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ON PRESERVING INTUITIONISTIC FUZZY gpr-CLOSED SETS

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Abstract. In this paper we introduce the concepts of intuitionistic fuzzy apr-closed and intuitionistic fuzzy *apr* - continuous mappings in intuitionistic fuzzy topological spaces and obtain several results concerning the preservation of intuitionistic fuzzy gpr-closed sets. Furthermore, we characterize intuitionistic fuzzy pre-regular $T_{\frac{1}{2}}$-spaces due to Thakur and Bajpai[13] in terms of intuitionistic fuzzy apr-continuous and intuitionistic fuzzy apr-closed mappings and obtain some of the basic properties and characterization of these mappings.

1. Introduction

After the introduction of fuzzy sets by Zadeh [15] in 1965 and fuzzy topology by Chang [4] in 1968, research was conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1, 2] as a generalization of fuzzy sets. In 2008 Thakur and Chaturvedi extended the concepts of fuzzy g-closed sets[9] and fuzzy g-continuity [7] in intuitionistic fuzzy topological spaces. Recently many generalizations of intuitionistic fuzzy *g*-closed sets[9] like intuitionistic fuzzy *rg*-closed sets [8], intuitionistic fuzzy *sg*-closed sets [12], intuitionistic fuzzy *w*-closed sets[10], intuitionistic fuzzy *rw*-closed sets [11], intuitionistic fuzzy *gpr*-closed sets[13] have appeared in the literature. In this paper we introduce the concepts of intuitionistic fuzzy apr-closed and intuitionistic fuzzy apr-continuous mappings using intuitionistic fuzzy gpr-closed sets. These definitions enable us to obtain conditions under which maps and inverse maps preserve intuitionistic fuzzy *gpr*-closed sets [13]. We also characterize intuitionistic fuzzy pre-regular $T_{\frac{1}{2}}$-spaces in terms of intuitionistic fuzzy *apr*-continuous and intuitionistic fuzzy *apr*-closed mappings. Finally some of basic properties of intuitionistic fuzzy *apr*-continuous and intuitionistic fuzzy *apr*-closed mappings are investigated.

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2. Preliminaries

Since we shall require the following known definitions, notations and some properties, we recall them in this section.

Definition 2.1. [1] Let X be a nonempty fixed set. An intuitionistic fuzzy set A is an object having the form

$$A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$$

Where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each element $x \in X$.

Definition 2.2. [1] Let X be a nonempty set and the intuitionistic fuzzy sets A and intuitionistic fuzzy set B be in the form $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$, $B = \{< x, \mu_B(x), \gamma_B(x) >: x \in X\}$ and let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic fuzzy sets in X.

Then:

(a) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$.

(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{< x, \gamma_A(x), \mu_A(x) >: x \in X\}$

(d) $\cap A_i = \{< x, \wedge\mu_A(x), \vee\gamma_A(x) >: x \in X\}$

(e) $\cup A_i = \{< x, \vee\mu_A(x), \wedge\gamma_A(x) >: x \in X\}$

(f) $\tilde{\mathbf{0}} = \{< x, 0, 1 >: x \in X\}$ and $\tilde{\mathbf{1}} = \{< x, 1, 0 >: x \in X\}$

Definition 2.3. [5] An intuitionistic fuzzy topology on a nonempty set X is a family τ of intuitionistic fuzzy sets in X, satisfying the following axioms:

(T_1) $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}} \in \tau$

(T_2) $G_1 \bigcap G_2 \in \tau$

(T_3) $G_1 \bigcup G_2 \in \tau$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set in X. The complement A^c of an intuitionistic fuzzy open set A is called an intuitionistic fuzzy closed set in X.

Definition 2.4. [5] Let (X, τ) be an intuitionistic fuzzy topological space and $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ be an intuitionistic fuzzy set in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by:

cl(A) = ∩ {K: K is an intuitionistic fuzzy closed set such that A ⊆ K }

int(A) = ∪ {K: K is an intuitionistic fuzzy open set such that K ⊆ A }

Definition 2.5. [6] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called:

(a) intuitionistic fuzzy pre-open if $A \subseteq int(cl(A))$ and intuitionistic fuzzy pre-closed if $cl(int(A)) \subseteq A$

(b) intuitionistic fuzzy regular open if $A = int(cl(A))$ and intuitionistic fuzzy regular closed if $A = cl(int(A))$.

