investigating the stability of such vortex solitons, we find very regular oscillations for lower values of TC, and dynamical instabilities for TC=5.

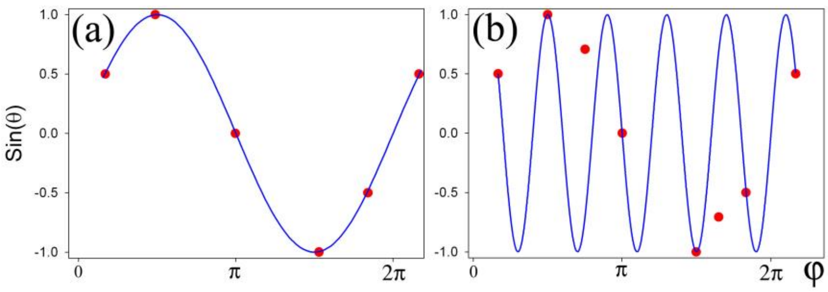

Fig. 3 Sin(θ) vs φ (azimutal angle for the lattice) diagram for the vortex solitons with: (a) TC=1 from Fig. 2(f), and (b) TC=5 from Fig. 2(j).

To clarify and analyze the observed phase distributions we take one contour where soliton lobes are located, and measure the phase. We plot sin of the lobe angle (sin(θ)) as a function of the corresponding lobe phase (φ). For five-lobe solution with TC=1 (Fig. 3(a)) we can see that the phase (red points) are perfectly fitted by the sinusoidal function (blue line) with one period. But for TC=5 solution (Fig. 3(b)) we have five periods. Also, two red points that are not fitted by blue line correspond to the additional 2 lobes of the vortex soliton with TC=5 (Fig. 2(e)). These graphs show the different topological charges contained in these two solutions, and they confirm a well-defined discrete vortex structure.

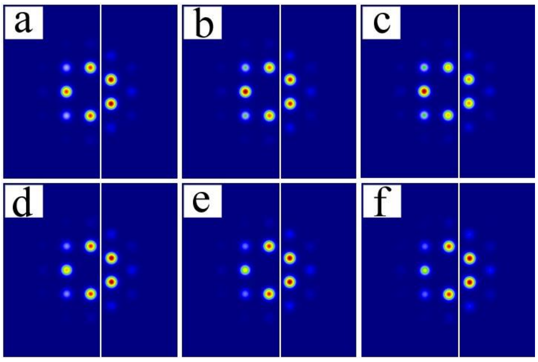

Fig. 4 Surface vortex solitons at different lattice interfaces: (a) V_{0s}=2.51, V_{0h}=2.5; (b) V_{0s}=2.53, V_{0h}=2.5; (c) V_{0s}=2.55, V_{0h}=2.5; (d) V_{0s}=2.5, V_{0h}=2.51; (e) V_{0s}=2.5, V_{0h}=2.53; (f) V_{0s}=2.5, V_{0h}=2.55.

Also, we want to put more attention to the investigation of different lattice interfaces (Fig. 4). We consider few interfaces with different intensities of square and hexagonal lattice parts. Again, we choose the input ring vortex beam to cover the lattice sites adjacent to the square lattice part of the interface (as in Fig. 1(a)).

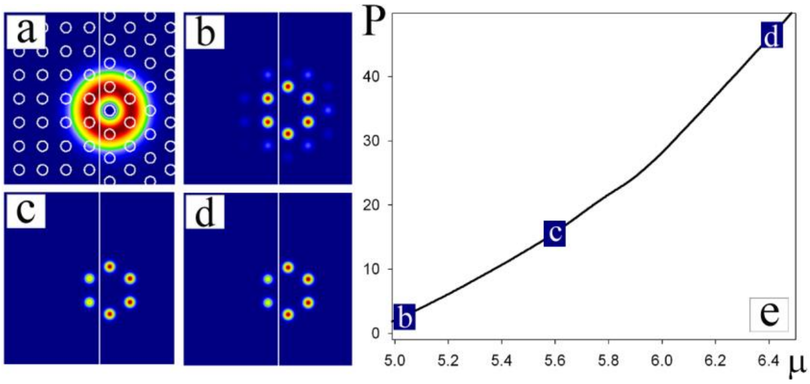

Fig. 5 Surface vortex solitons with six lobes. Input vortex beam is shown at the lattice interface (a), with the layout of the lattice beams indicated by open circles. (e) Power diagram for the existence of six-lobe surface vortex solitons. The corresponding intensity distributions for different vortex solitons are presented in (b), (c) and (d); the lines depict the lattice interface. Parameters are as in Fig. 1.

One can see that various lattice intensities in the square lattice part could drastically change the shape of the solution (Fig. 4 (a)-(c)), but it is not the case for various lattice intensities in the hexagonal lattice part (Fig. 4 (d)-(e)).

Next, we choose the input ring vortex beam to cover the lattice sites adjacent to the hexagonal lattice part of the interface (Fig. 5(a)). Figure 5 presents the six-lobe discrete vortex solutions at the interface separating square and hexagonal lattice: the characteristic outcomes are shown in Fig. 5 (b), (c) and (d). The corresponding power diagram is presented for six-lobe states in Fig. 5(e). The symmetric interface vortex solitons with six lobes can exist for lower values of the propagation constant μ. But increasing the values of propagation constant, one can observe asymmetric solutions with higher intensity lobes in the hexagonal lattice part of the interface. Investigating the stability of such solutions, we observed that symmetric kinds of six-lobe vortex solitons are stable during propagation and can exist for long propagation distances. The asymmetric solutions show oscillations or dynamical instabilities during propagation.

