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STUDY OF A HYPERBOLIC KAEHLERIAN MANIFOLDS EQUIPPED WITH A QUARTER-SYMMETRIC METRIC CONNECTION

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Abstract. This paper contains the study of a hyperbolic Kaehlerian manifold with different approaches. We consider a hyperbolic Kaehlerian manifold with a quarter-symmetric metric connection and obtained expressions for holomorphic conharmonic curvature tensor, conformal curvature tensor with respect to a quarter-symmetric metric connection. We have also studied holomorphic conharmonic recurrent, conformal recurrent and Weyl projective recurrent with respect to a quarter-symmetric metric connection.

Keywords: Kaehlerian manifold; curvature tensor; Riemannian connection; Ricci tensor.

1. Introduction

Let (M^n, g) , $(n > 2)$, be an even dimensional differentiable manifold with the structure F_i^h . If F_i^h satisfies the relation

$$(1.1) \quad F_j^i F_i^h = \delta_j^h,$$

$$(1.2) \quad F_{ij} = -F_{ji}, \quad (F_{ij} = g_{jk} F_i^k),$$

and

$$(1.3) \quad F_{i,j}^h = 0,$$

then the manifold is called hyperbolic Kaehlerian (space) manifold i.e. in a hyperbolic Kaehlerian manifold, equations (1.1), (1.2) and (1.3) hold. There, F_i^h is a tensor field of type (1.1) and $F_{i,j}^h$ is a covariant derivative of F_i^h with respect to Riemannian connection. Yano and Imai [3] considered a quarter-symmetric metric connection ∇ and Riemannian connection D with coefficients Γ_{ij}^h and $\{^h_{ij}\}$, respectively. According to them, if the torsion tensor T of the connection ∇ on (M^n, g) , $(n > 2)$, satisfies

$$(1.4) \quad T_{jk}^i = p_j A_k^i - p_k A_j^i$$

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the relation between the coefficients of quarter-symmetric metric connection ∇ and Riemannian connection D is given by

$$(1.5) \quad \Gamma_{jk}^i = \{^i_{jk}\} - p_k U_j^i + p_j V_k^i - p^i V_{jk},$$

where

$$(1.6) \quad U_{ij} = \frac{1}{2}(A_{ij} - A_{ji}), \quad V_{ij} = \frac{1}{2}(A_{ij} + A_{ji}),$$

$\nabla g = 0$ and p_i are the components of a 1-form. Also, A_j^i denotes the components of the tensor of the type (1,1). Equation (1.6) implies

$$(1.7) \quad A_{ij} = U_{ij} + V_{ij}.$$

In [4] a quarter-symmetric metric connection in a hyperbolic Kaehlerian manifold by taking $V_{ij} = g_{ij}$ and $U_{ij} = F_{ij}$ in (1.5) was constructed as the form

$$(1.8) \quad \Gamma_{jk}^i = \{^i_{jk}\} - p_k F_j^i + p_j \delta_k^i - p^i g_{jk}.$$

Also, it is shown [4] that the relation between the curvature tensor with respect to a quarter-symmetric metric connection and a Riemannian connection is given by

$$(1.9) \quad \begin{aligned} \bar{R}_{ijkh} &= R_{ijkh} - g_{ih} p_{kj} + g_{ik} p_{hj} - g_{jk} p_{hi} \\ &+ g_{hj} p_{ki} + p_j p_h F_{ik} + p_i p_k F_{jh} \\ &- p_j p_k F_{ih} - p_i p_h F_{jk}, \end{aligned}$$

where

$$(1.10) \quad p_{jk} = \nabla_j p_k - p_j p_k + p_k q_j + \frac{1}{2} p_s p^s g_{jk}.$$