Definition 2.6. [6] If A is an intuitionistic fuzzy set in intuitionistic fuzzy topological space (X, τ) then pcl(A) = ∩ {K: K is an intuitionistic fuzzy pre-closed set such that A ⊆ K }.

Definition 2.7. [13] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called:

(a) intuitionistic fuzzy *gpr*-closed if $pcl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular open.

(b) intuitionistic fuzzy *gpr*-open if and only if A^c is intuitionistic fuzzy *gpr* -closed.

Definition 2.8. [13] An intuitionistic fuzzy topological space (X, τ) is said to be intuitionistic fuzzy pre-regular $T_{\frac{1}{2}}$-space if every intuitionistic fuzzy *gpr*-closed set in X is intuitionistic fuzzy pre-closed in X.

Remark 2.1. [13] Every intuitionistic fuzzy regular closed set is intuitionistic fuzzy *gpr*-closed but its converse may not be true.

Remark 2.2. [13] Every intuitionistic fuzzy pre-closed set is intuitionistic fuzzy *gpr*-closed but its converse may not be true.

Theorem 2.1. *[13] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space is intuitionistic fuzzy gpr-open if and only if $F \subseteq pint(A)$ whenever F is intuitionistic fuzzy regular closed and $F \subseteq A$.*

Theorem 2.2. *[13] Let (X, τ) be an intuitionistic fuzzy topological space and IFPC (resp. IFRO(X)) be the family of all intuitionistic fuzzy pre-closed (resp. intuitionistic fuzzy regular open) sets of X. Then IFPC(X) = IFRO(X) if and only if every intuitionistic fuzzy set of X is intuitionistic fuzzy gpr-closed.*

Definition 2.9. [5] Let X and Y be two nonempty sets and $f : X \to Y$ be a mapping. Then:

(a) If $B = \{< (y, \mu_B(y), \gamma_B(y) >: y \in Y\}$ is an intuitionistic fuzzy set in Y, then the pre-image of B under f denoted by $f^{-1}(B)$, is the intuitionistic fuzzy set in X defined by $f^{-1}(B) = \{< x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) >: x \in X\}$.

(b) If $A = \{< x, \lambda_A(x), \nu_A(x) >: x \in X\}$is an intuitionistic fuzzy set in X, then the image of A under f denoted by $f(a)$ is the intuitionistic fuzzy set in Y defined by $f(a) = \{< y, f(\lambda_A)(y), f(\nu_A)(y) >: y \in Y\}$ where $f(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 2.10. Let (X, τ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let $f : X \to Y$ be a mapping. Then f is said to be:

(a). Intuitionistic fuzzy continuous [6] if the pre-image of each intuitionistic fuzzy open set if Y is an intuitionistic fuzzy open set in X.

(b). Intuitionistic fuzzy *gpr*-continuous [13] if the pre image of every intuitionistic fuzzy closed set in Y is an intuitionistic fuzzy *gpr*-closed set in X.

(c). Intuitionistic fuzzy irresolute [6] if the pre-image of every intuitionistic fuzzy semi-closed set in Y is an intuitionistic fuzzy semi-closed set in X.

(d). Intuitionistic fuzzy *gpr*-irresolute [15] if the pre-image of every intuitionistic fuzzy *gpr*-closed set in Y is an intuitionistic fuzzy *gpr*-closed set in X.

(e). Intuitionistic fuzzy pre-closed [6] if the image of each intuitionistic fuzzy closed set in X is an intuitionistic fuzzy pre-closed set in Y.

(f). Intuitionistic fuzzy pre-regular closed [8] if the image of each intuitionistic fuzzy regular closed set in X is an intuitionistic fuzzy regular closed set in Y.

(g). Intuitionistic fuzzy R mapping [8] if the pre-image of each intuitionistic fuzzy regular open set of Y is an intuitionistic fuzzy regular open set in X.

Remark 2.3. [13] Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy *gpr*-continuous, but the converse may not be true.

