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ON NUMERICAL EVALUATION OF THE PACKET-ERROR RATE FOR BINARY PHASE-MODULATED SIGNALS RECEPTION OVER GENERALIZED–K FADING CHANNELS *

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Abstract. We present a numerical evaluation of the packet error rate (PER) for digital binary phase modulations over wireless communication channels. The analysis is valid for a quasistatic fading communication channel, where multipath fading and shadowing appear simultaneously. The approach is based on a numerical evaluation of the signal-to-noise ratio threshold that is further used in PER computation. We analyze the threshold and PER dependence on signal power, multipath fading and shadowing severity, as well as packet length.

Keywords: bit error rate, packet error rate, wireless communication channel.

1. Introduction

The quality of service in communication systems is usually described by bit error rate (BER) and packet error rate (PER). The BER is a probability that a transmitted bit over a channel will be wrongly detected in the receiver due to the noise and interferences over the channel. The PER is a probability that a packet of bits (or symbols) will be wrongly detected. The packet is detected wrongly if at least one bit (or symbol) is wrongly detected. Both metrics are associated to the physical layer of communication systems. However, PER is a very important metric in designing across multiple protocol layers of wireless networks [11], [12].

It is very hard to calculate the exact value of PER, especially when encoding and decoding algorithms are implemented. Because of that, many efforts have been made in order to analytically or numerically approximate PER. Chatzigeorgiou at al. [3], [4] developed a threshold-based method for approximating PER over quasistatic fading channels. They examined both single-input single-output and

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multiple-input multiple-output channels. They observed a situation when a direct propagation component does not exist in the channel, i.e., fading is described by the Rayleigh probability density function (PDF). Xi at al. [14] proposed a novel analytical approach for evaluating the signal-to-noise (SNR) ratio threshold required for computation of PER. Their analysis is valid basically for the Rayleigh fading channel, but it was also extended for a more general case when besides scattering propagation components there is also a direct signal propagation component. In other words, their analysis is valid for the Nakagami-m fading channel, too. Wang et al. [11], [12] suggested an accurate approximation of the PER of diversity receivers over the Rayleigh fading channel when different error correction coding schemes are implemented.

All previously mentioned works were applicable in the situation when only multipath fading exists in the channel. However, very often, besides multipath fading, shadowing appears simultaneously during signal transmission [5, 9]. In this case, signal variations at the receiver input can be accurately described by the Gamma-shadowed Nakagami-m PDF. This PDF is also known as the generalized-K PDF [10, 2, 9]. The aim of this paper is to provide a numerical method for evaluating PER for binary phase shift keying over the composite fading channel. We give an approach for evaluating the SNR threshold and after that use this threshold for estimating PER. We examine the effect of signal power, packet length, multipath fading severity and shadowing sharpness on the numerical value of the SNR threshold, and consequently on PER. Approximate PER values are expressed in terms of Meijer's G functions [13] with appropriate arguments.

The paper is organized as follows. The system model is described in more detail in Section 2. Approximation for PER is discussed in Section 3, with preliminary numerical results indicating validity of the approximation. Section 4 examines a procedure for obtaining the threshold level value required for approximating PER, and proposes a simple method for its computation. An example is given for realistic system parameters. In Section 5, we present numerical results of the system analysis, and further validate the approximations made in the previous sections. Some concluding remarks are presented in the final section.

2. System model

BER represents the time-average of the ratio of wrongly decoded bits over a total number of transfered bits. If the process of the receiver operation is considered ergodic, as is the case for the most processes relevant in telecommunications, then the time-average is equal to the ensemble-average. Therefore, BER is equal to error probability P_e. The most important parameter on which BER depends is SNR. The most common model of noise treats it as having a zero-mean Gaussian probability density function, with the effective noise level being equal to variance of the distribution. In general, higher SNR values lead to lower BER, so BER is a strictly decreasing function of SNR.