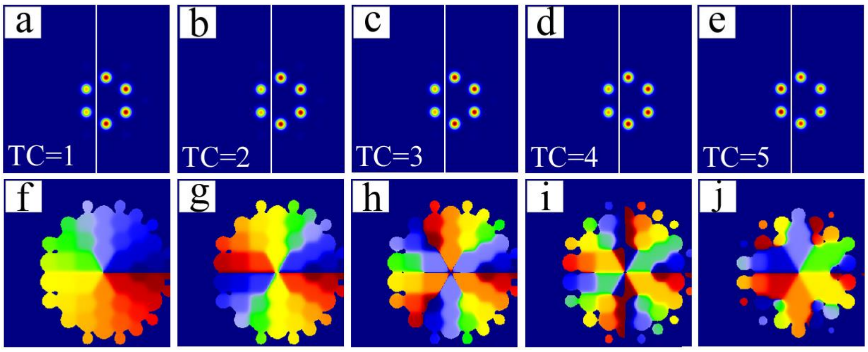

Fig. 6 Six-lobe surface vortex solitons with different topological charges: intensity distributions (the first row) and corresponding phase distributions (the second row). Parameters: $\mu=5.2$, other parameters are as in Fig.1.

The influence of various topological charges on the six-lobe interface vortex solitons is also considered (Fig. 6). The same input ring vortex beam as in Fig. 5(a) is used but with different topological charges TC. As before, we choose topological charges from 1 to 5. For the input vortices with TC=1-4 one can observe six-lobe vortex states with the same shape but different phase distributions. Different intensity distribution shape is visible for the vortex with TC=5. Investigating the stability of such vortex solitons, we observe regular oscillations for very long distances.

Also, we compare our results to the vortex solitons located in the bulk of the square and hexagonal lattices. Using the same vortex as the input we observed typical four-lobe in the square and six-lobe solitons in hexagonal lattices. If one compares power P for the corresponding values of propagating constant μ, we observed the lowest values for vortex solitons in square lattice, followed by the interface vortex solitons and then in the hexagonal lattice. This also means the same order of the P(μ) diagrams.

Finally, we discuss in more details the (in)stability of interface vortex solutions. Stable solutions are observed in the form of six-lobes for very long propagation distances, for lower values of propagation constant μ. Five-lobe solutions with lower powers are stable only for short propagation distance, but increasing power or TC they show regular oscillations such that neighboring lobes exchange power and then very irregular oscillations. The most illustrative cases of interface vortex solitons are presented in figure 7 along the propagation distance. Figure 7(a) shows typical behavior of symmetric six-lobe solutions during propagation. In the case of five-lobe solitons, we present the behavior of solutions for TC=1 (Fig. 7(b)) and TC=5 (Fig. 7(c)). During propagation, neighboring lobes exchange power and then irregular oscillations take place and they are more pronounced for solution with TC=5.

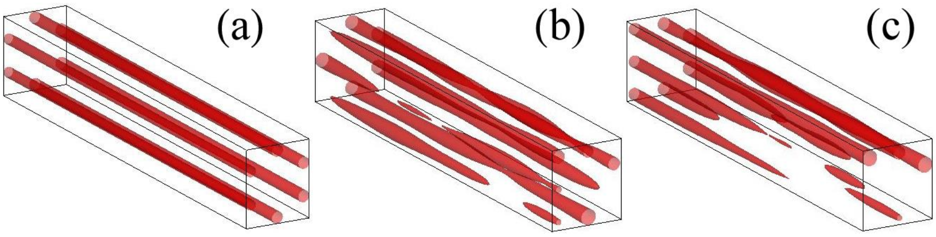

Fig. 7 Typical interface vortex solitons in propagation. The six- and five-lobe solitons are shown along the propagation direction for 200 mm. The parameters are the same as in: (a) Fig. 5(b), (b) Fig. 2(a), (c) Fig. 2(e).

4. CONCLUSIONS

We have studied surface vortex solitons at the interface separating square and triangular photonic lattices, and revealed the existence of novel types of discrete vortex surface solitons in the form of five-lobe solution. We have developed a concise picture of different scenarios of the vortex solutions behavior, and investigated their stability. Various vortices with different topological charges are considered, as well as various lattice interfaces with different lattice intensities. Beside the stable six-lobe discrete surface modes propagating for long distances, we have observed various oscillatory vortex surface solitons, as well as dynamical instabilities of different kinds of solutions. Dynamical instabilities occur for higher values of the propagation constant, or at higher beam powers.

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VORTEKSNI SOLITONI NA GRANICI FOTONSKIH REŠETKI

Korišćenjem numeričke analize pokazano je postojanje vorteksnih solitona na granici koja deli dve različite fotonske rešetke. Razmatrani su uslovi za postojanje diskretnih vorteksnih stanja na ovakvoj granici i izučavana je njihova stabilnost. Pronađen je novi oblik površinskih vorteksnih solitona sa pet pikova. Takođe su izučavana različita topološka naelektrisanja i fazne strukture ovakvih rešenja, kao i uticaj različitih intenziteta rešetki. Pronađena su i rešenja u obliku diskretnih solitona sa šest pikova. Za manje snage ovakva rešenja su stabilna tokom propagacije, ali za veće snage ona osciluju na taj način da razmenjuju snagu između susednih pikova, ili pokazuju dinamičke nestabilnosti.

Ključne reči: *vorteks, fotonska rešetka, površinski solitoni*