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Deteriorating Item Inventory Model with Shortages, Variable Holding Cost and Time Dependent Quadratic Demand: An Optimization Approach Using With and Without Controllable Rate of Deterioration

(With & without preservation technology variable holding cost and time dependent quadratic demand model)

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Abstract— The purpose of this study is to develop time dependent quadratic demand and variable holding cost, a model of inventory system, for instantaneous deteriorating items with the consideration of the facts that the deteriorating rate can be controlled by using the preservation technology (PT). A solution procedure is presented to find the optimal solution of the cost function. Shortages are allowed and partially backlogged. The backlogging rate is assumed to be dependent on the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Results have been validated with relevant examples. Sensitivity analysis is performed to show the effect of changes in the parameters on the optimum solution for both the cases that is with and without using the preservation technology respectively. The analysis of the model shows that the solution of the model is quite stable and can be applied for optimizing the inventory cost of deteriorating items for the business enterprises

Keywords: Inventory; Cost Optimization; Preservation Technology; Time dependent demand; Variable Holding Cost

I. INTRODUCTION

In reality, it is observed that during shortage period all demand are either backlogged or lost since some customers are willing to wait for the next replenishment Goyal and Giri [2001] explained the recent trends of modeling in deteriorating inventory. Ouyang, Wu and Cheng [2005] developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. Dye and Ouyang [2007] found an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. They assume that a fraction of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases. Singh and Singh [2007] presented an EOQ inventory model with Weibull distribution deterioration, Ramp type demand and Partial Backlogging. Ajanta Roy [2008] developed an inventory model for deteriorating items with time varying holding cost and price dependent demand. Huang and Hsu [2008] presented a simple algebraic approach to find the exact optimal lead time and the optimal cycle time in the constant markets demand situations. Sarala Pareek and Vinoda Kumar Mishra [2009] developed a deterministic inventory model for deteriorating items with salvage value and shortage. Nita Shah and Kunal Shukla [2009] developed a deteriorating inventory model for waiting time partial

backlogging when demand is constant and deterioration rate is constant. Singh, T.J., Singh, S.R. and Dutt, R. [2009] extended an EOQ model for perishable items with power demand and partial backlogging. Skouri, Konstantaras, Papachristos and Ganas [2009] developed an inventory models with ramp type demand rate, partial backlogging and Weibell's deterioration rate.

Mishra and Singh [2011] developed a deteriorating inventory model with partial backlogging when demand and deterioration rate is constant. Vinod kumar Mishra [2013] developed an inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost. J. Jagadeeswari and P. K. Chenniappan [2014] developed an order level inventory model for deteriorating items with time dependent quadratic demand and partial backlogging. Sarala Pareek and Garima Sharma [2014] developed an inventory model with Weibull distribution deteriorating item with exponential declining demand and partial backlogging. Kirtan Parmar and U. B. Gothi [2014] developed a deterministic inventory model for deteriorating items where time to deterioration has Exponential distribution and with time-dependent quadratic demand. In this model, shortages are not allowed and holding cost is time-dependent. Also, U.B. Gothi and Kirtan



Parmar [2015] have extended above deterministic inventory model by taking two parameter Weibull distribution to represent the distribution of time to deterioration and shortages are allowed and partially backlogged. Kirtan Parmar and U. B. Gothi [2015] developed an economic production model for deteriorating items using three parameter Weibull distribution with constant production rate and time varying holding cost.

The inventory system for deteriorating items has been an object of study for a long time, but a few authors have studied the effect of investing in reducing the rate of product deterioration. In practice, the deterioration rate of products can be controlled and reduced through various efforts such as procedural changes and specialized equipment acquisition etc. The consideration of preservation technology (PT) is important due to rapid social changes. There are huge number of paper in literature with shortage which is not possible to mention all here due to lack of space.

This paper has been formulated and is design and experimented with the empirical data by considering with or without using the preservation technology, influence by the survey of the literature since the Preservation technology can reduce the deterioration rate significantly by which one can reduce the economic losses, improve the customer service level and increase business competitiveness.

In reality, the demand and holding cost for physical goods may be time dependent. Time also plays and important role in the inventory system. An exponentially time-varying demand seems to be unrealistic because an exponential rate of change is very high and it is doubtful whether the market demand of any product may undergo with such a high rate of change as exponential. So, in this paper we consider time dependent quadric demand and holding cost is also time dependent

II. MATHEMATICAL MODEL

A deterministic inventory model for deteriorating items under time dependent quadratic demand and variable holding cost using an optimization approach considering with and without preservation technology investment have been analyzed where shortages are allowed and partially backlogged as Model-I. The backlogging rate is assumed to be dependent on the length of waiting time for the next replenishment. The following notations are used throughout the paper.