In cases where there are other random influences on the signal level, an important parameter for the receiver BER is signal level statistics. In general, BER is an average of its instantaneous value for a fixed signal level, over the signal-to-noise statistics, or BER = E{BER(SNR)}. The situation of signal level varying significantly is almost synonymous with modern wireless communications, be it mobile, cellular or Wi-Fi. It is almost universally recognized from the user's experience point that sometimes it is enough to move a few centimeters while talking over your mobile to suddenly lose the signal or encounter 'poor' signal levels. The effect is attributed to signal fading, which is a propagation effect of quasi-randomly interfering signal copies producing unpredictable signal levels. It is also accompanied by signal shadowing, which is a random process of the signal being attenuated through the obstacles in the propagation path. These two effects combined can significantly degrade the user experience with wireless technologies. The combined influence of multipath fading and signal shadowing can be described by the generalized-K fading model, which is the previously discussed relevant signal level statistics. Its probability density function is given by [7]:

$$
\begin{aligned}
p(m_m, m_s, \rho_0, \rho) \;=\; & 2\frac{\rho^{\frac{m_m+m_s-2}{2}}}{\Gamma(m_m)\Gamma(m_s)}\left(\frac{m_m m_s}{\rho_0}\right)^{\frac{m_m+m_s}{2}} \\
& \mathrm{K}_{m_m-m_s}\left(2\sqrt{\rho\frac{m_m m_s}{\rho_0}}\right),
\end{aligned}
\tag{2.1}
$$

where $\mathrm{K}_\nu(\cdot)$ is a modified Bessel function of the first kind, of order ν [6, (8.432)], and $\Gamma(\cdot)$ is a Gamma function. PDF is defined for positive values of ρ, and for $\rho_0 \geqslant 0, m_m > 0, m_s > 0$. It is suitable as a channel model when $m_m \geqslant 1/2$.

On the other hand, these types of data transfers are usually centered around the group transfer of bit packets, and are considered packet-radio communications. Therefore, in contrast to BER, the more important performance measure for these types of telecommunication systems is the packet-error-rate.

If individual bits in the packet are not mutually correlated, than PER can be expressed as $\mathrm{PER} = \mathrm{E}\{1-(1-\mathrm{BER})^{l_p}\}$:

$$
\mathrm{PER}(m_m, m_s, \rho_0) = \int_0^{+\infty} \left[1-\left(1-\mathrm{BER}(\rho)\right)^{l_p}\right] p(m_m, m_s, \rho_0, \rho)\,\mathrm{d}\rho, \tag{2.2}
$$

which is the ensemble-average of probability that the whole packet of l_p bits is received correctly without any errors. In the previous equation, the parameter m_m represents the multipath fading parameter, while m_s is the shadowing parameter, as discussed in the previous paragraph. The parameter ρ_0 represents the average value of SNR, over the signal level varying statistics.

Individual bits have BER that depends on the modulation format and demodulation operation of the receiver, and for the phase modulation formats of interest it can be expressed as:

$$\text{BER}(\rho) = \begin{cases} \dfrac{1}{2}\text{erfc}(\sqrt{\rho}), & \text{for BPSK modulation,} \\ \dfrac{1}{2}e^{-\rho/2}, & \text{for DBPSK modulation.} \end{cases} \tag{2.3}$$

Under the assumption that the signal level is constant, and the noise at the receiver is additive white Gaussian, the instantaneous SNR in the previous equation is designated as ρ.

As is obvious from the previous equations, one cannot express the receiver PER in a closed form by combining (2.2, 2.2, 2.3). Therefore, the performance analysis is limited to numerical computation of (2.2), or its approximations.