Remark 2.4. [13] Every intuitionistic fuzzy *gpr*-irresolute mapping is intuitionistic fuzzy *gpr* -continuous, but the converse may not be true. The concepts of intuitionistic fuzzy *gpr*-irresolute and intuitionistic fuzzy continuous mapping are independent.

3. Intuitionistic Fuzzy apr-Closed and Intuitionistic fuzzy apr-continuous mappings

Definition 3.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic fuzzy *apr*-closed provided that $f(F) \subseteq pint(A)$ whenever F is intuitionistic fuzzy regular closed set in X, A is an intuitionistic fuzzy *gpr*-open set in Y and $f(F) \subseteq A$.

Theorem 3.1. *Every intuitionistic fuzzy pre-regular closed mapping is intuitionistic fuzzy apr-closed.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy pre-regular closed mapping. Let F be intuitionistic fuzzy regular closed set in X and A is an intuitionistic fuzzy *gpr*-open set in Y such that $f(F) \subseteq A$. Since f is intuitionistic fuzzy pre-regular closed mapping, $f(a)$ is intuitionistic fuzzy regular closed set in Y. Now A is intuitionistic fuzzy *gpr*-open and $f(F) \subseteq A \Rightarrow f(F) \subseteq pint(A)$. Hence f is intuitionistic fuzzy *apr*-closed. □

Remark 3.1. The converse of Theorem 3.1 may not be true.

Example 3.1. Let $X = \{a, b\}$ and $U = \{< a, 0.6, 0.3 >, < b, 0.3, 0.6 >\}$ be an intuitionistic fuzzy set on$X$. Let $\tau = \{\ \tilde{\mathbf{0}}, \text{X}, \tilde{\mathbf{1}}\ \}$ be intuitionistic fuzzy topology on X. Then the mapping $f : (X, \tau) \rightarrow (X, \tau)$ defined by $f(a)$ = b and $f(b)$ = a is intuitionistic fuzzy *apr*-closed but it is not intuitionistic fuzzy pre-regular closed.

Definition 3.2. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be intuitionistic fuzzy *apr*-continuous provided that $pcl(F) \subseteq f^{-1}(O)$ whenever F is intuitionistic fuzzy *gpr*-closed set in X, O is an intuitionistic fuzzy regular open set in Y and $F \subseteq f^{-1}(O)$.

Theorem 3.2. *Every intuitionistic fuzzy R-mapping is intuitionistic fuzzy apr-continuous.*

Proof. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy R- mapping. Let O be an intuitionistic fuzzy regular open set of Y and F is an intuitionistic fuzzy *gpr*-closed set of X such that $F \subseteq f^{-1}(O)$. Now since f is intuitionistic fuzzy R -mapping, $f^{-1}(O)$ is intuitionistic fuzzy regular open set in X. Since F is intuitionistic fuzzy *gpr*-closed and $F \subseteq f^{-1}(O) \Rightarrow pcl(F) \subseteq f^{-1}(O)$. Hence f is intuitionistic fuzzy *apr*-continuous. □

Remark 3.2. The converse of Theorem 3.2 may not be true.

Example 3.2. Let $X = \{a, b\}$ and $U = \{< a, 0.3, 0.7 >, < b, 0.4, 0.6 >\}$ be an intuitionistic fuzzy set on$X$. Let $\tau = \{\ \tilde{\mathbf{0}}, \text{X}, \tilde{\mathbf{1}}\ \}$ be intuitionistic fuzzy topology on X. Then the mapping $f : (X, \tau) \rightarrow (X, \tau)$ defined by $f(a)$ = b and $f(b)$ = a is intuitionistic fuzzy *apr*-continuous but it is not intuitionistic fuzzy R-mapping.

Theorem 3.3. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a bijection, then f is intuitionistic fuzzy apr-closed if and only if f^{-1} is intuitionistic fuzzy apr - continuous.*

Proof. Obvious. □

4. Preserving Intuitionistic Fuzzy gpr-closed sets

In this section the concepts of intuitionistic fuzzy *apr*-continuous and intuitionistic fuzzy *apr*-closed mappings are used to obtain some results on preservation of intuitionistic fuzzy *gpr*-closed sets.

