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THE COMPARABLY ALMOST (S,T)- STABILITY FOR RANDOM JUNGCK-TYPE ITERATIVE SCHEMES

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Abstract. In this paper, we introduce the concept of generalized ϕ - weakly contractive random operators and study a new type of stability introduced by Kim [15] which is called a comparably almost stability and then prove the comparably almost (S,T)- stability for the Jungck-type random iterative schemes. Our results extend and improve the recent results in [15], [18], [32] and many others. We also give stochastic version of many important known results.

Keywords. Weakly contractive random operators; stability; Jungck-type random iterative schemes.

1. Introduction

The theory of random operator is an important branch of probabilistic analysis which plays a key role in many applied areas. The study of random fixed points forms a central topic in this area. Research of this direction was initiated by the Prague School of probabilists in connection with random operator theory [7, 8, 29]. Random fixed point theory has attracted much attention in recent times since the publication of the survey article by Bharucha-Reid [6] in 1976, in which the stochastic versions of some well-known fixed point theorems were proved. A lot of efforts have been devoted to random fixed point theory and applications (see e.g. [2, 3, 4, 5, 13, 24, 30]) and many others.

In (1953) Mann [16] introduced an iterative scheme and employed it to approximate the solution of a fixed point problem defined by non-expansive mapping where Picard iterative scheme failed to converge. After that in (1974) Ishikawa [12] introduced an iterative scheme and employed it to obtain the convergence of a Lipschitzian pseudo-contractive operator when Manns iterative scheme is not applicable. Later in (2000) Noor [17] introduced the iterative algorithm to solve variational inequality problems. Recently, Phuengrattana and Suantai [25] introduced

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SP iterative scheme and proved that it has a better convergence rate as compared to Mann, Ishikawa and Noor iterative schemes.
About Jungck iterative, in (1976), Jungck [14] introduced the Jungck iterative process as follows:
Suppose that X is a Banach space, Y an arbitrary set and $S, T : Y \to X$ are such that $T(Y) \subseteq S(Y)$. For $x_0 \in Y$, consider the iterative scheme:

$$Sx_{n+1} = Tx_n, n = 0, 1, \dots .$$

He used this iterative process to approximate the common fixed points of the mappings S and T satisfying the Jungck contraction. Clearly, this iterative process reduces to the Picard iteration when $S = I_d$ (identity mapping) and Y = X. Later, Singh et al. [28] introduced the Jungck- Mann iterative process as:

$$Sx_{n+1} = (1 - \alpha_n)Sx_n + \alpha_n Tx_n,\ \alpha_n \in [0, 1].$$

For $\alpha_n, \beta_n, \gamma_n \in [0, 1]$, Olatinwo [21] defined the Jungck-Ishikawa and Jungck-Noor iterative processes as follows:

$$\begin{aligned} Sx_{n+1} &= (1 - \alpha_n)Sx_n + \alpha_n Ty_n, \\ Sy_n &= (1 - \beta_n)Sx_n + \beta_n Tx_n. \end{aligned}$$

$$\begin{aligned} Sx_{n+1} &= (1 - \alpha_n)Sx_n + \alpha_n Ty_n, \\ Sy_n &= (1 - \beta_n)Sx_n + \beta_n Tz_n, \\ Sz_n &= (1 - \gamma_n)Sx_n + \gamma_n Tx_n. \end{aligned}$$

The concept of the ϕ- weak contraction was introduced by Alber and Guerre-Delabriere [1] in 1997, who proved the existence of fixed points in Hilbert spaces. Later Rhoades [27] in 2001, extended the results of [1] to metric spaces. In 2016, Xue [31] introduced a kind of generalized ϕ-weak contraction as follows:

Definition 1.1. [31]. Let (X,d) be a metric space. A mapping $T : X \to X$ is a generalized ϕ-weak contraction if there exists a continuous and nondecreasing function $\phi : [0, \infty] \to [0, \infty]$ with $\phi(0) = 0$ such that

$$d(Tx, Ty) \leqslant d(x, y) - \phi(d(Tx, Ty)), \forall x, y \in X. \tag{1.1}$$

The concept of stable fixed point iterative scheme was introduced and studied by Harder [9], Harder and Hicks [10, 11]. Many other stability results for several fixed point iterative schemes and various classes of nonlinear mappings were obtained.

Definition 1.2. [11] Let (X, d) be a metric space, $T : X \to X$ be a self-mapping and $x_0 \in X$. Assume that the iterative scheme

$$x_{n+1} = f(T, x_n), n \geq 0. \tag{1.2}$$

converges to a fixed point p of T. Let z_n be an arbitrary sequence in X and define

$$\varepsilon_n = d(z_{n+1}, f(T, z_n)), n \geq 0. \tag{1.3}$$

The iterative scheme defined by (1.2) is said to be T-stable or stable with respect to T if and only if

$$\lim_{n\to\infty} \varepsilon_n = 0 \Rightarrow \lim_{n\to\infty} z_n = p. \tag{1.4}$$

Osilike [23] introduced a weaker concept of stability.

Definition 1.3. [23] Let (X, d) be a metric space, $T : X \to X$ be a self-mapping and $x_0 \in X$. Assume that the iterative scheme (1.2) converges to a fixed point p of T. Let z_n be an arbitrary sequence in X and defined by (1.3). The iterative scheme defined by (1.2) is said to be almost T-stable or almost stable with respect to T if and only if

$$\sum_{n=0}^{\infty} \varepsilon_n < \infty \Rightarrow \lim_{n\to\infty} z_n = p. \tag{1.5}$$

Remark 1.1. It is obvious that any stable iterative scheme is also almost stable but the reverse is not true in general. For examples see [23].

The definition of (S, T)-stability can be found in Singh et al. [28].

Definition 1.4. [28] Let $S, T : Y \to X$ be non-self operators for an arbitrary set Y such that $T(Y) \subseteq S(Y)$ and p a point of coincidence of S and T. Let $\{Sx_n\}_{n=0}^{\infty} \subset X$ be the sequence generated by an iterative procedure

$$Sx_{n+1} = f(T, x_n), n = 0, 1, 2, ..., \tag{1.6}$$

where $x_0 \in X$ is the initial approximation and f is some functions. Suppose that $\{Sx_n\}_{n=0}^{\infty}$ converges to p. Let$\{Sy_n\}_{n=0}^{\infty} \subset X$ be an arbitrary sequence and set

$$\varepsilon_n = d(Sy_n, f(T, y_n)), n = 0, 1, 2, \ldots .$$

Then, the iterative procedure (1.6) is said to be (S,T)-stable if and only if $\lim_{n\to\infty} \varepsilon_n = 0$ implies $\lim_{n\infty} Sy_n = p$.