Additionally, the Ricci tensor and the scalar curvature are found as, [4], respectively

$$(1.11) \quad \bar{R}_{jk} = R_{jk} - (n-2)p_{kj} - g_{jk} p_m^m - p_j q_k + p_k q_j - p^s p_s F_{kj},$$

and

$$(1.12) \quad \bar{R} = R - 2(n-1)p_m^m.$$

We know that in a Kaehler manifold

$$(1.13) \quad (a) p_j = p^h g_{jh}, \quad (b) q_i = F_{ti} p^t, \quad (c) p^r = g^{ir} p_i.$$

2. Holomorphic conharmonic curvature tensor

We know that the holomorphic conharmonic curvature tensor in a Riemannian manifold is defined as

$$(2.1) \quad T_{ijkh} = R_{ijkh} + \frac{1}{n-2}(g_{ik} R_{jh} - g_{jk} R_{ih}).$$

Therefore the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection is given by

$$(2.2) \quad \bar{T}_{ijkh} = \bar{R}_{ijkh} + \frac{1}{n-2}(g_{ik}\bar{R}_{jh} - g_{jk}\bar{R}_{ih}),$$

where \bar{R}_{ijkh} and \bar{R}_{ih} denote the curvature tensor and the Ricci tensor with respect to a quarter-symmetric metric connection, respectively.

Using (1.9), (1.11) in (2.2), we get

$$(2.3) \quad \begin{aligned} \bar{T}_{ijkh} &= R_{ijkh} - g_{ih}p_{kj} + g_{ik}p_{hj} - g_{jk}p_{hi} + g_{hj}p_{ki} \\ &+ p_jp_hF_{ik} + p_i p_k F_{jh} - p_j p_k F_{ih} - p_i p_h F_{jk} \\ &+ \frac{1}{n-2}(g_{ik}(R_{jh} - (n-2)p_{hj} - g_{jh}p_m^m - p_j q_h \\ &+ p_h q_j - p_s p^s F_{hj}) \\ &- g_{jk}(R_{ih} - (n-2)p_{hi} - g_{ih}p_m^m - p_i q_h \\ &+ p_h q_i - p_s p^s F_{hi})). \end{aligned}$$

Using [(1.13(a, b))] in (2.3), we find

$$(2.4) \quad \begin{aligned} \bar{T}_{ijkh} &= R_{ijkh} + \frac{1}{n-2}(g_{ik}R_{jh} - g_{jk}R_{ih}) \\ &- \frac{n}{n-2}(p_j p_k F_{ih} - p_i p_k F_{jh}) \\ &+ \frac{n-1}{n-2}(p_h p_j F_{ik} - p_i p_h F_{jk}) \\ &- \frac{p_m^m}{n-2}(g_{ki}g_{hj} - g_{jk}g_{hi}) + (g_{hj}p_{ki} - g_{ih}p_{kj}). \end{aligned}$$

From (2.1) and (2.4), we obtain

$$(2.5) \quad \begin{aligned} \bar{T}_{ijkh} &= T_{ijkh} - \frac{n}{n-2}(p_j p_k F_{ih} - p_i p_k F_{jh}) \\ &+ \frac{n-1}{n-2}(p_h p_j F_{ik} - p_i p_h F_{jk}) \\ &- \frac{p_m^m}{n-2}(g_{ki}g_{hj} - g_{jk}g_{hi}) + (g_{hj}p_{ki} - g_{ih}p_{kj}). \end{aligned}$$

In this case, if

$$(2.6) \quad p_j F_{ih} = p_i F_{jh}, \quad p_{ki}g_{hj} = p_{kj}g_{hi}, \quad \text{and} \quad g_{ki}g_{hj} = g_{jk}g_{hi},$$

then from (2.5), we get

$$(2.7) \quad \bar{T}_{ijkh} = T_{ijkh}.$$

Thus, we conclude:

Theorem 2.1. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection will be equal to the holomorphic conharmonic curvature tensor with respect to a Riemannian connection if the following conditions hold:*

$$(1) p_j F_{ih} = p_i F_{jh}, \quad (2) p_{ki} g_{hj} = p_{kj} g_{hi}, \quad (3) g_{ki} g_{hj} = g_{jk} g_{hi}.$$