Notations

OC Ordering Cost C_d Deteriorating cost per unit per unit time h_1 Holding cost per unit time in the time interval $[0, \mu]$

 h_2 Holding cost per unit time in the time interval $[\mu, t_1]$

 θ_0 Constant deterioration rate

 ξ Preservation Technology (PT) cost

 $m_1(\xi)$ Reduced deterioration rate due to preservation technology

 $\hat{\theta}(\xi) = (\theta_0 - m_1(\xi))$ Resultant deterioration rate,

T Duration of cycle

 K_0 Setup Cost

 $I_0(t)$ Initial inventory level

 $I_i(t)$ Inventory level in the i^{th} interval $[t_i, t_{i-1}]$, where i = 1, 2, 3

The following criteria has been fulfilled to formulate the mathematical model

- i. The replenishment rate is finite.
- ii. Shortages are allowed and partially backlogged.
- iii. The inventory system deals with single item.
- iv. The planning horizon is finite.
- v. Lead time is zero
- vi. The demand function D(t) is considered as $D(t) = a + bt + ct^2$ where a, b and c are constant.
- vii. Holding cost HC is variable function that is $HC = HC_1 + HC_2$ where $HC_1 = h_1$; $0 \le t \le \mu$ and $HC_2 = h_2$; $\mu \le t \le t_1$
- viii. Preservation technology is used for controlling the deteriorating rate θ

III. MODEL FORMULATION & SOLUTION MODEL-WITH SHORTAGE



Figure-1: inventory behavior with time

During the period $[0, \mu]$ the inventory depletes due to demand only but during the period $[\mu, t_1]$ the inventory depletes due to deterioration as well as demand. At time t_1 the inventory reaches zero level, where the shortages start and, in the period, $[t_1, t_2 = T]$ shortages are allowed and backlogged partially as shown in Figure-1. The level of the inventory at time t is I(t). If $I_1(t)$ be the on hand



inventory level at any time $t \ge 0$ in the time interval $0 \le t \le \mu$ and $I_2(t)$ be the on hand inventory level at any time $t \ge 0$ in the time interval $\mu \le t \le t_1$ then at time $t + \Delta t$, the inventory in the interval $[0,\mu]$ and $[\mu,t_1]$ will be respectively

$$I_1(t + \Delta t) = I_1(t) - \left(a + bt + ct^2\right) \Delta t; 0 \le t \le \mu$$
(1)

$$I_2(t+\Delta t) = I_2(t) - \theta I_2(t)\Delta t - (a+bt+ct^2)\Delta t; \mu \le t \le t_1$$
⁽²⁾

Dividing equation (1) and (2) by Δt and then taking limit as $\Delta t \rightarrow 0$

$$\frac{dI_1(t)}{dt} = -\left(a + bt + ct^2\right); 0 \le t \le \mu \tag{3}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(a+bt+ct^2); \mu \le t \le t_1 \qquad (4)$$

Since initially the level of inventory is I_0 and the level reduces to zero at time $t = t_1$ hence the boundary conditions are

$$I_1(t=0) = I_0 \tag{5}$$

$$I_2(t = t_1) = 0 (6)$$

Solution of equation (3) & (4) using (5) and (6) respectively as follows:

$$I_{1}(t) = I_{0} - \left(at + \frac{bt^{2}}{2} + \frac{ct^{3}}{3}\right)$$
(7)

$$I_{2}(t) = \begin{pmatrix} k - (a + k\theta)t + (a\theta - b)\frac{t^{2}}{2} + \\ (b\theta - 2c)\frac{t^{3}}{6} + (c\theta)\frac{t^{4}}{12} \end{pmatrix}$$
(8)

where

k

$$= \begin{pmatrix} at_{1} + (a\theta + b)\frac{t_{1}^{2}}{2} + \\ (b\theta + c)\frac{t_{1}^{3}}{3} + (c\theta)\frac{t_{1}^{4}}{4} \end{pmatrix}$$
(9)

Using the continuity at time $t = \mu$, that is $I_1(\mu) = I_2(\mu)$ which gives

$$I_{0} - \left(a\mu + \frac{b\mu^{2}}{2} + \frac{c\mu^{3}}{3}\right) = \begin{pmatrix} k - (a + k\theta)\mu + (a\theta - b)\frac{\mu^{2}}{2} + \\ (b\theta - 2c)\frac{\mu^{3}}{6} + (c\theta)\frac{\mu^{4}}{12} \end{pmatrix}$$
(10)

