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A MODELING FRAMEWORK ON DISTANCE PREDICTING FUNCTIONS FOR LOCATION MODELS IN CONTINUOUS SPACE

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Abstract. Continuous location models are the oldest models in locations analysis dealing with the geometrical representations of reality, and they are based on the continuity of location area. The classical model in this area is the Weber problem. Distances in the Weber problem are often taken to be Euclidean distances, but almost all kinds of the distance functions can be employed. In this survey, we examine an important class of distance predicting functions (DPFs) in location problems all of practical relevance. This paper provides a review on recent efforts and development in modeling travel distances based on the coordinates they use and their applicability in certain practical settings. Very little has been done to include special cases of the class of metrics and its classification in location models and thus merit further attention. The new metrics are discussed in the well-known Weber problem, its multi-facility case and distance approximation problems. We also analyze a variety of papers related to the literature in order to demonstrate the effectiveness of the taxonomy and to get insights for possible research directions. Research issues which we believe to be worthwhile exploring in the future are also highlighted.

Keywords: Distance predicting functions, location, optimization, taxonomy.

1. Introduction

1.1. Location theory and DPFs

In [86], DPFs are affirmed to play an important role in many applications. The purpose of a DPF is to give an accurate measure of the distance between any two points in a space [76]. However, in physical location problems, this separation normally signifies the shortest travel distance between pairs of points in the transportation networks. Apart from location problems, applications of DPFs abound: they range from distribution problems, traveling salesman problem to the vehicle routing problem [4]. Other useful areas include approximation theory [61], statistical estimation problem [4, 58, 60], signal and image processing and engineering applications [71].

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There are four components that characterize location problems. These are
(1) customers who are presumed to be already located at points,
(2) facilities that will be located,
(3) a space in which customers and facilities are located, and
(4) a metric that indicated distances or times between customers and facilities [67]. For more details on location science or models, see e.g. [33, 67, 82]. The distance functions or metrics are the focus of this paper. Here we distinguish between different types of DPFs based on the type of coordinates they use and their applicability in diverse settings. We will discuss this issue in much more depth below; for now, it should suffice to say that there is more to DPFs than simple metrics or norms.

The choice of suitable distance predicting functions played a crucial role in a good estimation of travel distances in realistic environments, depending on the mode of travel and the type of problem considered. This indicates that different modes of transportation require different ways of distance estimation. Consequently, an increasing effort to estimate travel distances with high accuracy has arisen from the growing variety in the transportation sector [15, 16, 17, 19, 25, 32, 37, 46, 47, 48, 49, 50, 65]. Of course, it is a well-known parameter, but its determination in certain settings could be challenging and such a parameter that the decision makers seek to optimize in location analysis using the mean distance concept, median concept, and covering techniques [86]. Technical tools used to solve a location problem are consequences of its structure and in this way imply a close link to distance functions. Therefore, computational techniques (algorithms, heuristics, simulations, etc.), the use of convex analysis depend directly on the selected metric. Structure and stability of the solution set, coincidence conditions, are also affected by the choice one made [5].

1.2. A brief literature review and statistical findings

There are a variety of approaches in modeling travel distances between two points, ranging from shortest path calculations on a detailed street network to analytical approximations based on a few parameters [14]. Among the analytical approximate approaches proposed for the location problems with various or arbitrary distance functions transform the problem into a discrete location problem which takes many computational resources [40, 41] with no guarantee of the appropriate precision of the results. The influence of DPFs on location modeling, explicitly with respect to computational tractability and on the quality of solution obtained has been investigated by [5, 51]; while non-convex distance measures in barriers influenced location models are discussed in [46, 47]. Applications in urban environments and comparisons of distance functions and associated parameters can be found in Love and Morris [53, 55, 56], Berens [8], Berens and Koerling [9, 10] and Fernandez et al. [29]. For a detailed survey on the estimation of distances, reference is made to Brimberg and Love [12]. A non-parametric approach using neural networks for estimating actual distances on a real-world study using cities drawn from Turkey can be found in [4]. A recent article that studies the DPFs in location models includes

[86], but this is rather a general review whereas we focus on the systematic review in this study.

Various DPFs were studied in [86]; and their analysis and development of that work motivate many of the extensions in this paper. Another motivation is to give people new to the field an impression of what might be interesting to look at in location theory. The DPFs has not been as popular; only a limited number of papers on this issue have appeared in broad view. The majority of the literature focused on the specific type of distance measure. However, the aim of this paper is not to review all of the literature, but to develop a taxonomy framework which would help us identify the underlying characteristics and fundamental features of the concepts. Besides being a major component of location models, DPFs are utilized in several applications such as emergency control [48], crime control [74] and cluster analysis [58]. In what follows, we report some statistical findings.

We have looked at the academic literature in order to observe the general trend in terms of number of articles and journals. We used GOOGLE SCHOLAR for certain journal titles and topics. The journal titles and topics searched using different keyword combinations: “distance functions” OR “metrics”, OR “travel distances” OR “distance approximations” OR “objective”. The results are summarized as follows:

- About 230 source titles were discovered (in spite of the fact that we may be missing some relevant articles or those that would not directly fit in the framework that we are interested in, we believe that this list fairly reflects the general trend).