Theorem 4.1. *If a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy gpr-continuous and intuitionistic fuzzy apr-closed then $f^{-1}(A)$ is intuitionistic fuzzy gpr-closed set in X whenever A is intuitionistic fuzzy gpr-closed set in Y.*

Proof. Suppose that $f : (X, \tau) \rightarrow (Y, \sigma)$ is intuitionistic fuzzy gpr-continuous and intuitionistic fuzzy apr-closed. Let A be an intuitionistic fuzzy gpr-closed set in Y such that $f^{-1}(A) \subseteq O$, where O be an intuitionistic fuzzy regular open set in X. Then $O^c \subseteq f^{-1}(A^c)$ which implies that $f(O^c) \subseteq int(A^c) = (cl(A))^c$. Hence $f^{-1}(cl(A)) \subseteq O$. Since f is intuitionistic fuzzy gpr-continuous and $f^{-1}(cl(A))$ is intuitionistic fuzzy gpr-closed in X. Therefore $pcl(f^{-1(}cl(A))) \subseteq O$ which implies that $pcl(f^{-1}(A)) \subseteq O$. Hence $f^{-1}(A)$ is intuitionistic fuzzy *gpr*-closed set in X. □

Corollary 4.1. *If a mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy continuous and intuitionistic fuzzy apr-closed then $f^{-1}(A)$ is intuitionistic fuzzy gpr-closed set in X whenever A is intuitionistic fuzzy gpr-closed set in Y.*

Theorem 4.2. *If a mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy gpr-continuous and intuitionistic fuzzy apr-closed then $f^{-1}(A)$ is intuitionistic fuzzy gpr-open set in X whenever A is intuitionistic fuzzy gpr-open set in Y.*

Proof. Suppose that $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy *gpr*-continuous and intuitionistic fuzzy *apr*-closed mapping. Let A is intuitionistic fuzzy *gpr*-open in Y. Then by definition 2.7 A^c is intuitionistic fuzzy *gpr*-closed in Y. Hence by theorem 4.1 $f^{-1}(A^c)$ is intuitionistic fuzzy *gpr*-closed in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$ for every intuitionistic fuzzy set A of Y. Hence $(f^{-1}(A))^c$ is intuitionistic fuzzy *gpr* -closed set in X. Therefore $f^{-1}(A)$ is intuitionistic fuzzy *gpr*-open set in X. □

Corollary 4.2. *If a mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy continuous and intuitionistic fuzzy apr-closed then $f^{-1}(A)$ is intuitionistic fuzzy gpr-open set in X whenever A is intuitionistic fuzzy gpr-open set in Y.*

Theorem 4.3. *If $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy apr-continuous and intuitionistic fuzzy pre-closed mapping then the image of every intuitionistic fuzzy gpr-closed set of X is intuitionistic fuzzy gpr-closed in Y.*

Proof. Let B be an intuitionistic fuzzy *gpr*-closed set of X, and $f(B) \subseteq O$. where O is intuitionistic fuzzy regular open set in Y. Then $B \subseteq f^{-1}(O)$ and since f is intuitionistic fuzzy *apr*-continuous, $pcl(B) \subseteq f^{-1}(O)$ which implies that $f(pcl(B)) \subseteq O$. Since f is intuitionistic fuzzy pre-closed mapping and $pcl(B)$ is intuitionistic fuzzy pre-closed in X, $f(pcl(B))$ is intuitionistic fuzzy pre closed in Y. Hence we have $pcl(f(B)) \subseteq pcl(f(pcl(B)) = f(pcl(B)) \subseteq O$. Hence $f(B)$ is intuitionistic fuzzy *gpr* - closed in Y. □

5. A Characterization of Intuitionistic Fuzzy pre regular $T_{\frac{1}{2}}$ - spaces

In the following theorems we give a characterization of a class of intuitionistic fuzzy pre-regular $T_{\frac{1}{2}}$-spaces by using the concepts of intuitionistic fuzzy *apr*-closed and intuitionistic fuzzy *apr*-continuous mapping.