Now, we propose:

Theorem 2.2. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection satisfies the first Bianchi identity if*

$$p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0, \text{ and } p_{ij} g_{kh} = p_{ik} g_{jh},$$

i.e.

$$\bar{T}_{ijkh} + \bar{T}_{jkih} + \bar{T}_{kijh} = 0,$$

if

$$p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0, \text{ and } p_{ij} g_{kh} = p_{ik} g_{jh}.$$

Proof. Interchanging i, j and k in a cyclic order in (2.5), we get

$$\begin{aligned} \bar{T}_{ijkh} &= T_{ijkh} \\ &- \frac{n}{n-2}(p_j p_k F_{ih} - p_i p_k F_{jh}) \\ &+ \frac{n-1}{n-2}(p_h p_j F_{ik} - p_i p_h F_{jk}) \\ &- \frac{P_m^m}{n-2}(g_{ki} g_{hj} - g_{jk} g_{hi}) + (p_{ki} g_{hj} - p_{kj} g_{hi}), \end{aligned} \quad (2.8)$$

$$\begin{aligned} \bar{T}_{jkih} &= T_{jkih} \\ &- \frac{n}{n-2}(p_k p_i F_{jh} - p_j p_i F_{kh}) \\ &+ \frac{n-1}{n-2}(p_h p_k F_{ji} - p_j p_h F_{ki}) \\ &- \frac{P_m^m}{n-2}(g_{ij} g_{hk} - g_{ki} g_{hj}) + (p_{ij} g_{hk} - p_{ik} g_{hj}), \end{aligned} \quad (2.9)$$

and

$$\begin{aligned} \bar{T}_{kijh} &= T_{kijh} \\ &- \frac{n}{n-2}(p_i p_j F_{kh} - p_k p_j F_{ih}) \\ &+ \frac{n-1}{n-2}(p_h p_i F_{kj} - p_k p_h F_{ij}) \\ &- \frac{P_m^m}{n-2}(g_{jk} g_{hi} - g_{ij} g_{hk}) + (p_{jk} g_{hi} - p_{ji} g_{hk}). \end{aligned} \quad (2.10)$$

Adding equation (2.8), (2.9) and (2.10), we have

$$\begin{aligned}
 \bar{T}_{ijkh} + \bar{T}_{jkih} + \bar{T}_{kijh} &= T_{ijkh} + T_{jkih} + T_{kijh} \\
 &+ \frac{2(n-1)}{n-2} p_h(p_j F_{ik} + p_i F_{kj} + p_k F_{ji}) \\
 &+ (p_{ki} g_{hj} - p_{kj} g_{hi}) \\
 &+ (p_{ij} g_{hk} - p_{ik} g_{hj}) \\
 &+ (p_{jk} g_{hi} - p_{ji} g_{kh}).
 \end{aligned}
 \tag{2.11}$$

Since in a Riemannian manifold the holomorphic conharmonic curvature tensor satisfies the first Bianchi identity, i.e.

$$T_{ijkh} + T_{jkih} + T_{kijh} = 0. \tag{2.12}$$

Using (2.12) in (2.11), we have

$$\begin{aligned}
 \bar{T}_{ijkh} + \bar{T}_{jkih} + \bar{T}_{kijh} &= \frac{2(n-1)}{n-2} p_h(p_j F_{ik} + p_i F_{kj} + p_k F_{ji}) \\
 &+ (p_{ki} g_{hj} - p_{kj} g_{hi}) \\
 &+ (p_{ij} g_{hk} - p_{ik} g_{hj}) \\
 &+ (p_{jk} g_{hi} - p_{ji} g_{kh}).
 \end{aligned}
 \tag{2.13}$$

Now, assuming that

$$p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0, \text{ and } p_{ij} g_{hk} = p_{ik} g_{hj}, \tag{2.14}$$

then from (2.13), we get

$$\bar{T}_{ijkh} + \bar{T}_{jkih} + \bar{T}_{kijh} = 0. \tag{2.15}$$

This completes the proof. \square

Now, we propose:

Theorem 2.3. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the holomorphic conharmonic curvature tensor with respect to a quarter-symmetric metric connection satisfies*

$$\bar{T}_{ijkh} = -\bar{T}_{jikh}.$$

Proof. Interchanging i and j in (2.5), we have

$$\begin{aligned}
 \bar{T}_{jikh} &= T_{jikh} \\
 &- \frac{n}{n-2} (p_i p_k F_{jh} - p_j p_k F_{ih}) \\
 &+ \frac{n-1}{n-2} (p_h p_i F_{jk} - p_j p_h F_{ik}) \\
 &- \frac{P_m^m}{n-2} (g_{kj} g_{hi} - g_{ik} g_{hj}) + (g_{hi} p_{kj} - g_{jh} p_{ki}).
 \end{aligned}
 \tag{2.16}$$

Adding (2.5) and (2.16), we have

$$(2.17) \quad \bar{T}_{ijkh} + \bar{T}_{jikh} = T_{ijkh} + T_{jikh}.$$

Since in a Riemannian manifold the holomorphic conharmonic curvature tensor satisfies

$$(2.18) \quad T_{ijkh} + T_{jikh} = 0.$$

Then by using (2.18) in (2.17), we get

$$(2.19) \quad \bar{T}_{ijkh} = -\bar{T}_{jikh}.$$

□

3. Conformal curvature tensor

In the Riemannian manifold (M^n, g) , $(n > 2)$, the conformal curvature tensor of the type (0,4) is defined as

$$(3.1) \quad \begin{aligned} C_{ijkh} &= R_{ijkh} \\ &- \frac{1}{n-2}(R_{jk}g_{ih} - R_{ik}g_{jh} + R_{ih}g_{jk} - R_{jh}g_{ki}) \\ &+ \frac{R}{(n-1)(n-2)}(g_{ih}g_{jk} - g_{jh}g_{ki}). \end{aligned}$$

The conformal curvature tensor with respect to a quarter-symmetric metric connection is given by

$$(3.2) \quad \begin{aligned} \bar{C}_{ijkh} &= \bar{R}_{ijkh} \\ &- \frac{1}{n-2}(\bar{R}_{jk}g_{ih} - \bar{R}_{ik}g_{jh} + \bar{R}_{ih}g_{jk} - \bar{R}_{jh}g_{ki}) \\ &+ \frac{\bar{R}}{(n-1)(n-2)}(g_{ih}g_{jk} - g_{jh}g_{ki}). \end{aligned}$$

Using (1.9), (1.11) and (1.12) in (3.2), we have

$$(3.3) \quad \begin{aligned} \bar{C}_{ijkh} &= R_{ijkh} \\ &- g_{ih}p_{kj} + g_{ik}p_{hj} - g_{jk}p_{hi} + g_{hj}p_{ki} \\ &+ p_jp_hF_{ik} + p_i p_k F_{jh} - p_j p_k F_{ih} - p_i p_h F_{jk} \\ &- \frac{1}{n-2}(g_{ih}(R_{jk} - (n-2)p_{kj} - g_{jk}p_m^m - p_j q_k + p_k q_j - p^s p_s F_{kj}) \\ &- g_{jh}(R_{ik} - (n-2)p_{ki} - g_{ik}p_m^m - p_i q_k + p_k q_i - p^s p_s F_{ki}) \\ &+ g_{kj}(R_{ih} - (n-2)p_{hi} - g_{ih}p_m^m - p_i q_h + p_h q_i - p^s p_s F_{hi}) \\ &- g_{ik}(R_{jh} - (n-2)p_{hj} - g_{jh}p_m^m - p_j q_h + p_h q_j - p^s p_s F_{hj})) \\ &+ \frac{R - 2(n-1)p_m^m}{(n-1)(n-2)}(g_{ih}g_{jk} - g_{jh}g_{ik}). \end{aligned}$$