- It is interesting to note that there is growing interest in the subject after 2000.

- About 90% of the sources are journal articles and conference papers

- The articles span a range of areas including Operations Research, Regional Science, Management Science, Mathematics, Decision Science, Computer Science, Industrial Engineering, Civil Engineering, and Health Sciences. This is also an indication for a unified taxonomic scheme of the articles related to this topic.

- It is of note that only a few results were published for DPFs in broad view. References are made to [48, 83] and few dissertations [9, 76].

Table 1.1 shows the number of articles for each journal title for which there is at least an article. It is observed that the journals in this list have published more than 50% of the papers analyzed.

This paper seeks to present some useful classes of DPFs that are less studied in the literature and their formulations to handle many more location problems as well as to give insights for possible research directions. In particular, proposed methods of the review are as follows:

- Identify the key groups of the DPFs in facility location problems

Table 1.1: Numbers of articles in journals

European Journal of Operational Research (09)
Operations Research (05)
Annals of Operations Research (03)
Facta Universitatis, Series: Mathematics and Informatics (03)
Journal of the Operational Research Society (03)
SIAM Journals (02)
Advances in Operations Research (02)
Others including technical reports and preprints (24)

- Classify distance measures and discuss implementation for the models based on theoretical and practical studies.

In the remaining part of the paper, after a brief introduction of Weber problem and the traditional metrics, grouping of DPFs are discussed. In the next section 3, we introduce the literature search process and taxonomic scheme of DPFs in location problem literature. Section 4, provides recent results of the reviewed literature. Future directions will be presented in section 5 and the paper then ends with some concluding remarks in section 5.

2. Developing a framework for DPFs classification

The problem of locating a facility so as to serve optimally a given set of customers, where locations and demands are known is a very old one. The classical study of industrial location analysis was pioneered by Weber [81], who studied the location on a plane of a factory between two resources and a single market. After the work of Weber, interest in the location analysis has steadily increased and several extensions, modifications and generalizations have been reported. Formally, the problem may be stated as follows:

$$\min z = \sum_{i=1}^{n} w_i d(x, a_i) \tag{2.1}$$

where $x = (x_1, \ldots, x_n)^T$ is the unknown position of a new facility; $a_i = (a_{i1}, \ldots, a_{in})^T$ is the known position of the i^{th} fixed point, $i = 1, \ldots, m$; $w_i = 1, 2, \ldots, m$ are non-negative constant that translate distances into the cost under the assumption that flow costs are proportional to distances. One parameter that is of utmost priority in (2.1) is the distance function $d(.,.)$. For the estimation of road distances between points based purely on coordinates of the end points, a location scientist often focus on three cases.

- The rectilinear or manhattan or l_1 distance, derived in the case $p = 1$:

$$l_1 = \sum_{i=1}^{n} |x_i - y_i| \tag{2.2}$$

- the Euclidean (or straight line or l_2) metric, corresponding to the case $p = 2$:

$$l_2 = \sqrt{\sum_{i=1}^{n} |x_i - y_i|^2}; \tag{2.3}$$

- and the Tchebyshev (or "max", or l_∞) metric, defined by

$$l_\infty = \max |x_i - y_i| \tag{2.4}$$

However, the Weber location problem favored most the Euclidean metric [64] and a large part of the literature focuses on this type of Minkowski distance follow by the rectilinear distances. The Tchebychev distances have been less studied in modeling location problems, but shown to be useful in practical problem involving the location of items moved in and out of an automated storage and retrieval system ([30, 63]). However, these distance measures are not realistic for some applications, for instance, material handling in plants [86], systems which use rotating mechanisms (like telescopic booms, lifting cranes, manipulators, etc.) as transportation means [42, 43], spider's web like road network structure [62, 64], and distance measurement based on great circle distance which represents the shortest path on a spherical surface [6, 22, 23, 39, 59, 83].

There are other DPFs that yield better approximation in certain settings and also very useful in applications. Since each distance function has its own strengths and weaknesses; therefore location scientists must take extra care to select DPF to match their needs and applications. In our identification of the key groups, we shall adopt the grouping of the authors [4] with respect to the type of coordinates they use and subsequently follow by their applicability in various settings.

2.1. DPFs based on spherical coordinates

This distance function is a suitable model when the demand points are widely separated. The area covering such points can no longer be approximated by a plane. Spherical distance provides a good approximation for large region location problem applicable in the detection of station placement, placement of a radio transmitter (for long range communication), international headquarters location in defense systems, distribution and marketing centers, etc. In the past years, many researchers have dealt with location problems involving points on a sphere (for details, see [23, 83] and references therein). The authors of [39] have considered the problem where the destination and source points are restricted to be on the surface of the sphere, but the distance norm used is Euclidean. Elsewhere [6] presented a unified approach to multisource location problems on a sphere using Euclidean, square Euclidean, and great circle distances with its convergence properties. Dhar and Rao [22] have investigated a comparative study of three norms for the classical Weber problem on a spherical surface proposing a Weiszfeld-like algorithm for its solution. The recent effort in that direction can be found in [59] where an alternative approach was introduced to distance modeling for travel over water in place of Euclidean distance by developing an algorithm for deducing distances from geographical addresses defined by the grid of latitudes and longitudes and spherical trigonometry principles.