In 2017, Kim [15] introduced a new concept of stability which is called comparably almost T- stability defined as:

Definition 1.5. Let (X, d) be a metric space, $T : X \to X$ be a self-mapping and $x_0 \in X$. Assume that the iterative scheme (1.2) converges to a fixed point p of T. Let z_n be an arbitrary sequence in X and defined by (1.3). The iterative scheme defined by (1.2) is said to be comparably almost T-stable or comparably almost stable with respect to T if and only if

$$\sum_{n=0}^{\infty} (\theta_n + \varepsilon_n) < \infty, \theta_n \geq 0 \Rightarrow \lim_{n\to\infty} z_n = p, \lim_{n\to\infty} \theta_n = 0. \tag{1.7}$$

Also, he proved some convergence results of Mann and Ishikawa iterative schemes containing a generalized ϕ- weak contractive self maps defined as in (1.1).

Remark 1.2. 1. It is obvious that any almost stable iterative scheme is also comparably almost stable. See [15].

2. If $\theta_n = 0$ in (1.7), then (1.7) reduces to (1.5). So an almost stable iterative scheme is a special case of comparably almost stable iterative scheme.

The aim of this paper is to introduce the concept of generalized ϕ- weakly contractive random operators and study a new type of stability which is called comparably almost stability and then prove the comparably almost (S,T)- stability for the Jungck- type and SP-Jungck-type random iterative schemes. Our results extend, improve and unify the recent results in [15], [18], [32] and many others. We also give the stochastic version of many important known results.

2. Preliminaries

Let (Ω, Σ) be a measurable space, E be nonempty subset of a separable Banach space X. A mapping $\xi : \Omega \to E$ is called measurable if $\xi^{-1}(B \cap E) \in \Sigma$ for every Borel subset B of X. A mapping $T : \Omega \times E \to E$ is said to be random mapping if for each fixed $x \in E$, the mapping $T(., x) : \Omega \to E$ is measurable. A measurable mapping $\xi^* : \Omega \to E$ is called a random fixed point of the random mapping $T : \Omega \times E \to E$ if $T(\omega, \xi^*(\omega)) = \xi^*(\omega)$ for each $\omega \in \Omega$. Let $S, T : \Omega \times E \to E$ be two random self-maps. A measurable map ξ^* is called a common random fixed point of the pair (S,T) if $\xi^*(\omega) = S(\omega, \xi^*(\omega)) = T(\omega, \xi^*(\omega))$, for each $\omega \in \Omega$ and some $\xi^*(\omega) \in E$.
let $S, T : \Omega \times E \leftrightarrow E$ be two random operator defined on E and E a nonempty subset of a separable Banach space X. Let $x_0(w) \in E$ be arbitrary measurable mapping for $w \in \Omega, n = 0, 1, ...$ with $T(w, X) \subseteq S(w, X)$, S is injective.
The Jungck-Noor type random iterative scheme is a sequence $\{S(w, x_n(\omega))\}_{n=0}^{\infty}$ defined by

$$\begin{aligned} S(w,x_{n+1}(w)) &= (1-\alpha_n)S(w,x_n(w))+\alpha_n T(w,y_n(w)), \\ S(w,y_n(w)) &= (1-\beta_n)S(w,x_n(w))+\beta_n T(w,z_n(w)), \\ S(w,z_n(w)) &= (1-\gamma_n)S(w,x_n(w))+\gamma_n T(w,x_n(w)), \end{aligned} \tag{2.1}$$

where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are real sequences in (0,1).
The Jungck-SP type random iterative scheme is a sequence $\{S(w, x_n(\omega))\}_{n=0}^{\infty}$ defined by

$$\begin{aligned} S(w,x_{n+1}(w)) &= (1-\alpha_n)S(w,y_n(w))+\alpha_n T(w,y_n(w)), \\ S(w,y_n(w)) &= (1-\beta_n)S(w,z_n(w))+\beta_n T(w,z_n(w)), \\ S(w,z_n(w)) &= (1-\gamma_n)S(w,x_n(w))+\gamma_n T(w,x_n(w)), \end{aligned} \tag{2.2}$$

where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are real sequences in (0,1).

Remark 2.1. 1. If $\gamma_n = 0$ for each $n \in \mathbb{N}$ in (2.1), then the Jungck-Noor type random iterative scheme reduce to Jungck-Ishikawa type random iterative scheme.

$$\begin{aligned} S(w, x_{n+1}(w)) &= (1-\alpha_n)S(w, x_n(w)) + \alpha_n T(w, y_n(w)), \\ S(w, y_n(w)) &= (1-\beta_n)S(w, x_n(w)) + \beta_n T(w, x_n(w)), \end{aligned} \tag{2.3}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are real sequences in (0,1).

2. If $\beta_n = \gamma_n = 0$ for each $n \in \mathbb{N}$ in (2.1), then the Jungck-Noor type random iterative scheme reduce to Jungck-Mann type random iterative scheme.

$$S(w, x_{n+1}(w)) = (1-\alpha_n)S(w, x_n(w)) + \alpha_n T(w, x_n(w)), \tag{2.4}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is real sequence in (0,1).

Zhang et al. [32] in (2011), studied the almost sure T-stability and convergence of Ishikawa-type and Mann-type random iterative processes for certain ϕ- weakly contractive-type random operators in a separable Banach space. The following is the contractive condition studied by Zhang et al. [32].

Definition 2.1. [32] Let (Ω, Σ, μ) be a complete probability measure space and E be a nonempty subset of a separable Banach space X. A random operator $T : \Omega \times E \leftrightarrow E$ is called a ϕ- weakly contractive-type random operator if there exists a continuous and non- decreasing function $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\phi(t) > 0$ for each $t \in (0, \infty)$ and $\phi(0) = 0$ such that for each $x, y \in E, \omega \in \Omega$,

$$\int_\Omega \|T(w,x)-T(w,y)\|d\mu(w) \le \int_\Omega \|x-y\|d\mu(w) - \phi\left(\int_\Omega \|x-y\|d\mu(w)\right) \tag{2.5}$$

Recently, in (2015) Okeke and Abbas [18] introduced the concept of generalized ϕ- weakly contraction random operators and then proved the convergence and almost sure T-stability of Mann-type and Ishikawa-type random iterative schemes. Their results improved the results of Zhang et al. [32] and Olatinwo [22] and others. The generalized ϕ- weakly contraction is defined as follows:

Definition 2.2. [18] Let (Ω, Σ, μ) be a complete probability measure space and E be a nonempty subset of a separable Banach space X. A random operator $T : \Omega \times E \leftrightarrow E$ is called a ϕ- weakly contractive-type random operator if there exists $L(w) \ge 0$ and a continuous and non- decreasing function $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\phi(t) > 0$ for each $t \in (0, \infty)$ and $\phi(0) = 0$ such that for each $x, y \in E, \omega \in \Omega$,

$$\int_\Omega \|T(w,x)-T(w,y)\|d\mu(w) \le e^{L(w)\|x-y\|}\left(\int_\Omega \|x-y\|d\mu(w) - \phi\left(\int_\Omega \|x-y\|d\mu(w)\right)\right) \tag{2.6}$$

If $L(w) = 0$ for each $w \in \Omega$ in (2.6), then it reduces to condition (2.5).
Furthermore, Okeke and Kim in [19] introduced the random Picard-Mann hybrid iterative process. They established strong convergence theorems and summable almost T-stability of the random PicardMann hybrid iterative process and the random Mann-type iterative process generated by a generalized class of random operators in separable Banach spaces. Their results improved and generalized several well-known

deterministic stability results in a stochastic version. In addition, Okeke and Kim [20] proved some convergence and (S,T)- stability results for random Jungck-Mann type and random Ishikawa type iterative processes. Rashwan et al. [26] studied the convergence and almost sure (S,T)- stability for the random Jungck-Noor type and the random Jungck-SP type under some contractive conditions.
Keeping in mind the generalized ϕ-weakly contractive conditions (1.1) and (2.6), we introduce the following generalized ϕ-weakly contractive condition:

Definition 2.3. Let (Ω, Σ) be a measurable space and E be a nonempty subset of a separable Banach space X. Let $S, T : \Omega \times E \leftrightarrow E$ be random operators such that $T(w, X) \subseteq S(w, X)$. Then the random operators S and T are satisfying the following generalized ϕ- weakly contractive-type if there exist $L(w) \geq 0$ and a continuous and non- decreasing function $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\phi(t) > 0$ for each $t \in (0, \infty)$ and $\phi(0) = 0$ such that for each $x, y \in E, \omega \in \Omega$,

$$\|T(w,x)-T(w,y)\| \leq e^{L(w)\|S(w,x)-T(w,x)\|}(\|S(w,x)-S(w,y)\|-\phi(\|T(w,x)-T(w,y)\|)) \tag{2.7}$$

If $L(w) = 0$ for each $\omega \in \Omega$ and $S = I_d$ (identity random mapping) in the condition (2.7), then it reduces to the stochastic version of the condition (1.1).
Motivated by the definition of a comparably almost stability in [15] together with the definition of (S,T)-stability in [28], we state the stochastic version of the comparably almost (S,T)- stability as follows:

Definition 2.4. Let (Ω, Σ) be a measurable space and E be a nonempty subset of a separable Banach space X. Let $S, T : \Omega \times E \leftrightarrow E$ be random operators such that $T(w, X) \subseteq S(w, X)$ and $\xi^*(\omega)$ be a common random fixed point of S and T. For any given random variable $x_0 : \Omega \to E$. Define a random iterative scheme with the functions $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ as follows:

$$S(\omega, x_{n+1}(\omega)) = f(T; x_n(\omega)) \; n = 0, 1, 2, ..., \tag{2.8}$$

where f is some function measurable in the second variable.
Suppose that $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ converges to $\xi^*(\omega)$, and Let $\{S(\omega, \xi_n(\omega))\}_{n=0}^{\infty} \subset E$ be an arbitrary sequence of a random variable. Denote by

$$\varepsilon_n(\omega) = \|S(\omega, \xi_{n+1}(\omega)) - f(T; \xi_n(\omega))\|.$$

Then the iterative scheme (2.8) is a comparably almost (S,T)- stable or comparably almost stable with respect to (S,T) if and only if for $\omega \in \Omega$,

$$\sum_{n=0}^{\infty}(\theta_n(\omega) + \varepsilon_n(\omega)) < \infty, \; \theta_n(\omega) \geq 0 \Rightarrow S(\omega, \xi_n(\omega)) \to \xi^*, \; \theta_n(\omega) \to 0 \; as \; n \to \infty.$$

The following lemma is useful for proving our results

Lemma 2.1. *[1] Let $\{\lambda_n\}$ and $\{\gamma_n\}$ be two sequences of nonnegative real numbers and $\{\sigma_n\}$ be a sequence of positive numbers satisfying*

$$\lambda_{n+1} \leq \lambda_n - \sigma_n \phi(\lambda_n) + \gamma_n, \quad \forall n \geq 1,$$

where $\phi : [0, \infty) \to [0, \infty)$ is a continuous and nondecreasing function with $\phi(0) = 0$. If $\sum_{n=1}^{\infty} \sigma_n = \infty$ and $\lim_{n\to\infty} \frac{\gamma_n}{\sigma_n} = 0$, then $\{\lambda_n\}$ converges to 0 as $n \to \infty$.

3. Main Results

In this section, we present our main results. First, we prove the comparably almost (S,T)- stability of the Jungck-Noor type random iterative scheme.

Theorem 3.1. *Let (Ω, Σ) be a measurable space and E be a nonempty subset of a separable Banach space X and let $S, T : \Omega \times E \leftrightarrow E$ be two random operators defined on E satisfying a generalized ϕ- weakly contractive-type (2.7) with $T(w, X) \subseteq S(w, X)$. Let $\xi^*(\omega)$ be a common random fixed point of (S,T) and $\{S(\omega, x_n(\omega))\}_{n=0}^{\infty}$ be a Jungck-Noor type random iterative scheme defined by (2.1) converging strongly to $\xi^*(\omega)$, where $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$ are sequences of positive numbers in [0,1] satisfying*

- $\sum_{n=1}^{\infty} \alpha_n \beta_n \gamma_n = \infty$,
- $\alpha_n(1 + \beta_n + \beta_n \gamma_n) \leq 1$.

Let $\{S(w, \xi_n(w))\}_{n=0}^{\infty}$ be any sequence of random variable in E and define

$$\begin{aligned} \varepsilon_n &= \|S(w, \xi_{n+1}(w)) - (1 - \alpha_n)S(w, \xi_n(w)) - \alpha_n T(w, \eta_n(w))\|, \\ S(w, \eta_n(w)) &= (1-\beta_n)S(w,\xi_n(w))+\beta_n T(w,\zeta_n(w)), \\ S(w, \zeta_n(w)) &= (1-\gamma_n)S(w,\xi_n(w))+\gamma_n T(w,\xi_n(w)). \end{aligned}$$

Then

1. *If $\sum_{n=0}^{\infty}(\theta_n + \varepsilon_n) < \infty$, where*

$$\begin{aligned} \theta_n &= \phi(\|S(w, \xi_n(w)) - \xi^*(w)\|) - \alpha_n \beta_n \gamma_n \phi(\|T(w, \xi_n(w)) - \xi^*(w)\|) \\ &\quad -\alpha_n \beta_n \phi(\|T(w, \zeta_n(w)) - \xi^*(w)\|) - \alpha_n \phi(\|T(w, \eta_n(w)) - \xi^*(w)\|). \end{aligned}$$

   Then the Jungck-Noor type random iterative scheme $\{S(w, x_n(w))\}_{n=0}^{\infty}$ is a comparably almost (S,T)- stable.

