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KENMOTSU MANIFOLDS ADMITTING SCHOUTEN-VAN KAMPEN CONNECTION

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Abstract. The objective of the present paper is to study the Kenmotsu manifold admitting the Schouten-van Kampen connection. We study the Kenmotsu manifold admitting the Schouten-van Kampen connection satisfying certain curvature conditions. Also, we prove the equivalent conditions for the Ricci soliton in a Kenmotsu manifold to be steady with respect to the Schouten-van Kampen connection.
Keywords: Ricci solitons, Kenmotsu manifolds, Schouten-van Kampen connection, concircular curvature tensor, projective curvature tensor, conharmonic curvature tensor, shrinking.

1. Introduction

The Schouten-van Kampen connection has been introduced for studying non-holomorphic manifolds. It preserves - by parallelism - a pair of complementary distributions on a differentiable manifold endowed with an affine connection [2] [9] [17]. Then, Olszak studied the Schouten-van Kampen connection to adapt it to an almost contact metric structure [14]. He characterized some classes of almost contact metric manifolds with the Schouten-van Kampen connection and established certain curvature properties with respect to this connection. Recently, Gopal Ghosh [7] and Yildiz [24] studied the Schouten-van Kampen connection in Sasakian manifolds and f-Kenmotsu manifolds, respectively. Kenmotsu manifolds introduced by Kenmotsu in 1971[10] have been extensively studied by many authors [20] [15] [16]. In 1982, Hamilton [8] introduced the notion of Ricci flow to find a canonical metric on a smooth manifold. Since then the Ricci flow has become a powerful tool for the study of Riemannian manifolds. The Ricci soliton, considered to be a self-similar solution to the Ricci flow is a Riemannian metric g on a manifold M, together with a vector field V such that

$$(L_V g)(X,Y) + 2S(X,Y) + 2\lambda g(X,Y) = 0, \tag{1.1}$$

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where $L_{\rm V}$ denotes the Lie derivative along V, and S and λ are respectively the Ricci tensor and a constant. A Ricci soliton is said to be shrinking or steady or expanding depending on whether λ is negative, zero or positive. A Ricci soliton is said to be a gradient Ricci soliton if the vector field V is the gradient of some smooth function f on M. In [18], Sharma started the study of Ricci solitons in the K-contact geometry. In 2016, the authors in [21] explained the nature of Ricci solitons in f-Kenmotsu manifolds with a semi-symmetric non-metric connection. Ramesh Sharma et al. [18] [19], De et al. [4][1], and Nagaraja et al. [12] [11] [13] extensively studied Ricci solitons in contact metric manifolds in many different ways.

This paper is structured as follows. After a brief review of Kenmotsu manifolds in Section 2, in Section 3 we obtain the expressions of the curvature tensor, Ricci tensor and scalar curvature with respect to the Schouten-van Kampen connection, study the curvature properties of the Kemotsu manifold admitting the Schouten-van Kampen connection, and prove the conditions for the Kenmotsu manifold admitting the Schouten-van Kampen connection to be isomorphic to the hyperbolic space. In the last section we prove the equivalent conditions for the Ricci soliton in a Kenmotsu manifold admitting the Schouten-van Kampen connection to be steady.

2. Preliminaries

A $(2n+1)$-dimensional smooth manifold M is said to be an almost contact metric manifold if it admits an almost contact metric structure (ϕ, ξ, η, g) consisting of a tensor field ϕ of type $(1,1)$, a vector field ξ, a 1-form η and a Riemannian metric g compatible with (ϕ, ξ, η) satisfying

$$\phi^2 X = -X + \eta(X)\xi,\ \phi\xi = 0,\ g(X,\xi) = \eta(X),\ \eta(\xi) = 1, \eta \circ \phi = 0, \tag{2.1}$$

and

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y). \tag{2.2}$$

An almost contact metric manifold is said to be a Kenmotsu manifold [3] if

$$(\nabla_X \phi)Y = -g(X, \phi Y)\xi - \eta(Y)\phi X, \tag{2.3}$$

where ∇ denotes the Riemannian connection of g.