2.2. DPFs based on polar coordinates

The motivation for this group of coordinates is based on the observation that the roads in old cities are not usually planned according to a grid pattern and therefore distances can be approximated better by a star-shaped measure or ring-radial measure. Some of the DPFs in this category are presented below.

2.2.1. French metro metric

Several types of non-convex distance measures have been proposed in the context of location [66]. Perreur and Thisse [64] introduced metrics in the plane suitable for location situations around a center with either radial or circumferential structure network or combination of both. French metro metric is described by the following distance function [20, 21]:

$$d_F(X, A_i) = \begin{cases} \|X - A_i\|, & A_i = cX \\ \|A_i, 0\| + \|X, 0\|, & A_i \neq cX. \end{cases} \tag{2.5}$$

Here, c is a real coefficient, and $\| \; \|$ denotes the usual Euclidean metric. Transforming (2.5) using polar coordinates as in [62], formula (2.5) becomes

$$d_F(X, A_i) = \begin{cases} |x_{R1} - a_{R1}|, & x_{R2} = a_{R2,i} \\ a_{R1,i} + x_{R1}, & x_{R2} \neq a_{R2,i}. \end{cases} \tag{2.6}$$

This makes possible to evaluate the French metro distance in location contexts of business and service planning, transportation, etc., as it was studied in [62]. The approach seems to be very accurate especially for a spider's web like road network structure.

2.2.2. Moscow metric

Moscow distances have been employed in the construction of Voronoi diagrams [48] for cities like Moscow, Karlsruhe, and Amsterdam. A Voronoi diagram of a finite point set is a powerful mechanism directly related to distance measures, with strong application potential for location problems. Plastria [66] discussed that some researchers, such as Mittel and Palsule (1984), looked at the continuous version of Perreur and Thisse's circum-radial metric [64], by considering the "ring-radial" distance metric obtained when movement is restricted to straight lines emanating from a fixed center and circles centered at the same point. The streets in the cities like Moscow, Karlsruhe and Amsterdam are divided into two classes:

"rays" starting from the central place (Moscow Kremlin or central train station in Amsterdam) or "rings" around this central point. The "rings" do not cross each other, their form is close to a circle and the center of these circles is close to the central place.

2.2.3. Tower crane distance

A similar to ring-radial metric, but still different distance measure, is the crane distance. The crane distance is employed at the location of crane in construction sites to minimize the time necessary to move facilities from one position to another. Abdel-Khalel and Shawki [1] affirmed that the crane location models have evolved over forty years. Several researchers have been attempting to solve crane location problem. Warzawski and Peer [79] established a time-distance formula by which quantitative evaluation of location was possible. It was reported in [1] that Rodrigues-Ramos and Francis (1983) developed a mathematical model to establish the optimal location of a single tower within a construction site. The model of Choi and Harris (1991) optimized the single tower crane location by calculating total transportation times incurred. Zhang et al. (1991) developed a stochastic simulation model to optimize a single tower crane and then in 1992 developed a mathematical model for location optimization for a group of tower cranes by using the Monte Carlo simulation approach. Other efforts in this direction are those presented by the authors [3], who developed a new model for crane location optimization using genetic algorithms and that of the Irizarry and Karan [35] who introduced the use of GIS and BIM integration in crane location. A more recent result in crane optimization was reported in Kazakovtsev et al. [44], where four distance functions were employed for manipulator control and algorithms proposed for solving such problems. The results in [44] will be highlighted in the subsequent section.

2.3. DPFs based on simple functions of the Cartesian coordinates

This approach contains some simple functions of the Cartesian coordinates. These are mostly norms or norm-based functions. They are further sub-divided into the following two non-Euclidean functions reducible to l_1 metric and norm-based functions.

2.3.1. Non-Euclidean DPFs reducible to l_1 metric.

The results in this section have been partially published in [44, 72]. However, we will discuss in this section some properties and applications of these distance measures in two-dimensional cases (unless it is otherwise stated).

2.3.1.1 Lift Metric

Lift distances are a special case of norm distances which were introduced to location models by Stanimirović et al. [72]. Lift distances are used to model road networks with only one main street and the other crossing it at right angles. A similar situation occurs in a tier building, where the lift (in the role of y-axis) connects tiers. Also, the location problems in the mines and in the rack storages can be formulated as the problems with the lift metric. The lift metric for two points x and a_i in Cartesian coordinate system is denoted by $d_L(x, a_i)$ and defined in [21, 72]:

$$d_L(x, a_i) = \begin{cases} |x_1 - a_{i,1}| & x_2 = a_{2,i} \\ |x_1| + |x_2 - a_{2,i}| + |a_{1,i}|, & x_2 \neq a_{2,i}. \end{cases} \tag{2.7}$$

2.3.1.2 British Rail Metric

Given a norm in $\mathbb{R}^2$ (in general in $\mathbb{R}^n$), the British rail distance is a metric on $\mathbb{R}^2$, defined in [21] by

$$\|x\| + \|y\| \tag{2.8}$$

in the case $x \neq y$ (and it is equal to zero, otherwise). Thus, the distance between two points x and a_i is equal to

$$d_{BR}(x, a_i) = \begin{cases} X_r + A_r^i, & X \neq A_i \\ 0, & X = A_i \ \forall i = 1, \ldots, m \end{cases} \tag{2.9}$$

assuming that any path between two points include the central point (origin).