After simplification and neglecting higher power of θ the initial level of inventory is given as:

$$I_0 = k - k\theta\mu + a\theta\frac{\mu^2}{2} + b\theta\frac{\mu^3}{6} + c\theta\frac{\mu^4}{12}$$

or $I_0 = A + B\theta$ (11)

where $A = a(t_1 - \mu) + \frac{b(t_1^2 - \mu^2)}{2} + \frac{c(t_1^3 - \mu^3)}{3}$ and $p = \frac{aL}{2} + \frac{bM}{2} + \frac{cN}{2}$ with $L = (t_1 - \mu)^2$.

and
$$B = \frac{\mu L}{2} + \frac{\mu M}{6} + \frac{cN}{12}$$
 with $L = (t_1 - \mu)^2 (t_1 - \mu)^2 (2t_1 + \mu)$ and
 $N = (t_1 - \mu)^2 (3t_1^2 + 2t_1\mu + \mu^2)$ (12)

Total demand fulfilled in the time interval $0 \le t \le t_1$ is as follows:

$$\int_{0}^{t_{1}} \left(a+bt+ct^{2}\right) dt = at_{1} + \frac{bt_{1}^{2}}{2} + \frac{ct_{1}^{3}}{3}$$

During the stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. The backlogging rate is assumed to be $\frac{1}{1+\delta(T-t)}$ where the backlogging parameter is $\delta(0 < \delta < 1)$. If $I_3(t)$ be the on-hand inventory level at any time $t \ge 0$ in the time interval $t_1 \le t \le T$ then at time

t + Δ t, then the inventory will be as follows $I_3(t + \Delta t) = I_3(t) - \frac{a + bt + ct^2}{1 + \delta(t - t)} \Delta t; \quad t_1 \le t \le T$

Dividing equation (13) by Δt and then taking limit as $\Delta t \rightarrow 0$

$$\frac{dI_3(t)}{dt} = -\frac{a+bt+ct^2}{1+\delta(T-t)}; \ t_1 \le t \le T \quad (13)$$

With boundary conditions $I_3(T) = -S$ and $I_3(t_1) = 0$

Solution of equation (14) is given by

$$I_{3}(t) = \begin{pmatrix} \left(\frac{b\delta + c\delta T + c}{\delta^{2}}\right)t + \left(\frac{c}{2\delta}\right)t^{2} + \\ R\log(1 + \delta(T - t)) + k_{1} \end{pmatrix}$$
(14)

Where k_1 is the constant of integration and

$$R = \frac{\delta + bT + cT^{2}}{\delta} + \frac{b + 2cT}{\delta^{2}} + \frac{c}{\delta^{3}}$$

With the boundary condition $I_3(t_1) = 0$ which gives

$$k_{1} = -\frac{b\delta + c\delta T + c}{\delta^{2}}t_{1} + \frac{c}{2\delta}t_{1}^{2} + R\left[1 + \delta(T - t_{1})\right].(15)$$

Therefore

$$I_{3}(t) = \begin{pmatrix} \left(\frac{b\delta + c\delta T + c}{\delta^{2}}\right)(t - t_{1}) + \\ \left(\frac{c}{2\delta}\right)(t^{2} - t_{1}^{2}) + \\ R\log\left(\frac{1 + \delta(T - t)}{1 + \delta(T - t_{1})}\right) \end{pmatrix} (16)$$

The maximum shortage fulfilled S is given as

$$S = I_{3}(T) = \begin{pmatrix} \left(\frac{b\delta + c\delta T + c}{\delta^{2}}\right)(T - t_{1}) + \\ \left(\frac{c}{2\delta}\right)(T^{2} - t_{1}^{2}) - \\ R\log\left\{1 + \delta(T - t_{1})\right\} \end{pmatrix}$$
(17)



Cost Calculation

Case-1: With preservation technology investment

The cost of the model is the combination of only the setup cost, deteriorating cost, holding cost, purchase cost, preservation technology cost, lost sale cost and shortage cost as follows:

Setup Cost
$$OC$$
 is: $OC = K_0$ (18)

Deterioration Cost DC is: $DC = C_d$ {Initial level of inventory total domand fulfilled)

Holding Cost *HC* is:
$$HC = HC_1 + HC_2$$

$$DC = C_d \left\{ I_0 - \left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3}\right) \right\} = (19)$$

Where,

$$HC_{1} = h_{1} \int_{0}^{\mu} I_{1}(t) dt ; \quad HC_{2} = h_{2} \int_{\mu}^{t_{1}} I_{2}(t) dt$$