2. *If the sequence $\{S(w, \xi_n(w))\}_{n=0}^{\infty}$ converge to the fixed point $\xi^*(w)$ of (S,T), then $\lim_{n\to\infty} \varepsilon_n = 0$.*

Proof. Using the random Jungck-Noor iterative scheme (2.1) and the sequence $\{S(w,\xi_n(w))\}_{n=0}^{\infty}$ defined in (3.1), we have

$$\begin{aligned}
\|S(w,\xi_{n+1}(w))-\xi^*(w)\| &\leq \|S(w,\xi_{n+1}(w))-(1-\alpha_n)S(w,\xi_n(w))-\alpha_n T(w,\eta_n(w))\| \\
&+ (1-\alpha_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\alpha_n\|T(w,\eta_n(w))-\xi^*(w)\| \\
&= \varepsilon_n+(1-\alpha_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\alpha_n\|T(w,\eta_n(w))-\xi^*(w)\|
\end{aligned}$$

(3.1)

Now, we compute the last estimate of (3.1) by using (2.7) and (3.1)

$$\begin{aligned}
\|T(w,\eta_n(w))-\xi^*(w)\| &= \|T(w,\xi^*(w))-T(w,\eta_n(w))\| \\
&\leq e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|}\big(\|S(w,\xi^*(w))-S(w,\eta_n(w))\| \\
&- \phi(\|T(w,\xi^*(w))-T(w,\eta_n(w))\|)\big) \\
&= \|\xi^*(w)-S(w,\eta_n(w))\|-\phi(\|T(w,\xi^*(w))-T(w,\eta_n(w))\|) \\
&\leq (1-\beta_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\beta_n\|T(w,\zeta_n(w))-\xi^*(w)\| \\
&- \phi(\|\xi^*(w)-T(w,\eta_n(w))\|) \\
&\leq (1-\beta_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\beta_n[e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|} \\
&\big(\|S(w,\xi^*(w))-S(w,\zeta_n(w))\|-\phi(\|T(\omega,\xi^*(\omega))-T(w,\zeta_n(w))\|))] \\
&- \phi(\|\xi^*(w)-T(w,\eta_n(w))\|) \\
&= (1-\beta_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\beta_n\|\xi^*(w)-S(w,\zeta_n(w))\| \\
&- \beta_n\phi(\|\xi^*(w)-T(w,\zeta_n(w))\|)-\phi(\|\xi^*(w)-T(w,\eta_n(w))\|) \\
&\leq (1-\beta_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\beta_n[(1-\gamma_n)\|S(w,\xi_n(w))-\xi^*(w)\| \\
&+ \gamma_n\|T(w,\xi_n(w))-\xi^*(w)\|]-\beta_n\phi(\|\xi^*(w)-T(w,\zeta_n(w))\|) \\
&- \phi(\|\xi^*(w)-T(w,\eta_n(w))\|) \\
&\leq (1-\beta_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\beta_n(1-\gamma_n)\|S(w,\xi_n(w))-\xi^*(w)\| \\
&+ \beta_n\gamma_n[e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|}(\|S(w,\xi_n(w))-\xi^*(w)\| \\
&- \phi(\|T(w,\xi_n(w))-\xi^*(w)\|)]-\beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|) \\
&- \phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\
&= (1-\beta_n+\beta_n-\beta_n\gamma_n+\beta_n\gamma_n)\|S(w,\xi_n(w))-\xi^*(w)\| \\
&- \beta_n\gamma_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|)-\beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|) \\
&- \phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\
&= \|S(w,\xi_n(w))-\xi^*(w)\|-\beta_n\gamma_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|) \\
&- \beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)-\phi(\|T(w,\eta_n(w))-\xi^*(w)\|)
\end{aligned}$$

(3.2)

Applying (3.2) in (3.1), we obtain

$$\begin{aligned}\|S(w,\xi_{n+1}(w))-\xi^*(w)\| &\leq \varepsilon_n+(1-\alpha_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\alpha_n\|S(w,\xi_n(w))-\xi^*(w)\| \\ &- \alpha_n\beta_n\gamma_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|)-\alpha_n\beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|) \\ &- \alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\ &= \|S(w,\xi_n(w))-\xi^*(w)\|-\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)+(\varepsilon_n+\theta_n), \end{aligned} \tag{3.3}$$

where, $\theta_n=\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\alpha_n\beta_n\gamma_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|)-\alpha_n\beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)$ $-\ \alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|)$.
Now, we want to prove that $\theta_n \geq 0$, note that

$$\begin{aligned}\|T(w,\xi_n(w))-\xi^*(w)\| &\leq e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|}(\|S(w,\xi^*(w))-S(w,\xi_n(w))\| \\ &- \phi(\|T(w,\xi^*(w))-T(w,\xi_n(w))\|)) \\ &\leq \|S(w,\xi_n(w))-\xi^*(w)\|. \end{aligned} \tag{3.4}$$

Also, we have by (3.4)

$$\begin{aligned}\|T(w,\zeta_n(w))-\xi^*(w)\| &= \|T(w,\xi^*(w))-T(w,\zeta_n(w))\| \\ &\leq e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|}(\|S(w,\xi^*(w))-S(w,\zeta_n(w))\| \\ &- \phi(\|T(w,\xi^*(w))-T(w,\zeta_n(w))\|)) \\ &\leq \|S(w,\zeta_n(w))-\xi^*(w)\| \\ &\leq (1-\gamma_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\gamma_n\|T(w,\xi_n(w))-\xi^*(w)\| \\ &\leq (1-\gamma_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\gamma_n\|S(w,\xi_n(w))-\xi^*(w)\| \\ &= \|S(w,\xi_n(w))-\xi^*(w)\|. \end{aligned} \tag{3.5}$$