2.3.2. Norm-based DPFs

Most DPFs in continuous location problems belong to the family of norms [11]. Several of these norms and norm-based DPFs have had widespread use in facilities location applications.

2.3.2.1 Block distance

The block norms are norms whose contours are polytopes. For example l_1 and l_∞ are block norms. The block norms first time are used to solve the location problems by Witzgall [85] and Ward and Wendell [77, 78]. They showed a block norm can be characterized as follows:

$$k(x) = \|x\|_B = \min\left\{\sum_{g=1}^{r} |\lambda_g| : x = \sum_{g=1}^{r} \lambda_g b_g\right\}, \tag{2.10}$$

where the points b_g and $-b_g$ with $g = 1, \ldots, r$ are vectors which define the extreme points of the polytope corresponding to the unit contour. The block norm is also characterized as follows:

$$\|x\|_B = \max\{|xb_g^\circ| : g = 1, \ldots, r^\circ\}, \tag{2.11}$$

where b_g° and $-b_g^\circ$ with $g = 1, \ldots, r$ are extreme pints of the polar set

$$B^\circ = \{v : b_g v \leq 1 \forall g = \pm 1, \pm 2, \ldots, \pm r\}. \tag{2.12}$$

When the block norms are used for characterizing distance in facilities location models, such as the single facility location models, linear programming problems can be formulated [27, 76]. A block norm can be made to represent a round one as accurately as desired by increasing the number of extreme points (b_g's) of its polytope. In the limiting sense then, as $r \to \infty$, the block norm becomes a round one [76].

2.3.2.2 *Round norms*

The round norm has been used extensively in facility location problems (see e.g. [9]), where it is defined as follows:

$$l_{k,p} = d(x, y) = k\left[\sum_{i=1}^{2} |x_i - y_i|^p\right]^{\frac{1}{p}}, k > 0, p \geq 1, \forall x \in \mathrm{Re}^2 \tag{2.13}$$

This norm is called a round norm by Thisse, Ward and Wendell [73] because the unit contour u, for any $p \in (1, \infty)$ and $k > 0$ contains no flat spots. Round norms allow a generalization of known results in continuous minisum location models.

2.3.2.3 *Gauges*

The gauge distance can be applied when a non symmetric shipment exists, as with travel up and down streams or inclined planes, and travel along one-way streets in an urban area. Plastria [66] discussed that researchers such as Durier and Michelot (1985) studied theoretically asymmetric distance measures. While Hodgson, Wong and Honsaker [34] and Drezner and Wesolowsky [24] applied asymmetric distance measures using Weiszfeld-like algorithm. The authors of [34] formulated a minisum model to determine the optimal location of log harvesting problem on a slope and developed a Weiszfeld - type iterative solution procedure, proving convergence of their algorithm to the optimal site. Gauges defined by Minkowski functional of compact convex sets containing the origin in its interior have a very interesting property useful in facilities location problems (see [47]). The author of [86] pointed that in gauges, the distance between two points is the shortest path between them using only fundamental directions of the unit ball.

2.3.2.4 A-distance

Analogous to the symmetric polyhedral gauges, often called block norms, is the A-distance. This derived distance defined by Widmayer et al. [84] arises when movement is restricted to a finite set of (oriented) directions. The A-distance between two points $a_1, a_2 \in \mathbb{R}^2$ is defined as follows

$$d_A(a_1,a_2) = \begin{cases} |a_1 - a_2|, & \text{if } [a_1,a_2] \text{ is A-oriented,} \\ \min\limits_{a_3 \in \mathrm{Re}^2} \{d_A(a_1,a_3) + d_A(a_3,a_2)\} & \text{otherwise,} \end{cases} \tag{2.14}$$

wherein $[a_1, a_2]$ denote the line segment between $a_1, a_2 \in \mathbb{R}^2$ and $\|\,\|$ is the Euclidean norm.

The A-distance is used in many distance problems, such as Voronoi diagram, minimum spanning trees, minimax distances between convex polygons and other set of points [50]. The authors of [50] considered a minisum location problem under A-distance, and studied the properties of optimal solutions for the problem and proposed the edge tracing algorithm to find all optimal solutions for the problem. According to [74], Shiode and Ishii 1991 considered single facility stochastic location problem under A-metric. Recent results in this direction have been proposed in a competitive environment. Uno et al. [74] introduced A-distance in competitive location problem, reformulated the problem as a combinatorial optimization problem and solved using genetic algorithms.