Theorem 5.1. *An intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy pre-regular $T_{\frac{1}{2}}$-space if and only if every mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy apr-continuous.*

Proof. Necessity: Let $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy mapping. Let A is intuitionistic fuzzy *gpr*-closed set of X and $A \subseteq f^{-1}(O)$ where O is intuitionistic fuzzy regular open set of Y. Since X is intuitionistic fuzzy pre-regular $T_{\frac{1}{2}}$-space, A

is intuitionistic fuzzy pre-closed set in X. Therefore $pcl(A) = A \subseteq f^{-1}(O)$. Hence A is intuitionistic fuzzy *apr*-continuous.

Sufficiency: Let A be a nonempty intuitionistic fuzzy *gpr*-closed set in X and let Y is intuitionistic fuzzy topological space with the intuitionistic fuzzy topology $\sigma = \{ \tilde{\mathbf{0}}, A, \tilde{\mathbf{1}} \}$. Finally let $f : (X, \tau) \to (Y, \sigma)$ be identity mapping. By assumption f is intuitionistic fuzzy *apr*-continuous. Since A is intuitionistic fuzzy *gpr*-closed in X and intuitionistic fuzzy open in Y and $A \subseteq f^{-1}(A)$, it follows that $pcl(A) \subseteq f^{-1}(A) = A$, because f is identity mapping. Hence $A$is intuitionistic fuzzy pre-closed in X and therefore X is intuitionistic fuzzy pre-regular $T_{\frac{1}{2}}$-space. □

An analogous argument proves the following result for intuitionistic fuzzy apr-closed mapping.

Theorem 5.2. *An intuitionistic fuzzy topological space (X, τ) is intuitionistic fuzzy pre-regular $T_{\frac{1}{2}}$-space if and only if every mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy apr-closed.*

6. Properties of Intuitionistic Fuzzy apr - closed and Intuitionistic Fuzzy apr - continuous mappings

In this section we investigate some of the properties of intuitionistic fuzzy *apr*-closed and intuitionistic fuzzy *apr*-continuous mappings.

Theorem 6.1. *Every intuitionistic fuzzy gpr-continuous and intuitionistic fuzzy apr -closed mapping is intuitionistic fuzzy gpr-irresolute.*

Proof. Suppose that $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy *gpr*-continuous and intuitionistic fuzzy *apr*-closed mapping and A is intuitionistic fuzzy *gpr* - closed set in Y. Let $f^{-1}(A) \subseteq O$ where O be an intuitionistic fuzzy regular open set in X. Then $O^c \subseteq f^{-1}(A^c)$ which implies that $f(O^c) \subseteq int(A^c) = (cl(A))^c$. Hence $f^{-1}(cl(A)) \subseteq O$. Since f is intuitionistic fuzzy *gpr*-continuous $f^{-1}(cl(A))$ is intuitionistic fuzzy *gpr*-closed in X. Therefore $pcl(f^{-1}(cl(A))) \subseteq O$ which implies that $pcl(f^{-1}(A)) \subseteq O$. Hence $f^{-1}(A)$ is intuitionistic fuzzy *gpr*-closed set in X. Therefore f is intuitionistic fuzzy *gpr*-irresolute. □

Theorem 6.2. *Every intuitionistic fuzzy continuous and intuitionistic fuzzy apr-closed mapping is intuitionistic fuzzy gpr-irresolute.*

Proof. It follows from Remark 2.3 and Theorem 6.1. □

Theorem 6.3. *If $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy mapping for which $f(F)$ is intuitionistic fuzzy pre-open set in Y for every intuitionistic fuzzy regular closed set F of X then f is intuitionistic fuzzy apr-closed mapping.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be intuitionistic fuzzy mapping, F intuitionistic fuzzy regular closed in X, A intuitionistic fuzzy *gpr*-open in Y and $f(F) \subseteq A$. By hypothesis $f(F)$ is intuitionistic fuzzy pre-open in X. Therefore $f(F) = pint f(F) \subseteq pint(A)$. Hence f is intuitionistic fuzzy *apr*-closed. □