In a Kenmotsu manifold the following relations hold [6].

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.4}$$

$$(\nabla_X \eta)Y = g(\nabla_X \xi, Y), \tag{2.5}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.6}$$

$$S(X,\xi) = -2n\eta(X), \tag{2.7}$$

$$S(\phi X, \phi Y) = S(X,Y) + 2n\eta(X)\eta(Y), \tag{2.8}$$

for any vector fields X, Y, Z on M, where R denote the curvature tensor of type $(1,3)$ on M.

3. Kenmotsu manifolds admitting Schouten-van Kampen connection

Throughout this paper we associate $*$ with the quantities with respect to the Schouten-van Kampen connection. The Schouten-van Kampen connection ∇^* associated to the Levi-Civita connection ∇ is given by [14]

$$\nabla^*_X Y = \nabla_X Y - \eta(Y)\nabla_X \xi + (\nabla_X \eta)(Y)\xi, \tag{3.1}$$

for any vector fields X, Y on M.
Using (2.4) and (2.5), the above equation yields,

$$\nabla^*_X Y = \nabla_X Y + g(X,Y)\xi - \eta(Y)X. \tag{3.2}$$

By taking $Y = \xi$ in (3.2) and using (2.4) we obtain

$$\nabla^*_X \xi = 0. \tag{3.3}$$

We now calculate the Riemann curvature tensor R^* using (3.2) as follows:

$$R^*(X,Y)Z = R(X,Y)Z + g(Y,Z)X - g(X,Z)Y. \tag{3.4}$$

Using (2.6) and taking $Z = \xi$ in (3.4), we get

$$R^*(X,Y)\xi = 0. \tag{3.5}$$

On contracting (3.4), we obtain the Ricci tensor S^* of a Kenmotsu manifold with respect to the Schouten-van Kampen connection ∇^* as

$$S^*(Y,Z) = S(Y,Z) + 2ng(Y,Z). \tag{3.6}$$

This gives

$$Q^*Y = QY + 2nY. \tag{3.7}$$

Contracting with respect to Y and Z in (3.6), we get

$$r^* = r + 2n(2n+1), \tag{3.8}$$

where r^* and r are the scalar curvatures with respect to the Schouten-van Kampen connection ∇^* and the Levi-Civita connection ∇, respectively.

From the above discussions we state the following:

Theorem 3.1. *The curvature tensor R^*, the Ricci tensor S^* and the scalar curvature r^* of a Kenmotsu manifold M with respect to the Schouten-van Kampen connection ∇^* are given by (3.4), (3.6) and (3.8), respectively. Further, the curvature tensor R^* of ∇^* satisfies*
i) $R^*(X,Y)Z = -R^*(Y,X)Z$,
ii) $R^*(X,Y,Z,W) + R^*(Y,X,Z,W) = 0$,
iii) $R^*(X,Y,Z,W) + R^*(X,Y,W,Z) = 0$,
iv)$R^*(X,Y)Z + R^*(Y,Z)X + R^*(Z,X)Y = 0$,
v) S^* is symmetric.

From (3.6), it follows that

Theorem 3.2. *A Kenmotsu manifold M admitting the Schouten-van Kampen connection is Ricci flat with respect to the Schouten-van Kampen connection if and only if M is an Einstein manifold with respect to Levi-Civita connection.*

Now, if $R^*(X,Y)Z = 0$, then by virtue of (3.4), we get

$$R(X,Y,Z,U) = g(X,Z)g(Y,U) - g(Y,Z)g(X,U). \tag{3.9}$$

Thus, we state that

Theorem 3.3. *Let M be a Kenmotsu manifold admitting the Schouten-van Kampen connection. The curvature tensor of M with respect to the Schouten-van Kampen connection vanishes if and only if M with respect to the Levi-Civita connection is isomorphic to the hyperbolic space $H^{2n+1}(-1)$.*

An interesting invariant of the concircular transformation is concircular curvature tensor. The concircular curvature tensor [22] C^* with respect to the Schouten-van Kampen connection ∇^* is defined by

$$C^*(X,Y)Z = R^*(X,Y)Z - \frac{r^*}{2n(2n+1)}\{g(Y,Z)X - g(X,Z)Y\}, \tag{3.10}$$

for all vector fields X, Y, Z on M.
If C^* vanishes, the conditions in theorem (3.1) are satisfied.