3. Packet-error rate approximation

One of the adopted PER approximations concentrates on the following form:

$$\text{PER} = \int_0^{\gamma_\omega} p(x)\,\mathrm{d}x, \tag{3.1}$$

where $\text{PER} = \text{PER}(m_m, m_s, \rho_0)$, and $p(x) = p(m_m, m_s, \rho_0, x)$. The threshold value γ_ω that satisfies the equation is to be determined. The integral on the right-hand side is by definition the cumulative distribution function (CDF) corresponding to PDF $p(x)$, which can be expressed in the form of Meijer's G function [1] as:

$$\text{CDF}(m_m, m_s, \rho_0, \rho) = \frac{1}{\Gamma(m_m)\Gamma(m_s)} G_{1,3}^{2,1}\left(\rho\frac{m_m m_s}{\rho_0} \,\middle|\, \begin{matrix} 1 \\ m_s, m_m, 0 \end{matrix}\right), \tag{3.2}$$

where $G_{p,q}^{m,n}\left(x \,\middle|\, \begin{matrix} a1, \cdots, a_p \\ b1, \cdots, b_q \end{matrix}\right)$ denotes Meijer's G function defined by [6, (9.301)] [8, (16.17.1)].

In order to obtain the value of the threshold γ_ω, we have to solve the equation:

$$\text{PER}(m_m, m_s, \rho_0) = \text{CDF}(m_m, m_s, \rho_0, \gamma_\omega) \tag{3.3}$$

According to available literature, the motivation behind this approach is that the threshold γ_ω will not depend significantly on the values of m_m, m_s, and ρ_0, thus enabling one threshold to be used across the range of values of interest. As a consequence, PER is expressed in the aforementioned form of Meijer's G function, which can be used for further analytic or numerical manipulation.

In order to validate this assumption, we have computed numerical values of the threshold γ_ω for a realistic range of values $0.5 \leqslant m_m \leqslant 6$, $0.3 \leqslant m_s \leqslant 12$, and $-5 \leqslant \rho_0 \leqslant 25$, and the results are shown in Fig. 3.1. The results are computed

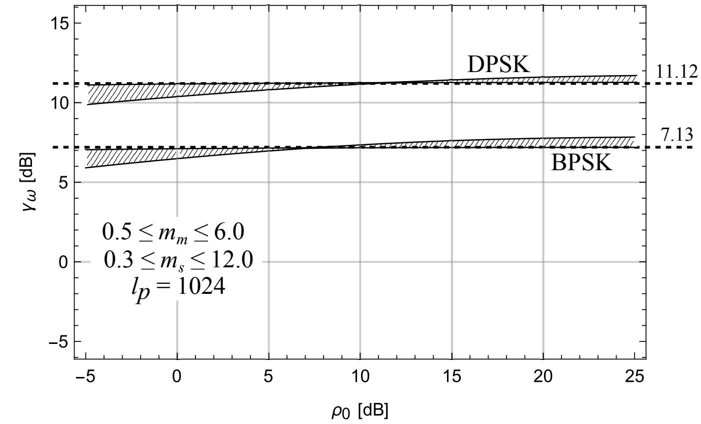

FIG. 3.1: Threshold values for a range of fading and shadowing propagation conditions. Patterned areas represent the range of threshold values as the propagation parameters are swept across their ranges.

for a fixed packet length of 1024 bits. Numerical values are obtained by using *Mathematica*'s *FindRoot* implementation of Newton's method for solving non-linear equations, and care has been taken to ensure the results are computed with enough working precision to justify the accuracy of the solutions. The figure shows that the threshold somewhat depends on a particular set of values, but it remains in a fairly narrow band of possible values. By averaging the threshold values over uniformly distributed points in m_m, m_s and ρ_0 that we computed in the simulation, we get an average value of $\gamma_\omega \approx 7.13$ dB for BPSK modulation, and $\gamma_\omega \approx 11.12$ dB for DBPSK.

4. Approximation for the threshold level

Formally, the solution to (3.3) can be written in the form of an inverse function:

$$\gamma_\omega = \mathrm{CDF}^{-1}(\mathrm{PER}), \tag{4.1}$$

since CDF is a monotonically increasing function.