2.4. Special purpose DPFs

The classical Weber location problem has been well researched over the years with the commonest used metrics which include Euclidean, rectangular and Tchebychev. Indeed, many results have been generalized for n-dimensional space; however, very little has been done to include special cases of the class of metrics in location models. These include some distance functions or measures which do not fit in completely in any of the above mentioned three groups. They can be included here as they are not always simple functions of the coordinates and require additional information such as rotating angle for the coordinate axes [44] or vectors for possible directions on a typical road [77, 78]. The authors of [77, 78] suggested the hybrid distance approximations such as weighted one-infinity norms which were later generalized to obtain a family of block norms. Another hybrid distance function presented is due to Brimberg and Love [12]. It is called weighted one-two norm since the rectilinear and Euclidean elements of the travel are presented by the weighted l_1 and l_2 norms.

2.4.1. Jaccard distance

In cluster analysis, DPFs are used to measure similarity or difference between data objects such as documents, images and signals; if the data objects are represented by vectors [31]. Spath [70] introduced Jaccard distance in minisum location problem otherwise known as Weber problem. The Jaccard distance was shown to be a metric on a set of binary vectors. Consider a metric space M and one its metric $d(.,.)$. Further, let $a_i \in M,\ i = (1, 2, \ldots, m \geq 2)$; the minisum location problem under Jaccard distance is defined as

$$\min z_j = \sum_{i=1}^{m} d_j^{(a)}(z, a_i), \tag{2.15}$$

where $d_j^{(a)}(.,.)$ is the Jaccard metric. [18] also studied the computational complexity of the Jaccard median problem while Watson [80] gave a vertex-descent algorithm for the minisum location problem using Jaccard median and showed that his algorithm terminates and always returns an optimal solution.

2.4.2. The Hamming distance

Richard Hamming introduced Hamming distance as a measure of errors that transform one string of a binary code into another in 1950. Since then it has found applications in several scientific disciplines, including information theory, coding theory, cryptography and combinatorial optimization etc., [38]. The Hamming distance is applied in network location models under the sum-type and bottleneck type objectives.

The center location improvement problem has been considered under Hamming distance. For the sum type objective; the problem (called the center location improvement problem under the sum-type Hamming distance, and denoted by (CLISH) is to improve the network such that the distance between vertex S and other vertices of the network cannot exceed the given upper bounds and the total cost of modifying edges is minimized under the Hamming measurement. Details of location models under Hamming distance with both the sum-type and bottleneck type objectives are contained in [87]. The corresponding unconstrained location problem under Hamming distance is stated as

$$\min z_H = \sum_{i=1}^{n} c_i d_H(w_i^{\phi}, w_i), \tag{2.16}$$

where $d_H(w_i^{\phi}, w_i) = \begin{cases} 1 \text{ if} & w_i^{\phi} \neq w_i \\ 0 \text{ if} & w_1^{\phi} = w_i. \end{cases}$

The only setback with the use of Hamming distance in the minisum location problem is that the Hamming distance $H(.,.)$ is discontinuous and non-convex which makes the known methods for l_p-norms unable to be applied directly to the problem under such distance measure [52].

Apart from these two special DPFs, other distances used in location problem include aisle distance, distance matrix, minimum length paths, Hilbert curve, Mahalanobis distance, Hausdoff distance, Levenshtein distance and a variety of distances [59, 86]. It was reported in [59] that these distances were classified into multi-parameter round norms, block norms and polyhedral distances in [47]. However, the authors of [45] presented another DPF called "taxi" metric and proposed an algorithm using results for solving the Weber problem by Weiszfield procedure as the initial point for a special local search procedure. In what follows, we present the summary of the references for the reviewed DPFs (Table 2.1) and recent results in location problems.

Table 2.1: Reference list for DPFs in location problems

S/N	DPFs	Type of problem	References
1.	Spherical distance	Large region location problems	Aykin and Babu [6], Dhar and Rao [22], Katz and Cooper [39], Mwemezi and Huang [59], Wesolowsky [83].
2.	French Metro metric	Single and multi-facility location problems, Design distribution system, Business planning location	Kazakovtsev and Stanimirovic [43], Osinuga et al. [62], Perreur and Thisse [64].
3.	Moscow metric	service location problem and crime control.	Kazakovtsev et al. [44], Klein [48], Perreur and Thisse [64].
4.	Tower crane distance	Single and group tower location problems in construction sites.	Abdel-Khalel et al. [1], Alkriz and Mangin [3], Irizarry and Karan [35],Warzawski [79]
5.	Lift metric	Single and multi-facility location problems, Location problem in mining and rack storage.	Kazakovtsev and Stanimirovic [43], Kazakovtsev et al. [44], Stanimirović et al. [69].
6.	British rail metric	Crane location optimization	Kazakovtsev et al. [44]
7.	Block distance	Barrier and restricted planar location problems	Brimberg [11], Fathali and Zaferanieh [27], Uster [75], Walter [76], Ward and Wendell [77, 78], Witzgall [85].
8.	Round norms	Barrier and restricted location problems	Brimberg [11], Thisse et al. [73], Walter [76], Zarimbal [86].