Definition 3.1. A Kenmotsu manifold with respect to the Schouten-van Kampen connection ∇^* is said to be ξ- concircularly flat if $C^*(X,Y)\xi = 0$.

In view of (3.4) and (3.8) in (3.10), we get

$$\begin{aligned} C^*(X,Y)Z &= R(X,Y)Z + g(Y,Z)X - g(X,Z)Y \\ &- \frac{r + 2n(2n+1)}{2n(2n+1)}\{g(Y,Z)X - g(X,Z)Y\}. \end{aligned} \tag{3.11}$$

By taking $Z = \xi$ in (3.11) and then using (2.1) and (2.6), we find

$$C^*(X,Y)\xi = \frac{r + 2n(2n+1)}{2n(2n+1)}R(X,Y)\xi. \tag{3.12}$$

Thus, from (3.4), (3.8), (3.11) and (3.12), we have the following theorem:

Theorem 3.4. *Let M be a Kenmotsu manifold admitting the Schouten-van Kampen connection. In M, the following three conditions are equivalent:*
i) M is ξ- concircularly flat,
ii) $r = -2n(2n+1)$,
iii) $r^* = 0$.

Definition 3.2. A Kenmotsu manifold is said to be ϕ-concircularly flat with respect to the Schouten-van Kampen connection ∇^* if

$$g(C^*(\phi X, \phi Y)\phi Z, \phi W) = 0, \tag{3.13}$$

for any vector fields X, Y, Z on M.

Using (3.10) in (3.13), we have

$$\begin{aligned} g(R^*(\phi X, \phi Y)\phi Z, \phi W) &= \frac{r^*}{2n(2n+1)}\{g(\phi Y, \phi Z)g(\phi X, \phi W) \\ &- g(\phi X, \phi Z)g(\phi Y, \phi W)\}. \end{aligned} \tag{3.14}$$

Let $\{e_1, e_2, e_3, ...e_{2n+1}\}$ be a local orthonormal basis of vector fields in M. Then $\{\phi e_1, \phi e_2, \phi e_3, ...\phi e_{2n+1}\}$ is also a local orthonormal basis. If we put $X = W = e_i$ in (3.14) and summing up with respect to $i, 1 \leqslant i \leqslant 2n+1$, we obtain

$$\begin{aligned} \sum_{i=1}^{2n} g(R^*(\phi e_i, \phi Y)\phi Z, \phi e_i) &= \frac{r^*}{2n(2n+1)}\sum_{i=1}^{2n}\{g(\phi Y, \phi Z)g(\phi e_i, \phi e_i) \\ &- g(\phi e_i, \phi Z)g(\phi Y, \phi e_i)\}. \end{aligned} \tag{3.15}$$

From (3.15), it follows that

$$S^*(\phi Y, \phi Z) = \frac{r^*(2n-1)}{2n(2n+1)}g(\phi Y, \phi Z). \tag{3.16}$$

Using (2.1), (3.6) and (3.8) in (3.16), we get

$$S(\phi Y, \phi Z) + 2ng(\phi Y, \phi Z) = \frac{(r + 2n(2n+1))(2n-1)}{2n(2n+1)}g(\phi Y, \phi Z). \tag{3.17}$$

By using (2.2) and (2.8) in (3.17), we obtain

$$S(Y, Z) + 2n\eta(Y)\eta(Z) + \{2n - \frac{(r + 2n(2n+1))(2n-1)}{2n(2n+1)}\}g(\phi Y, \phi Z) = 0. \tag{3.18}$$