Similarly, from (3.5), we get

$$\begin{aligned}\|T(w,\eta_n(w))-\xi^*(w)\| &= \|T(w,\xi^*(w))-T(w,\eta_n(w))\| \\ &\leq e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|}(\|S(w,\xi^*(w))-S(w,\eta_n(w))\| \\ &- \phi(\|T(w,\xi^*(w))-T(w,\eta_n(w))\|)) \\ &\leq \|S(w,\eta_n(w))-\xi^*(w)\| \\ &\leq (1-\beta_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\beta_n\|T(w,\zeta_n(w))-\xi^*(w)\| \\ &\leq (1-\beta_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\beta_n\|S(w,\xi_n(w))-\xi^*(w)\| \\ &= \|S(w,\xi_n(w))-\xi^*(w)\|. \end{aligned} \tag{3.6}$$

Now, we can study the sign of θ_n by using (3.4), (3.5), (3.6) and the condition

$\alpha_n(1+\beta_n+\beta_n\gamma_n) \leq 1$ as:

$$\begin{array}{rcl}\theta_n & = & \phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\alpha_n\beta_n\gamma_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|)\\ & - & \alpha_n\beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|)\\ & \geq & \phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\alpha_n\beta_n\gamma_n\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)\\ & - & \alpha_n\beta_n\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)\\ & = & [1-\alpha_n(1+\beta_n+\beta_n\gamma_n)]\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)\\ & \geq & 0.\end{array}$$

Since $\sum_{n=0}^{\infty}(\theta_n+\varepsilon_n)<\infty$, we have $\lim_{n\to\infty}(\theta_n+\varepsilon_n)=0$. Back to the relation (3.3) and by Lemma 2.1, we get

(3.7) $$\lim_{n\to\infty}\|S(w,\xi_n(w))-\xi^*(w)\|=0 \textit{ or } S(w,\xi_n(w))\to\xi^*(w) \textit{ as } n\to\infty.$$

From (3.4) and (3.7), we get

(3.8) $$0\leq\|T(w,\xi_n(w))-\xi^*(w)\|\leq\|S(w,\xi_n(w))-\xi^*(w)\|\to 0 \text{ as } n\to\infty.$$

Similarly, from (3.5), (3.6) and using (3.7)

(3.9) $$0\leq\|T(w,\zeta_n(w))-\xi^*(w)\|\leq\|S(w,\xi_n(w))-\xi^*(w)\|\to 0 \text{ as } n\to\infty.$$

(3.10) $$0\leq\|T(w,\eta_n(w))-\xi^*(w)\|\leq\|S(w,,\xi_n(w))-\xi^*(w)\|\to 0 \text{ as } n\to\infty.$$

Since ϕ is continuous, from (3.7)-(3.10), we obtain

$$\begin{array}{rl} & \lim_{n\to\infty}\theta_n=\lim_{n\to\infty}[\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\alpha_n\beta_n\gamma_n\phi(\|T(w,\xi_n(\omega))-\xi^*(w)\|)\\ - & \alpha_n\beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|)]\\ = & 0.\end{array}$$

Hence the Jungck-Noor type random iterative scheme $\{S(w,x_n(w))\}_{n=0}^{\infty}$ is a comparably almost (S,T)- stable.
Next, suppose that $S(w,\xi_n(w))\to\xi^*(w)\ as\ n\to\infty$, and using (3.6) and (3.7), then we can write

$$\begin{array}{rcl}\varepsilon_n & = & \|S(w,\xi_{n+1}(w))-(1-\alpha_n)S(w,\xi_n(w))-\alpha_nT(w,\eta_n(w))\|\\ & \leq & \|S(w,\xi_{n+1}(w))-\xi^*(w)\|+(1-\alpha_n)\|S(w,\xi_n(\omega))-\xi^*(w)\|\\ & + & \alpha_n\|T(w,\eta_n(w))-\xi^*(w)\|\\ & \leq & \|S(w,\xi_{n+1}(w))-\xi^*(w)\|+(1-\alpha_n)\|S(w,\xi_n(\omega))-\xi^*(w)\|\\ & + & \alpha_n\|S(w,\xi_n(w))-\xi^*(w)\|\\ & = & \|S(w,\xi_{n+1}(w))-\xi^*(w)\|+\|S(w,\xi_n(\omega))-\xi^*(w)\|.\end{array}$$

Hence, we get $\varepsilon_n\to 0$ as $n\to\infty$. □

From Theorem 3.1, we can present the following corollaries.

Corollary 3.1. *Let (Ω, Σ) be a measurable space and E be a nonempty subset of a separable Banach space X and let $S, T : \Omega \times E \leftrightarrow E$ be two random operators defined on E satisfying a generalized ϕ- weakly contractive-type (2.7) with $T(w, X) \subseteq S(w, X)$. Let $\xi^*(w)$ be a common random fixed point of (S,T) and $\{S(w, x_n(w))\}_{n=0}^{\infty}$ be a Jungck-Ishikawa type random iterative scheme defined by (2.3) converging strongly to $\xi^*(w)$, where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of positive numbers in [0,1] satisfying*

- $\sum_{n=1}^{\infty} \alpha_n \beta_n = \infty$,
- $\alpha_n(1 + \beta_n) \leq 1$.

Let $\{S(w, \xi_n(w))\}_{n=0}^{\infty}$ be any sequence of random variable in E and define

$$\begin{aligned} \varepsilon_n &= \|S(w, \xi_{n+1}(w)) - (1 - \alpha_n)S(w, \xi_n(w)) - \alpha_n T(w, \eta_n(w))\|, \\ S(w, \eta_n(w)) &= (1 - \beta_n)S(w, \xi_n(w)) + \beta_n T(w, \xi_n(w)). \end{aligned}$$

Then

1. *If $\sum_{n=0}^{\infty}(\theta_n + \varepsilon_n) < \infty$, where*

   $$\theta_n = \phi(\|S(w,\xi_n(w)) - \xi^*(w)\|) - \alpha_n\beta_n\phi(\|T(w,\xi_n(w)) - \xi^*(w)\|) - \alpha_n\phi(\|T(w,\eta_n(w)) - \xi^*(w)\|).$$

   Then the Jungck-Ishikawa type random iterative scheme $\{S(w, x_n(w))\}_{n=0}^{\infty}$ is a comparably almost (S,T)- stable.