Hence,

$$HC = \begin{bmatrix} h_{1} \left\{ \left(at_{1} + \frac{bt_{1}^{2}}{2} + \frac{ct_{1}^{3}}{3} \right) \mu + \\ B\theta\mu - \left(a\mu + \frac{b\mu^{2}}{2} + \frac{c\mu^{3}}{3} \right) \right\}^{+} \\ h_{2} \left\{ B + \theta \left(\frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60} \right) \right\}^{+} \end{bmatrix}$$
where $X = (t_{1} - \mu)^{3}, Y = (t_{1} - \mu)^{3} (3t_{1} + \mu)$

and $Z = (t_1 - \mu)^3 (6t_1^2 + 3\mu t_1 + \mu^2)$. (21)

Purchase Cost $PC = C_p * I_0 = C_p (A + B\theta)$ (22)

Preservation Technology Cost $PTC = \xi t_1$ (23) Lost sale cost

$$LSC = c_{l} \int_{t_{1}}^{t} \left[1 - \frac{1}{1 + \delta(T - t)} \right] (a + bt + ct^{2}) dt$$
(24)
$$= c_{l} \left\{ \frac{a\delta^{2} + c\delta T + b\delta + c}{\delta^{2}} (T - t_{1}) + \frac{b\delta + c}{2\delta} (T^{2} - t_{1}^{2}) + \frac{c}{3} (T^{3} - t_{1}^{3}) + \frac{b\delta + c}{2\delta} (T^{2} - t_{1}^{2}) + \frac{c}{3} (T^{3} - t_{1}^{3}) + \frac{b\delta + c}{2\delta} (T^{2} - t_{1}^{2}) + \frac{c}{3} (T^{3} - t_{1}^{3}) + \frac{b\delta + c}{2\delta} (T^{2} - t_{1}^{2}) + \frac{c}{3} (T^{3} - t_{1}^{3}) + \frac{b\delta + c}{2\delta} (T^{2} - t_{1}^{2}) + \frac{c}{3} (T^{3} - t_{1}^{3}) + \frac{b\delta + c}{2\delta} (T^{3} - t_$$

Shortage Cost

$$SC = c_{s} \int_{t_{i}}^{T} I_{3}(t) dt = c_{s} \begin{cases} \frac{3b\delta + 4c\delta T + 2c\delta t_{1} + 3c}{6\delta^{2}} (T - t_{i})^{2} - \\ R(T - t_{i}) + \frac{R}{\delta} \log\{1 + \delta(t - t_{i}) \end{cases}$$
(25)
$$= c_{s} \begin{cases} \left(\frac{b\delta + c\delta T + c}{\delta^{2}}\right) \left\{\frac{T(T - 2t_{i}) - t_{i}^{2}}{2}\right\} + \\ \left(\frac{c}{2\delta}\right) \left\{\frac{T(T^{2} - 3t_{i}^{2}) - 2t_{i}^{3}}{3}\right\} - \\ R\log(1 + \delta(T - t_{i}))(T - t_{i}) + \\ R\left[\frac{-t_{i}\log(1 + \delta(T - t_{i})) - \\ (T - t_{i}) + \frac{1 + \delta T}{\delta} \left[\log(1 + \delta(T - t_{i}))\right] \right] \end{cases}$$

The Cost of Model TC = OC + DC + HC+PC + PTC + SC + LSC is given as follow:

$$TC = \begin{cases} K_{0} + C_{d}B\theta - C_{d}\left(a\mu + \frac{b\mu^{2}}{2} + \frac{c\mu^{3}}{3}\right) + \\ \left(at_{1} + \frac{bt_{1}^{2}}{2} + \frac{ct_{1}^{3}}{3}\right)h_{1}\mu - \left(a\mu + \frac{b\mu^{2}}{2} + \frac{c\mu^{3}}{3}\right)h_{1} + \\ h_{1}B\theta\mu + Bh_{2} + \thetah_{2}\left(\frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60}\right) + \\ \xi t_{1} + C_{p}\left[\left(A + B\theta\right) + \left(\frac{b\delta + c\delta T + c}{\delta^{2}}\right)(T - t_{1}) + \\ \left(\frac{c}{2\delta}\right)(T^{2} - t_{1}^{2}) - R\log\left\{1 + \delta(T - t_{1})\right\}\right] \\ c_{s}\left\{\frac{3b\delta + 4c\delta T + 2c\delta t_{1} + 3c}{6\delta^{2}}(T - t_{1})^{2} - \\ R(T - t_{1}) + \frac{R}{\delta}\log\{1 + \delta(T - t_{1})\right\} + \\ c_{t}\left\{\frac{a\delta^{2} + c\delta T + b\delta + c}{2\delta}(T^{2} - t_{1}^{2}) + \frac{c}{3}(T^{3} - t_{1}^{3}) + \\ R\log\{1 + \delta(T - t_{1})\right\} \right\} \end{cases}$$
(26)