Hence by contracting (3.18), we get

$$r = -2n. \tag{3.19}$$

By substituting the equation (3.19) in (3.10), we get

$$C^*(X, Y)Z = R(X, Y)Z + \frac{1}{2n+1}\{g(Y, Z)X - g(X, Z)Y\}. \tag{3.20}$$

This leads to the following:

Theorem 3.5. *Let the Kenmotsu manifold M admitting the Schouten-van Kampen connection be ϕ-concircularly flat. Then M is of constant sectional curvature $-\frac{1}{2n+1}$ if and only if the concircular curvature tensor C^* vanishes.*

We consider

$$C^*.S^* = S^*(C^*(X,Y)Z,U) + S^*(Z,C^*(X,Y)U). \tag{3.21}$$

By making use of (3.10) and (3.6) in (3.21), we obtain

$$\begin{aligned} C^*.S^* &= S(R(X,Y)Z - \frac{r}{2n(2n+1)}\{g(Y,Z)X - g(X,Z)Y\},U) \\ &+ S(Z,R(X,Y)U - \frac{r}{2n(2n+1)}\{g(Y,U)X - g(X,U)Y\}). \end{aligned} \tag{3.22}$$

Suppose $C^*.S^* = 0$. Then we have

$$S^*(C^*(X,Y)Z,U) + S^*(Z,C^*(X,Y)U) = 0. \tag{3.23}$$

Taking $U = \xi$ in (3.23) and using (3.6), it follows that

$$S^*(Z,C^*(X,Y)\xi) = 0. \tag{3.24}$$

Making use of (2.1), (2.6) and (3.11) in (3.24), we get

$$\frac{r+2n(2n+1)}{2n(2n+1)}S^*(Z,\eta(X)Y - \eta(Y)X) = 0. \tag{3.25}$$

Replacing X by ξ in (3.25) and using (2.1) and (3.6), we see that

$$\frac{r+2n(2n+1)}{2n(2n+1)}\{S(Z,Y) + 2ng(Z,Y)\} = 0. \tag{3.26}$$

Contracting (3.26) with respect to Y and Z, we get

$$r = -2n(2n+1). \tag{3.27}$$

From (3.22) and (3.27), we obtain

$$S(Y,Z) = -2ng(Y,Z). \tag{3.28}$$

Thus M is an Einstein manifold.
Again, by substituting (3.27) in (3.11), we obtain

$$C^*(X,Y)Z = R(X,Y)Z + \{g(Y,Z)X - g(X,Z)Y\}. \tag{3.29}$$

Thus, from the above discussion and using (3.4), (3.8) and (3.12), we state the following:

Theorem 3.6. *Let M be a Kenmotsu manifold admitting the Schouten-van Kampen connection. Then $C^*.S^* = 0$ if and only if $S(Y,Z) = -2ng(Y,Z)$.*
Further if $C^* = 0$ then M is isomorphic to the hyperbolic space $H^{2n+1}(-1)$.

Theorem 3.7. *If in a Kenmotsu manifold M admitting the Schouten-van Kampen connection, $C^*.S^* = 0$ holds, then the following three conditions are equivalent:*
i) M is ξ- concircularly flat,
ii) $r = -2n(2n+1)$,
iii) $r^* = 0$.

The projective curvature tensor [23] P^* with respect to the Schouten-van Kampen connection ∇^* is defined by

$$P^*(X,Y)Z = R^*(X,Y)Z - \frac{1}{2n}\{S^*(Y,Z)X - S^*(X,Z)Y\}. \tag{3.30}$$

If the projective curvature tensor P^* with respect to the Schouten-van Kampen connection ∇^* vanishes, then from (3.30), we have

$$R^*(X,Y)Z = \frac{1}{2n}\{S^*(Y,Z)X - S^*(X,Z)Y\}. \tag{3.31}$$

Now in view of (3.4) and (3.6), (3.31) takes the form

$$\begin{aligned}&g(R(X,Y)Z,W) + g(Y,Z)g(X,W) - g(X,Z)g(Y,W) =\\ &\frac{1}{2n}[\{S(Y,Z) + 2ng(Y,Z)\}g(X,W) - \{S(X,Z) + 2ng(X,Z)\}g(Y,W)].\end{aligned} \tag{3.32}$$