2. *If the sequence $\{S(w, \xi_n(w))\}_{n=0}^{\infty}$ converge to the fixed point $\xi^*(w)$ of (S,T), then $\lim_{n\to\infty} \varepsilon_n = 0$.*

Proof. Putting $\gamma_n = 0$ in the Jungck-Noor type random iterative scheme in Theorem 3.1. Then we obtain the Jungck-Ishikawa type random iterative scheme and then can be prove the Corollary 3.1 by following the same steps of proofing of Theorem 3.1. □

Corollary 3.2. *Let (Ω, Σ) be a measurable space and E be a nonempty subset of a separable Banach space X and let $S, T : \Omega \times E \leftrightarrow E$ be two random operators defined on E satisfying a generalized ϕ- weakly contractive-type (2.7) with $T(w, X) \subseteq S(w, X)$. Let $\xi^*(w)$ be a common random fixed point of (S,T) and $\{S(w, x_n(w))\}_{n=0}^{\infty}$ be a Jungck-Mann type random iterative scheme defined by (2.4) converging strongly to $\xi^*(w)$, where $\{\alpha_n\}$ is a sequence of positive numbers in [0,1] such that $\sum_{n=1}^{\infty} \alpha_n = \infty$. Let $\{S(w, \xi_n(w))\}_{n=0}^{\infty}$ be any sequence of random variable in E and define*

$$\varepsilon_n = \|S(w, \xi_{n+1}(w)) - (1 - \alpha_n)S(w, \xi_n(w)) - \alpha_n T(w, \xi_n(w))\|,$$

Then

1. *If* $\sum_{n=0}^{\infty}(\theta_n+\varepsilon_n)<\infty$, *where*

$$\theta_n=\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|).$$

   Then the Jungck-Mann iterative scheme $\{S(w,x_n(w))\}_{n=0}^{\infty}$ *is a comparably almost (S,T)- stable.*

2. *If the sequence* $\{S(w,\xi_n(w))\}_{n=0}^{\infty}$ *converge to the fixed point* $\xi^*(w)$ *of (S,T), then* $\lim_{n\to\infty}\varepsilon_n=0$.

Proof. If $\gamma_n=\beta_n=0$ in the Jungck-Noor type random iterative scheme in Theorem 3.1. Then we obtain the Jungck-Mann type random iterative and then the proof of the Corollary 3.2 is similar to that of Theorem 3.1. □

Remark 3.1. If the random mapping $S=I_d$ (Identity random mapping) and $L(\omega)=0$ in Corollary 3.1 and Corollary 3.2. Then Corollary 3.1 and Corollary 3.2 are random versions of Theorem 3.2 and Corollary 3.3 respectively of Kim in [15].

Next, we prove that the Jungck- SP type random iterative scheme $\{S(w,x_n(w))\}_{n=0}^{\infty}$ is a comparably almost (S,T)- stable.

Theorem 3.2. *Let* (Ω,Σ) *be a measurable space and* E *be a nonempty subset of a separable Banach space* X *and let* $S,T:\Omega\times E\leftrightarrow E$ *be two random operators defined on* E *satisfying a generalized* ϕ*- weakly contractive-type (2.7) with* $T(w,X)\subseteq S(w,X)$. *Let* $\xi^*(w)$ *be a common random fixed point of (S,T) and* $\{S(w,x_n(w))\}_{n=0}^{\infty}$ *be a Jungck-SP type random iterative scheme defined by (2.2) converging strongly to* $\xi^*(w)$, *where* $\{\alpha_n\},\{\beta_n\}$ *and* $\{\gamma_n\}$ *are sequences of positive numbers in [0,1] satisfying*

- $\sum_{n=1}^{\infty}\alpha_n=\infty$ *or* $\sum_{n=1}^{\infty}\beta_n=\infty$ *or* $\sum_{n=1}^{\infty}\gamma_n=\infty$.
- $\alpha_n(1+\beta_n+\gamma_n)\le 1$.

Let $\{S(w,\xi_n(w))\}_{n=0}^{\infty}$ *be any sequence of random variable in E and define*

$$\begin{aligned}\varepsilon_n &= \|S(w,\xi_{n+1}(w))-(1-\alpha_n)S(w,\eta_n(\omega))-\alpha_nT(w,\eta_n(w))\|,\\ S(w,\eta_n(w)) &= (1-\beta_n)S(w,\zeta_n(w))+\beta_nT(w,\zeta_n(w)),\\ S(w,\zeta_n(w)) &= (1-\gamma_n)S(w,\xi_n(w))+\gamma_nT(w,\xi_n(w)).\end{aligned}\tag{3.11}$$

Then

1. *If* $\sum_{n=0}^{\infty}(\theta_n+\varepsilon_n)<\infty$, *where*

$$\theta_n=\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\alpha_n\gamma_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|)$$
$$-\alpha_n\beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|).$$

   Then the Jungck-SP iterative scheme $\{S(w,x_n(w))\}_{n=0}^{\infty}$ *is a comparably almost (S,T)- stable.*

2. *If the sequence* $\{S(w,\xi_n(w))\}_{n=0}^{\infty}$ *converge to the fixed point* $\xi^*(w)$ *of (S,T), then* $\lim_{n\to\infty}\varepsilon_n = 0$.

Proof. By the same steps of proofing of Theorem 3.1, using the random Jungck-SP iterative scheme (2.2) and the sequence $\{S(w,\xi_n(w))\}_{n=0}^{\infty}$ defined in (3.11), we have

$$\begin{aligned}\|S(w,\xi_{n+1}(w))-\xi^*(w)\| &\leq \|S(w,\xi_{n+1}(w))-(1-\alpha_n)S(w,\eta_n(w))-\alpha_n T(w,\eta_n(w))\|\\ &+ (1-\alpha_n)\|S(w,\eta_n(w))-\xi^*(w)\|+\alpha_n\|T(w,\eta_n(w))-\xi^*(w)\|\\ &= \varepsilon_n+(1-\alpha_n)\|S(w,\eta_n(w))-\xi^*(w)\|+\alpha_n\|T(w,\eta_n(w))-\xi^*(w)\|\end{aligned}$$

(3.12)

Using (2.7) to compute the following

$$\begin{aligned}\|T(w,\eta_n(w))-\xi^*(w)\| &= \|T(w,\xi^*(w))-T(w,\eta_n(w))\|\\ &\leq e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|}\big(\|S(w,\xi^*(w))-S(w,\eta_n(w))\|\\ &- \phi(\|T(\omega,\xi^*(w))-T(w,\eta_n(w))\|)\big)\\ &= \|S(w,\eta_n(w))-\xi^*(w)\|-\phi(\|T(w,\eta_n(w))-\xi^*(w)\|)\end{aligned}$$

(3.13)