Theorem 6.4. *If $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy mapping for which $f^{-1}(V)$ is intuitionistic fuzzy pre-closed in X for every intuitionistic fuzzy regular open set V of Y, then f is intuitionistic fuzzy apr-continuous mapping.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be intuitionistic fuzzy mapping. Let F be intuitionistic fuzzy gpr-closed set in X and V intuitionistic fuzzy regular open set of Y such that $F \subseteq f^{-1}(V)$. By hypothesis $f^{-1}(V)$ is intuitionistic fuzzy pre-closed in X. Hence $pcl(f^{-1}(V)) = f^{-1}(V)$. Therefore $pcl(F) \subseteq pcl(f^{-1}(V)) = f^{-1}(V)$. Hence f is intuitionistic fuzzy *apr*-continuous. □

Remark 6.1. Since the identity mapping on any intuitionistic fuzzy topological space is both intuitionistic fuzzy *apr*-continuous and intuitionistic fuzzy *apr*-closed, it is clear that the converse of Theorem 6.3 and Theorem 6.4 do not hold.

Theorem 6.5. *If $IFRO(Y) = IFPC(Y)$ where $IFRO(Y)$ (resp. $IFPC(Y)$) denotes the family of all intuitionistic fuzzy regular open (resp. intuitionistic fuzzy pre-closed) sets of Y, then the mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy apr-closed if and only if $f(F)$ is intuitionistic fuzzy pre-open set in Y, for every intuitionistic fuzzy regular closed set F of X.*

Proof. Necessity: Let$f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy *apr*-closed mapping. By Theorem 2.2 [13] every intuitionistic fuzzy set of Y is intuitionistic fuzzy *gpr*-closed and hence all are intuitionistic fuzzy *gpr*-open. Thus for any intuitionistic fuzzy regular closed set F of X, $f(F)$ is intuitionistic fuzzy *gpr*-open in Y. Since f is intuitionistic fuzzy *apr*-closed, $f(F) \subseteq pint(f(F))$ and then $f(F) = pint(f(F))$. Hence $f(F)$ is intuitionistic fuzzy pre-open.

Sufficiency: Let F be an intuitionistic fuzzy regular closed set of X and A be an intuitionistic *gpr*-open set of Y and $f(F) \subseteq A$. By hypothesis $f(F)$ is intuitionistic fuzzy pre-open in Y and $f(F) = pint(f(F)) \subseteq pint(A)$. Hence f is intuitionistic fuzzy *apr*-closed. □

Theorem 6.6. *If $IFRO(Y) = IFPC(Y)$ where $IFRO(Y)$ (resp. $IFPC(Y)$) denotes the family of all intuitionistic fuzzy regular open (resp. intuitionistic fuzzy pre closed) sets of Y then the mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy apr-closed if and only if f is intuitionistic fuzzy pre-regular closed.*

Proof. Necessity: Let O be an intuitionistic fuzzy regular closed set of X. Then by theorem 6.5 $f(O)$ is intuitionistic fuzzy pre-open in Y. Since every intuitionistic fuzzy pre-open set is intuitionistic fuzzy regular open, therefore $f(O)$ is intuitionistic fuzzy regular open in Y and hence by hypothesis $f(O)$ is intuitionistic fuzzy pre-closed in Y and therefore $f(O)$ is intuitionistic fuzzy regular closed in Y. Hence f is intuitionistic fuzzy pre-regular closed.

Sufficiency: Let F be an intuitionistic fuzzy regular closed set of X and A be an intuitionistic gpr-open set of Y and $f(F) \subseteq A$. Since f is intuitionistic fuzzy pre regular closed, $f(F)$ is intuitionistic fuzzy regular closed in Y and therefore $(f(F))^c$ is intuitionistic fuzzy regular open in Y. By hypothesis$(f(F))^c$ is intuitionistic fuzzy pre-closed in Y and hence $f(F)$ is intuitionistic fuzzy pre-open in Y which implies that $f(F) = pint(f(F)) \subseteq pint(A)$. Hence f is intuitionistic fuzzy *apr*-closed. □

Theorem 6.7. *If $IFRO(X) = IFPC(X)$ where $IFRO(X)$ (resp. $IFPC(X)$) denotes the family of all intuitionistic fuzzy regular open (resp. intuitionistic fuzzy pre-closed) sets of X, then the mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy apr-continuous if and only if $f^{-1}(O)$ is intuitionistic fuzzy pre-closed in X for every intuitionistic fuzzy regular open set O of Y.*