Now using [(1.13(a, b))] in (3.3), we have

$$\begin{aligned}
 \bar{C}_{ijkh} &= R_{ijkh} - \frac{1}{n-2}(R_{jk}g_{ih} - R_{ik}g_{jh} + R_{ih}g_{jk} - R_{jh}g_{ki}) \\
 &+ \frac{R}{(n-1)(n-2)}(g_{ih}g_{jk} - g_{jh}g_{ki}) \\
 &+ \frac{n+1}{n-2}(p_j p_h F_{ik} - p_j p_k F_{ih}) \\
 &+ \frac{n+1}{n-2}(p_i p_k F_{jh} - p_i p_h F_{jk}).
 \end{aligned}
 \tag{3.4}$$

Using (3.1) in (3.4), we have

$$\begin{aligned}
 \bar{C}_{ijkh} &= C_{ijkh} + \frac{n+1}{n-2}(p_j p_h F_{ik} - p_j p_k F_{ih}) \\
 &+ \frac{n+1}{n-2}(p_i p_k F_{jh} - p_i p_h F_{jk}).
 \end{aligned}
 \tag{3.5}$$

If we take $p_h F_{ik} = p_k F_{ih}$ then (3.5) reduces to the form

$$\bar{C}_{ijkh} = C_{ijkh}.
 \tag{3.6}$$

Thus, we conclude:

Theorem 3.1. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the conformal curvature tensor with respect to a quarter-symmetric metric connection will be equal to the conformal curvature tensor with respect to a Riemannian connection if and only if*

$$p_h F_{ik} = p_k F_{ih}.
 \tag{3.7}$$

Now, we propose:

Theorem 3.2. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the conformal curvature tensor with respect to a quarter-symmetric metric connection satisfies the first Bianchi identity if*

$$p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0.
 \tag{3.8}$$

Proof. Interchanging i, j and k in a cyclic order in (3.5), we find

$$\begin{aligned}
 \bar{C}_{ijkh} &= C_{ijkh} \\
 &+ \frac{n+1}{n-2}(p_j p_h F_{ik} - p_j p_k F_{ih}) \\
 &+ \frac{n+1}{n-2}(p_i p_k F_{jh} - p_i p_h F_{jk}),
 \end{aligned}
 \tag{3.9}$$

$$\begin{aligned}
\bar{C}_{jkih} &= C_{jkih} \\
&+ \frac{n+1}{n-2}(p_k p_h F_{ji} - p_k p_i F_{jh}) \\
(3.10) \quad &+ \frac{n+1}{n-2}(p_j p_i F_{kh} - p_j p_h F_{ki}),
\end{aligned}$$

and

$$\begin{aligned}
\bar{C}_{kijh} &= C_{kijh} \\
&+ \frac{n+1}{n-2}(p_i p_h F_{kj} - p_i p_j F_{kh}) \\
(3.11) \quad &+ \frac{n+1}{n-2}(p_k p_j F_{ih} - p_k p_h F_{ij}).
\end{aligned}$$

Adding (3.9), (3.10) and (3.11), we get

$$\begin{aligned}
\bar{C}_{ijkh} + \bar{C}_{jkih} + \bar{C}_{kijh} &= C_{ijkh} + C_{jkih} + C_{kijh} \\
(3.12) \quad &+ 2\frac{n+1}{n-2}p_h(p_j F_{ik} + p_i F_{kj} + p_k F_{ji}).
\end{aligned}$$

Since in a Riemannian manifold the conformal curvature tensor satisfies the condition

$$(3.13) \quad C_{ijkh} + C_{jkih} + C_{kijh} = 0,$$

by using (3.13) in (3.12), we find

$$(3.14) \quad \bar{C}_{ijkh} + \bar{C}_{jkih} + \bar{C}_{kijh} = 2\frac{n+1}{n-2}p_h(p_j F_{ik} + p_i F_{kj} + p_k F_{ji}).$$