From the previous experience in the evaluation of telecommunication systems performance, we are confident that, at least for a significant range of parameter values, the cumulative distribution function exhibits log–log behavior, i.e. it can be approximated to some extent by a straight line in the log–log scale. Therefore, we proceed by imposing the log–log scale for the cumulative distribution function. We further assume that the function can be expanded to power series:

$$\log\left[\mathrm{CDF}\left(e^x\right)\right] = \sum_{i=0}^{N} a_i x^i + R_N(x). \tag{4.2}$$

In the previous equation we used $\mathrm{CDF}(x) = \mathrm{CDF}(m_m, m_s, \rho_0, x)$ notation in order to avoid a lengthy and non-essential list of function arguments. Coefficients a_n can be determined, for example, as the coefficients in the Maclaurin series of the function:

$$a_n = \frac{1}{n!}\frac{\mathrm{d^n}}{\mathrm{d}x^n}\log\left[\mathrm{CDF}(e^x)\right]\Big|_{x=0}. \tag{4.3}$$

The first four a_n coefficients are given in Table 4.1. Meijer's G functions exhibit a closure in the sense that if the function argument is a constant multiple of the constant power of the argument, the derivatives and antiderivatives with respect to the argument are also expressible as G-functions. This is clearly reflected in the coefficients shown in Table 4.1, where we have expressed a_n in terms of $a_{n-1}, a_{n-2}, \cdots, a_1$. It should be possible to formulate a general expression for a_n, but we have not done so here. Instead we have focused on the cases $N \leqslant 4$, which enable a relatively simple symbolic representation of the approximate inverse function CDF^{-1}. In order to solve (3.3) using a series representation of (4.2), we write a nonlinear equation:

$$\sum_{i=0}^{N} a_i x^i - \log(\mathrm{PER}) = 0. \tag{4.4}$$

Let us assume that at least one real solution to this polynomial equation exists and can be expressed in a symbolic form as:

$$x = f(a_1, a_2, \cdots, a_N, \mathrm{PER}). \tag{4.5}$$

Then, the approximate inverse function CDF^{-1} can be expressed as:

$$\mathrm{CDF}^{-1}(x) \approx e^{f(a_1, a_2, \cdots, a_N, \mathrm{PER})}. \tag{4.6}$$

Let us examine a simple linear approximation, i.e. $N = 1$. We can directly write the function f:

$$f(a_0, a_1, \mathrm{PER}) = \frac{\log(\mathrm{PER}) - a_0}{a_1}. \tag{4.7}$$

From this solution and (4.6), it follows directly that the approximate threshold γ_{ω_1} is:

$$\gamma_{\omega_1} = \left(\frac{\mathrm{PER}}{\mathrm{CDF}(1)}\right)^{1/a_1}, \tag{4.8}$$

where a_1 is given in Table 4.1.

A numerical example for illustrative telecommunication system parameters can be the following: $m_m = 6$, $m_s = 12$, $\rho_0 = 3000$, modulation format - DBPSK.

Table 4.1: The first four coefficients a_n for the Maclaurin series of (4.2)

n	a_n
0	$\log[\mathrm{CDF}(1)]$
1	$\frac{\Gamma(m_m)\Gamma(m_s)}{\mathrm{CDF}(1)}\frac{m_m m_s}{\rho_0}G_{0,2}^{2,0}\left(\frac{m_m m_s}{\rho_0}\,\middle\vert\,\begin{matrix}-\\ m_m-1, m_s-1\end{matrix}\right)$
2	$\frac{a_1}{2}(1-a_1)+ \frac{\Gamma(m_m)\Gamma(m_s)}{2\,\mathrm{CDF}(1)}\left(\frac{m_m m_s}{\rho_0}\right)^2 G_{1,3}^{2,1}\left(\frac{m_m m_s}{\rho_0}\,\middle\vert\,\begin{matrix}-1\\ m_m-2, m_s-2, 0\end{matrix}\right)$
3	$a_1\left(\frac{a_1^2}{6}+\frac{a_1}{2}+a_2+\frac{1}{3}\right)-a_2+ \frac{\Gamma(m_m)\Gamma(m_s)}{6\,\mathrm{CDF}(1)}\left(\frac{m_m m_s}{\rho_0}\right)^3 G_{1,3}^{2,1}\left(\frac{m_m m_s}{\rho_0}\,\middle\vert\,\begin{matrix}-2\\ m_m-3, m_s-3, 0\end{matrix}\right)$
4	$\frac{a_1}{4}\left(\frac{a_1^3}{6}-a_1^2+\frac{11}{6}a_1-1\right)+\frac{a_2}{2}\left(a_1^2+a_2+a_1+\frac{11}{6}\right)+a_3\left(a_1-\frac{3}{2}\right)+ \frac{\Gamma(m_m)\Gamma(m_s)}{24\,\mathrm{CDF}(1)}\left(\frac{m_m m_s}{\rho_0}\right)^4 G_{1,3}^{2,1}\left(\frac{m_m m_s}{\rho_0}\,\middle\vert\,\begin{matrix}-3\\ m_m-4, m_s-4, 0\end{matrix}\right)$