Now taking $W = \xi$ in (3.32), we obtain

$$S(Y,Z)\eta(X) - S(X,Z)\eta(Y) = 2n\{g(X,Z)\eta(Y) - g(Y,Z)\eta(X)\}. \tag{3.33}$$

Again, setting $X = \xi$ in (3.33), we get

$$S(Y,Z) = -2ng(Y,Z). \tag{3.34}$$

Contracting the above equation (3.34), we get

$$r = -2n(2n+1). \tag{3.35}$$

Using (3.34) in (3.31), we have $R^* = 0$.
Thus we state the following:

Theorem 3.8. *Let M be a Kenmotsu manifold admitting the Schouten-van Kampen connection. In M, the vanishing of the projective curvature tensor with respect to the Schouten-van Kampen connection leads to the vanishing of the curvature tensor with respect to the Schouten-van Kampen connection.*

By making use of (3.4) and (3.6) in (3.30), we get

$$P^*(X,Y)Z = R(X,Y)Z - \frac{1}{2n}\{S(Y,Z)X - S(X,Z)Y\}. \tag{3.36}$$

Suppose $(P^*(X,Y).S^*)(Z,U) = 0$ holds in a Kenmotsu manifold M. Then we have

$$S^*(P^*(X,Y)Z,U) + S^*(Z,P^*(X,Y)U) = 0. \tag{3.37}$$

Taking $X = \xi$ in the equation (3.37), we get

$$S^*(P^*(\xi,Y)Z,U) + S^*(Z,P^*(\xi,Y)U) = 0. \tag{3.38}$$

By using (3.36), equation (3.38) turns into

$$S^*(Y,Z)\eta(U) + S^*(Y,U)\eta(Z) = 0. \tag{3.39}$$

In view of the equation (3.6), (3.39) becomes

$$S(Y,Z)\eta(U) + S(Y,U)\eta(Z) + 2n\{g(Y,Z)\eta(U) + g(Y,U)\eta(Z)\} = 0. \tag{3.40}$$

In (3.40), taking $U = \xi$ and contracting with respect to Y and Z, we get

$$S(Y,Z) = -2ng(Y,Z). \tag{3.41}$$

and

$$r = -2n(2n+1). \tag{3.42}$$

Again, by substituting (3.42) in (3.30), we obtain

$$P^*(X,Y)Z = R(X,Y)Z + \{g(Y,Z)X - g(X,Z)Y\}. \tag{3.43}$$

Thus we can state that

Theorem 3.9. *In a Kenmotsu manifold M admitting the Schouten-van Kampen connection, $P^*.S^* = 0$ if and only if $S(Y,Z) = -2ng(Y,Z)$.*
Further, if $P^* = 0$ then M is isomorphic to the hyperbolic space $H^{2n+1}(-1)$.

The conharmonic curvature tensor [5] K^* with respect to the Schouten-van Kampen connection ∇^* is defined by

$$\begin{aligned} K^*(X,Y)Z &= R^*(X,Y)Z - \frac{1}{2n-1}\{S^*(Y,Z)X - S^*(X,Z)Y \\ &+ g(Y,Z)Q^*X - g(X,Z)Q^*Y\}. \end{aligned} \tag{3.44}$$

If the conharmonic curvature tensor K^* with respect to the Schouten-van Kampen connection ∇^* vanishes, then from (3.44), we have

$$\begin{aligned} R^*(X,Y)Z &= \frac{1}{2n-1}\{S^*(Y,Z)X - S^*(X,Z)Y \\ &+ g(Y,Z)Q^*X - g(X,Z)Q^*Y\}. \end{aligned} \tag{3.45}$$