If we take $p_j F_{ik} + p_i F_{kj} + p_k F_{ji} = 0$ then from (3.14), we get

$$(3.15) \quad \bar{C}_{ijkh} + \bar{C}_{jkih} + \bar{C}_{kijh} = 0.$$

□

Now, we propose:

Theorem 3.3. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, the conformal curvature tensor with respect to a quarter-symmetric metric connection satisfies the following properties:*

- (1) $\bar{C}_{ijkh} = -\bar{C}_{jikh}$,
- (2) $\bar{C}_{ijkh} = -\bar{C}_{ijhk}$.

Proof. Interchanging i and j in (3.5), we find

$$\begin{aligned}
\bar{C}_{jikh} &= C_{jikh} \\
&+ \frac{n+1}{n-2}(p_i p_h F_{jk} - p_i p_k F_{jh}) \\
(3.16) \quad &+ \frac{n+1}{n-2}(p_j p_k F_{ih} - p_j p_h F_{ik}).
\end{aligned}$$

Adding (3.5) and (3.16), we obtain

$$(3.17) \quad \bar{C}_{ijkh} + \bar{C}_{jikh} = C_{ijkh} + C_{jikh}.$$

Since in a Riemannian manifold the conformal curvature tensor satisfies

$$(3.18) \quad C_{ijkh} + C_{jikh} = 0,$$

then by using (3.18) in (3.17), we get the expression (1). Now interchanging k and h in (3.5), we have

$$(3.19) \quad \begin{aligned} \bar{C}_{ijhk} &= C_{ijhk} \\ &+ \frac{n+1}{n-2}(p_j p_k F_{ih} - p_j p_h F_{ik}) \\ &+ \frac{n+1}{n-2}(p_i p_h F_{jk} - p_i p_k F_{jh}). \end{aligned}$$

Adding (3.5) and (3.19), we have

$$(3.20) \quad \bar{C}_{ijkh} + \bar{C}_{ijhk} = C_{ijkh} + C_{ijhk}.$$

Since in a Riemannian manifold the conformal curvature tensor satisfies

$$(3.21) \quad C_{ijkh} + C_{ijhk} = 0,$$

by using (3.21) in (3.20), we get expression (2). \square

Taking the covariant derivative of the conformal curvature tensor with respect to the Riemannian connection and quarter-symmetric metric connection, respectively, we get

$$(3.22) \quad \begin{aligned} D_m C_{ijkh} &= \partial_m C_{ijkh} - C_{rjkh} \{^r_{mi}\} - C_{irkh} \{^r_{mj}\} \\ &- C_{ijrh} \{^r_{mk}\} - C_{ijk r} \{^r_{mh}\}, \end{aligned}$$

and

$$(3.23) \quad \begin{aligned} \nabla_m C_{ijkh} &= \partial_m C_{ijkh} - C_{rjkh} \Gamma^r_{mi} - C_{irkh} \Gamma^r_{mj} \\ &- C_{ijrh} \Gamma^r_{mk} - C_{ijk r} \Gamma^r_{mh}. \end{aligned}$$

Subtracting (3.22) from (3.23), we get

$$(3.24) \quad \begin{aligned} \nabla_m C_{ijkh} - D_m C_{ijkh} &= C_{rjkh} (\{^r_{mi}\} - \Gamma^r_{mi}) \\ &+ C_{irkh} (\{^r_{mj}\} - \Gamma^r_{mj}) \\ &+ C_{ijrh} (\{^r_{mk}\} - \Gamma^r_{mk}) \\ &+ C_{ijk r} (\{^r_{mh}\} - \Gamma^r_{mh}). \end{aligned}$$