9.	Gauges	Business planning location, Barrier location problem	Drezner and Wesolowsky [24], Hodgson et al. [34], Witzgall [85], Zarimbal [86].
10.	A-distance	Design of VLSI, Distance problems such as Voronoi diagram, emergency minimax location problems, minimum spanning trees etc	Kon and Kushimoto [50], Uno et al. [71], Widmayer et al. [81].
11.	Jaccard distance	Single facility location problem and clustering analysis	Chierichetti et al. [18], Spath [70], Watson [80].
12.	Hamming distance	Center location improvement problem, Quadratic assignment problem	Kammerdiner et al. [38], Liu and Yao [52], Zhang et al. [87].
13.	Others	Single and multi facility location problems, The median shortest path problem, Assigning machine to locations.	Kazakovtsev and Stanimirovic [45], Zarimbal [86].

3. Location search process and taxonomic scheme

We scrutinize the articles that we need to produce the statistics in Section 1.2 in order to find the ones that would best fit to this typical framework. Finally, 63 papers were classified in the taxonomy in total. There are other important studies on DPFs that would not fit into this framework. For instance, in some articles the goal is to estimate the accuracy of Euclidean and Manhattan distances only [8, 9, 10, 29, 53, 55, 57] while in some references the authors only paraphrase the various DPFs in literature without any taxonomy [51, 86]. There are articles that suggest a taxonomy for various area of research [2, 7, 13, 26] whiles some provided a structured overview of the literature on location theory [66, 69]. We build our taxonomy tree with at most three levels in order to provide simplicity and an ability to understand a broad range of features. Since a paper may belong to several different subcategories under the same category, the first and second level classifications are not strictly differentiating. In the first level of classification tree, we discussed briefly the general properties of the DPFs under the "coordinate type". Then we have subcategories where we consider further basic components and models, as well as the applications. However, this does not apply to all categories. The whole classification trees can be seen in Fig. 3.1 below:

3. DPFs Type
    - 3.1 Spherical coordinates type (a)
    - 3.2 Polar coordinates type (b)
        - 3.2.1 French metro metric (c)
        - 3.2.2 Moscow metric (d)
        - 3.2.3 Tower crane distance (e)
    - 3.3 Cartesian coordinate type (f)
        - 3.3.1 non-Euclidean reducible to l_1 metric (g)
            - 3.3.1.1 Lift metric (h)
            - 3.3.1.2 British rail metric (i)
        - 3.3.2 Norm-based type (j)
            - 3.3.2.1 Block distance (k)
            - 3.3.2.2 Round norms (l)
            - 3.3.2.3 Gauges (m)
            - 3.3.2.4 A-distance (n)
    - 3.4 Special DPFs type (o)
        - 3.4.1 Jaccard Metric (p)
        - 3.4.2 Hamming distance (q)

Fig. 3.1: Taxonomy of the DPFs literature

We used 63 of the articles representing different DPFs in location problems. The articles used in the taxonomic review are listed in Tables 3.2 and **??**. Empty cells mean that the paper does not involve any information about the specified attribute. On the contrary, the mark "X" in a cell means that the corresponding paper can be associated with that attribute.

Table 3.1: Classification of the selected articles

Paper	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q
1					X												
3					X												
4	X	X				X			X	X	X	X			X		
6	X																
8						X											
9						X											
10						X											
11	X	X	X	X		X				X	X	X	X			X	
12																	
14						X											
15																	

Table 3.2: Classification of the selected articles

16						X										X	
17						X											
18																X	
19						X				X	X						
20																	
21																	
22	X																
23	X																
24						X				X			X				
27										X	X						
29						X				X							
31																X	
32															X		
34						X				X			X				
35					X												
38															X		X
39	X																
41						X				X							
42			X	X													
43		X	X			X	X	X									
44		X	X	X	X	X	X	X	X								
46						X				X	X	X	X				
47						X				X	X	X	X				
48		X		X													
50										X				X			
51						X				X							
52															X		X
53						X				X							
54						X				X							
55						X				X							
59	X																
62		X	X														
63						X				X							
64		X	X	X	X												
66	X	X				X				X	X	X	X	X	X	X	
68																	
70															X	X	
71																	
72						X	X	X									
73						X				X	X	X					
74						X				X			X				
75																	
76						X				X	X	X					
77						X				X							
78						X				X	X						
79		X			X												
80															X	X	
83	X																
84						X				X				X			
85						X				X							
86		X				X				X	X	X	X				X
87																	X

Among all the attributes, only four articles do not involve any information about the specified attribute. The unmarked articles constitute only a small percentage 6.34%, among all attributes. Also, the attributes marked only twice, and thrice times constitute 1.59%, 3.17% respectively. These values indicate that our taxonomy is robust enough to systematically identify the literature about DPFs in location theory by showing that 95.24% of all the attributes were studied in at least two articles. The attributes marked twice and thrice in the taxonomy are 3.3.1, 3.3.1.1, and 3.3.1.2. This shows that the lift metric and British rail metric are not studied deeply in literature. On the other hand, when we examine the attributes that are frequently marked we observe that most articles present a deterministic model with a single objective. In general, the number of articles discussing deterministic models is greater than the ones including stochastic ones.