By using (3.4), (3.6) and (3.7) in (3.45), we get

$$\begin{aligned} & g(R(X,Y)Z,W)+g(Y,Z)g(X,W)-g(X,Z)g(Y,W) \\ = & \frac{1}{2n-1}[\{S(Y,Z)+4ng(Y,Z)\}g(X,W) \\ - & \{S(X,Z)+4ng(X,Z)\}g(Y,W) \\ + & S(X,W)g(Y,Z)-S(Y,W)g(X,Z)]. \end{aligned} \tag{3.46}$$

Taking $W=\xi$ in (3.46), we obtain

$$S(Y,Z)\eta(X)-S(X,Z)\eta(Y)-2n\{g(X,Z)\eta(Y)-g(Y,Z)\eta(X)\}=0. \tag{3.47}$$

Taking $X=\xi$ in (3.47), we get

$$S(Y,Z)=-2ng(Y,Z). \tag{3.48}$$

Contracting the equation (3.48), we get

$$r=-2n(2n+1). \tag{3.49}$$

Using (3.48) in (3.45), we have $R^*=0$.

Thus we state the following :

Theorem 3.10. *Let M be a Kenmotsu manifold admitting the Schouten-van Kampen connection. In M, the vanishing of the conharmonic curvature tensor with respect to the Schouten-van Kampen connection leads to the vanishing of the curvature tensor with respect to the Schouten-van Kampen connection.*

4. Ricci solitons in Kenmotsu manifold admitting Schouten-van Kampen connection

Suppose the Kenmotsu manifold M admits a Ricci soliton with respect to the Schouten-van Kampen connection ∇^*. Then

$$(L^*_V g)(X,Y)+2S^*(X,Y)+2\lambda g(X,Y)=0. \tag{4.1}$$

If the potential vector field V is the structure vector field ξ, then since ξ is a parallel vector field with respect to the Schouten-van Kampen connection (from (3.3)), the first term in the equation (4.1) becomes zero, hence M reduces to an Einstein manifold. In this case, the results in Theorem (3.6) and (3.9) hold.
If V is pointwise collinear with the structure vector field ξ, i.e. $V=b\xi$, where b is a function on M, then the equation (1.1) implies that

$$\begin{aligned} & bg(\nabla^*_X\xi,Y)+(Xb)\eta(Y)+bg(X,\nabla^*_Y\xi)+(Yb)\eta(X)+ \\ & 2S^*(X,Y)+2\lambda g(X,Y)=0. \end{aligned} \tag{4.2}$$

Using (3.3) and (3.6) in (4.2), it follows that

$$(Xb)\eta(Y) + (Yb)\eta(X) + 2S(X,Y) + 2\{2n+\lambda\}g(X,Y) = 0. \tag{4.3}$$

By setting $Y = \xi$ in (4.3) and using (2.7), we obtain

$$(Xb) = -\{2\lambda + \xi b\}\eta(X). \tag{4.4}$$

Again replacing X by ξ in (4.4), we get

$$(\xi b) = -\lambda. \tag{4.5}$$

Substituting this in (4.4), we have

$$(Xb) = -\lambda\eta(X). \tag{4.6}$$

By applying d on (4.6), we get

$$\lambda d\eta = 0. \tag{4.7}$$

Since $d\eta \neq 0$ from (4.7), we have

$$\lambda = 0. \tag{4.8}$$

Substituting (4.8) in (4.6), we conclude that b is a constant. Hence it is verified from (4.3) that

$$S(X,Y) = -(2n+\lambda)g(X,Y) + \lambda\eta(X)\eta(Y). \tag{4.9}$$

This leads to the following:

Theorem 4.1. *If a Kenmotsu manifold with respect to the Schouten-van Kampen connection admits a Ricci soliton (g, V, λ) with V, pointwise collinear with ξ, then the manifold is an η-Einstein manifold and the Ricci soliton is steady.*

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REFERENCES

1. C. S. Bagewadi and G. Ingalahalli: *Ricci solitons in Lorentzian α-Sasakian manifolds*. Acta Mathematica. Academiae Paedagogicae Nyiregyhaziensis, **28** (2012), 59–68.
2. A. Bejancu and H. R. Farran: *Foliations and geometric structures.* Springer Science and Business Media, **580** (2006).
3. D. E. Blair: *Contact manifolds in Riemannian geometry.* Springer-Verlag, Berlin-New York, **509** (1976).
4. U. C. De and Y. Matsuyama: *Ricci solitons and gradient Ricci solitons in a Kenmotsu manifolds.* Southeast Asian Bulletin of Mathematics, **37** (2013), 691–697.