The cumulative distribution function value at argument value 1 for given system parameters is:

$$\mathrm{CDF}(1) = 7.94648 \times 10^{-19}.$$

The coefficient a_1 has the value of:

$$a_1 = 5.99589.$$

The numerical value of PER, obtained by a numerical evaluation of the integral (2.2) is:

$$\mathrm{PER} = 8.64275 \times 10^{-12}.$$

From the equation (4.8), we get the threshold value:

$$\gamma_{\omega_1} = 14.91247.$$

Once the threshold value is computed given the initial system parameters, one can approximate the PER values for different system parameters as:

$$\mathrm{PER}(m_m, m_s, \rho_0) \approx \mathrm{CDF}(m_m, m_s, \rho_0, \gamma_{\omega_1}). \tag{4.9}$$

5. Numerical results and discussion

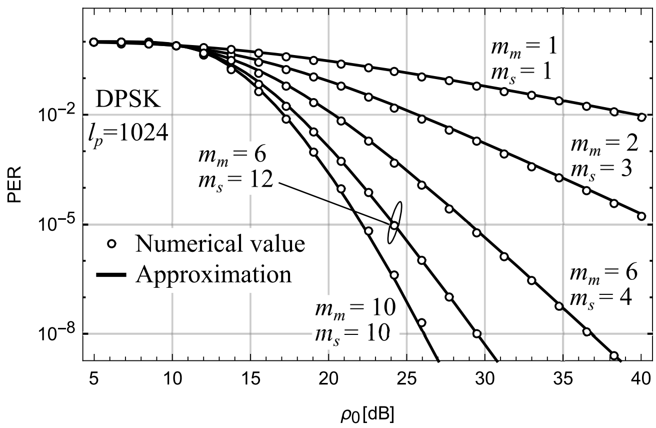

FIG. 5.1: Numerical values of DBPSK PER compared to approximate values obtained in the example for $N = 1$.

In order to further validate the assumption about a universal usage of the threshold value for different parameters, we have calculated numerical values for PER and compared them to the approximate values obtained using (4.9) for different system parameters. The results are shown in Fig. 5.1. The threshold value is the same as the one determined in the example in the previous section, and it is used in all obtained approximate PER values. For a wide range of the multipath fading parameter values m_m, the shadowing parameter m_s, and the average signal-to-noise ratio ρ_0, we can see very good agreement between the precisely computed numerical values and the approximate values of PER. It is not unexpected that the best match is achieved for the values m_m and m_s, for which we have calculated the threshold γ_ω. The practical values of PER that are of interest in telecommunications range from 10^{-1} to 10^{-9}, and Fig. 5.1 is shown in a logarithmic scale to better illustrate this magnitude range of PER values.

Fig. 5.2 shows the results of the approximation when applied to the BPSK modulation format. Linear approximation, i.e. $N = 1$, results in a threshold value of $\gamma_\omega = 6.112566$ when the modulation format is BPSK, and this value is used for all the curves shown in Fig. 5.2. Numerically obtained PER values are shown with circle marks, and the overall figure is similar to that for DBPSK. The exception is that, in general, the system performs better when using BPSK, compared to the case when the system uses DBPSK. The same value of PER is achieved in BPSK when SNR is lower, i.e. a lower SNR is required to make the system perform as well as in the DBPSK case. This can be viewed as increased receiver sensitivity. In the reverse sense, when we look at BPSK as a reference and than compare DBPSK with it, we usually say that DBPSK incurs power penalty for its lower complexity.