Proof. Necessity: Let $f : (X, \tau) \to (Y, \sigma)$ be intuitionistic fuzzy *apr*-continuous mapping. By Theorem 2.2 [13] every intuitionistic fuzzy set of X is intuitionistic fuzzy *gpr*-closed and hence all are intuitionistic fuzzy *gpr*-open. Thus for any intuitionistic fuzzy regular open set O of Y, $f^{-1}(O)$ is intuitionistic fuzzy *gpr*-closed in X. Since $f^{-1}(O) \subseteq f^{-1}(O)$ and f is intuitionistic fuzzy *apr*-continuous then $pcl(f^{-1}(O)) \subseteq f^{-1}(O)$. Hence $f^{-1}(O)$ is intuitionistic fuzzy pre-closed set in X.

Sufficiency: Let O be an intuitionistic fuzzy regular open set of Y and A be an intuitionistic fuzzy *gpr*-closed set of X such that $A \subseteq f^{-1}(O)$ then $pcl(A) \subseteq pcl(f^{-1}(O)) = f^{-1}(O)$ because by hypothesis $f^{-1}(O)$ is intuitionistic fuzzy pre-closed in X. Hence f is intuitionistic fuzzy *apr*-continuous. □

Theorem 6.8. *If $IFRO(X) = IFPC(X)$ where $IFRO(X)$ (resp. $IFPC(X)$ denotes the family of all intuitionistic fuzzy regular open (resp. intuitionistic fuzzy pre-closed) sets of X, then the mapping $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy apr-continuous if and only if it is intuitionistic fuzzy R-mapping.*

Proof. Necessity: Let $f : (X, \tau) \to (Y, \sigma)$ be intuitionistic fuzzy *apr*-continuous mapping. Let O is an intuitionistic fuzzy regular open set of Y, then by Theorem 6.7 $f^{-1}(O)$ is intuitionistic fuzzy pre-closed in X and so by hypothesis $f^{-1}(O)$ is intuitionistic fuzzy regular open in X. Hence f is an intuitionistic fuzzy R-mapping.

Sufficiency: Let O be an intuitionistic fuzzy regular open set of Y and A be an intuitionistic fuzzy gpr-closed set of X such that $A \subseteq f - 1(O)$. Since f is intuitionistic fuzzy R-mapping, $f^{-1}(O)$ is intuitionistic fuzzy regular open in X and thus by hypothesis $f^{-1}(O)$ is intuitionistic fuzzy pre-closed in X which implies that $pcl(A) \subseteq pcl(f^{-1}(O)) = f^{-1}(O)$. Hence f is intuitionistic fuzzy *apr*-continuous. □

Theorem 6.9. *If $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy pre-regular closed and $g : (Y, \sigma) \to (Z, \phi)$ is intuitionistic fuzzy apr-closed mapping, then $gof : (X, \tau) \to (Z, \phi)$ is intuitionistic fuzzy apr-closed.*

Proof. Let F be an intuitionistic fuzzy regular closed set of X and A is intuitionistic fuzzy *gpr*-open set of Z for which $gof(F) \subseteq A$ since $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy pre-regular closed mapping, $f(F)$ is intuitionistic fuzzy regular closed set of Y. Now $g : (Y, \sigma) \to (Z, \phi)$ is intuitionistic fuzzy *apr*-closed mapping, then $g(f(F)) \subseteq pint(A)$. Hence $gof : (X, \tau) \to (Z, \phi)$ is intuitionistic fuzzy *apr*-closed mapping. □

Theorem 6.10. *If* $f : (X, \tau) \to (Y, \sigma)$ *is intuitionistic fuzzy apr-closed and* $g : (Y, \sigma) \to (Z, \phi)$ *is intuitionistic fuzzy open and intuitionistic fuzzy gpr-irresolute then* $gof : (X, \tau) \to (Z, \phi)$ *is intuitionistic fuzzy apr-closed.*