5. U. C. De and A. A. Shaikh: *Differential Geometry of manifolds.* Narosa Publishing House, New Delhi, (2007).
6. U. C. De, A. Yildiz and F. Yaliniz: *On ϕ-recurrent Kenmotsu manifolds.* Turkish Journal of Mathematics, **33** (2009), 17–25.
7. G. Ghosh: *On Schouten-van Kampen connection in Sasakian manifolds.* Boletim da Sociedade Paranaense de Matematica, **36** (2018), 171–182.
8. R. S. Hamilton: *The Ricci flow on surfaces.* Contemp. Math., **71** (1988), 237–261.
9. S. Ianus: *Some almost product structures on manifolds with linear connection.* Kodai Math. Sem. Rep., **23** (1971), 305–310.
10. K. Kenmotsu: *A class of almost contact Riemannian manifolds.* Tohoku Mathematical Journal, Second Series, **24** (1972), 93–103.
11. H. G. Nagaraja, D. L. Kiran Kumar and V. S. Prasad: *Ricci solitons on Kenmotsu manifolds under D-homothetic deformation*, Khayyam J. Math., **4** (2018), 102–109.
12. H. G. Nagaraja and C. R. Premalatha: *Ricci solitons in f-Kenmotsu manifolds and 3-dimensional trans-Sasakian manifolds.* Progress in Applied Mathematics, **3** (2012), 1–6.
13. H. G. Nagaraja and K. Venu: *Ricci Solitons in Kenmotsu manifolds.* Journal of Informatics and Mathematical Sciences, **8** (2016), 29–36.
14. Z. Olszak: *The Schouten-van Kampen affine connection adapted to an almost (para) contact metric structure.* Publications de l'Institut Mathematique, **94** (2013), 31–42.
15. G. Pathak and U. C. De: *On a semi-symmetric connection in a Kenmotsu manifold.*Bull. Calcutta Math. Soc., **94** (2002), 319–324.
16. D. G. Prakasha, A. T. Vanli, C. S. Bagewadi and D. A. Patil: *Some classes of Kenmotsu manifolds with respect to semi-symmetric metric connection.* Bull. Acta Mathematica Sinica, English Series, **29** (2013), 1311–1322.
17. J. A. Schouten and E. R. Van Kampen: *Zur Einbettungs-und Krmmungstheorie nichtholonomer Gebilde.* Mathematische Annalen, **103** (1930), 752–783.
18. R. Sharma: *Certain results on K-contact and (k, μ)-contact manifolds.* Journal of Geometry, **89** (2008), 138–147.
19. B. B. Sinha and R. Sharma: *On para-A-Einstein manifolds.* Publications De L'Institut Mathematique. Nouvelle Serie, **34** (1983), 211–215.
20. S. Sular, C. Ozgur and U. C. De: *Quarter-symmetric metric connection in a Kenmotsu manifold.* SUT. Journal of mathematics, **44** (2008), 297–306.
21. D. Tolga, E. Cumali and G. Ali: *Ricci Solitons in f-Kenmotsu Manifolds with the semi-symmetric non-metric connection.* NTMSC, **4** (2016), 276–284.
22. K. Yano: *Concircular geometry I. Concircular transformations.* Proceedings of the Imperial Academy, **16** (1940), 195–200.
23. K. Yano and M. Kon: *Structures on manifolds.* Series in Pure Mathematics, **3** (1984).
24. A. Yildiz: *f-Kenmotsu manifolds with the Schouten-van Kampen connection.* Publi. Inst. Math. (N. S), **102** (2017), 93–105.

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