Applying (3.13) in (3.12), we obtain

$$\begin{aligned}
\|S(w,\xi_{n+1}(w))-\xi^*(w)\| &\leq \varepsilon_n+(1-\alpha_n)\|S(w,\eta_n(w))-\xi^*(w)\| \\
&+ \alpha_n[\|S(w,\eta_n(w))-\xi^*(w)\|-\phi(\|T(w,\eta_n(w))-\xi^*(w)\|)] \\
&= \varepsilon_n+\|S(w,\eta_n(w))-\xi^*(w)\|-\alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\
&\leq \varepsilon_n+(1-\beta_n)\|S(w,\zeta_n(w))-\xi^*(w)\|+\beta_n\|T(w,\zeta_n(w))-\xi^*(w)\| \\
&- \alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\
&\leq \varepsilon_n+(1-\beta_n)\|S(w,\zeta_n(w))-\xi^*(w)\|+\beta_n[e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|} \\
&(\|S(w,\xi^*(w))-S(w,\zeta_n(w))\|-\phi(\|T(\omega,\xi^*(w))-T(w,\zeta_n(w))\|))] \\
&- \alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\
&= \varepsilon_n+(1-\beta_n)\|S(w,\zeta_n(w))-\xi^*(w)\|+\beta_n\|S(w,\zeta_n(w))-\xi^*(w)\| \\
&- \beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\
&= \varepsilon_n+\|S(w,\zeta_n(w))-\xi^*(w)\|-\beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|) \\
&- \alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\
&\leq \varepsilon_n+(1-\gamma_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\gamma_n\|T(w,\xi_n(w))-\xi^*(w)\| \\
&- \beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\
&\leq \varepsilon_n+(1-\gamma_n)\|S(w,\xi_n(w))-\xi^*(w)\| \\
&+ \gamma_n[e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|}(\|S(w,\xi^*(w))-S(w,\xi_n(w))\| \\
&- \phi(\|T(w,\xi^*(w))-T(w,\xi_n(w))\|))]-\beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|) \\
&- \alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\
&= \varepsilon_n+\|S(w,\xi_n(w))-\xi^*(w)\|-\gamma_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|) \\
&- \beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|) \\
&= \|S(w,\xi_n(w))-\xi^*(w)\|-\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)+(\theta_n+\varepsilon_n) \qquad (3.14)
\end{aligned}$$

where

$$\theta_n=\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\gamma_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|)$$
$$-\beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|).$$

Note that,

$$\begin{aligned}
\|T(w,\xi_n(w))-\xi^*(w)\| &\leq e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|}(\|S(w,\xi^*(w))-S(w,\xi_n(w))\| \\
&- \phi(\|T(w,\xi^*(w))-T(w,\xi_n(w))\|)) \\
&\leq \|S(w,\xi_n(w))-\xi^*(w)\|. \qquad (3.15)
\end{aligned}$$

Also, from (3.15) , we get

$$\begin{aligned}\|T(w,\zeta_n(w))-\xi^*(w)\| &= \|T(w,\xi^*(w))-T(w,\zeta_n(w))\|\\ &\leq e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|}(\|S(w,\xi^*(w))-S(w,\zeta_n(w))\|\\ &- \phi(\|T(w,\xi^*(w))-T(w,\zeta_n(w))\|))\\ &\leq \|S(w,\zeta_n(w))-\xi^*(w)\|\\ &\leq (1-\gamma_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\gamma_n\|T(w,\xi_n(w))-\xi^*(w)\|\\ &\leq (1-\gamma_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\gamma_n\|S(w,\xi_n(w))-\xi^*(w)\|\\ &= \|S(w,\xi_n(w))-\xi^*(w)\|\end{aligned}$$

(3.16)

Similarly, from (3.16), we get,

$$\begin{aligned}\|T(w,\eta_n(w))-\xi^*(w)\| &= \|T(w,\xi^*(w))-T(w,\eta_n(w))\|\\ &\leq e^{L(w)\|S(w,\xi^*(w))-T(w,\xi^*(w))\|}(\|S(w,\xi^*(w))-S(w,\eta_n(w))\|\\ &- \phi(\|T(w,\xi^*(w))-T(w,\eta_n(w))\|))\\ &\leq \|S(w,\eta_n(w))-\xi^*(w)\|\\ &\leq (1-\beta_n)\|S(w,\zeta_n(w))-\xi^*(w)\|+\beta_n\|T(w,\zeta_n(w))-\xi^*(w)\|\\ &\leq (1-\beta_n)\|S(w,\xi_n(w))-\xi^*(w)\|+\beta_n\|S(w,\xi_n(w))-\xi^*(w)\|\\ &= \|S(w,\xi_n(w))-\xi^*(w)\|\end{aligned}$$

(3.17)

Using (3.15), (3.16) and (3.17) with the condition $\alpha_n+\beta_n+\gamma_n\leq 1$ we obtain,

$$\begin{aligned}\theta_n &= \phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\gamma_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|)\\ &- \beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|)\\ &\geq \phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\gamma_n\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)\\ &- \beta_n\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)\\ &= [1-(\alpha_n+\beta_n+\gamma_n)]\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)\\ &\geq 0\end{aligned}$$

Since $\sum_{n=0}^{\infty}(\theta_n+\varepsilon_n)<\infty$, then $\lim_{n\to\infty}(\theta_n+\varepsilon_n)=0$ and by Lemma 2.1, we get

(3.18) $\lim_{n\to\infty}\|S(w,\xi_n(w))-\xi^*(w)\|=0 \ or \ S(w,\xi_n(w))\to\xi^*(w) \ as \ n\to\infty.$

Also, we have by using (3.15), (3.16), (3.17) and (3.18)

(3.19) $0\leq\|T(w,\xi_n(w))-\xi^*(w)\|\leq\|S(w,\xi_n(w))-\xi^*(w)\|\to 0$ as $n\to\infty.$

(3.20) $0\leq\|T(w,\zeta_n(w))-\xi^*(w)\|\leq\|S(w,\xi_n(w))-\xi^*(w)\|\to 0$ as $n\to\infty.$

(3.21) $0\leq\|T(w,\eta_n(w))-\xi^*(w)\|\leq\|S(w,\xi_n(w))-\xi^*(w)\|\to 0$ as $n\to\infty.$

Since ϕ is continuous, from (3.18)- (3.21), we obtain

$$\begin{aligned}\lim_{n\to\infty}\theta_n &= \lim_{n\to\infty}[\phi(\|S(w,\xi_n(w))-\xi^*(w)\|)-\gamma_n\phi(\|T(w,\xi_n(w))-\xi^*(w)\|)\\ &- \beta_n\phi(\|T(w,\zeta_n(w))-\xi^*(w)\|)-\alpha_n\phi(\|T(w,\eta_n(w))-\xi^*(w)\|)]\\ &= 0.\end{aligned}$$