4. Results in continuous location problems

Recently, several interesting results on DPFs are used to find solutions of the continuous minisum location problems. In most of the cases the basic tools has been the non-Euclidean metrics reducible to rectangular distances. A good deal of work has been associated with these metrics.

The following results dealt with the generalized metrics employed in the study of multi-facility location [43].

The new generalized metric defined by

$$G(A_i, X) = \begin{cases} |x_1 - a_{1,i}|, & x_2 = a_{2,i} \\ |x_1| + v|x_2 - a_{2,i}| + |a_{1,i}|, & x_2 \neq a_{2,i} \end{cases} \tag{4.1}$$

where v is a non-negative real coefficient which determines the cost of movement along second coordinates in comparison with movement along the first coordinate.

Obviously, if $v = 1$, then this metric coincides with the lift metric (2.7) while the metric coincides with the transformed French metro metric (2.6) if $v = 0$.

The multi-facility problem is a further generalization of Weber problem (2.1). In the Euclidean metric the problem may be stated as follows:

$$\begin{aligned} \min \; & \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ji} \sqrt{(x_j - a_i)^2 + (y_j - b_i)^2} \\ & + \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} v_{jk} \sqrt{(x_j - x_k)^2 + (y_i - y_k)^2}, \end{aligned} \tag{4.2}$$

where:
- n is the number of new facility to be located;
- m is the population points;
- (x_j, y_j) are the coordinates of the jth facility to be located;

- w_{ji} is a constant converting the distance between the *jth* facility and the *ith* population point into cost.
- v_{jk} is a constant converting the distance between the *jth* facility and the *kth* population point into cost.

There are different publications addressing multi-facility location problems numerous to mention. The researchers [68] proposed two algorithms to solve single source capacitated multi-facility location problems with rectilinear distances while authors [54] described a method for approximating non-differentiable convex minimization problem occurring in location theory by differentiable problem. In this case distances are generalized to l_p distances (including rectangular and Euclidean as special cases). The authors utilized a hyperbolic distance function that is uniformly convergent to each l_p distances in their method. Interested readers can refer to [4] for multi-facility location problems on a spherical surface. An extension of multi-facility Weber problem under the assumption of special or 'tailored' metric has been the major focus of the authors [43]. In particular, the authors proposed a solution algorithm similar to Trubin (1978) to multi-facility Weber problem with a new metric (4.1) which generalizes the lift, French metro metrics and other analogous metrics. As evident from the previous sections that the metric for distance measurement may be different in various instances due to specific applications. The following example from [44] is worth mentioning.

For the mechanism with a rotating telescope boom (lifting cranes, manipulators etc.), the Euclidean distance between two points does not reflect the cost of moving objects between two points. In [44], the authors considered four strategies of optimal control for such systems with corresponding distance functions and proposed an algorithm for its solution. In general, the problem is formulated as:

$$f(X) = \sum_{i=1}^{k} w_i r_{ad}(X, P_i) \tag{4.3}$$

where $r_{ad}(X, P_i)$, is the distance function, which determines the cost of moving goods from one point to another (herein, assume the use of different distance functions). Since the expenses (energy, time, cost etc) of the mechanism are not proportional to Euclidean distance. Herein, the results were summarized case by case to describe the cost of mechanism motions as follows:

Case 1: Minimization of the cost of the lifting mechanism movement along the boom, boom rotation and vertical movement. Then the distance between X and P_i is defined by

$$r_{ad1}(X, P_i) = C_r|x_r - a_r^i| + C_\varphi \delta_\varphi(x_\varphi, a_\varphi^i) + C_h|x_h - a_h^i| \ \forall i = 1, \ldots, k. \tag{4.4}$$

The metric used here is termed 'lifting crane' metric analogous to lift metric.

Case 2: Minimization of the freight path provided that only one type of movement (vertical movement, boom rotation or radius change) allowed to be performed in a single unit of time. Then the distance between X and P_i is defined by

$$r_{ad2}(X, P_i) = |x_h - a_h^i| + \begin{cases} x_r + a_r^i, \delta_\varphi(a_\varphi^i, x_\varphi), \geq 2 \ \forall i = 1, \ldots, k \\ \min(x_r, a_r^i)\delta_\varphi(a_\varphi^i, x_\varphi) + |x_r - a_r^i|, \delta_\varphi(a_\varphi^i, x_\varphi) < 2. \end{cases} \tag{4.5}$$

Here the horizontal components illustrate the use of the Moscow-Karlsruhe metric.