Case-2: Without preservation technology investment

The cost of the model without preservation technology investment is the combination of only setup cost, deteriorating cost, holding cost, purchase cost, lost sale cost and shortage cost as follows:

Setup Cost
$$OC$$
 is: $OC = K_0$ (27)

Deterioration Cost *DC* is: $DC = C_d$ {Initial level of inventory – total demand fulfilled}

level of inventory – total demand fulfilled }
$$\left(\begin{pmatrix} bt^2 & ct^3 \end{pmatrix} \right)$$

$$DC = C_d \left\{ I_0 - \left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \right) \right\} =$$

$$C_d \left\{ B\theta_0 - \left(a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) \right\}$$
(28)

Holding Cost HC is: $HC = HC_1 + HC_2$

Hence,

$$HC = \begin{bmatrix} h_1 \left\{ \left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} \right) \mu + \\ B\theta_0 \mu - \left(a\mu + \frac{b\mu^2}{2} + \frac{c\mu^3}{3} \right) \right\} + \\ h_2 \left\{ B + \theta_0 \left(\frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60} \right) \right\} \end{bmatrix}$$
(29)

Purchase Cost $PC = C_p * I_0 = C_p (A + B\theta_0)$ (30)

Lost sale cost

$$LSC = c_{l} \int_{t_{1}}^{t} \left[1 - \frac{1}{1 + \delta(T - t)} \right] (a + bt + ct^{2}) dt$$
(31)
$$= c_{l} \left\{ \frac{a\delta^{2} + c\delta T + b\delta + c}{\delta^{2}} (T - t_{1}) + \frac{b\delta + c}{2\delta} (T^{2} - t_{1}^{2}) + \frac{c}{3} (T^{3} - t_{1}^{3}) + \frac{b\delta + c}{2\delta} (T^{2} - t_{1}^{2}) + \frac{c}{3} (T^{3} - t_{1}^{3}) + \frac{b\delta + c}{2\delta} (T - t_{1}) + \frac{b\delta + c}{2\delta} (T - t_{1}) + \frac{b\delta + c}{2\delta} (T^{2} - t_{1}^{2}) + \frac{c}{3} (T^{3} - t_{1}^{3}) + \frac{b\delta + c}{2\delta} (T - t_{1}) + \frac{b\delta + c}{2\delta} (T - t_{1})$$



Shortage Cost

$$SC = c_s \int_{t_1}^{T} I_3(t) dt = c_s \begin{cases} \frac{3b\delta + 4c\delta T + 2c\delta t_1 + 3c}{6\delta^2} (T - t_1)^2 - \\ R(T - t_1) + \frac{R}{\delta} \log\{1 + \delta(t - t_1) \end{cases}$$
(32)
$$= c_s \begin{cases} \left(\frac{b\delta + c\delta T + c}{\delta^2}\right) \left\{\frac{T(T - 2t_1) - t_1^2}{2}\right\} + \\ \left(\frac{c}{2\delta}\right) \left\{\frac{T(T^2 - 3t_1^2) - 2t_1^3}{3}\right\} - \\ R\log(1 + \delta(T - t_1))(T - t_1) + \\ R \\ \left[\frac{-t_1 \log(1 + \delta(T - t_1)) - \\ (T - t_1) + \frac{1 + \delta T}{\delta} \left[\log(1 + \delta(T - t_1))\right] \right] \end{cases}$$

The Cost of the system is given as follow:

$$TC = OC + DC + HC + PC + SC + LSC$$

$$\begin{bmatrix} K_{0} + C_{d}B\theta - C_{d}\left(a\mu + \frac{b\mu^{2}}{2} + \frac{c\mu^{3}}{3}\right) + \\ \left(at_{1} + \frac{bt_{1}^{2}}{2} + \frac{ct_{1}^{3}}{3}\right)h_{1}\mu - \left(a\mu + \frac{b\mu^{2}}{2} + \frac{c\mu^{3}}{3}\right)h_{1} + \\ h_{1}B\theta\mu + Bh_{2} + \thetah_{2}\left(\frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60}\right) + \\ C_{p}\left[\left(A + B\theta\right) + \left(\frac{b\delta + c\delta T + c}{\delta^{2}}\right)(T - t_{1}) \\ + \left(\frac{c}{2\delta}\right)(T^{2} - t_{1}^{2}) - R\log\left\{1 + \delta(T - t_{1})\right\} \right] \\ C_{s}\left\{ \frac{3b\delta + 4c\delta T + 2c\delta t_{1} + 3c}{6\delta^{2}}(T - t_{1})^{2} - \\ R(T - t_{1}) + \frac{R}{\delta}\log\{1 + \delta(T - t_{1}) + \\ c_{l}\left\{ \frac{a\delta^{2} + c\delta T + b\delta + c}{2\delta}(T^{2} - t_{1}^{2}) + \frac{c}{3}(T^{3} - t_{1}^{3}) + \\ R\log\{1 + \delta(T - t_{1}) + \\ \end{array} \right\}$$
(33)