Hence the Jungck-SP type random iterative scheme $\{S(w, x_n(w))\}_{n=0}^{\infty}$ is a comparably almost (S,T)- stable.
Next, suppose that $S(w, \xi_n(w)) \to \xi^*(w)\ as\ n \to \infty$, and using (3.21), then we obtain

$$\begin{aligned}\varepsilon_n &= \|S(w,\xi_{n+1}(w))-(1-\alpha_n)S(w,\eta_n(w))-\alpha_nT(w,\eta_n(w))\|\\ &\leq \|S(w,\xi_{n+1}(w))-\xi^*(w)\|+(1-\alpha_n)\|S(w,\eta_n(\omega))-\xi^*(w)\|\\ &+ \alpha_n\|T(w,\eta_n(w))-\xi^*(w)\|\\ &\leq \|S(w,\xi_{n+1}(w))-\xi^*(w)\|+(1-\alpha_n)\|S(w,\xi_n(\omega))-\xi^*(w)\|\\ &+ \alpha_n\|S(w,\xi_n(w))-\xi^*(w)\|\\ &= \|S(w,\xi_{n+1}(w))-\xi^*(w)\|+\|S(w,\xi_n(\omega))-\xi^*(w)\|\\ &\to 0\ as\ n\to\infty.\end{aligned}$$

□

REFERENCES

1. Y. I. Alber and S. Guerre-Delabriere: *Principle of weakly contractive maps in Hilbert spaces.* In: Gohberg, I, Lyubich, Y(eds.), New Results in Operator Theory and Its Applications .Birkhuser, Basel **98** 1997, 7-22.
2. I. Beg: *Approximaton of random fixed points in normed spaces.* Nonlinear Anal. **51** 2002, 1363-1372.
3. I. Beg: *Minimal displacement of random variables under Lipschitz random maps.* Topol. Methods Nonlinear Anal. **19** 2002, 391-397.
4. I. Beg and M. Abbas: *Iterative procedure for solutions of random operator equations in Banach spaces.* J. Math. Appl. **315** 2006, 181-201.
5. I. Beg and N. Shahzad: *Random fixed point theorems for nonepansive and contractive type random operators on Banach spaces.* J. Appl. Math. Stochastic Anal. **7** 1994, 569-580.
6. A. T. Bharucha-Reid: *Fixed point theorems in probabilistic analysis.* Bull. Amer. Math. Soc. **82** 1976, 641-657.
7. O. Hans: *Reduzierende zulliallige transformaten.* Czechoslovak Math. J. **7** 1957, 154-158.
8. O. Hans: *Random operator equations.* Proceedings of the fourth Berkeley Symposium on Math. Statistics and Probability **II** 1961, 185-202.
9. A. M. Harder: *Fixed point theory and stability results for fixed points iteration procedures.* Ph. D. Thesis, University of Missouri- Rolla, 1987.

10. A. M. Harder and T. L. Hicks: *A stable iteration procedure for nonexpansive mappings*. Math. Japonica **33(5)** 1988, 687-692.
11. A. M. Harder and T. L. Hicks: *Stability results for fixed point iteration procedures*. Math. Japonica **33(5)** 1988, 693-706.
12. S. Ishikawa: *Fixed points by a new iteration method*. Proceedings of the American Mathematical Society **44** 1974, 147-150.
13. S. Itoh: *Random fixed point theorems with an application to random differential equations in Banach spaces*. J. Math. Anal. Appl. **67** 1979, 261-273.
14. G. Jungck: *Commuting mappings and fixed points*. The American Mathematical Monthly **83** 1976, 261-263.
15. K. S. Kim: *Convergence and stabililty of generalized ϕ-weak contraction mapping in CAT(0) Spaces*. Open Math. **15** 2017, 1063-1074.
16. W. R. Mann: *Mean Value methods in iteration*. Proceedings of the American Mathematical Society **4** 1953, 506-510.
17. M. A. Noor: *New approximation schemes for general variational inequalities*. J. Math. Anal. Appl. **251** 2000, 217-229.
18. G. A. Okeke and M. Abbas: *Convergence and almost sure T-stability for a random iterative sequence generated by a generalized random operator* Journal of Inequalities and Applications **146** 2015, 1-11.
19. G. A. Okeke and J. K. Kim: *Convergence and summable almost T-stability of the random Picard-Mann hybrid iterative process*. Journal of Inequalities and Applications **290** 2015.
20. G. A. Okeke and J. K. Kim: *Convergence and (S,T)-stability almost surely for the random Jungck-type iteration processes with applications*. Congent Mathematics **3:1258768** 2016, 1-15.
21. M. O. Olatinwo: *Some stability and strong convergence results for the Jungck-Ishikawa iteration process*. Creative Mathematics and Informatics **17** 2008, 33-42.
22. M. O. Olatinwo: *Some stability results for two hybrid fixed point iterative algorithms of Kirk-Ishikawa and Kirk-Mann type*. Journal of Advanced Mathematical Studies **1** 2008, 514.
23. M. O. Osilike: *Stability of the Mann and Ishikawa iteration procedures for ϕ-strong pseudo-contractions and nonlinear equations of the ϕ- strongly accretive type*. J. Math. Anal. Appl. **227(2)** 1998, 319-334.
24. N. S. Papageorgiou: *Random fixed point theorems for measurable multifunction in Banach spaces*. Proc. Amer. Math. Soc. **97** 1986, 507-514.
25. W. Phuengrattana and S. Suantai: *On the rate of convergence of Mann, Ishikawa, Noor and SP iterations for continuous functions on an arbitrary interval*. J. Comp. Appl. Math. **235** 2011, 3006-3014.
26. R. A. Rashwan, H. A. Hammad and G. A. Okeke: *Convergence and almost sure (S,T)-stability for random iterative schemes*. International Journal of Advances in Mathematics **2016 NO.1** 2016, 1-16.
27. B. E. Rhoades: *Some theorems on weakly contractive maps*. Nonlinear Anal. **47**(4) 2001, 2683-2693.
28. S. L. Singh, C. Bhatnagar and S. N. Mishra: *Stability of Jungck-type iterative procedures*. International Journal of Mathematics and Mathematical Sciences **19** 2005, 3035-3043.

29. A. Spacek: *Zufallige gleichungen*. Czechoslovak Math. J. **5** 1955, 462-466.
30. H. K. Xu: *Some random fixed point theorems for condensing and nonexpansive operators*. Proc. Amer. Math. Soc. **110** 1990, 103-123.
31. Z. Xue: *The convergence of fixed point for a kind of weak contraction*. Nonlinear Func. Anal. Appl. **21(3)** 2016, 497-500.
32. SS. Zhang, XR. Wang and M. Liu: *Almost sure T-stability and convergence for random iterative algorithms*. Appl. Math. Mech. **32(6)** 2011, 805-810.

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