After reviewing Figs. 5.1 and 5.2, we conclude that the assumptions made in writing PER approximation (4.9) are not unfounded. We further investigate numerically the influence of using better than linear approximations in obtaining threshold

Table 5.1: Threshold values obtained by N-th order approximation, for $m_m = 6, m_s = 12, \rho_0 = 30$ dB, and packet size $l_p = 1024$

N	γ_{ω_N}[DBPSK]	γ_{ω_N}[BPSK]
1	14.912471	6.112566
2	14.949862	6.119436
3	14.983723	6.123588
4	15.006688	6.125465
7	15.026646	6.126391

values γ_ω. If we try to obtain the threshold for $N = 2$, and for the same system parameters as in the linear example, we get the following closed form:

$$\gamma_{\omega_2} = \exp\left[-\frac{a_1}{2a_2} - \sqrt{\left(\frac{a_1}{2a_2}\right)^2 + \frac{1}{a_2}\log\frac{\text{PER}}{\text{CDF}(1)}}\,\right], \tag{5.1}$$

which evaluates to: $\gamma_{\omega_2} = 14.949862$.

Approximations of higher order are also possible and obviously expressible in a closed form for $N = 3$ and $N = 4$, but we have not developed the expressions due to their complexity. After the fourth order, the polynomial equation is solvable numerically in the general case, and the results are not of great interest to wireless engineers. The results shown in Table 5.1 summarize the results we have obtained for both modulation formats and for the same system parameters. We clearly see that the approximation order has only secondary influence on the numerically evaluated values of γ_ω, which is not particularly significant when used in the approximate expression for PER.

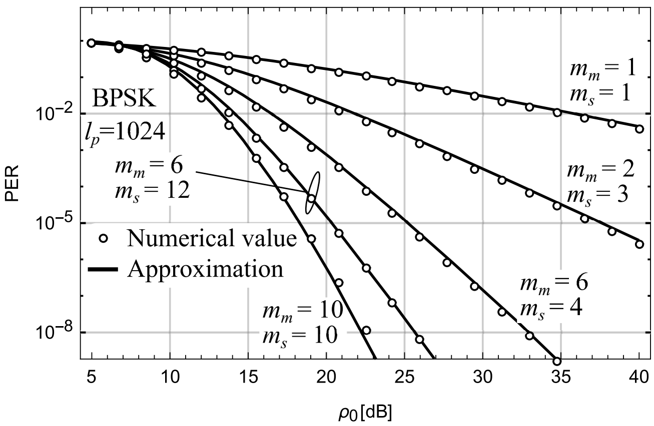

FIG. 5.2: Numerical values of BPSK PER compared to the approximate values obtained in the example for $N = 1$.

Having in mind good agreement of approximation to performance under different system parameters, we conclude that the small variations of the threshold γ_ω hardly justify the use of higher order approximations, especially when used in a wide range of system parameters.

Dependence of the threshold value on fading and shadowing parameters is shown in Fig. 5.4. The figure clearly indicates that the threshold depends on the propagation parameters. However, in the parameter range of interest, this variation of threshold has relatively low influence on wireless system performance. On the other hand, if one of the parameters, either m_m or m_s, is larger than the other one, the threshold value 'saturates' and its variation is significantly lower. This indicates that the system performance may be limited mainly by the lower of the two parameters, $\min(m_m, m_s)$. Fig. 5.4 also shows approximate threshold values obtained via approximations of order 1 and 2. Linear approximation slightly underestimates the threshold in cases where the propagation parameter values are close to each other, and in such cases it would be a better choice to use the second-order approximation whose results are closer to the numerical results.

Fig. 5.5 shows the dependence of SNR threshold on the packet length l_p. This dependence is stronger than the dependencies on the propagation parameters and SNR, and this behavior is expected. In general, as the performance of DBPSK is somewhat poorer than the performance of BPSK, from (3.1) it follows that the corresponding DBPSK threshold is always larger. On the other hand, when the packet length increases, so does the probability of packet errors, which is again reflected in the corresponding threshold increase, as shown in Fig. 5.5.