IV. OBJECTIVE

The objective of the study is to determine the optimal value of the preservation cost ξ for the model where preservation has been done. The optimal value of ξ which minimizes the total cost TC that is ξ^* will be calculated from equation (34) using Mathematica-software as given by, $\frac{dTC}{d\xi} = \left[-\alpha \theta_0 e^{-\alpha \xi} \begin{pmatrix} C_d B + h_1 B \mu \\ +h_2 \left(\frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60} \right) + C_p B \end{pmatrix} + t_1 \right] = 0$ (34)

If and only if the second derivative with respective to ξ of the cost TC is positive in nature or as follows:

$$\frac{d^2 TC}{d\xi^2} = \left[\alpha^2 \theta_0 e^{-\alpha\xi} \left(\frac{C_d B + h_l B\mu}{+h_2 \left(\frac{aX}{6} + \frac{bY}{24} + \frac{cZ}{60} \right) + C_p B} \right) \right] > 0$$
(35)

The second objective of the study is to determine the optimal value of the time μ when the deterioration start which minimizes TC. Now differentiating *TC* with respect to μ , where μ is a discrete variable twice as follows:

$$\frac{\partial TC}{\partial \mu} = \begin{bmatrix} C_{d}\theta \frac{\partial B}{\partial \mu} - C_{d}(a+b\mu+c\mu^{2}) + \\ \left(at_{1} + \frac{bt_{1}^{2}}{2} + \frac{ct_{1}^{3}}{3}\right)h_{1} - \left(a+b\mu+c\mu^{2}\right)h_{1} + h_{1}B\theta \\ + \left(h_{1}\theta\mu+h_{2}\right)\frac{\partial B}{\partial \mu} + \thetah_{2}\left(\frac{a}{6}\frac{\partial X}{\partial \mu} + \frac{b}{24}\frac{\partial Y}{\partial \mu} + \frac{c}{60}\frac{\partial Z}{\partial \mu}\right) \\ + C_{p}\left(\frac{\partial A}{\partial \mu} + \theta\frac{\partial B}{\partial \mu}\right) \end{bmatrix}$$

$$\frac{\partial^{2}TC}{\partial \mu^{2}} = \begin{bmatrix} C_{d}\theta \frac{\partial^{2}B}{\partial \mu^{2}} - C_{d}(b+2c\mu) \\ - \left(b+2c\mu\right)h_{1} + 2h_{1}\theta\frac{\partial B}{\partial \mu} + \left(h_{1}\theta\mu+h_{2}\right)\frac{\partial^{2}B}{\partial \mu^{2}} + \\ \thetah_{2}\left(\frac{a}{6}\frac{\partial^{2}X}{\partial \mu^{2}} + \frac{b}{24}\frac{\partial^{2}Y}{\partial \mu^{2}} + \frac{c}{60}\frac{\partial^{2}Z}{\partial \mu^{2}}\right) \\ + C_{p}\left(\frac{\partial^{2}A}{\partial \mu^{2}} + \theta\frac{\partial^{2}B}{\partial \mu^{2}}\right) \end{bmatrix}$$

$$(37)$$

If and only if $\frac{\partial^2 TC}{\partial \mu^2} > 0$ then the optimal value of

 μ will be calculated from equation (36) using Mathematica-software.