Proof. Let F be an intuitionistic fuzzy regular closed set of X and A is intuitionistic fuzzy *gpr*-open set of Z for which $gof(F) \subseteq A$. Then $f(F) \subseteq g^{-1}(A)$. Since g is gpr-irresolute, $g^{-1}(A)$ is intuitionistic fuzzy *gpr*-open in X and $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy *apr*-closed mapping. It follows that $f(F) \subseteq pint(g^{-1}(A))$. Thus $(gof)(F) = g(f(F)) \subseteq g(pint(g^{-1}(A)) \subseteq pint(g(g^{-1}(A))) \subseteq pint(A)$. Hence $gof : (X, \tau) \to (Z, \phi)$ is intuitionistic fuzzy *apr*-closed. □

Theorem 6.11. *If* $f : (X, \tau) \to (Y, \sigma)$ *is intuitionistic fuzzy apr-continuous and* $g : (Y, \sigma) \to (Z, \phi)$ *is intuitionistic fuzzy R-mapping then* $gof : (X, \tau) \to (Z, \phi)$ *is intuitionistic fuzzy apr-continuous.*

Proof. Let A be an intuitionistic fuzzy *gpr*-closed set of X and V is intuitionistic fuzzy regular open set of Z for which $A \subseteq (gof)^{-1}(V)$. Now since $g : (Y, \sigma) \to (Z, \phi)$ is intuitionistic fuzzy R-mapping ,$g^{-1}(V)$ is intuitionistic fuzzy regular open set of Y. Since $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy *apr*-continuous, $pcl(A) \subseteq f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$. Hence $gof : (X, \tau) \to (Z, \phi)$ is intuitionistic fuzzy *apr*-continuous mapping. □

REFERENCES

[1] K. Atanassov and S. Stoeva: *Intuitionistic Fuzzy Sets*. on Interval and Fuzzy Mathematics , Poznan, **Aug**(1983), 23-26.

[2] K. Atanassov: *Intuitionistic Fuzzy Sets*. Fuzzy Sets and Systems, **20** (1986), 87-96.

[3] C. W. Baker: *On Preserving g-closed sets*. Kyungpook Math. J. **36** (1996), 195-199.

[4] C.Ł. Chang: *Fuzzy Topological Spaces*. J. Math. Anal. Appl. **24** (1968), 182-190.

[5] D. Coker: *An Introduction to Intuitionistic Fuzzy Topological Spaces*. Fuzzy Sets and Systems, **88** (1997), 81-89.

[6] H. Gurcay, D. Coker and A. Es. Hayder: *On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces*. J. Fuzzy Math. , **5(2**)(1997), 365-378.

[7] S. S. Thakur and R. Chaturvedi: *Generalized Continuity in intuitionistic fuzzy topological spaces*. Notes on IFS, **12(1)** (2006), 38-44.

[8] S. S. Thakur and R. Chaturvedi:*Regular generalized closed sets in intuitionistic fuzzy topological spaces*. Stud. Cercet. Stiint. Ser. Mat. Univ. Bacau, **16** (2006) 257-272.

[9] S. S. Thakur and R. Chaturvedi:*Generalized closed sets in intuitionistic fuzzy topology*. J. Fuzzy Math. **16(2)**(2008), 559-572.

[10] S. S. Thakur and J. P. Bajpai : *Intuitionistic fuzzy w-closed set and intuitionistic fuzzy w- continuity*. International Journal of contemporary Advanced Mathematics (IJCM) **1** (2010), 1-15.

[11] S. S. Thakur and J. P. Bajpai: *Intuitionistic Fuzzy rw-Closed Sets and Intuitionistic Fuzzy rw-Continuity*. Notes on IFS **17(2)** (2011), 82-96.

[12] S. S. Thakur and J. P. Bajpai: *sg-closed sets in Intuitionistic Fuzzy Topological Spaces*. International Review of Fuzzy Mathematics, **6(2)**, (2011), 59-76.

[13] S. S. Thakur and J. P. Bajpai: *Intuitionistic Fuzzy gpr-Closed Sets and Intuitionistic Fuzzy gpr-Continuity*. Fuzzy Inf. Eng. (Springer) **4** (2012), 425-444.

[14] S. S. Thakur and J. P. Bajpai: *On Preserving Intuitionistic Fuzzy w-closed sets*. J. Fuzzy Math. **20(3)**, (2012), 635-646.

[15] L. A. Zadeh: *Fuzzy Sets*. Information and Control, **8** (1965), 338-353.

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