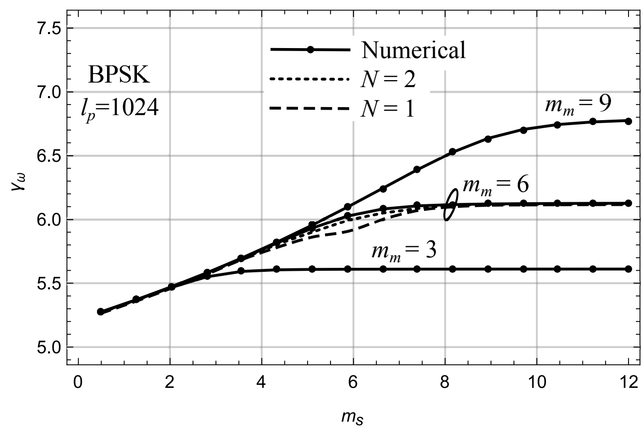

FIG. 5.3: Dependence of threshold values on the multipath fading parameter m_m and the shadowing parameter m_s, while SNR is fixed at 30 dB. Full line curves are obtained numerically, the long-dash is the first-order, while the short-dash curve is the second-order approximation.

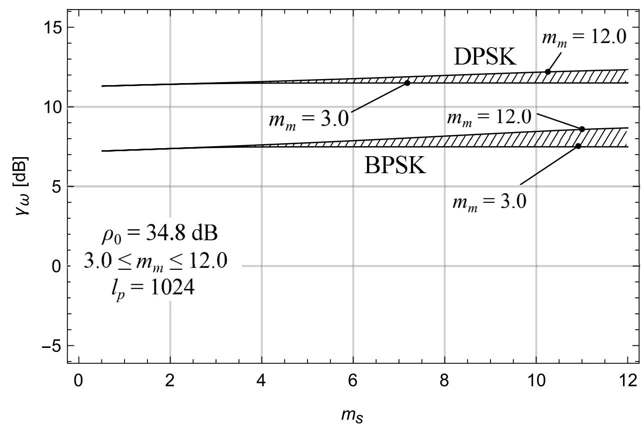

Fig. 5.4: Dependance of threshold γ_ω on the propagation parameters, with same decibel scale as in Fig. 3.1, for comparison.

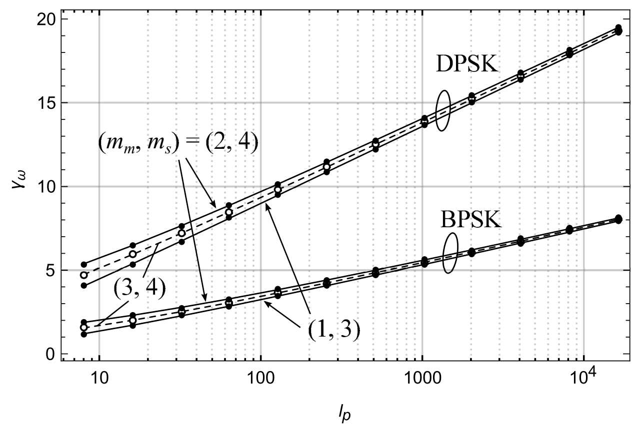

Fig. 5.5: Threshold value γ_ω versus packet length for both the modulation formats and the practical values of fading and shadowing parameters.

6. Conclusion

In this paper, we have analyzed PER in detecting BPSK and DBPSK signals transmitted over the quasistatic Gamma-shadowed Nakagami-m wireless channel. A numerical approach has been proposed for determining SNR threshold required for approximate PER evaluation. The resulting method provides means for determining the threshold level in a symbolic form that is suitable for analysis of different system parameter influence. The results illustrate that the threshold strongly depends on the packet length and much less so on the propagation parameters and SNR. The numerical values of PER indicate that a single threshold value may be used for PER calculation in a wide range of system parameters. Even better results can be obtained when the threshold is calculated separately for specific fading severity and shadowing sharpness.

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