Proposition 1: The remark (35) that is double derivative of the cost TC with respect to ξ is convex in nature if

$$B > 0, X > 0, Y > 0$$
 and $Z > 0$

Proof: The value of **B** will be positive since $t_1 > \mu$. By using equation (12)

$$B = \frac{aL}{2} + \frac{bM}{6} + \frac{cN}{12}$$

where $L = (t_1 - \mu)^2 > 0$, $M = (t_1 - \mu)^2 (2t_1 + \mu) > 0$ and $N = (t_1 - \mu)^2 (3t_1^2 + 2t_1\mu + \mu^2) > 0$ hence B > 0. Moreover X > 0, Y > 0 and Z > 0, can be shown directly since $t_1 > \mu$ by using equation (21) where $X = (t_1 - \mu)^3$,

$$Y = (t_1 - \mu)^3 (3t_1 + \mu) \text{ and}$$

$$Z = (t_1 - \mu)^3 (6t_1^2 + 3\mu t_1 + \mu^2)$$

Proposition 2: The remark (37) $\frac{\partial^2 TC}{\partial \mu^2} > 0$ that

is double derivative of the total cost TC of the model with respect to μ is convex in nature if $h_2 > 2h_1$ and a > b > 2c in such a way that overall $ah_2 > b(h_1 + C_d + C_p)$ and also $bh_2 > 2c(h_1 + C_d + C_p)$

Proof: Equation (37) has been rewritten after substituting the derivative of B, X, Y and Z with respect to μ as given by:



$$\frac{\partial^2 TC}{\partial \mu^2} = \begin{bmatrix} -(C_a + h_1 + C_p)(b + 2c\mu) + (a + b\mu + c\mu^2)h_2 \\ \left[2h_1 \begin{cases} \frac{a}{2}(-2t_1 + 2\mu) + \frac{b}{6}(-3t_1^2 + 3\mu^2) \\ + \frac{c}{12}(-4t_1^3 + 4\mu^3) \end{cases} \right] \\ \theta \\ \left[(C_a + h_1\mu + C_p)(a + b\mu + c\mu^2) \\ + h_2 \begin{cases} \frac{a}{6}6(t_1 - \mu) + \frac{b}{24}12(t_1^2 - \mu^2) \\ + \frac{c}{60}20(t_1^3 - \mu^3) \end{cases} \right] \end{bmatrix}$$

Rearranging as follows:

$$\frac{\partial^{2}TC}{\partial\mu^{2}} = \left[\begin{pmatrix} a + \\ b\mu + \\ c\mu^{2} \end{pmatrix} h_{2} - \begin{pmatrix} C_{d} + \\ h_{1} + \\ C_{p} \end{pmatrix} \begin{pmatrix} b + \\ 2c\mu \end{pmatrix} \right]$$
(38)
$$+ \theta \left[\begin{pmatrix} h_{2} - \\ 2h_{1} \end{pmatrix} \begin{cases} a(t_{1} - \mu) + \\ \frac{b}{2}(t_{1}^{2} - \mu^{2}) \\ + \frac{c}{3}(t_{1}^{3} - \mu^{3}) \end{cases} + \begin{pmatrix} a + \\ b\mu + \\ c\mu^{2} \end{pmatrix} \begin{bmatrix} C_{d} \\ + C_{p} \\ + h_{1}\mu \end{bmatrix} \right]$$

Second part will be positive if and only if $h_2 > 2h_1$. Moreover the first part after properly rearranging gives

 $\begin{bmatrix} ah_2 - b(C_d + h_1 + C_p) \end{bmatrix} + \mu \begin{bmatrix} bh_2 - 2c(C_d + h_1 + C_p) \end{bmatrix} + c\mu^2 h_2$ This part is positive if a > b > 2c in such a way that overall $ah_2 > b(h_1 + C_d + C_p)$ and

 $bh_2 > 2c(h_1 + C_d + C_p)$. Hence the proposition-2 is proved.

Numerical ANALYSIS

The proposed model for validation has been experimented with the empirical data, keeping in view of the proposition-1 and proposition-2 as follows:

Ordering cost OC is 300, deteriorating cost per unit per unit time C_d is 5, holding cost per unit time h_1 in the time interval $0 \le t \le \mu$ is 1, holding cost per unit time h_2 in the time interval $\mu \le t \le t_1$ is 3, constant deterioration rate θ_0 is 0.2, the constant a is 10, b is 8 and C is 5 in the demand function D(t), preservation parameter α is 2, the purchasing cost per unit C_p is 15, shortage cost per unit c_s is 2, lost sale cost per unit c_l is 2, duration of a cycle with shortage T is 12, and the backlogging parameter $\delta(0 < \delta < 1)$ is 0.03.

The optimal value of the preservation technology cost ξ^* for reducing the deterioration rate in order to preserve the product, the optimal time when the deterioration starts μ^* and the optimal cost TC^* with and without preservation technology investment respectively has been calculated using equation (34), (36), (26), (33) as follows in table -1.