Now using (1.8) in (3.24), we find

$$\begin{aligned}
 \nabla_m C_{ijkh} - D_m C_{ijkh} &= C_{rjkh} (p_i F_m^r - p_m \delta_i^r + p^r g_{mi}) \\
 &\quad + C_{irkh} (p_j F_m^r - p_m \delta_j^r + p^r g_{mj}) \\
 &\quad + C_{ijrh} (p_k F_m^r - p_m \delta_k^r + p^r g_{mk}) \\
 &\quad + C_{ijk r} (p_h F_m^r - p_m \delta_h^r + p^r g_{mh}).
 \end{aligned}
 \tag{3.25}$$

Using [1.13(c)] in (3.25), we get

$$\begin{aligned}
 \nabla_m C_{ijkh} - D_m C_{ijkh} &= (C_{rjkh} p_i + C_{irkh} p_j \\
 &\quad + C_{ijrh} p_k + C_{ijk r} p_h) F_m^r.
 \end{aligned}
 \tag{3.26}$$

Considering that the expression

$$C_{rjkh} p_i + C_{irkh} p_j + C_{ijrh} p_k + C_{ijk r} p_h = 0
 \tag{3.27}$$

is satisfied, then it is finally obtained that

$$\nabla_m C_{ijkh} = D_m C_{ijkh}.
 \tag{3.28}$$

Thus, we conclude:

Theorem 3.4. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, if the conformal curvature tensor with respect to a Riemannian connection is recurrent with respect to the Riemannian connection then it is also recurrent with respect to the quarter-symmetric metric connection if and only if*

$$C_{rjkh} p_i + C_{irkh} p_j + C_{ijrh} p_k + C_{ijk r} p_h = 0.$$

Taking the covariant derivative of the holomorphic conharmonic curvature tensor with respect to the Riemannian connection and quarter-symmetric metric connection, respectively, we have

$$\begin{aligned}
 D_m T_{ijkh} &= \partial_m T_{ijkh} - T_{rjkh} \{^r_{mi}\} \\
 &\quad - T_{irkh} \{^r_{mj}\} - T_{ijrh} \{^r_{mk}\} - T_{ijk r} \{^r_{mh}\},
 \end{aligned}
 \tag{3.29}$$

and

$$\begin{aligned}
 \nabla_m T_{ijkh} &= \partial_m T_{ijkh} - T_{rjkh} \Gamma_{mi}^r \\
 &\quad - T_{irkh} \Gamma_{mj}^r - T_{ijrh} \Gamma_{mk}^r - T_{ijk r} \Gamma_{mh}^r.
 \end{aligned}
 \tag{3.30}$$

Subtracting (3.29) from (3.30), we get

$$\begin{aligned}
 \nabla_m T_{ijkh} - D_m T_{ijkh} &= T_{rjkh} (\{^r_{mi}\} - \Gamma_{mi}^r) \\
 &\quad + T_{irkh} (\{^r_{mj}\} - \Gamma_{mj}^r) \\
 &\quad + T_{ijrh} (\{^r_{mk}\} - \Gamma_{mk}^r) \\
 &\quad + T_{ijk r} (\{^r_{mh}\} - \Gamma_{mh}^r).
 \end{aligned}
 \tag{3.31}$$

Now, using (1.8) in (3.31), we find

$$\begin{aligned}
 \nabla_m T_{ijkh} - D_m T_{ijkh} &= T_{rjkh} (p_i F_m^r - p_m \delta_i^r + p^r g_{mi}) \\
 &+ T_{irkh} (p_j F_m^r - p_m \delta_j^r + p^r g_{mj}) \\
 &+ T_{ijrh} (p_k F_m^r - p_m \delta_k^r + p^r g_{mk}) \\
 &+ T_{ijk r} (p_h F_m^r - p_m \delta_h^r + p^r g_{mh}).
 \end{aligned}
 \tag{3.32}$$

Using [1.13(c)] in (3.32), we obtain

$$\begin{aligned}
 \nabla_m T_{ijkh} - D_m T_{ijkh} &= (T_{rjkh} p_i + T_{irkh} p_j \\
 &+ T_{ijrh} p_k + T_{ijk r} p_h) F_m^r.
 \end{aligned}
 \tag{3.33}$$