Case 3: : Minimization of the path of the hook under the assumption that the rotation is allowed with the zero spread of the lifting mechanism only (when the boom is shortened). This case arise when the demand point cannot be reached from the current point without rotation of the boom then the spread of the lifting mechanism must be shortened first (i.e a load moves to the origin), the boom rotates and then the spread increases to reach the demand point. The case is similar to location models with French metro metric and the distance between two points is given by

$$r_{ad3}(X, P_i) = \begin{cases} x_r + a_r^i, a_\varphi^i \neq x_\varphi \ \forall i = 1, \ldots, k \\ |x_r - a_r^i|, a_\varphi^i = x_\varphi. \end{cases} \tag{4.6}$$

Case 4: This case is similar to case 3 except for one important condition that the load moves to the zero point in any cases, no matter whether the rotation of the boom is required or not, so that the distance between two points is defined by

$$r_{ad4}(X, P_i) = \begin{cases} x_r + a_r^i, X \neq P_i \ \forall i = 1, \ldots, k \\ 0, X = P_i. \end{cases} \tag{4.7}$$

The last case illustrate the use of British rail metric in angular distance problem.

The DPF is also closely associated with the problem of locating a circle with respect to existing facilities in the plane such that the sum of weighted distances between the circle and the facilities is minimized. The corresponding weighted minisum problem may be formulated as follows [58]:

$$\min f(C) = f(X, r) = \sum_{j=1}^{n} w_j d_j(C) = \sum_{j=1}^{n} w_j |d(X, A_j) - r| \tag{4.8}$$

where $A_j = (x_j, y_j)$ is the known location of existing facilities in the plane, the circle C to be located is determined by its center $X = (x, y)$ and its radius r, thus we write $C = (X, r)$.

A significant advance is reported in [58], who consider multiple-circle detection problem in data clustering based on different measures and various criteria for defining a 'closest' circle:

$$\arg \min_{C_1, \ldots, C_k \subset \mathbb{R}^2} F(C_1, \ldots, C_k) \tag{4.9}$$

where

$$F(C_1, \dots, C_k) = \sum_{j=1}^{n} \min_{h=1,\dots,k} d_h(C) = \sum_{j=1}^{n} \min_{h=1,\dots,k} |d(X_h, A_j) - r|.$$

The author is able to propose a k-closest circle algorithm (*KCC-algorithm*) that enables one to see a certain dependence of results of circle reconstruction on different criteria implemented for fitting circles. The circle detection problem demonstrates the usefulness of norm based DPFs in location models. In general, optimization problem (4.9) is a non-convex and non-smooth problem and it could have several local minima. Therefore a global optimization problem is required.

Other related work on the circle detection problem include Fathali et al. [28] and Jamalian and Fathali [36] who studied location problems with the minimum absolute and square errors respectively. A radius, r_i, and value of demand, w_i, correspond to every point p_i is considered. Under the assumption that the transportation cost is proportional to distance traveled, the general version of the problem may be formulated as follows:

$$\min f(x) = \sum_{i=1}^{n} w_i e(x, p_i). \tag{4.10}$$

If the measure in (4.10) of the distance (error) between two points x and p_i has the form

$$e(x, p_i) = |d(x, p_i) - r_i| \tag{4.11}$$

it is possible to talk about *location problem with minimum absolute error*.

If the measure of the distance (error) between points x and p_i has the form

$$e(x, p_i) = (d(x, p_i) - r_i)^2 \tag{4.12}$$

then we have *location problem with minimum square error*.

5. Conclusion

In this paper, we presented a taxonomic framework for the distance predicting functions. Although some classification schemes have been proposed for the general location problem no article in the literature provided a systematic classification of the DPFs in location problems. Our taxonomy categorizes the DPFs based on their coordinates and applicability in diverse settings. We saw the literature on DPFs has been growing increasingly. The growing attention and interest in these problems is due to recognition of the need to consider more DPFs in order to achieve closer solution to reality. In this paper, we reviewed some of the existing and recent works on DPFs in four categories of spherical coordinates, polar coordinates, Cartesian coordinates and special distances. Statistically, we observed that most of the articles appeared in the European Journal of Operations research, Operations Research, Annals of Operations Research, Journal of the Operational Research Society, Advances in Operations Research, SIAM Journals, Facta Universitatis and others including preprints and technical reports. To analyze the taxonomy we selected 63 papers that fairly reflected the attributes of the DPFs under consideration. The taxonomy is robust in the sense that all attributes were marked and 1.59% of the attributes were marked twice.

The results revealed that norm-based DPFs have been extensively studied, whereas the special distances have been less studied. This may be due to the fact that these DPFs involve other characteristics and features that make it difficult to study them from location theory perspective. We attempt to introduce unfamiliar readers to the rich history of distance functions that spans several decades following the initial work of the authors [86]. As such, their applications are highlighted from different perspectives for the advancement of logistics and location analysis so as to broaden the scope of the set of knowledge from which the logistics discipline borrows.

As a final note, we suggest that the relevance of the DPFs in minisum models in the real world is increasing and will continue to do so. One of the reasons is that new applications are evolving where the continuous minisum model is ideally suited. On the other hand, it is the technology involvement such as the GIS, GPS etc., that enable much larger problem instances to be dealt with in practice. A case in point occurs in the area of dynamic and path location problems. Another area deserving more attention is the area of robotic movement where other quite complicated distance concepts abound, notwithstanding their importance in automatic control systems such as chemical and manufacturing processes. It is hoped that this review will stimulate more research in theory and applications of distance functions in location analysis.

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