With preservation technology			Without preservation technology		
ξ*	μ^{*}	With Shortage TC*	μ^{*}	With Shortage TC*	
3.92639	1.67518	75929.16	3.43187	43393.3	

Table-1: Optimal values of ξ , μ and TC

V. SENSITIVITY ANALYSIS

The effect of changes in various parameters of the proposed model, which may happen due to uncertainties in any decisive situation, that is the sensitivity analysis is carried out by changing the specified parameter a, b, c, h_1 , h_2 and θ_0 by -20%, -10%,+10% and +20% keeping the remaining other parameter at their standard value. Table-2 shows the sensitiveness of the various parameters on optimal value of ξ^* , μ^* , total cost TC^* with and without preservation technology investment respectively as follows:

Parameter	%	% With preservation				Without	
	change	technology investment			preservation		
					technology		
					investment		
		ξ*	<i>u</i> *	Total	u*	Total cost	
		2	<i>C</i>	cost	~		
				With		With	
				shortage		shortage	
				тс		тс*	
а	+20	-0.02	4.21	-0.41	0.42	0.20	
	+10	-0.01	2.11	-0.39	0.21	0.10	
	-10	0.01	-2.14	0.39	-0.21	0.10	
	-20	0.02	-4.32	0.41	-0.41	0.20	
b	+20	-0.10	0.23	-0.38	16.00	-3.63	
	+10	-0.05	0.12	-0.39	16.13	-2.23	
	-10	0.05	-0.13	0.41	16.42	3.11	
	-20	0.10	-0.27	0.38	16.56	3.54	
с	+20	-0.30	-4.08	-0.20	-0.95	-17.08	
	+10	-0.19	-2.14	-0.35	-0.51	-8.54	
	-10	0.19	2.39	0.38	0.59	8.54	
	-20	0.30	5.07	0.24	2.26	17.05	
h,	+20	-0.47	9.53	-0.31	2.54	2.03	
	+10	-0.23	4.66	-0.36	1.27	1.10	
	-10	0.24	-4.82	0.44	-1.27	-0.83	
	-20	0.49	-9.01	0.40	-2.55	-1.84	
h ₂	+20	-0.50	-23.50	4.48	-3.78	13.00	
	+10	-0.25	-12.55	2.04	-2.00	6.39	
	-10	0.26	14.63	-2.85	2.27	-6.89	
	-20	0.52	32.07	-5.29	4.90	-13.58	
$\theta_{\rm o}$	+20	-2.32	-0.04	-0.40	-2.18	15.48	
	+10	-1.21	-0.02	-0.32	-1.15	7.67	
	-10	1.34	0.02	0.30	1.32	-7.93	
	-20	2.84	0.04	0.43	2.86	-8.07	

Table-2: Sensitivity analysis of ξ , μ and TC



The study manifested the following facts:

- Optimal value of ξ^{*} changes slightly with the change in the value of parameters a, b, c, and θ₀, and moderately with change in h₁, h₂
- Optimal value of µ^{*} (with preservation technology) changes slightly with the change in b and θ₀, moderately with a and c, and highly with h₁, h₂. Whereas, optimal value of µ^{*} (without preservation technology) changes slightly with the change in h₁, moderately with a, c, h₂, θ₀ and highly with the change in b.
- Optimal value of TC^* (with preservation technology) changes moderately with the change in a, b, c, h_1, h_2 and θ_0 . Whereas, optimal value of TC^* (without preservation technology) changes slightly with a and h_1 , moderately with b, highly with c, h_2 and θ_0 .

Graphical analysis

The graphical representation of the optimal total cost TC^* with respect to the time μ \and duration of cycle that is T has been shown in figure-2 with preservation technology and in figure-3 without preservation technology. Also, the graphical representation of TC^* with respect to μ and ξ has been shown in figure-4



Figure-2: Graph of μ vs T vs TC







Figure-4: Graph of μ vs ξ vs TC

VI. CONCLUSION

It is obvious that, the demand rate cannot be considered as a fixed in the business world. Hence researcher allows the demand rate of the item to be a function of time. The items (like food grains, fashion apparels and electronic equipments etc.) have fixed shelf-life which decreases with time during the end of the season. The products with high deterioration rate are always crucible to the retailer's business. However, the tradeoff between the increased cost due to investment on improving deterioration rate and the increased profit due to decreased deterioration rate is complex. Based on this a new inventory model for deteriorating item with shortages is design and experimented with the empirical data. This study develops a deteriorating inventory model where the retailer may invest in the PT cost. The results give management managerial insights in the amount of investment on PT equipments or facilities. In the future study, it is hoped to further extend the proposal model into several situations such as the consideration of multiitem problem or for items having stock dependent demand, price dependent demand or power demand.

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