If we take the condition

$$T_{rjkh} p_i + T_{irkh} p_j + T_{ijrh} p_k + T_{ijk r} p_h = 0,
 \tag{3.34}$$

then

$$\nabla_m T_{ijkh} = D_m T_{ijkh}.
 \tag{3.35}$$

Thus, we conclude:

Theorem 3.5. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, if the holomorphic conharmonic curvature tensore with respect to a Riemannian connection is recurrent with respect to the Riemannian connection then it is also recurrent with respect to the quarter-symmetric metric connection if and only if the following condition holds*

$$T_{rjkh} p_i + T_{irkh} p_j + T_{ijrh} p_k + T_{ijk r} p_h = 0.$$

Taking the covariant derivative of the Weyl projective curvature tensor with respect to a Riemannian connection and quarter-symmetric metric connection, respectively, we can write

$$\begin{aligned}
 D_m W_{ijkh} &= \partial_m W_{ijkh} - W_{rjkh} \{_{mi}^r\} - W_{irkh} \{_{mj}^r\} \\
 &- W_{ijrh} \{_{mk}^r\} - W_{ijk r} \{_{mh}^r\},
 \end{aligned}
 \tag{3.36}$$

and

$$\begin{aligned}
 \nabla_m W_{ijkh} &= \partial_m W_{ijkh} - W_{rjkh} \Gamma_{mi}^r - W_{irkh} \Gamma_{mj}^r \\
 &- W_{ijrh} \Gamma_{mk}^r - W_{ijk r} \Gamma_{mh}^r.
 \end{aligned}
 \tag{3.37}$$

Subtracting (3.36) from (3.37), we get

$$\begin{aligned}
 \nabla_m W_{ijkh} - D_m W_{ijkh} &= W_{rjkh} (\{_{mi}^r\} - \Gamma_{mi}^r) + W_{irkh} (\{_{mj}^r\} \\
 &- \Gamma_{mj}^r) + W_{ijrh} (\{_{mk}^r\} - \Gamma_{mk}^r) \\
 &+ W_{ijk r} (\{_{mh}^r\} - \Gamma_{mh}^r).
 \end{aligned}
 \tag{3.38}$$

Now, using (1.8) in (3.38), we find

$$\begin{aligned}
 \nabla_m W_{ijkh} - D_m W_{ijkh} &= W_{rjkh} (p_i F_m^r - p_m \delta_i^r + p^r g_{mi}) \\
 &+ W_{irkh} (p_j F_m^r - p_m \delta_j^r + p^r g_{mj}) \\
 &+ W_{ijrh} (p_k F_m^r - p_m \delta_k^r + p^r g_{mk}) \\
 &+ W_{ijkr} (p_h F_m^r - p_m \delta_h^r + p^r g_{mh}).
 \end{aligned}
 \tag{3.39}$$

Using [1.13(c)] in (3.39), it is obtained that

$$\begin{aligned}
 \nabla_m W_{ijkh} - D_m W_{ijkh} &= (W_{rjkh} p_i + W_{irkh} p_j \\
 &+ W_{ijrh} p_k + W_{ijkr} p_h) F_m^r.
 \end{aligned}
 \tag{3.40}$$

If we consider that the expression

$$W_{rjkh} p_i + W_{irkh} p_j + W_{ijrh} p_k + W_{ijkr} p_h = 0
 \tag{3.41}$$

holds, then we find

$$\nabla_m W_{ijkh} = D_m W_{ijkh}.
 \tag{3.42}$$

Thus, we conclude:

Theorem 3.6. *In a hyperbolic Kaehlerian manifold equipped with a quarter-symmetric metric connection, if the Weyl projective curvature tensor with respect to a Riemannian connection is recurrent with respect to the Riemannian connection then it is also recurrent with respect to a quarter-symmetric metric connection if and only if*

$$W_{rjkh} p_i + W_{irkh} p_j + W_{ijrh} p_k + W_{ijkr} p_h = 0.$$

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